Critical Coupling and Its Control in Optical Waveguide-Ring Resonator Systems

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***Abstract*—The coupling of optical waveguides to ring resonators holds the promise of a new generation of switches (modulators) which employ orders of magnitude smaller switching (modulation) voltages (or control intensities). This requires a means for voltage (or intensity) control of the coupling between the waveguide and the resonator. Schemes for achieving such control are discussed.**

***Index Terms*—Electrooptic modulation, integrated optics, optical resonators, optical waveguides, optoelectronic devices.**

RECENTLY, we have been witnessing an ever-increasing pace of activity in the area of coupling between optical waveguides and ring or micro resonators [1]–[3]. Devices based on this coupling hold the promise of a new modality of light switching, amplification, and modulation.

Before this field can proceed to the realm of scientific and technological applications, we must develop methods, preferably electrical or optical, to precisely control the coupling. This is the main concern of this note.

The generic geometry is illustrated in Fig. 1. A waveguide and a ring resonator, both enter and emerge from a coupling region (the dashed box) where power exchange takes place. This exchange is describable in terms of universal relations which are independent of the specific embodiment [4]. Some key results of that analysis needed here are stated in what follows.

If the coupling is limited only to waves traveling in one sense i.e., no reflection takes place, and if the total powers entering and leaving the coupling region ("box") are equal (lossless case), then the coupling can be described by means of two constants κ and t and a unitary scattering matrix

$$\begin{vmatrix} b_1 \\ b_2 \end{vmatrix} = \begin{vmatrix} t & \kappa \\ \kappa^* & -t^* \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \end{vmatrix} \tag{1}$$

$$|t|^2 + |\kappa|^2 = 1. \tag{2}$$

Equations (1) and (2) are supplemented by the circulation condition in the ring

$$a_2 = b_2 \alpha e^{i\theta} \tag{3}$$

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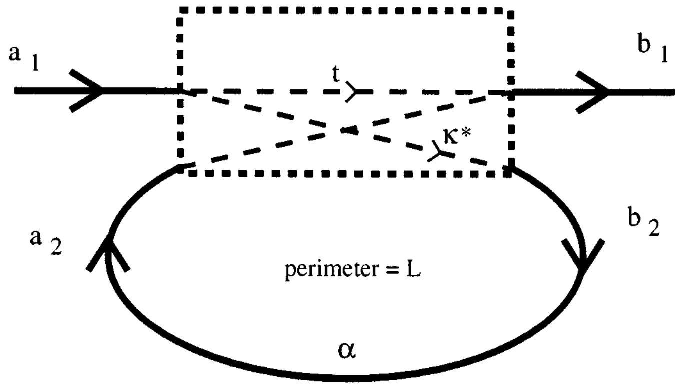

Fig. 1. The generic geometry for waveguide ring resonator coupling.

where α and θ, real numbers, give respectively, the loss (or gain) and the phase shift per circulation. The above equations are solved to yield the transmission factor in the input waveguide

$$\left|\frac{b_1}{a_1}\right|^2 = \frac{\alpha^2 + |t|^2 - 2\alpha|t|\cos\theta}{1 + \alpha^2|t|^2 - 2\alpha|t|\cos\theta}. \tag{4}$$

In the above, we use power normalization so that $|a_i|^2$, $|b_i|^2$ are the respective traveling wave powers. We will, without loss of generality, take the incident power $|a_1|^2$ to be unity. At resonance $\theta = m\,2\pi$, m an integer, and

$$|b_1|^2 = \frac{(\alpha - |t|)^2}{(1 - \alpha|t|)^2}. \tag{5}$$

This simple universal relation, plotted in Fig. 2, has two very important features which are the key for most of the proposed applications. 1) The transmitted power $|b_1|^2$ is zero at a value of coupling $\alpha = t$, "critical coupling," and 2) For high Q resonators, α near unity, the portion of the curve to the right of the critical coupling point is extremely steep. "Small" changes in α for a given t, or vice versa, can control the transmitted power, $|b_1|^2$, between unity and zero. If we can learn how to control α and/or t, we have a basis for a switching technology. If we can do it sufficiently rapid, we have the basis of a new type of an optical modulator.

The first of the proposed coupling control schemes is illustrated in Fig. 3. It incorporates a Mach–Zehnder interferometer (MZI) sandwiched between two 3-dB couplers (the "composite" interferometer, CI) into the ring resonator. The MZI introduces a differential phase shift $\Delta\phi$ between its two arms. A light circulating, say in a clockwise sense, in the ring need enter one arm

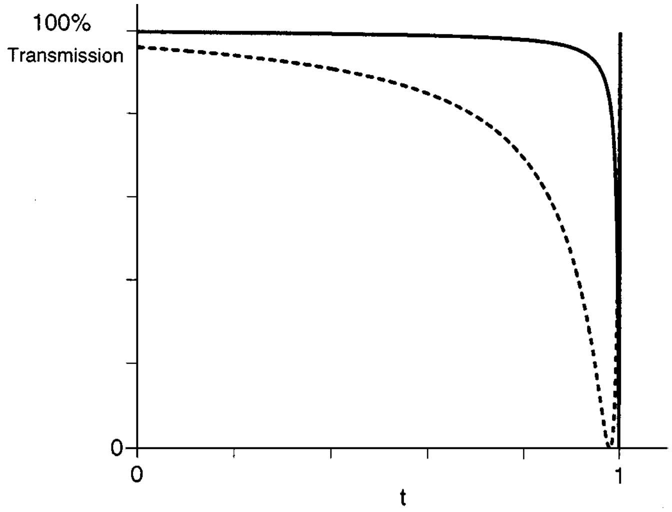

Fig. 2. The universal transmission plot for the configuration of Fig. 1. —— $\alpha = 0.999$, — — $\alpha = 0.98$.

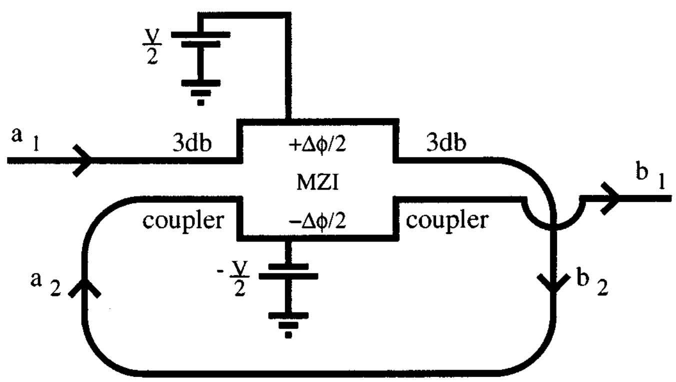

Fig. 3. A composite interferometer for achieving voltage (or light) control of the coupling between a waveguide and a ring resonator.

of the CI on the left and exit it on the right. Using the same wave designation a_i, b_i ($i = 1, 2$), as in Fig. 1, the CI is described by [4]

$$\begin{vmatrix} b_1 \\ b_2 \end{vmatrix} = \begin{vmatrix} t & \kappa \\ \kappa^* & -t^* \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} -i\cos\dfrac{\Delta\phi}{2} & -i\sin\dfrac{\Delta\phi}{2} \\ i\sin\dfrac{\Delta\phi}{2} & -i\cos\dfrac{\Delta\phi}{2} \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \end{vmatrix} \tag{6}$$

so that

$$t = -i\cos\frac{\Delta\phi}{2}, \quad \kappa = -i\sin\frac{\Delta\phi}{2}. \tag{7}$$

We note, for example, that if $\Delta\phi$ is zero $|t| = 1$, $\kappa = 0$ and $|b_1| = |a_1|$, i.e., unity transmission. When $\Delta\phi = \pi$, $t = 0$, $|\kappa| = 1$. Critical coupling and in general coupling control are, thus, achieved by controlling $\Delta\phi$. Using (7) in (4) leads directly to the transmission expression

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \left|\frac{b_1}{a_1}\right|^2 = \frac{\alpha^2 + \cos^2\dfrac{\Delta\phi}{2} - 2\alpha\left|\cos\dfrac{\Delta\phi}{2}\right|\cos\theta}{1 + \alpha^2\cos^2\dfrac{\Delta\phi}{2} - 2\alpha\left|\cos\dfrac{\Delta\phi}{2}\right|\cos\theta}. \tag{8}$$

If the two arms of the MZI consist of an electrooptic material the differential phase shift $\Delta\phi$ is proportional to the applied voltage V.

$$\Delta\phi = \frac{\pi}{V_\pi} V$$

where V_π is the voltage causing a differential phase shift $\Delta\phi = \pi$ in the MZI. Using the last relation and (7) in (5) leads to the following expression for the transmission at resonance ($\theta = m2\pi$, $m = 1, 2, 3, \ldots$)

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\left(\alpha - \left|\cos\dfrac{V}{2V_\pi}\pi\right|\right)^2}{\left(1 - \alpha\left|\cos\dfrac{V}{2V_\pi}\pi\right|\right)^2}. \tag{9}$$

The power transmission $|b_1|^2$ in the "through" fiber can, thus, be controlled via the applied voltage. When $V = 0$ the transmission is unity. When $V/V_\pi = (2/\pi)\cos^{-1}\alpha$ critical coupling occurs and the transmission is zero. For $\alpha \approx 1$ (high Qring resonator) the voltage V_c needed to turn the transmission off (from a transmission of unity at $V = 0$) is

$$\frac{V_c}{V_\pi} \sim \sqrt{1-\alpha}. \tag{10}$$

For a value of $\alpha = 0.999$, $V_c/V_\pi \approx 1/32$. Since in conventional electrooptic modulators the "on" to "off" voltage is approximately V_π, this reduction by nearly two orders of magnitude is, potentially, one of the most important features of this approach since, using present day materials, it points the way to modulation voltages measured in tenths of millivolts.

A simple directional coupler can be used instead of the CI proposed above. The geometry is the same as that depicted in Fig. 3 except that the MZI section is eliminated and $\Delta\phi$ is now equal to $\Delta\beta L$ i.e., the electrooptically induced phase mismatch between the two waveguides. This results in a value of $V_c/V_\pi \approx (1-\alpha^2)^{1/4}$.

The control of the phase shift $\Delta\phi$, and thus, switching can be achieved, instead of electrooptically, by injecting an optical signal into one arm of the MZI of Fig. 3 and utilizing the Kerr effect. The reductions in the ratio V_c/V_π will be reflected here in similar reductions in the switching light intensity compared to nonresonant geometries. Related resonant reductions have been predicted by Heebner and Boyd [5].

A plot of the transmission as given by (4) as a function of optical frequency ω (or equivalently $\theta = \omega nL/c$, where L is the optical path) is shown in Fig. 4. We note the critical coupling at $\alpha = |t|$ in the lower solid trace. We also note net transmission gain results if gain is provided [6] $(\alpha > 1)$ when

$$1 < \alpha < \frac{1}{|t|}. \tag{11}$$

We note from Fig. 4 that coupling control can also be achieved by controlling the internal loss parameter α. An experiment demonstrating such control is described in [6].

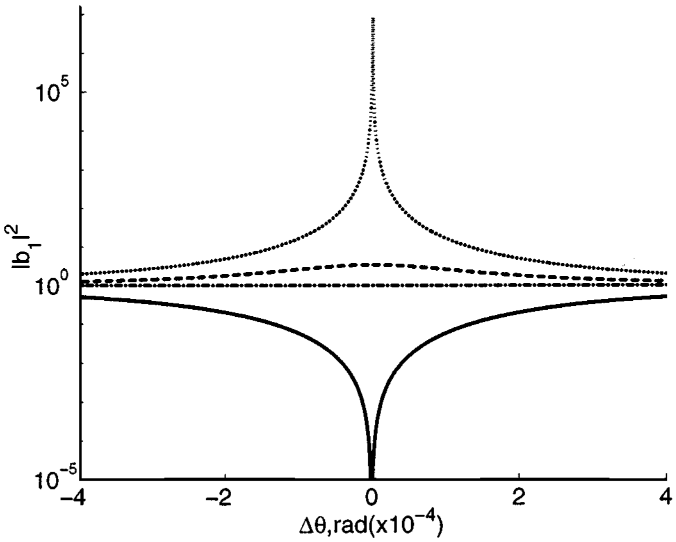

Fig. 4. Waveguide power transmission against frequency ($\theta = \omega L/c$) for the geometry of Fig. 1 with internal loss factor α as a parameter. $|t| = 0.9998$. —— $\alpha = 0.9998$ critical coupling, $|b_1|^2 = 0$, - · - · - · $\alpha = 1$ transparency, $|b_1|^2 = 1$, - - - - $\alpha = 1.000 06$ $1 < \alpha < 1/t$ transmission gain, $|b_1|^2 > 1$, · · · · · · $\alpha = 1.200 02$ $\alpha = 1/t$ laser oscillation.

The modulation bandwidth is limited to $\Delta f = f/Q$, where f is the optical frequency and Q the (loaded) figure of merit of the resonator

$$Q = \frac{2\pi n^2}{1 - t^2}\left(\frac{R}{\lambda}\right) \tag{12}$$

in the case of a ring resonator of radius R near critical coupling. The bandwidth can also be estimated from Fig. 4. In the case of a ring resonator with $2\pi R = 0.1$ cm, $\alpha^2 = t^2 = 0.99$, $\lambda = 1.5$ μm, $n = 1.5$, we obtain from (12) $Q = 3 \times 10^5$ and $\Delta f = f/Q = 10^9$ Hz.

Conclusion

The control of coupling between optical resonators and waveguides opens up new possibilities of modulating and switching light. Some generic schemes for achieving this control by means of applied voltages or injected optical (control) power have been proposed and discussed.

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