Critical hysteresis from random anisotropy

Rava A. da Silveira[1] and Stefano Zapperi[2]
[1]*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*INFM UdR Roma 1 and SMC, Dipartimento di Fisica, Università "La Sapienza," P. le A. Moro 2, 00185 Roma, Italy*
(Received 2 March 2004; published 17 June 2004)

Critical hysteresis in ferromagnets is investigated through a N-component spin model with random anisotropies, more prevalent experimentally than the random fields used in most theoretical studies. Metastability, and the tensorial nature of anisotropy, dictate its physics. Generically, random-field Ising criticality occurs, but other universality classes exist. In particular, proximity to $O(N)$ criticality may explain the discrepancy between experiment and earlier theories. The uniaxial anisotropy constant, which can be controlled in magnetostrictive materials by an applied stress, emerges as a natural tuning parameter.

DOI: 10.1103/PhysRevB.69.212404 PACS number(s): 75.60.−d, 64.60.−i, 75.40.−s, 75.80.+q

Hysteretic properties of ferromagnetic materials have long fueled applied research and, more recently, much theoretical interest.[1] As a manifestation of the nonequilibrium dynamics of a disordered system with many degrees of freedom, hysteresis is described naturally in the language of statistical mechanics. A central aim of theoretical studies of hysteresis is to elucidate the ways in which microstructural details, such as domain configurations, lattice structure, impurities or defects, affect macroscopic properties such as the shape of the hysteresis loop and the Barkhausen noise statistics.

A nonequilibrium version of the zero-temperature random-field Ising model (RFIM) has served to illustrate the competing effects of disorder and (ferromagnetic) exchange interaction involved in hysteresis.[2] In three and higher dimensions, at weak disorder the model exhibits a discontinuous hysteresis loop, which becomes continuous at strong disorder. These two phases are separated by a critical loop for a given value of the disorder; as the latter is approached from the weak disorder side, the macroscopic discontinuity vanishes continuously, resulting in a critical point characterized by universal scaling laws.[2–4] The corresponding critical exponents were obtained within a mean-field approximation,[2] perturbatively in a renormalization group treatment,[3] and exactly on the Bethe lattice.[5] (While the model was originally suggested in part to relate this disorder induced critical scaling to Barkhausen noise measurements,[2,3] it seems that in most experiments the statistics of the noise is controlled instead by the depinning transition of domain walls,[6,7] which do not emerge simply from an analysis in terms of a RFIM.)

Direct experimental evidence of disorder induced transitions in ferromagnets was obtained only recently. A temperature controlled transition was reported for Co-CoO bilayers[8] and a similar transition was observed in Gd/W films subjected to different annealing procedures, which induce variations in the disorder through variations of the grain size.[9] A study of hysteresis loops of Cu-Al-Mn alloys for different Mn concentrations and temperatures also identified a transition,[10] and the measured scaling exponents are consistent with those observed for Co-CoO bilayers. They do not agree, however, with predictions of the RFIM. A natural explanation proposal for this discrepancy focuses on the nature of disorder; indeed, while random fields are convenient for theoretical exploration, they are seldom present in real ferromagnets, which display more complicated forms of disorder. Prominently, random anisotropies are present in most ferromagnets, including soft materials, and are believed to be particularly relevant in amorphous rare earth alloys.[11]

A disorder induced phase transition was observed numerically in a random *infinite* anisotropy model, with exponents close to the RFIM ones,[12] supporting general symmetry arguments that were put forth in favor of universality.[3] However, infinite anisotropies pin the spins to given (random) directions, making each spin Ising-like on its own; as a result, the model is equivalent to a random-field, random bond model. Furthermore, within a non-equilibrium context symmetry arguments ought to be taken with a grain of salt; it is known, for example, that the magnetization may point away from the applied field out of equilibrium, while in equilibrium minimization of the free energy requires alignment of the two. Such phenomena are a consequence of the presence of many metastable states (involved in the dynamics), and more systematic analyses that clarify their role and substantiate the symmetry arguments are worthwhile. Along these lines, a renormalization group study of a random field vectorial ($\mathcal{O}(N)$) model (RFVM), taking metastability into account, showed that while one is justified in expecting a critical behavior identical to that of the RFIM generically, by tuning additional parameters different universality classes may be visited.[13]

Here, we analyze a non-equilibrium random anisotropy vectorial model (RAVM), in which N-component spins are subjected to ferromagnetic interactions and random (finite) anisotropies. The $N=2$ case was proposed in the past as a model of rare-earth alloys, and its hysteretic behavior was studied numerically.[14] While in these studies the anisotropy averages to zero, here we allow for a fixed uniaxial component in addition to a random background. From the zero-temperature spin dynamics, we construct the appropriate non-equilibrium effective action[13] which describes the evolution of the magnet along the hysteresis loop. The most notable consequence of *metastability* is the generation of a "*random field* term" in the action; in addition, the latter breaks the rotational symmetry verified by the *equilibrium* action. As a result, random anisotropy magnets indeed generically display usual, RFIM exponents (at least within the domain of validity of the perturbative analysis) for given

values of the applied field and disorder strength. However, there exists a number of additional universality classes, and in particular a critical point with $\mathcal{O}(N)$ exponents which is reachable upon tuning of an additional parameter. The higher (tensorial) nature of the disorder in the RAVM provides such an additional parameter, namely, the uniaxial anisotropy constant, in a natural fashion. As explained below, proximity to a vectorial ($\mathcal{O}(N)$) critical point may help explain the discrepancy between experimentally measured exponents and Ising ones.

In the RAVM, N-component spins $\vec{s}$ on a d dimensional lattice interact *via* ferromagnetic nearest-neighbor couplings J_0 and a spin at site i couples to an anisotropy tensor $K_i^{\alpha\beta}$. In addition, the spins are subjected uniformly to an applied magnetic field $\vec{H}$, which varies (adiabatically) in time and hence forces the system out of equilibrium, through a Hamiltonian

$$\mathcal{H} = -J_0 \sum_{<ij>} \vec{s}_i \cdot \vec{s}_j - \sum_i (\vec{s}_i \cdot K_i \cdot \vec{s}_i + \vec{H} \cdot \vec{s}_i), \tag{1}$$

where $\vec{s}_i \cdot K_i \cdot \vec{s}_i$ stands as a shorthand for $\Sigma_{\alpha,\beta=1}^{N} s_i^{\alpha} K_i^{\alpha\beta} s_i^{\beta}$. (Latin indices label lattice sites while Greek indices label spin components.) The anisotropy tensor may be decomposed into non-random and random components, and in the simplest (uniaxial) case

$$K_i^{\alpha\beta} = K_0 n^{\alpha} n^{\beta} + \delta K_i^{\alpha\beta}, \tag{2}$$

where K_0 is the *uniaxial anisotropy constant* and $\hat{n}$ a unit vector lying along the easy magnetization axis. The random components $\delta K^{\alpha\beta}$ are uncorrelated Gaussian random numbers with vanishing mean and standard deviation R, so that the anisotropy tensors are distributed according to the density

$$\rho(K^{\alpha\beta}) = \frac{1}{\sqrt{2\pi}R} \exp\left(- \frac{(K^{\alpha\beta} - K_0 n^{\alpha} n^{\beta})^2}{2R^2} \right). \tag{3}$$

The parameter R plays an analogous role here to that of the width of the disorder distribution (also called R) in random field models.[3,13] Finally, for the sake of calculational simplicity, instead of fixed length spins (with, e.g., $|\vec{s}_i|^2 = 1$ for each site i) we consider ''soft spins'' whose lengths can take any values. Following Refs. 3 and 13, for stability we add to the Hamiltonian a sum of single site terms $\Sigma_i V(\vec{s}_i)$, so that, at each site, a Mexican hat potential $V(\vec{s}_i) = -a|\vec{s}_i|^2/2 + b|\vec{s}_i|^4/4$ prevents the spin from diverging. In the appropriate limit ($a = b \to \infty$), soft spins reduce back to unit spins, but as the length of spins is modified under renormalization, the specific (bare) values of a and b are irrelevant.

As mentioned, the applied field $\vec{H}$ varies (adiabatically) in time and forces the spins through a non-equilibrium trajectory. In order to study the critical behavior of the system, we may confine ourselves to the simple zero-temperature relaxational dynamics

$$\Gamma \frac{\partial \vec{s}_i}{\partial t} = -\frac{\partial \mathcal{H}}{\partial \vec{s}_i}, \tag{4}$$

where Γ is an effective damping coefficient. We point out, though, that Eq. (4) is certainly not the most realistic choice of dynamics, which in general includes precession of spins and a more complicated damping factor, better described by a Landau-Lifshitz-Gilbert-like equation.[1] Nevertheless, we expect the critical behavior not to bear crucially on the specifics of the microscopic dynamics, and Eq. (4) appears as the simplest candidate for an analytic treatment. In the same vein, in what follows we make a final, customary simplification in replacing the lattice spins $\vec{s}_i$ by a continuum vector field $\vec{s}(\vec{x})$.

In the continuum approximation, Eq. (4) becomes

$$\Gamma \frac{\partial \vec{s}(\mathbf{x},t)}{\partial t} = J\nabla^2 \vec{s} + \vec{H}(t) + K(\mathbf{x}) \cdot \vec{s} + a\vec{s} - b|\vec{s}|^2 \vec{s}, \tag{5}$$

where the constant J results from the continuum expansion of the exchange interaction,[13] $K(\mathbf{x}) \cdot \vec{s}$ is a shorthand for the vector field with components $\Sigma_{\beta=1}^{N} K^{\alpha\beta}(\mathbf{x}) s^{\beta}$, and higher orders (in derivatives and possibly fields) have been neglected. Aspects of the hysteretic critical behavior are more transparent in the language of a generating functional[15] than directly through the equation of motion. In the usual fashion,[3,13] we introduce an auxiliary field $\vec{\phi}$ to exponentiate a δ function that forbids any trajectory that does not obey the equation of motion, resulting, up to some constant prefactors, in a functional

$$Z = \int \mathcal{D}s \mathcal{D}\phi \exp\left(\int dt d^d x \vec{\phi} \cdot \left(-\Gamma \frac{\partial \vec{s}}{\partial t} + J\nabla^2 \vec{s} + \vec{H} + K \cdot \vec{s} + a\vec{s} - b|\vec{s}|^2 \vec{s} \right) \right) \tag{6}$$

which captures the possible histories of the system. The advantage of this procedure is that now one can easily average the generating functional over the distribution $\rho(K^{\alpha\beta})$ for anisotropy tensors $K^{\alpha\beta}$ at all positions, as

$$\overline{Z} = \int \mathcal{D}K^{\alpha\beta} \rho(K^{\alpha\beta}) Z = \int \mathcal{D}s \mathcal{D}\phi \exp(S_{\text{eff}}[s(\mathbf{x},t), \phi(\mathbf{x},t)]), \tag{7}$$

with an effective action

$$S_{\text{eff}} = \int dt d^d x \left\{ \vec{\phi} \cdot \left[-\Gamma \frac{\partial \vec{s}}{\partial t} + J\nabla^2 \vec{s} + \vec{H} + K \cdot \vec{s} + a\vec{s} - b|\vec{s}|^2 \vec{s} + K_0 \hat{n}(\hat{n} \cdot \vec{s}) \right] + \frac{R^2}{2} \int dt' (\vec{\phi} \cdot \vec{\phi}')(\vec{s} \cdot \vec{s}') \right\}, \tag{8}$$

where $\vec{s}, \vec{\phi}$ and $\vec{s}', \vec{\phi}'$ are evaluated at times t and t', respectively. The effective action encodes the averaged solutions of Eq. (5) and avoids one the complication of solving a stochastic equation first and then averaging. In carrying out the average, one trades the stochastic (anisotropy) term with new

terms coupling $\vec{s}$ and $\vec{\phi}$, and the quenched nature of the disorder is reflected in the presence of a double integral over time (over t and t'). In the RAVM, the two terms with $(\vec{\phi}\cdot\hat{n})(\hat{n}\cdot\vec{s})$ and $(\vec{\phi}\cdot\vec{\phi}')(\vec{s}\cdot\vec{s}')$ replace random field terms of the form $(\vec{\phi}\cdot\vec{\phi}')$ in the RFIM[3] and the RFVM.[13]

As mentioned, the effective action in Eq. (8) encompasses all the solutions and as a result is invariant under the transformation $(\vec{H},\vec{s},\vec{\phi})\rightarrow(-\vec{H},-\vec{s},-\vec{\phi})$. The hysteresis curve, however, is *not* symmetric in general upon inversion of the magnetic field and magnetization; in particular, the value of the magnetization at zero field (remanent magnetization), and *vice versa* that of the field when the magnetization is zero (coercive field), do not vanish. This is because the system follows in reality a *given metastable state* which evolves along with $\vec{H}(t)$ (more precisely, each branch of the hysteresis loop corresponds to a metastable trajectory). Following a trick of Ref. 13, we get rid of the unwanted solutions by shifting the field $\vec{s}$ by a quantity $\vec{\sigma}(t)$, which represents the averaged sum of all the ''unphysical minima''; the resulting effective action, $S_{\text{eff, metastable}}$, encapsulates the magnetization and response function along a branch of the hysteresis loop. For the sake of simplicity, we consider first the case in which the field $\vec{H}$ is applied along the easy axis of magnetization, i.e., $\vec{H}=H\hat{n}$; we comment below on the general case. We then expect the magnetization, and hence the vector $\vec{\sigma}$, to lie along $\hat{n}$ too. Shifting the spin field according to $\vec{s}\rightarrow\vec{s}+\sigma(t)\hat{n}$, we obtain

$$
\begin{aligned}
S_{\text{eff, metastable}} = \int dt d^d x \Bigg\{ & \phi_\parallel \Bigg[-\Gamma \frac{\partial s_\parallel}{\partial t} - \Gamma \frac{\partial \sigma}{\partial t} + J\nabla^2 s_\parallel + H \\
& + (s_\parallel + \sigma)(a - b|\vec{s}+\vec{\sigma}|^2 + K_0) \Bigg] \\
& + \vec{\phi}_\perp \cdot \Bigg[-\Gamma \frac{\partial \vec{s}_\perp}{\partial t} + J\nabla^2 \vec{s}_\perp + \vec{s}_\perp (a - b|\vec{s}+\sigma\hat{n}|^2) \Bigg] \Bigg\} \\
& + \frac{R^2}{2} \int dt dt' d^d x (\phi_\parallel \phi_\parallel' + \vec{\phi}_\perp \cdot \vec{\phi}_\perp')[(s_\parallel + \sigma(t)) \\
& \times (s_\parallel' + \sigma(t')) + \vec{s}_\perp \cdot \vec{s}_\perp'], \qquad (9)
\end{aligned}
$$

where we have decomposed $\vec{s}=(s_\parallel,\vec{s}_\perp)$ and $\vec{\phi}=(\phi_\parallel,\vec{\phi}_\perp)$ into longitudinal and transverse components with respect to the direction given by $\hat{n}$.

A number of results may be deduced from the form of the corrected action $S_{\text{eff, metastable}}$. In Eq. (9), the bare longitudinal ''mass'' a (the coefficient of the $\phi_\parallel s_\parallel$ term) is dressed into $\tilde{a}_\parallel = a - 3b\sigma^2 + K_0$ and the bare field H into $\tilde{H}=H-\Gamma\partial_t\sigma+\sigma(a-b\sigma^2+K_0)$. As, in general, $\tilde{a}_\parallel$ and $\tilde{H}$ do not become small (or vanish) simultaneously at $H=0$, criticality does *not* occur at vanishing field. This reflects the nonequilibrium nature of the trajectory, chosen among many metastable states generated by the disorder. The more remarkable manifestation of metastability is, however, the generation of an effective ''random field'' $\sigma(t)\sigma(t')\vec{\phi}\cdot\vec{\phi}'$ term. As a result, the effective action in Eq. (9) differs from its random field counterpart by the presence of additional terms of the form $\phi\phi' s^{(\prime)}$ and $\phi\phi' ss'$. Without these terms the action exhibits a nontrivial (non-Gaussian) critical point below the upper critical dimension $d_c=6$, which can be characterized by a perturbative renormalization group treatment in $d=6-\varepsilon$ dimensions.[3,13] Now, power counting predicts that this critical point is stable with respect to the extra random anisotropy terms: from the natural rescalings $x\rightarrow bx$, $t\rightarrow b^2 t$, $s\rightarrow b^{2-d/2}s$, and $\phi\rightarrow b^{-2-d/2}\phi$, we find a scaling dimension of $(4-d)/2$ for the $\phi\phi' s^{(\prime)}$ terms and of $4-d$ for the $\phi\phi' ss'$ terms, which are thus *irrelevant* close to $d_c=6$ dimensions. Consequently, at least within the perturbative domain, criticality in the RAVM is identical to that in the RFVM[13] with, generically, RFIM exponents[3] reflecting ''massless'' fluctuations of the longitudinal component $s_\parallel$.

By symmetry, as in the RFVM[13] there must exist here a $\mathcal{O}(N-1)$ critical point representing ''massless'' fluctuations of the transverse components $\vec{s}_\perp$, corresponding to spontaneous magnetization in the transverse direction. Then, upon appropriate tuning of an additional parameter, these two critical points may merge, resulting in a rotationally invariant vectorial $(\mathcal{O}(N))$ critical point. The latter occurs for a symmetric action, in particular the effective longitudinal mass $\tilde{a}_\parallel$ and the effective transverse mass $\tilde{a}_\perp = a - b\sigma^2$ must become small simultaneously. In the RFVM, simultaneous vanishing of the effective masses and applied field is possible only if the magnetization vanishes at $H=0$, i.e., for very ''thin'' hysteresis loops (with small area).[13] Here, crucially, the higher (tensorial) nature of anisotropy alters this picture: vector criticality occurs generically at *nonvanishing* values of the applied field and magnetization. Indeed, since K_0 modifies both the field *and* the longitudinal mass, the values of $\tilde{H}$, $\tilde{a}_\parallel$, and $\tilde{a}_\perp$ may become critical simultaneously at nonvanishing values of H and σ. As a result, in the RAVM the hysteresis loop need *not* be ''thin'' to display vectorial criticality. The uniaxial anisotropy constant K_0 may be tuned instead of the disorder width to reach Ising criticality, or along with the disorder width to reach vectorial criticality.

We emphasize that this picture is a direct consequence of metastability. By contrast, the *equilibrium* random anisotropy model[16] displays a lower critical dimension of $d_c=4$, which is also, if naive power counting is to be believed, the upper critical dimension. Systematic studies of the behavior about $d_c=4$ are plagued with a number of technical difficulties[17] and, although the weak disorder phase below d_c was captured in a recent analytical treatment,[18] agreement with experiments[19] and numerics[20] is still controversial. While it is certainly legitimate to ask whether similar difficulties arise out of equilibrium away from $d_c=6$, we note that, curiously, at least in the perturbative domain, metastability simplifies the problem.

So far, we have considered the particular case in which the system is magnetized along its easy axis. A similar analysis may be applied to the general case, in which the magnetization lies along a direction $\hat{\mu}$ intermediate between those of $\hat{n}$ and $\vec{H}$. As H increases (or is varied) in time, $\hat{\mu}$ rotates

in space. This, however, does not affect the analysis significantly (calculationally, because different times effectively decouple in the action). The fact that $\hat{\mu}$ does not lie along $\hat{n}$ or $\vec{H}$ changes the symmetries, in particular because additional transverse terms are generated by the shift in σ. These seem to allow for the possibility of $\mathcal{O}(2)$ and $\mathcal{O}(N-2)$ critical points, provided additional parameters may be tuned.

In sum, we have shown that critical hysteresis in the RAVM is described, generically, by RFIM exponents. Thus, we expect the conclusions of simulation studies of the random *infinite* anisotropy model[12] to extend to the *finite* anisotropy case in general. However, we also expect it to be easier to identify vectorial critical points in the presence of anisotropy than in the presence of random fields, and a potential explanation for the discrepancy between experiments and theory lies in a putative proximity of the regime in which experiments are carried out to such a vectorial critical point. An experimental study in which various parameters are scanned systematically should reveal whether proximity to a vectorial critical point is verified. If it is the case, one expects $\mathcal{O}(N)$ exponents, that cross over to RFIM ones only above some scale which may be rather large.[3,13]

For practical reasons, the standard experimental tuning parameter is temperature, either annealing or measured temperature, but a number of interpretation problems are associated with the corresponding techniques. In the first case, one loses control over changes in microstructure and one is bound to repeat the experiment on different samples, with possibly large sample-to-sample variations. In the second case, varying the temperature modifies simultaneously several physical quantities. In the RAVM, the natural tuning parameter is the uniaxial anisotropy constant K_0, rather than the temperature; it may be used both to reach the RFIM and, along with a second tuning parameter, to look for vectorial criticality. Tuning K_0 seems a good experimental possibility for magnetostrictive materials in which, in the simplest description, an applied stress τ along the easy axis shifts the value of the uniaxial anisotropy from K_0 to $K_0+3\lambda\tau/2$, where λ is the magnetostriction constant.[1]

A more drastic reason for the discrepancy between experiment and theory may be, of course, that the experimentally predominant form of disorder is neither of the random field type nor of the random anisotropy type. In particular, in materials used at present in experiments,[8–10] the presence of random bonds and demagnetizing fields might alter the theoretical picture,[5] and so would putative strong dipolar forces.[5] Nevertheless, we expect our results to be relevant for a wide class of amorphous ferromagnets, and, in particular, it would be interesting to check them against experiments on polycrystals, in which dipolar forces are weak and anisotropy is the dominant form of disorder.

We thank G. Durin, D. S. Fisher, M. Kardar, and M. A. Muñoz for useful conversations. R.A.S. is grateful for the hospitality at the Center for Statistical Mechanics of Complexity of the INFM, at *La Sapienza*, where this work was pursued in part. Support from the Harvard Society of Fellows, the Milton Fund, and the *Fonds national suisse* (R.A.S.) is acknowledged.

[1] G. Bertotti, *Hysteresis in Magnetism* (Academic, San Diego, 1998).
[2] J.P. Sethna, K. Dahmen, S. Kartha, J.A. Krumhansl, B.W. Roberts, and J.D. Shore, Phys. Rev. Lett. **70**, 3347 (1993).
[3] K. Dahmen and J.P. Sethna, Phys. Rev. B **53**, 14 872 (1996).
[4] O. Perkovic, K. Dahmen, and J.P. Sethna, Phys. Rev. B **59**, 6106 (1999).
[5] D. Dhar, P. Shukla, and J.P. Sethna, J. Phys. A **30**, 5259 (1997).
[6] S. Zapperi, P. Cizeau, G. Durin, and H.E. Stanley, Phys. Rev. B **58**, 6353 (1998).
[7] G. Durin and S. Zapperi, Phys. Rev. Lett. **84**, 4705 (2000).
[8] A. Berger, A. Inomata, J.S. Jiang, J.E. Pearson, and S.D. Bader, Phys. Rev. Lett. **85**, 4176 (2000).
[9] A. Berger, A.W. Pang, and H. Hopster, J. Mater. Process. Manuf. Sci. **9**, 131 (2000).
[10] J. Marcos, E. Vives, L. Mañosa, M. Acet, E. Duman, M. Morin, V. Novák, and A. Planes, Phys. Rev. B **67**, 224406 (2003).
[11] R. Alben, J.J. Becker, and M.C. Chi, J. Appl. Phys. **49**, 1653 (1978).
[12] E. Vives and A. Planes, Phys. Rev. B **63**, 134431 (2001).
[13] R. da Silveira and M. Kardar, Phys. Rev. E **59**, 1355 (1999).
[14] B. Dieny and B. Barbara, Phys. Rev. B **41**, 11 549 (1990); R. Dickman and E.M. Chudnovsky, *ibid.* **44**, 4397 (1991); R. Ribas, B. Dieny, B. Barbara, and A. Labrata, J. Phys.: Condens. Matter **7**, 3301 (1994).
[15] P.C. Martin, E. Siggia, and H. Rose, Phys. Rev. A **8**, 423 (1973); R. Bausch, H.K. Janssen, and H. Wagner, Z. Phys. B: Condens. Matter **24**, 113 (1976).
[16] R. Harris, M. Plischke, and M.J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973).
[17] A. Aharony, Phys. Rev. B **12**, 1038 (1975); R.A. Pelcovits, E. Pytte, and J. Rudnick, Phys. Rev. Lett. **40**, 476 (1978); **48**, 1297 (1982); D.S. Fisher, Phys. Rev. B **31**, 7233 (1985).
[18] D.E. Feldman, Phys. Rev. Lett. **84**, 4886 (2000); Phys. Rev. B **61**, 382 (2000).
[19] B. Barbara, M. Coauch, and B. Dieny, Europhys. Lett. **3**, 1129 (1987); A.C. Hannon, M. Hagen, R.A. Cowley, H.B. Stanley, and N. Cowlam, Physica B **180-181**, 230 (1992).
[20] R. Fisch, Phys. Rev. B **58**, 5684 (1998); M. Itakura, Phys. Rev. B **68**, 100405(R) (2003).