SLAC-PUB-708
January 1970
(ACC)

Critical Power Dissipation in a Superconductor*

Mario Rabinowitz
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

The magnetic breakdown field, H_p', is calculated which, if applied at the surface of a superconductor, produces a critical power dissipation leading to a steep rise in the power loss. Generally $H_p' < H_c$, the critical field of the material. The functional dependence of the Q of a microwave cavity for values of H_p near H_p' is also found.

As the magnetic field, $H_p \cos \omega t$, applied to the surface of a superconductor is increased, a critical value of $H_p = H_p'$ is reached at which the power dissipation rises sharply. Generally $H_p' < H_c$, the critical field of the material. The object of this paper is to calculate H_p'.

The fact that H_p' can be well below H_c (H_{c1} for type II) in both low and high frequency measurements has been blamed on surface defects such as impurities and dislocations; protrusions which cause local magnetic field enhancement; and trapped flux. Halbritter[1] realized that a suggestion by Easson _et al._,[2] for the low frequency case might be applicable to superconducting microwave cavities. Easson _et al._,[2] suggested that in type II the transition to the normal state is caused by a temperature rise above T_c due to ac losses rather than by the peak

*This work was supported by the U. S. Atomic Energy Commission.

current in the sample exceeding the thermodynamic critical value. However it appears that neither Halbritter, Easson et al., nor anyone else has pursued this suggestion in terms of a theoretical analysis which relates the magnetic breakdown field to the thermal and electrical properties of a superconductor.

For simplicity, let us consider a cylindrical normal region of radius a within the penetration depth, λ, which may be a fluxoid, etc. The power dissipation per unit area of the cylinder is

$$P_a = \frac{1}{2} R(FH_p)^2 \tag{1}$$

where FH_p represents the proper spatial average of the magnetic field across the cylinder whose center lies a distance d from the conducting surface. R is the effective normal state surface resistance of the cylinder. $R = R_0 + R(T) \doteq R_0$, as the temperature excursion is not great.

Assuming cylindrical symmetry as a fair approximation, and using cylindrical coordinates centered at the fluxoid, the heat flow equation is $P_a\left(\frac{a}{r}\right) = -K\frac{dT}{dr}$, where the superconducting thermal conductivity for many metals is roughly given by:

$$K = \begin{cases} k_1(T - T_1) \\ k_2T^{-3/2} + C_2 \\ k_3T^3 \end{cases} \doteq \begin{cases} k_1T & T_a \geq T \geq T_m \\ k_2T^{-3/2} & T_m \geq T \geq T_M \\ k_3T^3 & T_M \geq T \geq T_b \end{cases} \tag{2}$$

where T_a is the temperature at the periphery of the cylinder, T_b is the temperature of the outer surface of the conductor which is approximately the bath temperature neglecting Kapitza resistance, T_M is where K is maximum, and T_m is where K is minimum. (It makes only a small difference whether $T^{-3/2}$ or $\exp\left(\frac{g}{T}\right)$ is used.)

If the cylinder center is a distance b from the outer surface, the approximate solution is:

$$H_p = \frac{1}{F}\left[\frac{k_1\left(T_a^2 - T_m^2\right) + 2C_1}{NRa\,\ell n\left(\frac{b}{a}\right)}\right]^{1/2} \tag{3}$$

where N is a factor to correct for the departure from cylindrical symmetry, and

$$C_1 = 2k_2\left(T_M^{-\frac{1}{2}} - T_m^{-\frac{1}{2}}\right) + \frac{1}{4}k_3\left(T_M^4 - T_b^4\right) . \tag{4}$$

For

$$k_1T_a^2 \gg k_1T_m^2 - 2C_1,\ H_p \doteq \frac{1}{F}\left(\frac{k_1}{NRa\,\ell n\left(\frac{b}{a}\right)}\right)^{\frac{1}{2}} T_a\left[1 - \frac{1}{2}\left(\frac{k_1T_m^2 - 2C_1}{k_1T_a^2}\right) - \frac{1}{8}\left(\frac{k_1T_m^2 - 2C_1}{k_1T_a^2}\right)^2 - \ldots\right] \tag{5}$$

So at large T_a, H_p is approximately linear with T_a. If the superconductor is a microwave cavity, when $k_1T_a^2 \gg k_1T_m^2 - 2C_1$ and $T_a > \frac{1}{2}T_c$, then the cavity $Q \propto e^{\epsilon/2k_BT}$ where ϵ is the energy gap and k_B is the Boltzmann constant. If the cavity Q is dominated by the power loss around the cylinder (there may be more than one), $T = fT_a \propto H_p$, $(f < 1)$,

$$\Rightarrow Q \propto e^{D/H_p} \text{ for } H_p \text{ near } H_p' . \tag{6}$$

where

$$D \doteq \frac{\epsilon f}{2k_BF}\left[\frac{k_1}{NRa\,\ell n\left(\frac{b}{a}\right)}\right]^{1/2} . \tag{7}$$

When $T_a < \frac{1}{2}T_c$ and $k_1T_a^2 \gg k_1T_m^2 - 2C_1$, then for H_p near H_p',

$$Q \propto T e^{\epsilon/2k_BT} \propto H_p e^{D/H_p} . \tag{8}$$

Equation (8) gives a good representation of the data of Turneaure and Viet.[3]

When H_p reaches a point where a critical magnetic field is reached in the neighborhood, then the material surrounding the cylinder will go normal, leading to a sharp rise in the power dissipation and ultimately a run-away situation. In the case of a cavity, the Q will drop precipitously. The magnetic field in the neighborhood is $F\vec{H}_p \cos \omega t + \vec{H}_a$, where $\vec{H}_a$ is the magnetic field which exists at radius a due to the contribution from all sources besides the current in the cylinder. $\vec{H}_a$ may be an applied dc field, and/or the field penetration from a fluxoid. The worst case is when H_a and $F H_p \cos\omega t$ add together algebraically at peak value.

$$H_a + F H_p = H_c = H_0 \left[1 - \left(\frac{T_a}{T_c}\right)^2\right] \tag{9}$$

Combining Eq. (3) and (9) yields

$$H_p' = \frac{-\frac{1}{2}k_1 \frac{T_c^2}{H_0} + \left[\left(\frac{1}{2}k_1 T_c^2/H_0\right)^2 - 2NR\, a\ln\left(\frac{b}{a}\right)\left\{\frac{1}{2}k_1\left[T_m^2 - T_c^2\left(1 - \frac{H_a}{H_0}\right)\right] - C_1\right\}\right]^{1/2}}{NRF\, a\ln\left(\frac{b}{a}\right)} \tag{10}$$

This is the peak magnetic field at the surface of a superconductor which produces a critical power dissipation leading to a steep rise in the power loss. With the possible exception of R and F, all the parameters can be determined experimentally. The frequency dependence of H_p' may help to determine R.

In the case of Pb or Nb, if the normal region is a fluxoid, 2a can easily be $\gtrsim \lambda$, and an rf field might penetrate it fully if it is located at the conducting surface. The current in a conductor flows in such a manner as to minimize resistive losses for dc or low frequencies. However, at high frequencies, as in a GHz cavity, the stored electromagnetic energy is minimized and the current tends to flow more uniformly through the conducting surfaces. For an oscillating fluxoid which results when a high frequency current is perpendicular to the fluxoid, Gittleman and Rosenblum[3] have shown that the effective resistivity of the fluxoid is $\sim\rho_n$, the normal bulk resistivity. From the Wiedemann-Franz law $L/\rho_n = k_{1n} \doteq k_1$, where $L = 2.45 \times 10^{-8}$ W-Ω/$^{o}K^2$, and $k_{1n}T$ is the normal state thermal conductivity.

We are now in a position to make a predictive comparative estimate (see Table I) of the relative importance of the various parameters as they affect the magnetic breakdown field H_p' at which, for example, the Q of a niobium cavity would be seriously degraded. Take $T_c = 9.5^{o}K$, $N = 2$ and $\ell n\left(\frac{b}{a}\right) \sim 12$. The thermal conductivity values for group A were obtained from the highest values of Calverly et al.,[4] group B from Styles and Weaver,[5] and group C from H. Brechna.[6]

Turneaure and Weissman[7] have made measurements on niobium cavities. They report magnetic breakdown fields which range from 290 to 436 Oe. Similarly Nb measurements by Turneaure and Viet[3] have extended the breakdown field in the range from 710 to 1080 Oe, primarily by improved vacuum heat treating of the Nb cavities. The theoretical predictions of Table I are consistent with this entire range, as well as the fact that the thermal conductivity improves with vacuum heat treating.

I wish to thank Bruce Rosenblum for indirectly providing an incentive for this analysis, Perry Wilson and Dave Farkas for stimulating discussions, and Matthew Allen for his encouragement.

TABLE I

Comparative Predicted Values of H'_p for Nb

	H'_p (oersted)	H_0 (oersted)	H_a (oersted)	T_b °K	k_1 $\frac{W}{cm\,{}^oK^2}$	C_1 $\frac{W}{cm}$	T_a °K
A	1000	4000	2400	1.0	0.10	28.8	3.6
	960	4000	2400	1.8	0.10	20.9	3.8
B	690	4000	2400	1.0	0.045	~ 0	4.5
	660	4000	2400	1.8	0.045	~ 0	4.6
C	300	4000	2400	1.0	0.016	~ 0	5.4
	290	4000	2400	1.8	0.016	~ 0	5.4

REFERENCES

1. J. Halbritter, KFZ - Karlsruhe, Externer Bericht 3/69-6 (1969).
2. R. M. Easson, P. Hlawiczka, and J. M. Ross, Phys. Letters 20, 465 (1966).
3. J. P. Turneaure and N. T. Viet, Report No. HEPL-612, Stanford University (1969).
4. A. Calverly, K. Mendelssohn, and P. M. Rowell, Cryogenics 2, 26 (1961).
5. J. B. Styles and J. N. Weaver, Report No. HEPL-TN-68-12, Stanford University (1968).
6. H. Brechna, SLAC (private communication).
7. J. P. Turneaure and I. Weissman, J. Appl. Phys. 39, 4417 (1968).