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Crime, Sanctions and Scientific Explanation

Thomas Orsagh

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There is no dearth of theoretical and empirical literature dealing with the relation between crime and sanctions. In the last decade, interest in the subject seems to have quickened, no doubt prompted to some extent by a well advertised and substantial rise in rates of reported crime. Recent legalistic developments,[1] because of their presumed influence on crime rates, have also heightened our interest in the subject. Public pressure to "do something" about crime and the political responses to that pressure have provided additional stimuli. Yet despite a good deal of scholarly attention and society's compelling need to know what relation, if any, exists between crime and sanctions, we cannot honestly say that we know much about the subject.

To quantify the relation between the two variables we need data, and it is undoubtedly true that the quality of the data available for empirical investigation is unusually poor.[2] Criminal statistics impose constraints on the questions that can be asked, handicap the research effort and interject an annoying degree of imprecision in the statistical results. Yet, these deficiencies are troublesome nuisances, not roadblocks, in the path of scientific inquiry. The data are not so bad that the truth cannot be wrung out of them.[3] Our poor scholarly performance has another explanation. There is good reason to suspect that our problem in understanding the crime-sanctions relation lies in the nature of the relation itself. The purpose of this study is to demonstrate that past empirical analysis

of the Crime-Sanctions (C-S) relation has been incorrect.

It is generally accepted that the relation of crime to sanctions is likely to be quite complex. Current theory and empirical research suggest that the two variables probably interact with each other and with any number of other variables. But knowing this is one thing, coping with it something else. How does one evaluate a highly complex relation? Instinct says to simplify and to adopt a procedure which is at once persuasively obvious and disarmingly direct and easy to apply: analyze one relation at a time. In this case one might try to determine the effect of sanctions on crime as one independent research effort, and then the effect of crime rates on sanctions as another.

The decision to consider one relation at a time may not be improper *per se*, but the consequences of seeking a quantitative measure of the relation through conventional statistical procedures can be improper. Consider the first relation. In order to determine the effect of sanctions on crime, a probable formulation for the relation would be:

(1)   CRIME = f(SANCTIONS, X),

wherein X represents a collection of control variables. Specifically, we might evaluate the effect of sanctions on crime by hypothesizing the existence of a linear relation of the form

(2)   CRIME = f(β_0 + β_1 SANCTIONS

+ β_2 AGE + β_3 POVERTY + μ),

wherein μ represents the conventional error term. We would recognize, of course, that (2) is a simplification of (1) and hence a gross simplification of our basic conception of the crime-sanctions relation, and that Age and Poverty are but two of many variables subsumed in X which, *a priori*, deserve consideration. If there is reason to believe that the other X variable produces only small and largely self-canceling effects within the universe from which our observations were drawn, we might be inclined to believe that the derived coefficients are essentially correct. Hence, if b_1, our regression

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[1] The right to counsel, the *de facto* elimination of capital punishment and the lengthening time interval between the criminal act and the initiation of punishment come immediately to mind.

[2] For the crime variable the point is well documented. *See, e.g.,* Biderman, *Social Indicators and Goals,* in SOCIAL INDICATORS 68 (A. Bauer ed. 1966); 2 PRESIDENT'S COMMISSION ON FEDERAL STATISTICS, FEDERAL STATISTICS (1971).

[3] Criminal statistics are probably no worse than the data being used, often with little critical comment, in certain other specialized areas of social science research. Consider, for example, the quality of the sociological, demographic and economic data utilized in studies of underdeveloped regions.

estimate of β_1, is sufficiently large relative to its standard error, we would be willing to conclude that variations in Crime are associated with variations in Sanctions; or, in other words, a change in Sanctions, holding Age, Poverty and other variables constant, induces a change in Crime.

The foregoing is a fair description of the best practice in the field at the present time. It is not always the correct practice. There are a class of relations for which the above procedure is improper even though the usual regression equation conditions are met.[4] If the true state of the world is not characterized by a single relation between two variables but by a *system* of relations, the coefficients we estimate within a single regression equation such as (2) can be consistently incorrect not only in magnitude but even as to the sign of the coefficient. A mathematical proof of the proposition that single equation estimates are biased in such instances is easily developed,[5] but for our purposes is unnecessary. The flaw in the single equation approach to the C-S relation can be demonstrated rather easily.

Suppose the true C-S relation has the form given by Model 3:

(3a)   CRIME $= \beta_0 + \beta_1$ SANCTIONS
$+ BX + \mu$

(3b)   SANCTIONS $= \alpha_0 + \alpha_1$ CRIME
$+ AZ + \nu,$

wherein X and Z represent appropriate collections of independent variables with their respective sets of coefficients, B and A, and where μ and ν are the conventional error terms. Suppose, in the interests of simplification, we proceed with a regression analysis as we did with (2). What we might find is a configuration of observations in the C-S plane such as is illustrated by the five observation points in Figure 1. The Poverty and Age dimensions are not shown in the diagram. One can regard the observations in Figure 1 as having been "corrected" for Age and Poverty or as having come from a universe in which Age and Poverty are invariant. The estimated (partial) linear regression between Crime and Sanctions is also shown.
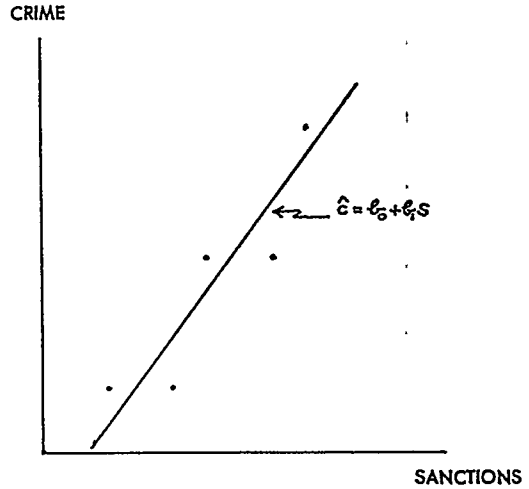
CRIME

SANCTIONS

FIGURE 1.

Assuming that b_1 is large relative to its standard error, what may we infer from Figure 1? If b_1 is statistically significant by our criterion, we must conclude $\beta_1 > 0$. However such a conclusion is troubling, since present belief, provisional and imperfect though it is, strongly suggests $\beta_1 \leq 0$.[6] No doubt our impulse would be to try a more refined model, such as introducing more of the X variables or trying non-linear functions. Yet, no matter what we do, if Model 3 is a correct description of the world, we are destined to obtain biased estimates of β_1 except by the purest of accidents.

To illustrate: Suppose, in fact, the conventional hypothesis is true, *viz.* $\beta_1 < 0$; that is, holding all X variables constant, more severe sanctions depresses the crime rate. Suppose it is also true that, holding all Z variables constant, an increase in crime rates leads to a rise in sanctions. (For example, society may react to a rising crime rate by imposing increasingly severe penalties.) From an empirical point of view, what would we observe? If the world is nonstochastic and all X and Z variables are constant, we would have the result depicted in Figure 2a. The downward sloping line shows Crime as a function of Sanctions, holding X and all random variation constant. The Sanctions function is similarly interpreted. The interaction of the two functions provides a unique equilib-

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[4] The disturbance term must be a random variable with zero expectation, constant variance and no inner correlation, no exact linear relation must exist between any two or more independent variables, etc.

[5] *See* the Technical Appendix *infra.*

[6] I refer to "average" situations, and readily concede that aberrant cases may exist for which $\beta_1 < 0$; as, for example, the case of civil rights workers who committed a crime because legal sanctions would be imposed.
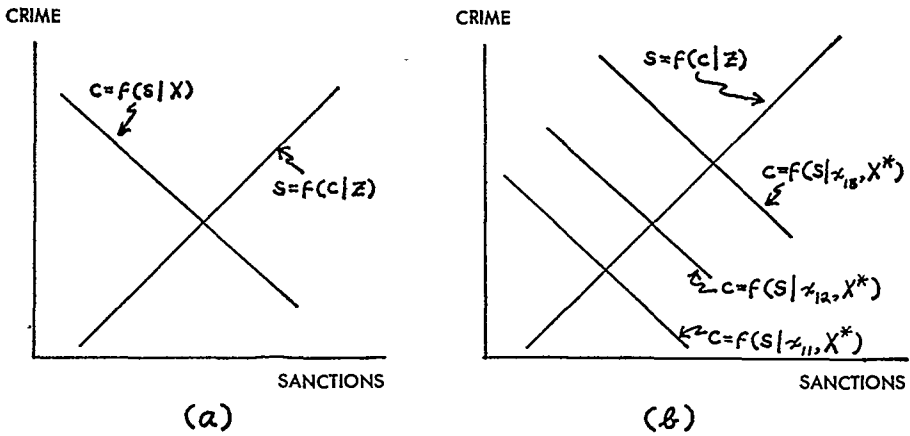
FIGURE 2.

rium solution: a crime rate of c_1 and a sanctions level s_1. Under the conditions we have stipulated, we would observe only one data point, (c_1, s_1).

Now suppose one and only one variable is permitted to assume different values, x_1 in X. Figure 2b illustrates the configuration of functions and data points that might arise. (X* is X with x_1 omitted.) Empirical observation would reveal three data points or observations, and since we disallow random variation, these observations would produce an exact linear relation which we might write as follows:

(4)   CRIME $= b_0 + b_1$ SANCTIONS, $b_1 > 0$.

But note: b_1 is not an estimator of β_1 but of α_1, and indeed is exactly equal to α_1! Thus, in this simplified scheme, if the Sanctions function remains constant while the Crime function varies, the Sanctions function is estimated. Conversely, if the Crime function is to be estimated, it must remain constant, and the Sanctions function must vary. Variation in the latter will trace out the pattern of the former.

With these considerations in mind, let us return to the main thread of the argument. Let Model 3 accurately describe the world. Let the research worker see only Figure 1. (In a very real sense, Model 3 is always essentially unknowable.) What can this investigator make of Figure 1? If he thinks that there is a likelihood that Model 3 correctly describes the world, he will recognize that the configuration of points in Figure 1 does not necessarily describe either a Crime or a Sanction function and that there are a number of possible interpretations for Figure 1. To pick two examples: (1) each observation may have arisen from varia-

tions in both the Crime and Sanctions functions as shown in Figure 3a; and (2) there may have been three upward shifts in the Sanctions function—with Sanctions being unresponsive to Crime, incidentally—and a continuous outward shift in the Crime function, as shown in Figure 3b. Thus, with Figure 1 subject to so many possible interpretations, single equation estimating procedures are clearly inappropriate.

The conceptual problem which we have outlined above is one familiar to economists. If Figure 2a is relabeled so that price and quantity are measured on the axes, and if the two functions are named Demand and Supply, the attempt to estimate one or both of the functions would confront us with the economist's classic "identification" problem. Becker,[7] an economist, appears to be the first person to give theoretical recognition to the fact that C-S relation involves interrelated functions, and therefore that one must seek a joint solution. More recently, Hahn[8] has applied conventional price theory analysis to an interrelated system involving "production set" and "social welfare" functions. He also refers to "crime and anticrime supply and demand functions." Since Becker is not interested in empirically determining the parameters of his model, the identification problem never arises in his study. On the other hand, Hahn makes explicit reference to the identification problem, but is pessimistic about our ability to devise a model whose parameters can be estimated in the near future.

[7] Becker, *Crime and Punishment: An Economic Approach*, 76 J. POL. ECON. 169 (1968).
[8] Hahn, *Crime and the Cost of Crime: An Economic Approach*, 9 J. RES. CRIM. & DELINQUENCY 12 (1972).
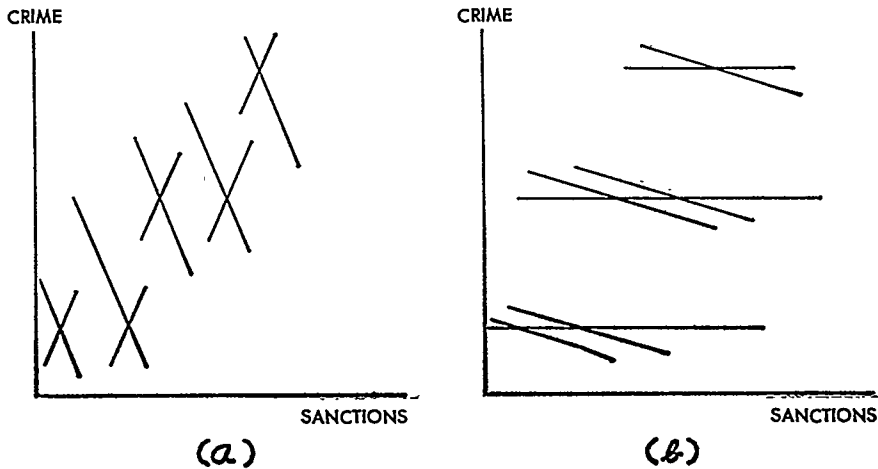
FIGURE 3.

The relevant empirical literature deals with the deterrent influence of sanctions on criminal activity. An important debate, initiated by Gibbs, and (temporarily) ending with Logan,[9] because of its empirical orientation, provided a natural forum for the discussion of the identification problem. Only Logan has recognized the problem, but he saw no way of surmounting it.[10] Phillips and Votey[11] treat the deterrence question, but without explicit reference to the Gibbs-Logan controversy. They do recognize the presence of the identification problem and provide estimates for an automobile theft, clearance rate relation, but the means by which they devised simultaneous estimates is not clear.[12]

Blumstein and Larsen[13] have dealt with models involving an interrelationship between crime and sanctions. Their models involve a chain of Markov-

process probabilities, the parameters of which are readily estimated. However, their models assume away interactions of the kind posited in Figure 2a. The models generate point estimate solutions. They do not and cannot yield estimates of the slope coefficients of the C-S relation.[14] Thus, with the single exception of the paper just published by Phillips and Votey,[15] the empirical literature dealing with the C-S relation is fatally flawed.

In recent years econometricians have provided us with several techniques for simultaneous least squares regression equation estimation. Under appropriate conditions these methods are capable of providing unbiased estimates of both the Crime and Sanctions functions. "Appropriate conditions" is, however, an important caveat. In order that the least squares estimates be trustworthy, the following minimum conditions must be satisfied.[16]

(1) Model 3 must truly describe the world as it is. For example, if the true Crime-Sanctions relation is characterized by a structure of the form

(4a)   CRIME = f(SANCTIONS, X)

(4b)   PROTECT = f(CRIME, Y)

(4c)   SANCTIONS = f(PROTECT, Z),

wherein PROTECT refers to the volume or type of protective service personnel, then Model 3 is inappropriate.[17]

[9] *See* Gibbs, *Crime, Punishment, and Deterrence*, 48 SOUTHWEST. SOC. SCI. QUART. 315 (1968); Gray & Martin, *Punishment and Deterrence: Another Analysis of Gibb's Data*, 50 Soc. SCI. Q. 189 (1969); Chiricos and Waldo, *Punishment and Crime: An Examination of Some Empirical Evidence*, 18 Soc. PROB. 200 (1970); Logan, *General Deterrent Effects of Imprisonment*, 51 Soc. FORCES 64 (1972).
    [10] *See* 51 Soc. FORCES, *supra* note 9, at 68.
    [11] Phillips & Votey, *An Economic Analysis of the Deterrent Effect of Law Enforcement on Criminal Activity*, 63 J. CRIM. L. C. & P.S. 330 (1972).
    [12] In their paper the specific technique used is not specified. The technique apparently is described in *Economic Crimes: Their Generation, Deterrence and Control*, 41, 23a, 27a (NATIONAL TECHNICAL INFORMATION SERVICE PB 194 984, 1969).
    [13] Blumstein & Larsen, *Problems of Modeling and Measuring Recidivism*, 8 J. RES. CRIM & DELINQUENCY 124 (1971); and *Models of a Total Criminal Justice System*, 17 OPERATIONS RES. 199 (1969).

[14] *Id.* at 207.
    [15] *See* note 11 *supra*.
    [16] For more detail, see the Technical Appendix *infra*.
    [17] Note, however, that simpler models need not be inconsistent with their more complex counterparts. With appropriate values for its coefficients, Model 4

(2) All important X and Z variables must appear in Model 3's regressions.

(3) The usual statistical requirements for ordinary single equation least squares estimation must be met.

Obviously, the first condition asks the impossible.[18] The validity of our results depends upon our picking the correct model of the world. But which model is correct? Since we cannot know the unknowable, how can we select the correct model from an infinite number of possibilities? We make an educated guess based upon the best information available. If informed opinion indicates that Model A best describes the world, then the desired parameters of the world ought to be estimated within the framework of Model A. To do otherwise is methodologically incorrect and has a greater likelihood of seriously misrepresenting the relation we seek to evaluate.

At this general level one more comment is relevant: I have chosen as my example—Model 3 and Figure 1—a particularly strong case, one in which ordinary least squares produces a totally incorrect estimate of the coefficients we wish to estimate. It is my impression, based on a cursory examination of the coefficients produced by ordinary least squares as compared to those produced by one of the simultaneous equation methods, that the two sets of coefficients often produce broadly similar results. But this is only a personal impression and, because of the difficult interpretive questions involved, is not documented in this paper.[19] In any event, large divergences can arise to the discredit of the single equation model. Hence, ordinary least squares estimates must always be suspect when there is a reasonable likelihood that the world is characterized by a complex relation among the variables which we wish to explain.

The following is a concrete example of the use of simultaneous equation estimation and a comparison of its results with that of ordinary least squares. The relation between the rate of reported felony crime and the risk of legal punishment has been selected for investigation. This relation is given the following structure:

$$(5a) \quad CRIME = \beta_0 + \beta_1 \, RISK + \beta_2 \, AGE$$
$$+ \beta_3 \, POOR + \beta_4 \, CITY + \mu$$

$$(5b) \quad RISK = \alpha_0 + \alpha_1 \, CRIME + \alpha_2 \, AGE$$
$$+ \alpha_3 \, POOR + \alpha_4 \, COP + \nu,$$

wherein the variables are defined as follows:

CRIME: Per capita reported felony crime for the conventional seven major felonies —the F.B.I.'s index crimes.

RISK: Number of persons convicted of a felony relative to number of felonies reported. (This is all reported felonies.)

AGE: Persons between the ages of 15 and 30 relative to all persons.

POOR: An index of poverty. Combines the proportion of all families who have incomes of less than $3000—a conventional index of poverty—with the infant mortality rate. Both variables, alone, are seriously flawed indicators of poverty. Their combined value may be more representative of true poverty levels.

COP: Number of police and sheriffs, personnel, sworn and civilian, per capita.

A brief justification for the variables included in the model is probably desirable, though not essential, since we are primarily concerned with methodological considerations. Age, poverty and risk have often been cited as affecting the crime rate, hence their inclusion in (5a).[20] CITY serves in a multiple capacity: it may be correlated with the degree to which attitudes tend to be asocial, if not anti-social, with the per capita number of possibilities (targets) for property crime, and with the expected payoff per crime. Inclusion of AGE in (5b) derives from the well known hypotheses that younger people are less likely to be arrested, to be charged if arrested, to have the charge reduced to a misdemeanor, or to be convicted if charged with a felony. The converse applies to POOR. The *raison d'être* for COP is obvious. Finally, the inclusion of

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reduces to Model 3 and Model 3 to a single equation system.

[18] In a way, so does the second. But practically speaking it seems easier to single out potential variables than to specify the correct structure for the variables.

[19] Several attempts at strictly controlled comparisons have been made, but they necessarily involve such simplified models that their applicability to the more complex, practical situations of applied research is questionable. For a summary of such comparisons, see C. CHRIST, ECONOMETRIC MODELS AND METHODS 474 (1966).

[20] The ages 15 to 30 represent a somewhat arbitrary truncation of a continuous age-arrest function which peaks in the early twenties. Alternative reasonable ranges for the crime susceptible ages would not yield significantly different values, however.

TABLE 1

ORDINARY AND TWO-STAGE LEAST SQUARES ESTIMATES OF THE CRIME-SANCTIONS RELATION

(t Values in Parentheses)

Ordinary Least Squares[a]

(1) CRIME = −.34 RISK − .07 AGE + .20 POOR + .66 CITY R^2 = .54
                (3.1)   (.7)   (1.8)   (7.0)

(2) RISK   = −.38 CRIME − .09 AGE + .53 POOR + .09 COP R^2 = .35
                (2.7)   (.8)   (4.8)   (.6)

Two Stage Least Squares[a]

(3) CRIME = −2.30 RISK − .22 AGE + 1.22 POOR + .42 CITY R^2 = .50
                (2.3)   (1.8)   (2.3)   (2.7)

(4) RISK   = −.06 CRIME − .09 AGE + .51 POOR − .11 COP R^2 = .26
                (.2)   (.8)   (4.3)   (.5)

[a] R^2 and t values are adjusted for degrees of freedom.

Source of the data:

CRIME, RISK, COP: CALIFORNIA DEPARTMENT OF JUSTICE, CRIME IN CALIFORNIA 1960.

AGE, POOR, CITY: U.S. BUREAU OF THE CENSUS, CENSUS OF POPULATION 1960.

CRIME in (5b) derives from the hypothesis that, with a given level of police activity (COP), the risk of apprehension will diminish because police services are spread more thinly.

No doubt Model 5 can be faulted on both its structure and its choice of variables. The model oversimplifies reality: the structure represented by Model 4 would probably get closer to the truth. Furthermore, it is not clear that the correct set of variables has been chosen for inclusion in the model, or that the variables have received their correct formulation. (For example, should protective service personnel other than police be included in COP?) Yet, despite these deficiencies, Model 5 will serve quite nicely. Our concern is with methodology, not with the substantive question of the magnitude and expected sign of certain regression coefficients. To ferret out the "correct" system of equations and the "correct" set of variables would require an extensive analysis of the literature and a more thoughtful, more closely reasoned set of hypotheses concerning the interrelatedness of the variables suggested by the literature, and is not the proper subject of this paper.

The sample data to evaluate Model 5 are drawn from California for the year 1960. The 41 largest counties as measured by the number of felonies reported, were used and have been combined with the remaining 17 counties into a single region. All cities having a population in excess of 100,000 persons—there were 13 of these—have been withdrawn from their respective counties and have been treated as separate regions. Thus, the sample size is 55. All variables were transformed into standard

normal deviates—zero mean and unit standard deviation—to facilitate comparisons among coefficients. The coefficients of Model 4 were estimated by ordinary least squares (OLS) and by two-stage least squares (TSLS). For a variety of technical reasons, TSLS seems to be preferred to the alternative multiple equation methods, and is today the most commonly used of these methods.[21]

The statistical results are displayed in Table 1. What can be said about the effect of Risk on Crime? The first TSLS equation shows that a one standard deviation increase in risk leads to a 2.3 standard deviation decrease in reported crime, whereas the corresponding OLS equation shows a decline of only 0.34 standard deviations. To revert to the original, nontransformed Crime and Risk units, this means that an increase in the ratio of convictions to reported felony crimes from, say, 92 per 1000 (the mean ratio for the 55 regions) to 101 per 1000 (a ten percent increase) leads to an estimated decline in per capita rates of reported crime from 10.2 per 1000 (the mean crime rate) to 8.4 per 1000 based on TSLS (an 18 percent decrease) but a decline to only 9.9 per 1000 based on OLS (a 2.6 percent decrease). Since both estimates seem to be large relative to their own standard errors, they seem to offer contradictory estimates of β_1.[22] The difference in results for the Risk equa-

[21] For some experimental results with alternative estimating techniques see CHRIST, *supra* note 19, at 474.

[22] The TSLS t values in Table 1 are not exact. They are only meant to suggest a general level of statistical significance. The probability distribution for TSLS coefficients is, indeed, known for two equation systems—but only two equation systems!—but the
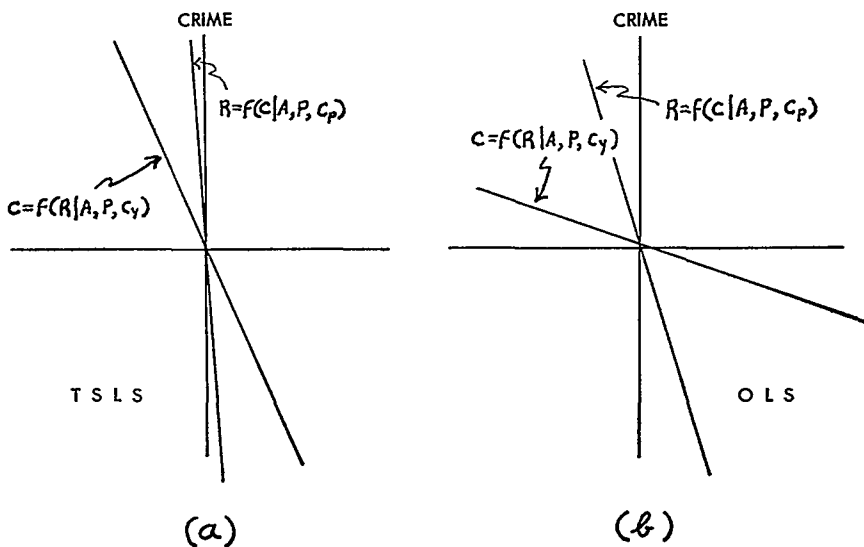
FIGURE 4.

tion are even more marked. The TSLS equation suggests that crime rates do not affect risk levels, whereas the OLS equation suggests that a rise in the crime rate induces a large (in a statistical sense) decline in the risk of punishment. A summary view of the differences in the estimates of α_1 and β_1 can be had by recourse to a two dimensional representation of the Crime-Risk relation. In Figure 4a the TSLS equations are presented, in Figure 4b the OLS equations.[23]

It is hoped that these California data have made the necessary point, that crime and risk must be treated as belonging to an interdependent system of relations. *A priori* we would have expected an interaction between Crime and Risk. The data support the hypothesis, since the treatment of Crime and Risk as belonging to a system of relations yields estimates that depart markedly from single equation estimates. In particular, OLS undervalues the impact of Sanctions on Crime. While we cannot be confident that our particular TSLS estimates of α_1 and β_1 are the correct estimates for California for 1960, we would do well to

prefer these estimates over those derived by OLS. Moreover, even though extrapolation from one model and one body of data is necessarily hazardous, the implication of the foregoing analysis is that past empirical studies have tended to underestimate the negative influence of Sanctions on Crime.[24]

A word about the other coefficients in Table 1. The variables of Model 5 were not chosen to get at the true Crime-Risk relation, but rather to illustrate two final points. First, the coefficients of the independent variables in a TSLS system are often statistically indistinguishable from their OLS counterparts, but not always: in the Risk function note the very different values for the POOR variable. The second point is more important. It concerns the statistically nonsignificant value of the COP coefficient. It is possible, of course, that the estimate, $\hat{\alpha}_4$, derives from a world in which $\alpha_4 \doteq 0$ and hence accurately represents the real world, but there are solid reasons for believing in $\alpha_4 > 0$, and no plausible reason for believing in a world of $\alpha_4 \leq 0$. An obvious and more palatable explanation for our inability to rule out $\alpha_4 = 0$ is readily available. It could be that COP varies systematically with some factor, x, and that x varies inversely with RISK. If x does not appear explicitly in the Risk function, the COP coefficient would reflect the net effect of the two variables rather than a "pure" COP effect.

computation of the correct standard errors is quite tedious and not worth the effort involved, given an unwillingness to accept the model as being an accurate description of reality. For the probability distribution of the TSLS coefficients, see Sawa, *The Exact Sampling Distribution of Ordinary Least Squares and Two-Stage Least Squares Estimators*, 64 J. AMER. STAT. ASSOC. 923 (1969).

[23] The equations go through the origin since all variables have been expressed as deviations from their arithmetic means.

[24] The analysis in the Technical Appendix lends further credence to this expectation.

There is another, more interesting possibility. It could be that the Cop-Risk relation is itself misspecified. Perhaps the Cop-Risk relation goes two ways. There is the obvious direction: the more police, the higher the risk of apprehension. But what about the other direction, that is, the higher the risk, the fewer the police? Could it not be that the community has its own conception of the appropriate level of police activity, and that this level may be a function of the risk level? If too many crimes go unpunished, might the community not hire more police? Since this hypothesis is plausible, we are obliged to reconsider the merits of Model 5. If there is a Cop-Risk interaction, (5a) is inappropriate and a more complex model, involving at least three equations, will have to be developed. Thus, we see how treacherous it is to adopt a single equation view of the world.

TECHNICAL APPENDIX

The diagrammatic demonstration which showed that OLS procedures can result in biased estimators, as well as the numerical demonstration, using California data for 1960, which showed that the error can be substantial, invite a more general proof of the existence of OLS bias and a more general appraisal of the magnitude of the OLS bias. Let us posit a two-equation model containing two exogenous variables:

(6a) $C = \beta_0 + \beta_1 S + \beta_2 X + \mu$

(6b) $S = \alpha_0 + \alpha_1 C + \alpha_2 Z + \nu$

We solve for C and S, obtaining

(7a) $C = (\alpha_0\beta_1 + \alpha_2\beta_1 Z + \beta_0 + \beta_2 X$
$+ \beta_1\nu + \mu)/(1 - \alpha_1\beta_1)$

(7b) $S = (\alpha_0 + \alpha_1\mu + \alpha_1\beta_0 + \alpha_1\beta_2 X$
$+ \alpha_2 Z + \nu)/(1 - \alpha_1\beta_1)$

The OLS estimate of β_1, written $\hat{\beta}_1$, with all variables expressed in terms of deviations from their means, is

(8) $\hat{\beta}_1 = [\Sigma \, cs \, \Sigma \, x^2 - \Sigma cx \, \Sigma sx] \, / \, [\Sigma \, s^2 \, \Sigma \, x^2$
$- (\Sigma \, sx)^2]$

Now place Equation (7), in deviation form, in (8) and allow the sample size to become infinitely

large. Assuming that X and Z are independent of μ and ν and independent of each other, we obtain

(9) $\hat{\beta}_1 = \dfrac{(1 + \alpha_1\beta_1)\sigma_{\mu\nu} + \alpha_1\sigma_{\mu}^2 + \alpha_2^2\beta_1\sigma_z^2 + \beta_1\sigma_\nu^2}{2\alpha_1\sigma_{\mu\nu} + \alpha_1^2\sigma_{\mu}^2 + \alpha_2^2\sigma_z^2 + \sigma_\nu^2}$

where $\sigma_{\mu\nu}$, σ_{μ}^2, σ_ν^2 and σ_Z^2 are the covariance of μ and ν and the variances of μ, ν and Z, respectively. We now isolate β_1 so as to compare the discrepancy between $\hat{\beta}_1$ and β_1:

(10) $\hat{\beta}_1 = \beta_1 + \dfrac{(1 - \alpha_1\beta_1)\sigma_{\mu\nu} + (1 - \beta_1)(\alpha_1\sigma_{\mu}^2 + \alpha_2^2\sigma_z^2)}{2\alpha_1\sigma_{\mu\nu} + \alpha_1^2\sigma_{\mu}^2 + \alpha_2^2\sigma_z^2 + \sigma_\nu^2}$

The right hand expression is, of course, the least squares bias ascribable to the use of OLS with Model 6. This expression will be zero only if $\alpha_1 = \alpha_2 = 0$ and $\sigma_{\mu\nu} = 0$. Note that $\alpha_1 = 0$ is necessary but not sufficient. Thus my discussion in the text dwelt on just one of the sources of OLS bias.

Intuition suggests that this bias will be positive. We believe that $\alpha_1 \geq 0$ and $\beta_1 \leq 0$. Hence all terms in the expression for bias are positive, with the possible exception of $\sigma_{\mu\nu}$. A priori there is little reason to suppose that μ and ν will be strongly negatively correlated. Hence our guess is that OLS will tend to understate the negative effect of Sanctions on Crime.

The foregoing results are special in a number of ways. If X and Z are not independent, σ_z^2 must be replaced in (10) by the variance of $(Z - \hat{Z})$, where $\hat{Z}$ is derived from the regression of Z on X. The effect of this should be transparent. I have estimated only the asymptotic bias. The small sample properties of the bias expression are not known, but it is doubtful that our conclusions would be changed by an examination of the expected value of the bias for finite samples. We have examined the OLS bias only for the case of two equations and two exogenous variables, one per equation. Bronfenbrenner[25] has evaluated the asymptotic bias for the general linear model, i.e. a model involving any number of equations and variables. Unfortunately, the bias expression is quite complex, and the results are so opaque as to be devoid of practical significance, except that they demonstrate that the bias exists.

[25] Bronfenbrenner, *Sources and Size of Least-Squares Bias in a Two-Equation Model*, in STUDIES IN ECONOMETRIC METHOD 221 (W. C. Hood and T. C. Koopmans, eds. 1953).