Crises and Capital Requirements in Banking: On Line Appendix

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Final Part of Proof of Proposition 3

Recall from the paper that the expected payment r_b from investment in a bank is given by

$$r_b = \alpha_b (R - Q) p_H + (1 - \alpha_b) \left\{ (1 - \lambda)(R - Q) p_L + \lambda R p_L \frac{k}{k-1} \right\},$$

where $\alpha_b = a + (1 - a) \frac{\mu}{b}$, and that the proportion of wealth which a depositor will invest in a bank given that the regulator is bad is given by

$$R_b = \frac{(\mu - 1)\mu}{N - 2\mu + \frac{\mu^2}{b}}.$$

Then straightforward manipulation yields:

$$\begin{aligned} & \frac{\partial}{\partial b} \left[R_b (r_b - R p_L) + R p_L \right]_{a=0} \\ & \quad = \frac{\mu^2 (k-1)}{b^2 \left(N - 2\mu + \frac{\mu^2}{b} \right)} \left\{ \left[(R - Q)(\Delta p + p_L) - p_L R \lambda \frac{k}{k-1} \right] \left(\frac{\mu^2}{(N - 2\mu) b + \mu^2} - 1 \right) \right. \\ & \qquad \left. + \frac{\mu}{N - 2\mu + \frac{\mu^2}{b}} \left[(1 - \lambda)(R - Q) p_L - \frac{R p_L}{k-1} (k(\lambda + 1) - 1) \right] \right\}. \end{aligned}$$

Since $N > 2\mu$, the first of these terms is clearly negative. The second term has the same sign as $(1 - \lambda)(R - Q) p_L - \frac{R p_L}{k-1} (k(\lambda + 1) - 1) < (R - Q) p_L - R p_L \frac{k}{k-1} < 0$: this concludes the proof.

Final Part of Proof of Proposition 4

Recall from the paper that

$$\begin{aligned} R^{IR}(a, k) &\equiv R \frac{(aN - 2a\mu + \mu)\Delta p + \frac{\lambda p_L}{k-1}(1 - a)(N - \mu)}{p_H (aN - 2a\mu + \mu) + p_L (1 - a)(N - \mu)(1 - \lambda)}; \\ B^P(a, k) &\equiv \frac{R\left((k-1)\,\mu\,(a\,(N - 2\,\mu) + \mu)\,\Delta p + \lambda\,(N - \mu)\,(N - a\,\mu)\,p_L\right)}{(k-1)\,(\mu\,(a\,(N - 2\,\mu) + \mu)\,p_H + (1 - \lambda)\,(N - \mu)\,(N - a\,\mu)\,p_L)}. \end{aligned}$$

Differentiation of these expressions with respect to a and manipulation yields:

$$\begin{aligned}
\frac{\partial}{\partial a} R^{IR}(a,k) &= \frac{R(N-\mu)^2 p_L \left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)}{\left((a(N-2\mu)+\mu)p_H + (1-a)(1-\lambda)(N-\mu)p_L\right)^2}; \\
\frac{\partial}{\partial a} B^{P}(a,k) &= \frac{R(N-\mu)^3 \mu p_L \left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)}{\left(\mu(a(N-2\mu)+\mu)p_H + (1-\lambda)(N-\mu)(N-a\mu)p_L\right)^2}.
\end{aligned}$$

As required, both of these expressions are positive multiples of $\left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)$.

Recall further that

$$\begin{aligned}
B^{O}(k) &\equiv \frac{R\left(\mu \Delta p + \lambda p_L \left(\frac{N-\mu}{k-1}\right)\right)}{N(1-\lambda)p_L + \mu(p_H - (1-\lambda)p_L)}; \\
B^{OP}(a,k) &\equiv \frac{R\lambda}{(k-1)(1-\lambda)}.
\end{aligned}$$

Then manipulations yield the following:

$$\begin{aligned}
B^P - B^{OP} &= \frac{R\mu(a(N-2\mu)+\mu)\left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)}{(1-\lambda)\left(\mu(a(N-2\mu)+\mu)p_H + (1-\lambda)(N-\mu)(N-a\mu)p_L\right)}; \\
B^O - B^P &= \frac{(1-a)R(N-\mu)^2 \mu p_L \left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)}{(\mu p_H + (1-\lambda)(N-\mu)p_L)\left(\mu(a(N-2\mu)+\mu)p_H + (1-\lambda)(N-\mu)(N-a\mu)p_L\right)}; \\
R^{IR} - B^O &= \frac{aR(N-\mu)^2 p_L \left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)}{(\mu p_H + (1-\lambda)(N-\mu)p_L)\left((a(N-2\mu)+\mu)p_H + (1-a)(1-\lambda)(N-\mu)p_L\right)}.
\end{aligned}$$

Once again, each of these expressions is a positive multiple of $\left((1-\lambda)\Delta p - \frac{\lambda p_H}{k-1}\right)$. Note moreover that $\left(R^{IR} - B^O\right)\big|_{a=0} \equiv 0$.

Detailed Manipulations from the Proof of Proposition ***5***

Recall from the paper that the welfare difference between tightly and loosely regulated economies when pessimistic expectations obtain is:

$$G(a) = \left(k^P - 1\right) - \left(k^M - 1\right)\frac{\mu + a(N-\mu)}{N}.$$

Furthermore,

$$\begin{aligned}
k^P &= 1 + \frac{R\lambda(N-\mu)(N-a\,\mu)p_H p_L}{C(1-\lambda)\,(N-\mu)(N-a\mu)p_L - (\mu(a(N-2\mu)+\mu)p_H(R\Delta p - C))}; \\
k^{BM} &= 1 + \frac{(1-a)R\lambda(N-\mu)p_H p_L}{C(1-\lambda)(N-\mu)(1-a)p_L - p_H((a\,(N-2\mu)+\mu)(R\Delta p - C))}; \\
k^{MM} &= \frac{R(N-a\mu)p_L(\Delta p + \lambda p_L)}{(1-a)\;C\;(1-\lambda)\;(N-\mu)\;p_L - (a\;(N-2\mu)+\mu)\;(R\Delta p(\Delta p + \lambda p_L) - Cp_H)}.
\end{aligned}$$

We use these to determine the welfare gap. Firstly, when $k^{BM} < k^{MM}$, substitution and extensive manipulation yields

$$G(a)|_{k^{BM}<k^{MM}} = \frac{R\lambda\,(N-\mu)\,(N-a\,\mu)\,p_H\,p_L}{C\,(1-\lambda)\,(N-\mu)\,(N-a\,\mu)\,p_L - (\mu\,(a\,(N-2\,\mu)+\mu)\,p_H\,(R\Delta p - C))} - \left(\frac{(1-a)\,R\lambda\left(1-\frac{\mu}{N}\right)(a\,(N-\mu)+\mu)\,p_H p_L}{C\,(1-a)\,(1-\lambda)\,(N-\mu)\,p_L - ((a\,(N-2\mu)+\mu)\,p_H\,(R\Delta p - C))}\right).$$

Whence further manipulation yields, when $k^{BM} < k^{MM}$,

$$(1)\quad G'(a)\big|_{k^{BM}<k^{MM}} = R\lambda(N-\mu)^2 p_H p_L \times \Bigg\{-\left(\frac{a\,(a\,(N-2\mu)+2\mu)\,p_H\,(R\Delta p - C) + (1-a)^2 C\,(1-\lambda)\,(N-\mu)\,p_L}{N(C\,(1-a)\,(1-\lambda)\,(N-\mu)\,p_L - ((a\,(N-2\mu)+\mu)\,p_H\,(R\Delta p - C)))^2}\right) + \frac{(N-\mu)\,\mu p_H\,(R\Delta p - C)}{(C\,(1-\lambda)\,(N-\mu)\,(N-a\mu)\,p_L - (\mu\,(a\,(N-2\mu)+\mu)\,p_H\,(R\Delta p - C)))^2}\Bigg\}.$$

When $k^{BM} > k^{MM}$, manipulation again yields the following:

$$G(a)|_{k^{BM}>k^{MM}} = \frac{R\lambda\,(N-\mu)\,(N-a\mu)\,p_H p_L}{C\,(1-\lambda)\,(N-\mu)\,(N-a\,\mu)\,p_L - (\mu\,(a\,(N-2\mu)+\mu)\,p_H\,(R\Delta p - C))} - \frac{a\,(N-\mu)+\mu}{N} \times \Bigg\{-1 + \frac{R\,(N-a\mu)\,p_L\,(\Delta p + \lambda p_L)}{C\,(1-a)\,(1-\lambda)\,(N-\mu)\,p_L - (a\,(N-2\mu)+\mu)\,(R\Delta p\,(\Delta p + \lambda p_L) - Cp_H)}\Bigg\},$$

and

$$(2)\quad G'(a)\big|_{k^{BM}>k^{MM}} = \frac{N-\mu}{N} + \frac{R\lambda(N-\mu)^3\mu {p_H}^2\,(R\Delta p - C)\,p_L}{(C\,(1-\lambda)\,(N-\mu)\,(N-a\mu)\,p_L - (\mu\,(a\,(N-2\mu)+\mu)\,p_H\,(R\Delta p - C)))^2} - \frac{(a\,(N-\mu)+\mu)}{N}\frac{R(N-\mu)^2\,(R\Delta p - C)\,p_L\,(\Delta p + \lambda p_L)^2}{(C\,(1-\lambda)\,(1-a)\,(N-\mu)\,p_L - (a\,(N-2\mu)+\mu)\,(R\Delta p\,(\Delta p + \lambda p_L) - Cp_H))^2} - \frac{(N-\mu)}{N}\frac{R\,(N-a\mu)\,p_L\,(\Delta p + \lambda p_L)}{C\,(1-\lambda)\,(1-a)\,(N-\mu)\,p_L - (a\,(N-2\mu)+\mu)\,(R\Delta p\,(\Delta p + \lambda p_L) - Cp_H)}.$$

Note that when $a = 0$, R^{IR} coincides with B^O, and hence that $k^{BM} < k^{MM}$. Hence:

$$G(0) = \frac{R\lambda\mu\left(1-\frac{\mu}{N}\right)p_H p_L}{C\,(1-\lambda)\,(N-\mu)\,p_L - \mu p_H\,(R\Delta p - C)} + \frac{R\lambda\,(N-\mu)\,p_H p_L}{C\,(1-\lambda)\,(N-\mu)\,p_L - \mu\frac{\mu}{N}p_H\,(R\Delta p - C)},$$

as in the paper.

Substituting into equations (1) and (2) yields the following in both cases:

$$G'(a)\big|_{C=R\Delta p} = -\frac{\lambda\,(N-\mu)\,p_H}{N\,(1-\lambda)\,\Delta p} < 0,$$

as required.