# Critical Infrastructure Protection – Modeling of Domino and Synergy Effects

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***Abstract*—Critical infrastructure protection is presently seen as an important aspect of society's maintenance of functional continuity from an economic and social perspective. This fact is seen as a motivation for the development of relevant approaches and methodologies, which have significant impact to the critical infrastructure protection and resilience level. The interactions of various critical infrastructure subsystems have a major relation to the domino and synergy effect assessment and on the overall critical infrastructure protection resilience level. The domino and synergy effect impact to critical infrastructure resilience will therefore be described in the article by a selected mathematical model. The model will present the assessment approach of inter-connections modeling between critical infrastructure subsectors, allowing the evaluation of the potential domino effect in the context of Czech Republic critical infrastructure.**

***Keywords- critical infrastructure; model; domino effect; resilience; protection; leontief's economy model;***

## I. INTRODUCTION

Increasing technological dependence increased the need to identification and designation of elementary State system, whose malfunction will have a major impact on the society functional continuity maintenance in all its social aspects. In the context of the previous argument, the elementary State system is considered critical infrastructure. Perceptions of critical infrastructure are not a modern phenomenon, although it can be stated that there is an increased attention on the protection of critical infrastructure since 2001, which is strongly influenced by the turbulence in the security environment. Influences in the security environment crate a discussion about relevancy of modeling of domino and synergy effects. There are many relevant research works which are addressing topics and issues of modeling, where the most proper were presented by Y., Haimes, and P., Jiang, (2001) [9], R. Santos, (2006) [10], Y., Haimes, (2015) [11], Rehak [12], T., Macaulay (2008) [13]. Relevant outcomes of above mentioned research works include developing the theoretical and, in some cases, also practical framework for modeling and simulation of critical infrastructure in a wider context. The framework was filled in this article by practical application of selected methodologies and models in connection with the Czech Republic critical infrastructure. The second part of the article discusses about theoretical baseline of structures and principles of modeling. The third part therefore theoretically explains the selected model's possibilities and application potential. The practical use and application of selected mathematical model is presented in the fourth section.

## II. STRUCTURES AND BASIC PRINCIPLES OF MODELING

In everyday language, the term modelling has different meanings and interpretations. It depends on the nature of the expertise, the degree of knowledge and education, culture, purpose, and many other attributes. Usually, in the model we are developing a kind of reality copy, or create a "prototype", or simulation, how something should be. Always we observe "something" from a certain point of view or interest for a purpose. It is essential to correctly define the elements of interest, abstracted from the whole and develop a system where the basic elements are incorporated together with the key factors influencing the elements. Relations between elements should be clearly visible, quantifiable so we can say that the elements of the system are structured. This structure "links tightness" between the elements is the criterion which defines whether the element is included or not in a given system and it is essential for the identification of impacts, and their level decides the success or failure of the system understanding. Ultimately, this leads to the abstraction of the real nature of the elements in their formal substitution variables and maintaining the relations between them have been observed in the corresponding real elements, i.e. developing a mathematical model. The model is already formal relations between quantities expressed by features, causes and consequences dependent respectively independent variables that come into corresponding quantitative values. This means that the real system is represented in a different form, which is clearer, simpler and more understandable and should give an idea about the future direction of the system. The model is in a way a copy of reality to which it corresponds (it is isomorphic) [1].

The explanation of the model can be expressed in two aspects. In particular, the model provides the knowledge of necessary consequences. What will be, how it will be, what, can be expected if "nothing" affects the system from outside we call this projection. The second aspect is that the model presents an idea of the direction. That is, the model gives an answer for a possible state in the future, provided that "now" is something that happened. We call this prediction. The theoretical basis will then be applied in the context of the mathematical modelling in a broader context.

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## III. LEONTIEF'S ECONOMY MODEL, STATIC INTERPRETATION

An extensively studied model applied to the problem of Critical Infrastructure is based on Leontief's economic model that addresses the description of the n companies' production steady state X 1, X 2. . . Xn of the selected system. If Xi company for its 1 dollar production needs from Xj company to purchase production worth ai, j, dollar and the matrix A = (i, j) n degree represents the supplier - customer companies relations X1, X2,. . . Xn (Leontief's matrix of technical coefficients, which sum's in each column are less than or equal to 1). Let U = [U1, U2 . . . UN] be the vector of external requirements for final consumption for customers outside the system. The question is, how much every company must make in order to satisfy the requirements with external customers (external, consumption outside the system) and the supplier - customer organizations requirements of the system (internal, consumption system). If we searched the entire output denoted x = [x1, x2…Xn], then it is obviously that x - for the internal consumption (production for the system). It can however also be expressed by the product Ax. Then the equation $x - u = Ax$ describes the equilibrium between production companies. It describes their mutual requirements and the requirements of external customers (produced just as much as they need. They do not overproduction or deficiency). Since the totals of each column of the matrix A are less than or equal to 1, the equation always has the only solution as x = (I - A) -1U where I is the identity matrix. The diagram for the three companies is presented in Figure 1. [2][3][4].

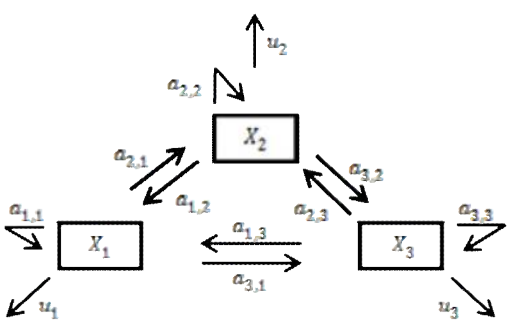


Figure 1. Diagram for three Critical Infrastructure elements [8]

### A. *Domino and synergy effect and Critical Infrastructure elements degradation spread*

To describe the internal state, we take into account the relationships between elements of the system, their response to input u and momentary state x. If the system is linear, status change can be described by the equation

$$dx/dt=Ax+Bu \qquad (1)$$

or in the discrete form by the equation

$$x(k+1)=Ax(k)+Bu(k). \qquad (2)$$

Leontief's economic model can be used in the context of Critical Infrastructure, wherein x is a Critical Infrastructure element degradation importance, u is an input variable to cause a primary Critical Infrastructure element degradation, A is a matrix of pairwise dependence, which features describe the tag container relation between two elements, and reflect the transfer of degradation from first element to second. Let paired relationship between elements Xi and Xj be expressed graphically Xi → Xj with transfer coefficient ai,j. If the value of the element degradation XJ is xj, its transfer to the element Xi is the value of ai, j · xj. This is a first degree transfer Figure 2.

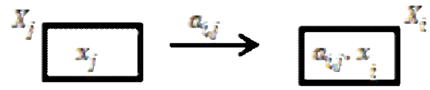


Figure 2. First degree transfer [4]

The value ai,j·xj spreads from Xi element to another and the second degree transfer is created. Next transfer creates a third transfer and so on. The following figure 3 shows the pair dependence of 5 Critical Infrastructure elements, which defines the first degree transfer. Second degree transfers are for example X1 → X2 → X3, X1 → X2 → X4, X2 → X3 → X5, X2 → X4 → X5. Third degree transfers are for example X1 → X2 → X3 → X5, X1 → X2 → X4 → X1, Fig. 3. For example, if the element X1 was degraded, the transfer X1 → X2 → X4 → X1 increase its degradation.

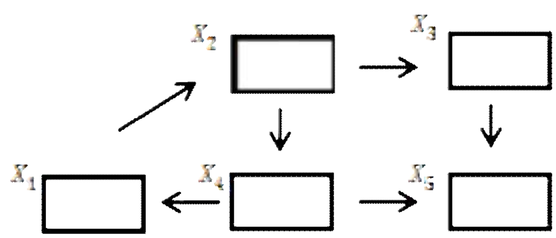


Figure 3. Third degree degradation [8]

The mentioned transfers are only possible passage as degradation can spread. It depends which element has been degraded, which element is "active" at being degraded and which way the degradation will spread. Figure 4 shows the initial degradation of the first element with the value x1 = 0:15 ("active element").

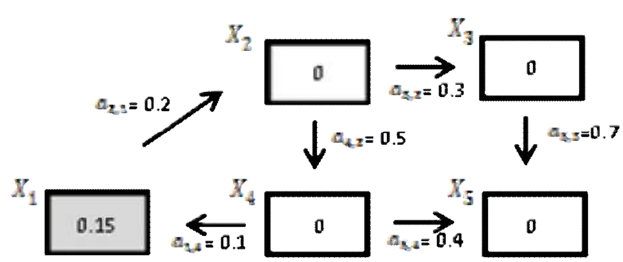


Figure 4. First degree degradation [8]

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From Figure 5, the first degree transfer degradation is transferred to the second element in the 0.03 value. The other elements degradation did not transfer to and for the next transfer will be "active" second element.

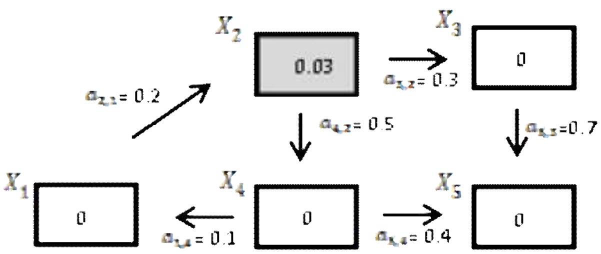


Figure 5. Second element degradation [8]

The second element delegate the second-degree transfer to third element degradation in the value of 0:03 · 0.3 = 0.009 and the fourth element transferred degradation in the value of 0:03 · 0.5 = 0015th "active" elements are X3 and X4 (see Figure 6).

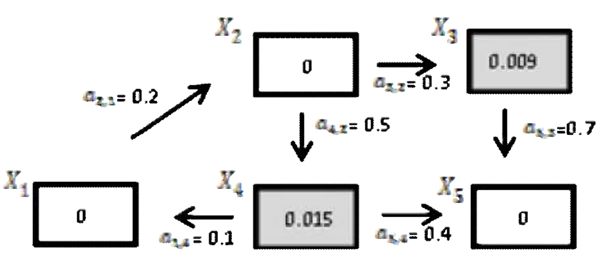


Figure 6. Degradation to third element [8]

From the third and fourth element is third degree degradation transferred to the elements X5 and X1 in the values 0009 · 0015 · 0.7 + 0.4 = 0.0123 to the element X5 valued at 0.015 · 0.1 = 0.0015 to X1. "Active" elements are X1 and X5 but the transfer from X5 does not exist, and the cycle is repeated with the "active" element X1. Figure 7.

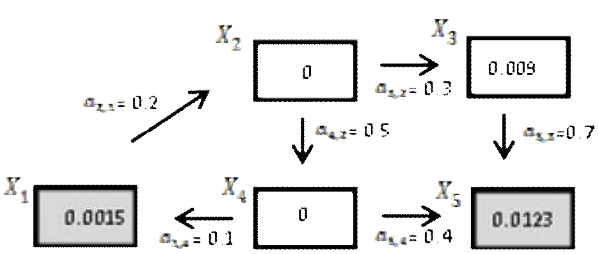


Figure 7. Third degree degradation [8]

In general, the input to the system resulting to primary elements degradation and by their mutual linkage spreading from one element to the other, from the second to the third and so on. The way degradation spreads through the system can be called a domino effect. If x0 = u is the first degree degradation, then by paired dependence will be extended to subsequent degradation, which increased by Δ1 = Ax0, which generates the subsequent amendment of Δ2 = AΔ1 = A (Ax0) = A2x0, etc. After n steps we get an increased value Dn = A (Dn-1) = Anx0. As with Leontief's economic model it forms the analogous question of whether the transfer of degradation would settle to the final value x, which will also vary, e.g. or reaches a steady state. That occurs when the resulting value of the degradation x is decomposed into the sum of the initial input degradation and degradation generated by domino effect. This is represented by equality x = Ax + u. In this case, the totals of each column of the matrix A may be greater than 1 and the equation may not have a solution. When equilibrium occurs, the resulting degradation is x = ((I-A)^τ)U [5], [6], [7].

For the above example, the pair-wise dependency matrix has the form:

$$A = \begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.1 & 0.0 \\ 0.2 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.3 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.4 & 0.0 \end{pmatrix}$$

and input degradation is given by the vector x0 = u = [0.15,0,0,0,0]. Additions degradation from "zero" (initial) to ninth are equal to:

Δ0 = A0x0 = Ix0 = x0 = [0.15,0,0,0,0],

Δ1 = Ax0 = [0,0.03,0,0,0],

Δ2 = AΔ1 = A(Ax0) = A2x0 = [0,0,0.009,0.015,0],

Δ3 = AΔ2 = A(A(A(x0)) = A3x0 = [0.0015,0,0,0,0.0123],

Δ4 = AΔ3 = A(A(A(A(x0))) = A4x0 = [0,0.0003,0,0,0],

Δ5 = AΔ4 = A5x0 = [0,0.00009,0.00015,0,0],

Δ6 = AΔ5 = A6x0 = [0.000015,0,0,0,0.000123],

Δ7 = AΔ6 = A7x0 = [0,0.000003,0,0,0],

Δ8 = AΔ7 = A8x0 = [0,3x10-6,0,0,0],

Δ9 = AΔ8 = A9x0 = [0,0,9x10-7, 1.5x10-6,0].

The resulting value of elements degradation is given by the n matrix equations solution Ax = ax + u as x = (I - A) - 1U = [0.151, 0.030, 0.009, 0.151, 0.012] T, Figure 8 - Graph 1.

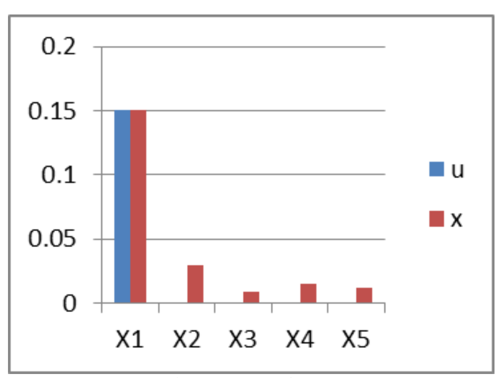


Figure 8. Graph 1[8]

For the input of degradation of the first and second elements in the values u1 = 0:15, 0:25 u2 = (vector u = [0.15, 0.25, 0, 0, 0]), the resulting degradation of the components of the vector x = (I - A) = -1U are [0.164, 0.283, 0.085, 0.141, 0.116] Figure 9 - Graph 2.

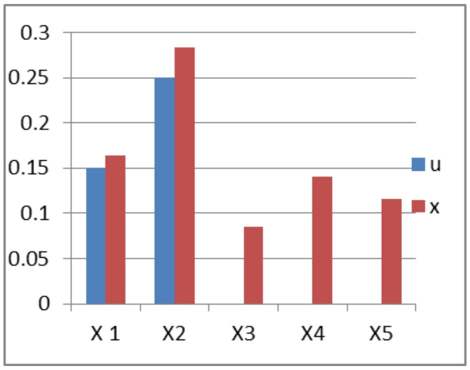


Figure 9. Graph 2 [8]

If all the elements are equally degraded, for example to 0.15 (vector u = [0.15, 0.15, 0.15, 0.15, 0.15] T) and the resulting degradation is given by the vector x = (I - A) -1U = [0.174, 0.185, 0.205, 0.242, 0.391] T. The percentage of degradation increased in steps of 16%, 23%, 36%, 61% and 160%. From these values it is clear that the most vulnerable elements are X3, X4, X5. X5 is the element which should to be protected as much as possible, Figure 10 [8].

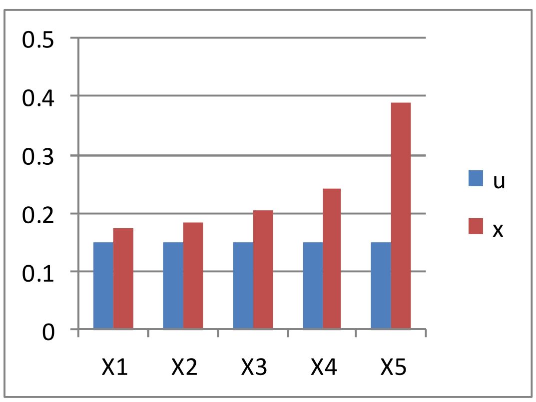


Figure 10. Graph 3 [8]

The theoretical framework will then be applied to the critical infrastructure protection in the Czech Republic. Within the scope of application model will be used in practical knowledge of individual sub-systems (the areas) of the Czech Republic critical infrastructure linkages assessment.

## IV. Outputs of the Czech Republic critical infrastructure practical modelling

The critical infrastructure in the Czech Republic is divided into the 27 areas (sectors) that have significant input into societal functions. Within the modelling they were established reciprocal links in terms of the coefficient of activity and passivity, which allows to determine the linkage value Activity coefficient expresses the potential of a critical infrastructure area affecting the other areas (electricity for district heating) and the passivity coefficient ex-pressed the impact of other areas to selected critical infrastructure area (electricity) - (district heating - electricity). The quantitative expression of these coefficients is implemented through Qualitative Risk Correlation Analysis [8]. Determination of linkages has been implemented through the 27x27 matrix. After determining the values of linkages, Leontief's model was applied up to the third degree of degradation, which expresses dominoes and then synergistic effects in three time intervals 24, 48, 72h for 25% of an electricity power failure (see Figure 11) .

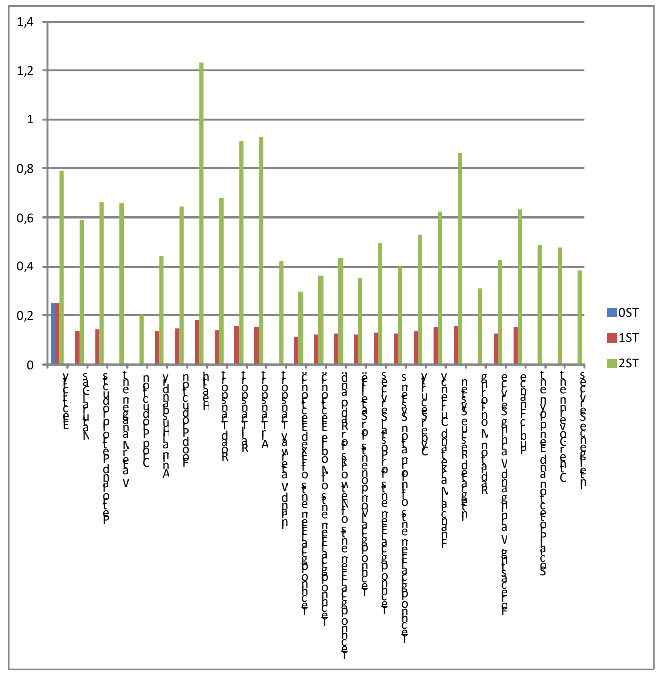


Figure 11. Impact of 25% of electricity power failure

For a better illustration, a 3D graph was generated, which expresses the impact of electricity power failure to the other areas of the Czech Republic critical infrastructure. From the graph you can read the most vulnerable areas of critical infrastructure and the overall spread of dominoes and synergistic effects. This approach allowed us to identify secondary and tertiary linkages below the area of the electricity power failure in the context of the time frame. These facts will then be implemented in the process of crisis preparedness of critical infrastructure operators. Another benefit will be the use of the conclusions in the project Security Research Project - VI20152019049 - RESILIENCE 2015 Dynamic Resilience Evaluation of Interrelated Critical Infrastructure Subsystems.

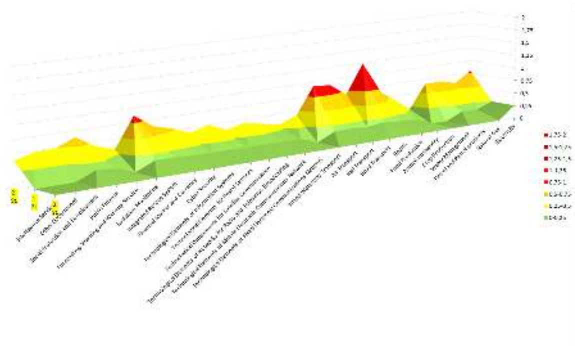

Figure 12. Impact of electricity power failure on other critical infrastructure subsystems

Figure 12 shows the impact o 25% electricity power failure on other critical infrastructure subsystems.

## V. CONCLUSION

The facts and knowledge presented in this paper pointed to the possibilities of mathematical modelling in the context of the needs of critical infrastructure protection and resilience. The first section of the article presented a definition of critical infrastructure as a fundamental system affecting the society functional continuity.. Another part of the text dealt with the general approach to the model development that has been further developed within the application of Leontief's economic model to the issues of determining the interactions and evaluation of domino and synergy effect within the system elements. In the end of this publication, we present partial results from the application of the model for Czech Republic critical infrastructure system and security research project RESILIENCE 2015 issues.

## ACKNOWLEDGMENT

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