Critical fluctuation of wind reversals in convective turbulence

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The irregular reversals of wind direction in convective turbulence are found to have fluctuating intervals that can be related, under certain circumstances, to critical behavior. In particular, by focusing on its *temporal* evolution, the net magnetization of a two-dimensional Ising lattice of finite size is observed to fluctuate in the same way. Detrended fluctuation analysis of the wind reversal time series results in a scaling behavior that agrees remarkably well with that of the Ising problem. The specific properties found here, as well as the lack of an external tuning parameter, also suggest that the wind reversal phenomenon exhibits signs of self-organized criticality.

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I. INTRODUCTION

In fully developed turbulent convection in confined containers, the most important dynamical parameter characterizing the motion is the Rayleigh number, Ra. For present purposes, it is enough to regard Ra as the nondimensional measure of the temperature difference between the bottom and the top plates of the container. At high Rayleigh numbers, two dynamical features can be detected prominently. One is the emission of plumes from the top and bottom boundary layers [1]; they occur at random locations and at random times in varying sizes. The other is the existence of large scale circulation called the mean wind, whose size is roughly that of the container itself [2–5]. For low Ra (say, below 10^9), the wind is not sufficiently impeded by the random emission of plumes and continues unimpeded in one direction. At high Ra, the cumulative effect of the many plumes is strong enough to reverse the wind direction abruptly at seemingly random intervals [4,6], and proceed as before until another reversal occurs. The reversals have been observed in other contexts as well [7]. A comprehensive view of the dynamics and statistics of the reversal problem is described in Ref. [8] and further consideration of the specific manner in which reversals may be manifested in experiments will be discussed below. Other relevant theoretical work is described in Ref. [9].

Despite considerable effort [8,9], it is reasonable to say that the physical origin of wind reversals is still not fully understood. We investigate in this paper the statistical properties of the reversal and draw analogies to critical phenomena. Specifically, by performing the analysis of detrended fluctuations of the wind velocity, we demonstrate that aspects of wind reversal correspond to those of a system undergoing second-order phase transition. If the system is at a critical state, whether self-organized or not, the competition between the ordered and disordered motions leads to the wind switching directions at irregular intervals of all scales. The probability of occurrence of the wind duration τ between reversals should satisfy a power law

$$p(\tau) \sim \tau^{-\gamma} \quad (1)$$

as a manifestation of criticality. Such a power law has been found in Ref. [8].

The possible connection between self-organized criticality (SOC) and turbulence was considered in Ref. [11], and the correspondence of wind reversal with SOC was pointed out in Ref. [12]. The interesting element here is the provocative possibility of comparisons between an equilibrium system and one that is far from equilibrium. We do not claim that there is any firm reason to expect a close relationship between the wind reversal problem and critical behavior but simply wish to show that a quantitative similarity exists between wind reversal with the Ising model.

II. EXPERIMENTAL FEATURES

The experimental data that we analyze are the same as those reported in Ref. [4] and studied in Ref. [8]. By varying the pressure and temperature of an enclosed low temperature helium gas sample, the Rayleigh number could be varied between 10^6 and 10^{16}. Further details of the apparatus can be found in Ref. [4]. We focus on the data that give the wind speed and direction for a continuous period of up to one week at $\mathrm{Ra}=1.5\times10^{11}$. Figure 1 shows a small segment of the wind velocity data for 6.5 h, starting at an arbitrary time. Note how the wind changes direction suddenly in the time scale of that figure. We are interested in the abrupt change in magnitude of $V(t)$.

Wind reversals in cylindrical containers have been inferred generally from measurements using a small number of probes. It is possible to interpret the measurements as a slow azimuthal rotation of the entire flow circulation, but recent observations with larger number of probes [10] show that both reversals and rotations occur. Certainly, while the cylindrical symmetry of the cell geometry might suggest the rotation in the azimuthal plane as more likely, the abruptness of the wind switching and its relatively unvarying magnitude [4,6,8] are more indicative of a flow cessation and reversal (see Fig. 1). It is important to point out, however, that this

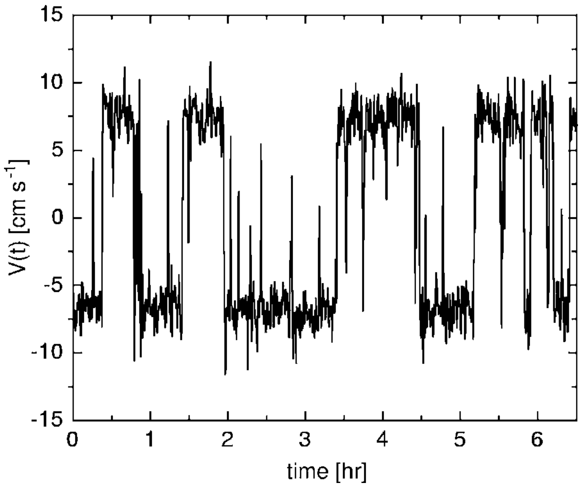

FIG. 1. A segment of the data on wind velocity fluctuation at Ra $=1.5\times 10^{11}$. The data were obtained from a cylindrical container of 50 cm diameter and 50 cm height, filled with cryogenic helium gas. Measurements were taken outside the boundary layer on the sidewall of the container at mid-height.

discussion has no bearing on the analysis considered here, and so we shall simply refer to flow reversals with no prejudice toward their precise dynamics.

III. CRITICAL BEHAVIOR

In the Ising system of near-neighbor interactions without external magnetic field the lattice spins tend to align in the same direction except for the random disorientation due to thermal fluctuation. For a finite lattice the net magnetization M is nonvanishing. For $T<T_c$, the critical temperature, M is likely to persist in the same direction for longer time in lattice-spin updating than at higher T. At $T>T_c$ the thermal interaction dominates, and M is more likely to flip sign more frequently upon updating. The fluctuation of the signs of M is therefore a property that reflects the tension between the ordered and disordered interactions of the whole system.

Since the mean wind is a global phenomenon in a vessel of finite volume, it is sensible for us to associate the wind direction with the sign of M of the Ising lattice of finite size. We can then map wind reversal to the reversal of M upon updating the lattice spins in a simulation. The plumes are the disordered fluctuations that correspond to the spin fluctuations due to thermal agitation, and the wind is the ordered motion that can change direction just as the magnetization can change sign when enough lattice spins change directions. The key connection between the two problems is the mapping of the real time in turbulence to the time of updating the Ising configurations. It is therefore crucial that each configuration has some memory of the previous configuration before updating; hence we employ the Metropolis algorithm that accomplishes precisely this task. It should be noted that we are entering a rather unexplored territory where the process of computer simulation itself is endowed with some physical significance, quite unrelated to the large body of analytical work that has been devoted to the Ising model of infinite lattice. This step enables us to compare a nonequilibrium problem in turbulence to a standard problem in critical phenomenon whose time dependence of fluctuations is not usually studied. Our task is to show that the fluctuations in the wind reversal problem correspond to those of the Ising problem of finite lattice at the critical temperature.

To be more specific, we consider a square lattice of size L^2, where L is taken to be 255, an odd number. We start with the L^2 site spins having a random distribution of ± 1 values. We then visit each site and determine from the usual near-neighbor interaction whether its spin should be reversed: yes, if the energy is lowered by the flip; if not lowered, the flip can still take place according to a thermal distribution specified by temperature T. One time step is taken by the whole system when all sites are updated. We take 3×10^5 time steps in total, and divide the whole series into 30 segments. The values of M at each of the 10^4 time points in each segment are discretized to ± 1, according to $M\gtrless 0$. A continuous string of M of one sign, either $+1$ or -1, forms a duration that is analogous to the mean wind rotating in one direction. The reversals of M correspond to the reversals of wind. Near the critical point, durations of all lengths can occur.

Before considering the issue of criticality for a finite lattice, let us discuss the measure that we shall use for quantifying the duration fluctuations appropriate for both the wind and Ising problems. The experimental data on wind consist of eight segments, each having $\mathcal{T}=10\,282$ time points. For the Ising case we have 30 segments, each having $\mathcal{T}=10^4$, roughly the same as wind data. Let N denote the number of reversals in a segment. With the locations of the reversals denoted by t_i, $i=1,\ldots,N$, define $\tau_i=t_{i+1}-t_i$ to be the ith duration (or gap), where t_0 and t_{N+1} are assigned to be the left and right ends of the segment, respectively. Now, define the moment [13]

$$G_q=\frac{1}{N+1}\sum_{i=0}^{N}\left(\frac{\tau_i}{\mathcal{T}}\right)^q, \tag{2}$$

where q is any positive integer. G_q is a measure that quantifies the pattern of reversals in each segment. For large q, G_q is a small number, since $\tau_i/\mathcal{T}$ is small. Its value can be dominated by a few large gaps, as when $T<T_c$, or may become cumulatively significant from the sum over many small contributions due to many small gaps, as when $T>T_c$. For a measure of the fluctuations of G_q from segment to segment, we define an entropylike quantity [13,14]

$$S_q=-\langle G_q \ln G_q\rangle, \tag{3}$$

where $\langle\cdots\rangle$ implies an average over all segments. For brevity we shall refer to the study of the time series in terms of S_q as the gap analysis. In Fig. 2 we show by filled circles the result of the gap analysis on the wind data at Ra $=1.5\times 10^{11}$. It is evident that for $q\geqslant 2$ the points can be well fitted by a straight line, shown by the solid line, exhibiting an exponential behavior for S_q,

$$\ln S_q=-\lambda q+\lambda_0, \quad \lambda=0.264. \tag{4}$$

For the Ising simulation we must first decide on the proper value of the critical temperature T_c for a finite lattice.

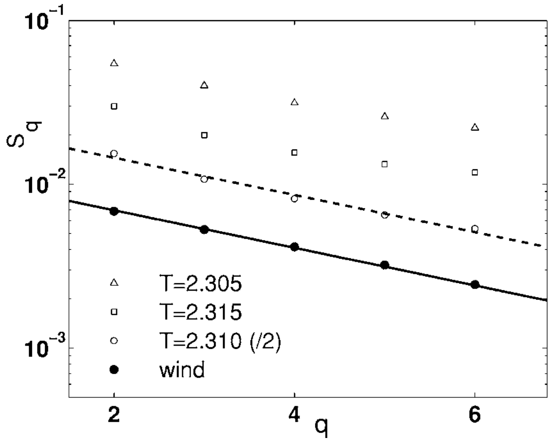

FIG. 2. Moments in the gap analysis of wind reversal (filled circles) and magnetization reversal in Ising lattice (open symbols) for different temperatures. The open circles are lowered by a factor of 2 to give space for clarity. The solid line is a linear fit of filled circles, and the dashed line is a linear fit of open circles.

For an infinite lattice its value has been determined analytically to be 2.269 in units of J/k_B, where J is the coupling strength of near-neighbor interaction and k_B the Boltzmann constant [15]. For a finite lattice T_c is no longer precisely defined, but we can still consider a transition occurring at a slightly higher value of temperature defined by a distinct property of S_q to be described below that separates the two regions of T above and below T_c. We have performed the simulation of our Ising system at three values of T, and determined the properties of M reversal. In Fig. 2 we show the results of our calculated values of S_q at $T=2.305$, 2.310, and 2.315. Only the one at $T=2.310$ (lowered by a factor of 2 for clarity) shows a nearly linear dependence in the plot. The dashed line is a linear fit of the points in open circle, giving a slope of $\lambda=0.261$. At the two neighboring values of T, the q dependencies of $\log_{10} S_q$ (shown by triangles and squares) are not linear, the values at high q being higher than at $T=2.310$. The linear behavior at $T=2.310$ is essentially the same as in the wind reversal problem, since λ differs from that in Eq. (4) by only 1%. We regard $T=2.31$ as the critical temperature T_c in our Ising system, since it has the unique property of being different from those of the neighboring T on both sides. When $T<T_c$, the gaps are longer and G_q is larger at large q (but still $\leqslant 1$) with the consequence that S_q is larger. When $T>T_c$, the gaps are shorter, but many gaps can contribute in the sum in Eq. (2), resulting in G_q still being larger at large q with the consequence that S_q is also larger. It is only at the critical point that gaps of all sizes can occur, resulting in G_q to be smaller and therefore S_q also smaller at large q. Thus the exponential decrease of S_q is a signature of criticality. The behavior of S_q for $q>6$ is unimportant, since they probe the tail of the distribution of τ_i. The value of T_c obtained here is in accord with the result of another calculation in a related finite-size lattice problem, for which T_c is found to be 2.315 [16].

We have applied the gap analysis to the time series of random walk, where a forward (backward) step is given a value of +1 (−1). The result is an exponential behavior of S_q with $\lambda=7.11$, which is grossly different from that in Eq. (4) and thus totally negligible. Clearly, the large gap behavior shown in Fig. 2 is a consequence of fluctuations near the critical point, not random fluctuations. This exercise also shows that the first six moments of S_q is sufficient to characterize the properties of fluctuations. The higher q values of S_q probe the details of the tail of a distribution that are unimportant.

The normalizations of S_q for the wind and Ising problems are not the same, since the average numbers N of reversals are different. However, the exponential behaviors are remarkably identical. The q dependence of S_q is a quantitative measure of the fluctuation behavior of the reversals. The fact that the slope λ is the same for both the wind and magnetization problems suggests strongly that the wind reversal in convective turbulence at high Ra is a critical phenomenon. The behavior is insensitive to the value of Ra, so long as $\mathrm{Ra}>10^9$. Since we have not tuned any adjustable parameter in the wind problem to bring the system to the critical point, as we have done for the Ising system by varying T, we conclude that the wind reversal phenomenon is a manifestation of self-organized criticality (SOC) [17].

It should be understood that the necessity for using a tunable parameter (i.e., T) in the Ising problem to bring the system to the critical point does not negate the possibility that the wind reversal problem is a SOC system. It only affirms the conjecture that the latter exhibits criticality.

IV. POWER-LAW BEHAVIOR

We now search for a power-law behavior that characterizes changes in the wind direction. (For other such efforts, see Ref. [8]). Our method is the detrended fluctuation analysis (DFA), which has been found to reveal the scale-independent nature of time series in a variety of problems, ranging from heartbeat irregularity [18] and EEG [19] to economics [20]. In that analysis we look for scaling behavior in the RMS deviation of the wind velocity from local linear trends. Given the time series of the wind velocity $V(t)$ over a total range of $\mathcal{T}_{\max}$, we divide it into B equal bins of width k, discarding the remainder $\mathcal{T}_{\max}-Bk$. Let $\bar{V}_b(t)$ denote the linear fit of $V(t)$ in the bth bin. The variance of the deviation of $V(t)$ from the local trend $\bar{V}_b(t)$ in bins of size k is defined by

$$F^2(k)=\frac{1}{B}\sum_{b=1}^{B}\frac{1}{k}\sum_{t=t_1}^{t_2}[V(t)-\bar{V}_b(t)]^2, \tag{5}$$

where $t_1=1+(b-1)k$ and $t_2=bk$, measured in units of $\Delta t=5$ sec, so that the values of t are dimensionless integers that count the time points in the data. The goal is to study the behavior of the RMS fluctuations $F(k)$, as k is varied. If there is no characteristic scale in the problem, then $F(k)$ should have a scaling behavior

$$F(k)\propto k^{\alpha}. \tag{6}$$

This power law cannot be valid for arbitrarily large k because the series $V(t)$ is bounded, so for very large k the linear trend is just the $V(t)=0$ line, and the RMS fluctuation $F(k)$

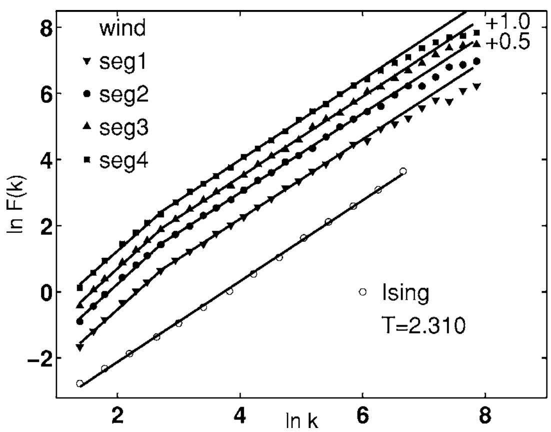

FIG. 3. Scaling behaviors of $F(k)$ in DFA of wind reversal (filled symbols) and magnetization reversal in Ising lattice at the critical temperature (open circles). Lines are linear fits.

must become independent of k. Thus we expect $\ln F(k)$ to saturate and deviate from Eq. (6) at some large k. We note parenthetically that we have applied DFA to the unintegrated time series $V(t)$, which is a departure from the usual practice.

We remark that in DFA the linear detrending is done for every chosen time scale k because fluctuation is defined with reference to a meaningful baseline. What appears as a sharp spike when k is large may seem like a smooth peak when k is small. Without local detrending, the scaling behavior of fluctuations over a wide range of k is meaningless.

In Fig. 3 we show $F(k)$in a log-log plot for four equal segments of the complete wind data in solid symbols. The segment seg1 is for time running from 0 to 116 435 s, corresponding to $\mathcal{T}_{\max}=23\,287$; other segments all have the same length. We have limited the maximum bin size to 2580, so that even for the largest bin the fluctuations can be averaged over nine bins. Evidently, there is a good scaling for each segment. The points for seg3 and seg4 are shifted upwards by the quantities indicated in order to give clarity without overlap. Note that the seg1 data do not have the same magnitude of $F(k)$ as the other segments; yet the scaling exponents are essentially the same. The deviation from the straight lines at the upper end is the saturation effect already discussed. There is another short region of scaling with a higher slope at low k. It is a consequence of fluctuations of the velocity within one direction of the wind, whose presence is evident in Fig. 1. Since the critical behavior identified here refers to wind reversals, and not to fluctuations of the wind velocity within one direction, we should ignore the lower short scaling region.

In the scaling region to which we pay attention here, the slopes are $\alpha=1.20$, 1.20, 1.21, and 1.22, for seg1 to seg4, respectively. The deviations among the segments are obviously small. The average value is $\alpha=1.21$. This large value of α implies a smoother landscape compared to the rough time series of white noise that is characterized by complete unpredictability [18]. Indeed, the fluctuations of the wind reversal time series has gaps of all sizes, the signature of critical behavior that is characterized by $1/f$ noise [17]. It is interesting to compare our result with the properties of the power spectral density for the velocity found in Ref. [21], where a scaling behavior is shown to exist with a slope roughly $-7/5$ (not by fitting) in the region $-3<\log_{10}f<-1.8$. If we identify the values of k in DFA to the time scale $1/f$, then that range of $\ln(1/f)$ corresponds to the range of $\ln k$ in Fig. 3. The scaling behavior found in DFA uses shorter segments of the whole data and exhibits the power law more precisely, from which the value of α can be more accurately determined.

We now apply DFA to the Ising problem. We consider ten segments of the M reversal time series of the Ising lattice set at T_c, each segment having 10^4 time points. From the $F(k)$ determined in each segment, we average over all segments and show the resultant dependence on k in Fig. 3 by the open circles. Clearly, the points can be well fitted by a straight line. The slope is $\alpha_M=1.22$, which is essentially the same as the value $\alpha=1.21$ for wind reversal. With the equivalence of these two scaling behaviors established, we have found stronger evidence that the wind reversal problem is a critical phenomenon.

V. CONCLUSIONS

To summarize, we have studied the time series of wind reversal in convective turbulence by two methods (gap analysis and detrended fluctuation analysis) and applied the same methods to the time series of the reversal of the net magnetization of a two-dimensional Ising lattice. We have shown that there exist similarities between the fluctuation properties in our nonequilibrium system and those of the Ising system that is well known to exhibit critical behavior—when appropriate measures are used to compare them. Since the Ising problem is generally understood to be in equilibrium, we investigated it in a novel way in order to exhibit the time-dependent behavior of its evolution. Indeed, the way in which the net magnetization changes with time exhibits relevant properties of critical fluctuation, and provides an important aspect of the so-called equilibrium system that is useful for us to relate to the convection problem. Perhaps at this fledgling stage of analogy, we can only say that the remarkable agreement of the results, obtained for both problems, suggests that the wind reversal exhibits all the essential properties characteristic of a critical behavior *when its temporal evolution is considered*. Certainly, this is not expected, but we feel it is an intriguing result nonetheless; one that is hard to ignore and that we hope will generate some further, systematic inquiry.

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