CRITICAL STUDY OF DA COSTA'S FOUNDATIONS OF LOGIC

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Introduction

Professor da Costa's *Ensaio sobre os fundamentos da lógica*[1] constitutes a major contribution to the philosophy of logic. It's a stimulating, provocative book, putting forward many original ideas, tackling a number of important if intricate issues in the field, and, to myself at least, suggesting how to open new ways in metalogical research. The book does not set forth any metalogical system. In fact, it is more a collection of essays than a systematic account of problems in the philosophy of logic. As the author himself puts it in the foreword, there may be incongruities between different parts of the book, since it has been written over a number of years and has originated in notes used for courses and seminars. The reader may well regret that the great Brazilian logician should have gone about editing those notes without first devoting a more strenuous effort to reshaping those outlines. and somehow making them into a more encompassing, and cohesive treatise on the subject. Nevertheless, I think there is a deeper ground for da Costa's failing to do so: he regards the philosophy of logic, and philosophy in general, as a field wherein we lack clear-cut and sure criteria and principles, so much so that, for us to aim at a systematic, well-organized, formalizable account of philosophical problems would be to embark in a speculative philosophy, turning our backs on the unavoidably piecemeal task of setting up a constructive, positive philosophy, which contents itself with tentative, partial elucidations, tied up with results of other sciences.

§1.— Verificationist positivism

The underlying doctrine which — not consequently enough to my mind — emerges throughout the book is a verificationist view, according to which to be true is to be ascertainable and nothing else. This is why da Costa shows himself highly sympathetic towards intuitionism. On the other hand, though, he cleaves to excluded

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[1] Newton C.A. da Costa, *Ensaio sobre os fundamentos da lógica*, São Paulo: Editorial Hucitec, 1979.

middle, groping not very successfully, I'm afraid — at a justification of this principle from the standpoint of a pragmatistic verificationism.

A telling illustration of da Costa's verificationism is his conception of objects of a mathematical theory as having only those properties they are assigned in virtue of the axioms and inference-rules of the theory under consideration. This is a recurrent claim (cf. particularly pp. 87-93). So, e.g., the axioms of ZF serve to define the notion of set. Nevertheless, there are other definitions of that notion, which are provided by alternative set-theories. Similarly — as we're going to see below, in §4 — negation can be defined in several, alternative ways, incompatible with one another. But that alternativeness doesn't bother da Costa: there's a plurivocity of 'negation', of 'set', and so on, and, when passing from some theory to an alternative one, we are in fact defining something different, even if closely related or akin. The reader may find such accounts of discrepancies between theories fairly puzzling; for, cannot a similar account be given as regards alternative physical or sociological theories? Then there would be no discrepancy proper, and, as a result, scientific discussions would be pointless.

As against that kind of indeterminacy — inter-systematic indeterminacy — there's an indeterminacy, though, which, according to da Costa, is provably uneliminable: some expected properties of sets are not just unprovable, but provably unprovable in ZF — e.g. the axiom of choice, the cardinality of the continuum and so on — (provided ZF is consistent), even though they also happen to be provably irrefutable within ZF (with the same proviso, of course). Let me call it 'intra-systematic indeterminacy'.

I find, though, some wavering in da Costa's position. On the one hand, he seems to uphold an indeterminacy thesis — Zermelian sets neither would comply with the axiom of choice nor would fail to do so —; on the other, he seems to favour an ontological relativism according to which for some extensions of ZF, Zermelian sets comply with the axiom of choice, whereas for other extensions they don't (cf. p. 88, where what is at issue is the Löwenheim-Skolem theorem's bearing on determinacy of Zermelian sets). Probably, da Costa identifies indeterminacy with relativity, since he thinks that the principle of excluded middle is needful in most contexts — and therefore part and parcel of systems of logic adequate to most scientific purposes. Accordingly, indeterminacy only means relativity: instead of cleaving to ZF as such, we ought to choose either ZF + axiom of choice, or else ZF + the negation of that axiom. and as much would to be said concerning any other indeterminacy result. Were my interpretation of da Costa's opinion correct, then all indeterminacy would after all reduce to the inter-systematic kind thereof.

Related to that issue is da Costa's discussion of Kreisel's comment on the significance of theorems of indeterminacy concerning the continuum hypothesis. For Kreisel, there's no objective indeterminacy, since, once we have fixed the cardinality of any normal model of ZF, that model is isomorphic with any other model of the same cardinality; hence, in any two such models there is just one aleph equal to the continuum's cardinal. Da Costa replies that the objective indeterminacy still stays,

since there remains some relativity with respect to the chosen model's cardinality. Da Costa's point is probably well taken, but I cannot subscribe to his conclusion. To myself, all that has been proved is, not that Zermelian sets are indeterminate with respect to certain properties or that they have, or respectively lack, those properties only relatively to some particular extension or other of ZF, but that, if all we know about sets is what ZF tells us, then we don't know whether sets have those properties or not. Nonetheless, da Costa would reject that remark, since it relies on an 'extreme platonism' as he calls it. We're going to see (in §3 below) that his own platonism is far from extreme.

I find another ground for regarding da Costa's view as verificationist in his rejection of realism: against Aristotle, da Costa somehow espouses (pp. 77-8) Kant's opinion that. instead of being governed by the objects, our knowledge is governed by the subject's a priori 'puttings', although, unlike Kant, da Costa thinks that those a priori postulations are not unchangeable, but vary a good deal according to the intellectual circumstances — ultimately, pragmatic ones. (Moreover, our knowledge is not merely our creation, but stems from an interaction between the external world and a priori cognitive structure; see p. 121.) So, it is us who determine, through our postulations, such theses as we want to call 'logical truths'. and this seems also to hold for metalogical theses, also. This is why da Costa says (on p. 95) that logical realism or platonism is unscientific and speculative, in that we lack any more or less sure criterion allowing us to know whether or not objects postulated by platonism have certain properties independently of their being assigned them by some particular theory.

Owing to that dominant trend towards verificationism, it's far from surprising that da Costa's view reveals itself positivist (cf. pp. 5, 7 ff., 60 ff., 225). Da Costa's positivism contains these five tenets:

(1) no method is allowed except reason, analysis, hypothetical models and appeals to real sciences and to historical criticism;

(2) problems dealt with must be amenable to objective criteria — a criterion being objective insomuch as it yields a clear-curt test or decision procedure;

(3) philosophy has no real content of this own (i.e. philosophy says nothing about reality which is not said by some particular science or other), its only task being a research about problems of particular sciences;

(4) philosophers ought to keep their researches independent of politics, religion, speculative philosophy or, for that matter, any other human activity except science;

(5) no metaphysical doctrine about reality is to be countenanced — particularly, no metaphysical theory is to be drawn from either language or logic (cf. p. 177): da Costa identifies — mistakenly, I dare say — every logical or mathematical theory with a language, and argues that use of language is compatible with conflicting metaphysical views, whence it would, according

to him, follow that logical and mathematical theories are ontologically neutral.

Moreover, da Costa's positivism is 'dialectical' — that is the word he uses — in that he grants no definitive value to any statement or theory, not even to scientific methods; when we assert something, we ought to do so in such a way that me take our assertion to be provisional, not true as such, but a mere approximation to truth.

I find all those claims highly exceptionable. Claim (1) seems to me too ambiguous and therefore hard to assess in any satisfactorily rigorous or accurate way. Claim (2) stems from verificationism which, among many other difficulties, faces this one: were it true, the claim could not be verified — there being no decision procedure for us to know whether or not it is true. Claim (3) is belied by da Costa's own metaphysics, platonistic after a fashion (see below, §3 of this paper); moreover, the problem of whether or not philosophy has real content of its own is dealt with by no particular science at all, and then the claim itself is non-positive — which by the way is the fate befalling all those claims. Claim (4) is by no means uncontroversial, and it can be contended that, willy-nilly, every philosopher — or even any scientist, for that matter — pursues his research taking as his start-point an assumption framework, an outlook which contains, or is conditioned by, values, attitudes, as well as staunch beliefs which we call 'intuitions' (see below, §2).

And claim (5) seems to me mistaken, since a theory is not a language, and when asserting some theory's theorems we commit ourselves to the existence or nonexistence of whatever a theorem says exists or, respectively, fails to exist. The latter point is of foremost significance, since, when I state 'There is nothing such that…', I commit myself to the fact that nothing is such that…, which is a negative-existential commitment. However, people have become worried with positive-existential commitments alone, which has resulted in carelessness about the ontological import of statements of the form 'All entities are such that…' (i.e. 'There doesn't exist anything such that not…'). Therefore, it's not enough to give up the EG rule in order to set rid of any ontological commitment, for, by stating universally quantified sentences, you commit yourselves ontologically, too; only your commitment is negative, which notwithstanding it remains an ontological commitment all the same.

As for the so-called dialectization of theories, I deem da Costa's a little cloudy statements are self-refuting, not because the statements themselves become liable to revision — that doesn't make them false —, but because, if I understand rightly what da Costa means, no statement can be asserted as true, but only as more or less near to truth (notice, though, that being more or less near to truth is by no means the same as being more or less true; degrees of truth are not degrees of proximity to truth). Da Costa's view of dialectics seems to me to mistake lack of truth guaranties for lack of truth — a confusion ensuant upon verificationism. For, our being unable to bear out our statements with such clinching arguments as rely only on premises whose evidence does undeniably lie beyond any reasonable dispute or

challenge nowise shows our statements to be false; it only shows them to be not radically justified. (Now, what we might draw as a conclusion from that lack of radical justification is that knowledge is, merely, true belief, and nothing else.)

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§2.— Da Costa's Pragmatist Intuitivism

According to da Costa, the ultimate justification of rational principles and methods of operation is pragmatic: without resorting to them, we should go irrational (cf. pp. 221-2 & p. 233), i.e. we should be stranded on situations pragmatically absurd (cf. p. 234). But why should we endeavour to escape that fate? Da Costa does not try to answer that question; in fact he does not even bring it up. Probably he thinks questions of that kind lie outside the scope of a positive philosophy: since there can be no verifiable answers to them, the questions themselves lack sense.

So, our metalogical research is, according to da Costa, conditioned by an unshakable presupposition, which we cannot tare to question: the need for rational principles and methods. But those principles, which have an unchangeable core, are not theses of logic, but mere rules of thought operation. There are three of them: systematization, unity, and adequacy.

The principle of systematization lays down that reason works and proceeds only through some system of logic or other. (But I've found no available definition of 'logic' in the book). Da Costa explains the principle like this (p. 46): reason has some game nature, and, accordingly, the rules of the game ought to be explicitly laid out. Thus I take it that for da Costa logic is the set of the rules of the rational game, which probably means that logic concerns itself more with inference rules (conceived of as — relatively at least — conventionally chosen for the sake of a thought game which we cannot give up) than with logical truths or theses (da Costa says, on p. 157, that finding inference rules in a wide sense is 'the ultimate purpose of logic'; true, he also says, on p. 20, that logic is much more than a theory of valid inferences; whether those apparently conflicting statements constitute one of the incongruities candidly avowed by da Costa himself I don't know; but I take it that, anyway, the theory of inference, in a large sense, is the core of logic for him).

The principle of unity lays down that, in any given context, just one system of logic can be used. So, even if there are sundry systems of logic, each of them having its point and its legitimate use in some context or other, no clash or chaotic mingle of alternative logical systems can be allowed once it comes to coping with some definite problems in a definite research environment.

The principle of adequacy lays down that the logical system chosen in a given situation ought to be the one that befits the situation best. This principle puts a limit on the game character of human reason: we are not altogether free in choosing the system of logic we want to use; for, experience as well as pragmatic factors comp el us to choose a particular system out of a gamut of systems. Those

pragmatic factors seem able to be subsumed under the principle of thought economy or simplicity, which is in turn what justifies the principle of adequacy itself.

Even though, according to da Costa, those three principles are the only permanent features of reason — and, moreover, somewhat vague ones at that — they might eventually be waived, without loss of rationality. What can be asserted on that score is that we have thus far found no alternative to them, and, therefore, relinquishing them would bring about a far-fetched or bizarre science (p. 48). I take it that, for da Costa, what would be wrong in a bizarre 'science' would be that it would fly in the face of deeply engrafted or ingrained wonts, without which we nowadays don't know how to proceed.

Within the moving range marked out by those three principles, nothing is fixed. Patterns of rationality are changing; however, we in principle ought to comply with those in operation in our environment, unless there provably are greater advantages ensuant upon replacing them with others. (Let me put in an objection: with which patterns are we to comply then proving the greater advantages of patterns of rationality different from the ones that have hitherto been enacted within our milieu?)

Against such a pragmatist and probably relativistic background, intuitivism would be hardly to be expected, but da Costa's somehow eclectic philosophy of logic has many a string in its fiddle; thus, we find throughout the book a defence of a 'dialectisized' intellectual intuition (in a sense of 'dialectisize' to be explained shortly).

In fact, in what we have set forth as yet, some link seems to be needed, in order to bind the empirical material with the three metalogical principles, or we shall incur an infinite regress: if we justify some theses or inference-rules on the base of their resulting from application of the three metalogical principles to the extant experience, how can we justify the very claim that those theses and inference-rules result from application of the three metalogical principles to the extant experience? Da Costa could have sidestepped the difficulty through a bold leap, but he seems to jib at doing so. Intuition is then needed: we know such theses as our intellectual intuition shows to be true, and such inference-rules as it shows to be valid or truth-preserving.

Admittedly, intuitivism and intuitionism are only accidental bedfellows (a coherentist constructivism, rid of the doctrine of intellectual intuition, could be bolstered up with arguments nowise contemptible from a largely verificationist viewpoint — and verificationism, however largely conceived, is the core of a constructivist view of logic; on the other hand, most advocates of intellectual intuition spurn constructivism). Still, da Costa's upholding intellectual intuition as an epistemological base and criterion in logic brings him closer to intuitionism.

Da Costa assesses the controversies pursued inside the intuitionist movement between followers of orthodox Brouwerianism and people such as Griss and Essenin-Volpin who stand up for a stricter variety of intuitionism. Da Costa agrees with

those ultra-intuitionists that Brouwerian mathematics does not rely on intuition such as it really takes place, but on a fairly idealized intuition. He contends, though, that 'an intuition at least as wide as the intuitionist one turns out indispensable for formal sciences' (p. 57), and, accordingly, ought to be accepted in virtue of the principle of adequacy, even though for some particular purposes and contexts ultra-intuitionism remains a useful logical approach.

Da Costa makes much of intellectual intuition and devotes much space to canvassing issues related thereto. However, as he sees it, the intellectual intuition available in logic and mathematics is formal rather then material (pp. 190-1); which means that what we grasp through intellectual intuition is, not an object, not even a categorical proposition, but only an hypothetical one. This is of course a new version of a Russellian view. The main difficulty surrounding the Russellian view is that many theorems of mathematics and logic don't have the form 'If p, then q'. Still, such an objection doesn't arise concerning da Costa's conception of intuition — since he could easily reply that not every theorem of logic and mathematics is intuitively grasped as true. For, da Costa says (p. 19) that logic and mathematics originate in an interaction between formal intuition and language, an interaction which is furthermore conditioned — but only genetically and heuristically — by empirical sciences. The arguments put forward by da Costa in support of the existence of intellectual intuitions seem to me weak. One of them is (p. 225) that without immediate knowledge there could be no mediate one. I deem that claim challengeable. The other argument is a Cartesian point: even an inference process is made up by steps, each of them being an immediate or intuitive grasping of the consequence. This claim, too, can be either disputed or, at least, duly qualified, so as to make intuition needless. An inference is a mental act or operation, something one performs. It consists in passing or shifting from asserting, or entertaining, statements making up a set of premises to asserting, or entertaining, another statement, which is the conclusion. Admittedly, that passage or shift is not a belief, even though it can legitimately be carnet out only if one believes that, if all the premises are truthfully assertible, so is the conclusion, too. Therefore, the passage in question is not an intuitive grasping of anything; for, a grasping is, I reckon, an act by which one asserts something while at the same time purportedly seeing that that something is true. As for the belief that the inference holds — that is, that the rule is sound or valid, and that it has been correctly applied (or, in other words, that, if the conjunction of the premises is truthfully assertible, so is the conclusion) —, that belief may be bolstered up with other reasonings, instead of being intuitively grasped as true. That by pursuing such a line of thought we are ultimately faced either with infinite regress (never completed, of course) or else with a circle (which only question-beggingly would be styled 'vicious') by no means invalidates that process insomuch as we have given up any hope for radical justification.

Furthermore, the intellectual intuition da Costa postulates is worked over, furbished, revamped time and again (cf. p. 224). Its justification power lies, not just in evidence, but in resistance to criticisms. Thus, da Costa's intuitivism is, in a way,

of a quasi-coherentist or non-foundationalist cast; for, what according to him justifies our asserting a mathematical, or logical, statement is not just this statement's being intuited as true, but its being so intuited in spite of strenuous attempts at having it undergo apparently cogent criticisms and at replacing it with some alternative or other.

However, if all that is the case, is there really something deserving the title of 'intuition'? What makes it different from mere conviction? For, if there is any difference between them, the difference is to be found in that intuition, should it exist, would be an immediate contact with the object, bearing a warrant for assertability 'on its sleeve'. Moreover, since da Costa is prepared to agree that evidence does not justify a theory on its own, why not require that, in order to be rendered justifiably assertible, with respect to some given background, the theory under consideration should be both fruitful to and coherent with a more far-reaching system, yielding observation statements — i.e. with science either as a whole or, anyway, as encompassing at least some empirical disciplines? Da Costa cannot accept that; for, according to him, logical and mathematical statements are, as such, wholly independent of experience and of empirical reality or knowledge.

My final comment on da Costa's pragmatism is that, by laying it down, he incurs an obvious circle: he bears out the three pragmatic principles with an argument showing that, without them, we should go irrational; and he takes for granted that we mustn't go irrational. Still, the argument itself undoubtedly needs further premises and further rules of inference. Should they be justified in turn by application of the pragmatic principle to the experience and to a body of beliefs whose truth is taken for granted (an application which needs some inference-rules, too, in order to yield any conclusion), then we obviously would enter a circle. But, if we tare to accept circular justification — which doesn't entail circular arguments, each argument remaining non circular, even if the chain they make up is circular —, then why are pragmatic considerations of paramount importance? For, beyond those considerations, a deeper issue arises: what is the objective ground for those principles? What is there in reality that makes it necessary for us to hew to those principles if we want to reach knowledge, i.e. to reflect reality? But that leads us to the relationship between thought and reality.

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§3. — Logic and reality

Da Costa regards the epistemological status of logical theses as twofold. On the one hand, they are formal truths, independent of experience, which, in order for them to be true, depend only on their being proved within a particular formal system (syntactical approach) or, whenever possible, on their holding in all models of that system (semantical approach). On the other hand, those theses are also scientific hypotheses about reality (cf. pp. 112, 113 ff.), and, as such, they are not relative to formal systems or models thereof, but they are as uncertain as any other empirical hypotheses may be. Considered as hypotheses about reality, their assertion relies on

induction (da Costa says that there is on that score a difference of degree rather than nature). I find very puzzling, though, that, while saying all that, da Costa goes on to claim that, as empirical hypotheses, logical principles are 'game rules' (p. 114): Is a!l science made up by game rules? Overwhelming difficulties, I daresay, are heaped on a position like that.

What da Costa seems to mean is that every natural law, even if true, has a limited range or scope, beyond which it does not hold; this is why by ignoring that limitation, we use it like a game rule. and the same holds, he thinks, as regards logical laws, such as the identity principle, which cannot be countenanced in general, unless we resort to a substantialist metaphysics (see pp. 115 ff). Da Costa alleges Schrodinger's claim that identity cannot be applied to elementary particles. and I take it that, according to da Costa, even within the limited range wherein they hold, scientific — and hence also logical — laws do not exactly reflect reality 'as such', but are mere approximations thereto, which owe much to the subject's a priori cognitive structures — which nevertheless vary from a period to another.

Still, if the epistemological status of logical laws as regards empirical reality is rather flabby or shaky, da Costa grants them a steadier — albeit by no means wholly un-problematic — status with respect to a realm of nonempirical or 'abstract' entities.

Da Costa bolsters up the claim that there are abstract entities with two arguments. The first one is a straightforward use of Quine's criterion (in one of its versions): when we say that there exist prime numbers greater than 10, we commit ourselves to postulating the existence of numbers. 'To deny that much would be dishonest' (p. 188). Even though I wholeheartedly agree with that claim, I should like to have found in the book a more thorough discussion of the issue, and particularly an assessment of substitutional reading proposals.

Da Costa's second argument buttressing up his defence of abstract entities is that once a system of axioms — and inference-rules as well, I suppose — has been laid down, we are, willy-nilly, compelled to accept whatever ensues thereon, regardless of whether we like it or not. 'Hence it seems reasonable to acknowledge that something exists beyond the axioms' (p. 189) something whose being so and so compels us to assert the theorems. Mathematics — da Costa goes on to say — is a game, but a game that, once its rules have been established, we cannot change at will. I think there's much to that argument; still, it seems to me unsatisfactory. That we in honesty cannot but assert a theorem which can be proved from some axioms through some inference rules once we have asserted those axioms, bound ourselves to those rules, and ascertained that the proof is correct does not by itself show that something or other corresponds to the theorem under consideration, and is or behaves as the theorem says. For one thing, we can devise inferential rules without any meaning whatever. For another, what solely follows from our being in honesty unable to change the theorems at choice is that, if the axioms are truthfully assertible, and if the inference rules preserve truthful assertability, then the theorems, too, are true. However, as it stands, da Costa's argument yields the conclusion that

any deductive system's theorems are true of some entities. This of course tallies with his ontological relativism: there are in fact infinitely many realms or spheres of abstract entities, and each complete deductive system would reflect one of them. Now, if that is true, every system's theorems are false unless the system's bound variables and quantifiers are read as if they were ranging only over entities belonging to the realm reflected by the system in question; which seems to be extremely untoward and make a theory thus interpreted a set of mere truisms. Any coherent theory will be true — true, that is, of its own objects —, but no theory will be true tout court, true of any object. All domains of quantification will perforce be restricted. For, needless to say, for any theorem of any given theory, you can set up a coherent deductive system wherein the overnegation of that theorem (i.e. the result of prefixing to the theorem the overnegation functor: 'It's not at all the case that') is a theorem.

Furthermore, are those purportedly abstract entities related to the concrete things which make up the empirical world? Which of them are and which of them are not? Do es that relationship — whenever it exists — bear on the usefulness of some formal systems for setting up and developing empirical science? Da Costa says (on p. 26) that many mathematical theories are not directly tied up with sense experience. But I think he is bound to make a much stronger claim, to wit: that most mathematical theories — as he regards them — are neither directly nor indirectly related to experience. But then, what ground is there for us to say that such abstract entities as are wholly unrelated to concrete ones — and whose study nowise bears on empirical science — exist all the same?

What is more, if — as da Costa contends — we only know those abstract entities through intellectual intuition, and intellectual intuition is — as he claims — formal, rather than material, then, I reckon, all we know about 'them' is that, if they exist and are so and so, as the axioms say, they also have whatever characteristics the theorems say 'they' have.

Da Costa's platonism is doubly qualified. The first qualification makes da Costa's peculiar platonism idealistic: not only is any theory true relatively to some particular domain of entities alone, but that domain's existence seems in turn to be relative to the subject that states it. This is why da Costa insistently says that the objects of our logical and mathematical theories are created by us. I take it that, for him, those objects exist only insomuch as we postulate them by laying down systems of axioms which define them implicitly. The second qualification makes da Costa's platonism relativistic: abstract objects exist and are what they are relatively not just to the conceiving subject, but to the theory by dint of postulating which the subject somehow creates the objects. Accordingly, da Costa's platonism agrees with the typical anti-platonist stance of regarding so-called abstract entities as mind-dependent and theory-dependent.

Furthermore, I find a number of difficulties surrounding da Costa's relativistic platonism. First, da Costa's main ground for arriving at it is his regarding sets of axioms as implicit definitions. An old controversy on that matter, originating

in the debate between Frege and Hilbert and pursued by Carnap, Quine, Priest and others, has, according to may lights, clarified the issue. But I find no discussion of the topic, or of the arguments put forward by either side, in da Costa's book. What could be replied to da Costa on behalf of a Fregean standpoint is that natural numbers, e.g., are not whatever entities satisfy a set of axioms or theorems. That we can figure out a great many different ontological reductions of numbers through alternative proxy functions, and deal with them in a number of conflicting ways, nowise proves that numbers 'as such' — whatever 'as' or 'qua' phrases may mean, if they mean anything at all — are indeterminate, or that there are natural numbers of several kinds — Zermelian, Neumannian, and so on. There are ontological — metaphysical, if you like — arguments bestowing more or less plausibility upon some of those accounts and reductions; and we need to assess those arguments' cogency. Moreover, were sets of axioms implicit definitions, then there would be no disagreements, e.g., among upholders of alternative set-theories. Yet, there are such disagreements.

My second ground for taking exception at da Costa's view on the relationship between logic and reality is that his own peculiar platonism, in addition to being idealistic and relativistic, overpopulates reality beyond any manageable boundaries. For, in accordance to that view, there are Zermelian sets, Neumannian sets, Quinean (ML) sets, Quinean (NF) sets, fuzzy sets of sundry, unrelated kinds, and so on and so forth.

My third objection is that I feel sure that in logical systems and straightforward extensions thereof, like set-theories, the range of bindable variables is universal or unrestricted: in 'everything', 'thing' ranges over entities in general. This is why no logical system is ontologically neutral; and this is also why logic is useful, and, as I see it, talk of regional logics — wherein da Costa indulges — is either mistaken or misleading. Of course we might fail to know which logical principles are true; however, when we propound a system of logic, we are regarding its theorems as true, even if we either are not altogether sure they are so or are unable to substantiate our claim beyond reasonable doubt; but, regarding them as true, we think that they apply to all and every thing without exception. If I assert the principle of excluded middle, I'm saying, not that it applies to whatever it applies to, but that it applies to anything whatever.

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§4. — Negation, fuzziness, and paraconsistent logic

In accordance with his pluralistic view of logic, da Costa, who is a distinguished founder of paraconsistent logic, goes about canvassing paraconsistent logics and backing them up with philosophical arguments. (Regrettably, though, he says nothing about fuzzy contradictorial logics and set-theories.)

First, da Costa contends that there are many alternative negations, each of them being defined through the axioms of a logical system (cf. p. 33, p. 106). Classical negation is all right, and ought to keep its own role and position; indeed

da Costa goes much further in defending classical negation: as regards 'simple judgments of experience', classical negation is the only right negation, for on that lev el all classical principles governing negation hold.

Da Costa contends, though, that, as regards certain abstract objects of some mathematical theories, there are true contradictions, i.e. mutually contradictory truths. But he insists that there are no true contradictions within empirical reality. Da Costa buttresses that claim up by remarking (on p. 109 and again on p. 128) that the negations correlated with perception judgments are not perceptible themselves; hence we could set at some contradictory truth only through inference, not by means of empirical ascertainment. This is, da Costa claims, why all purported real contradictions are open to exception. I disagree. We perceive negative states of affairs as well as positive ones. I see that there is no book on the bed, or that my cat is not brown. Da Costa replies (on p. 128) that what I perceive is that my cat is black, e.g. whence I draw the conclusion that he is not brown. I remain unconvinced. Which are the 'positive' states of affairs that I perceive when I find out that there are no noises, or that no birds are now flying before my window? Of course, something or other can be contrived as an answer to any such question. But such answers would anyway be at least as controversial as my perceiving a so-called negative fact — i.e. a fact which is denoted by a negative statement. When I go about finding out whether there are birds flying before my window, and I see that there is none, I may have failed to notice which other things are now to be seen before my window. Let's consider, as another example, a sorites case: I see that the man yonder is not bald and I also see that he is not not bald. In that connection, it's worth taking into account what da Costa has to say about fuzzy systems: for him (see p. 130), fuzziness or vagueness is not an objective feature of the world, but arises from the relationships we enter into with the environment. That claim does once more betoken da Costa's nonrealistic slant. Now, the same thing might be contended about apparently nonfuzzy features of the world. Knowing a man's baldness, or a house's bigness involves neither more nor less subjective moulding elements than knowing the man's having exactly 1008 hairs or the house's being exactly 8341 mm high. Yet, what we usually report to have seen is a fuzzy fact. As regards everyday experience, fuzzy properties, rather than crisp ones, serve to characterize the objects of perception. Still, da Costa claims (on p. 130) that in everyday experience there's no vagueness, or else it can be dodged. I think he is right on the second point: fuzziness could be skirted by substituting crisp properties for fuzzy ones. However, the price would be high. Most, if not all, of our everyday experience reports would become meaningless. This argument is an epistemological one, but it has an ontological bearing, if we wonder why our ordinary experience reports are useful: either they reflect the world, and then there are fuzzy situations, or else they don't reflect it at all; supposing there to be no objective fuzziness, and no degrees of truth, it would be senseless to say that, by reporting that the house is ten metres high, I happen to be nearer the truth than by reporting it to be six metres high, supposing the house is nine metres high: the mere notion of approximation to the truth would become void and idle; for, if 'being ten metres high' is understood as 'being exactly ten metres high', as a crisp predicate

that anything either definitely and completely satisfies or else doesn't satisfy at all, then by making the former statement I am as far from the truth as by making the latter one. My second argument against dodging fuzzy reports is that, if we hold a realistic view of sets and faces, we are unavoidably faced with this question: what faces, and what sets, exist? Is tallness a set (or property)? Or there is no such property at all, there being, instead, only properties such as being-1814-mm-tall and the like? Nevertheless I agree that, were we to regard sets and faces as mere fictions — were we to make do with virtual sets and 'virtual faces' alone —, then choosing either fuzzy or crisp predicates would be a matter of convenience; but, if there are sets, or faces, or both, then even pragmatic choices of certain predicates instead of others are to be explained and retraced to their ontological grounds.

Surprisingly, though, da Costa almost admits, in a couple of passages, that there is objective fuzziness: he quotes (on p. 144) a well-known discussion by Rosser and Turquette, and goes on to contend that in some cases we seem unable to overcome indeterminacy, so as to say, e.g., of something either definitely that it is inside a room, or definitely that it is outside. In another passage (on p. 217), da Costa says that in a continuum of colours there is a fringe such that we cannot either say that it is red or say that it is not red; we certainly can reduce the fringe's span, but we are unable to eliminate it. So, da Costa seems after all to concede that there is objective vagueness even at the macroscopic lev el of everyday experience. Whence he concludes that in some cases the principle of excluded middle fails. I want to make several comments on those passages.

First, I don't find da Costa's argument in support of objective fuzziness cogent: such as cleave to crisp or classical approaches may retort that that pointed inability is ours only instead of its stemming from the nature of the world. (Admittedly, that reply wouldn't carry conviction, according to da Costa's lights, since he regards objects of knowledge, and even reality, as somehow constructed by human reason; cf. p. 215 and passim; still, what that shows is that da Costa's acceptance of objective or real fuzziness is to be taken cum grano salis: what, according to him, is fuzzy is not the thing itself, as it is independently or our knowing it, but the object of knowledge somehow produced by us.)

My second, more important, comment is that the existence of fuzzy situations does not necessarily entail failure of the principle of excluded middle; it doesn't entail that failure unless we add an additional premise: that only what is altogether (i.e. entirely, wholly, one hundred percent) true is true. Such premise is the principle of maximisation, and the claim that it's true is alethic maximalism. Da Costa seems to hold on to alethic maximalism, and this is why he says (e.g. on p. 111) that all many-valued logics give up excluded middle. But not all of them do. Such many-valued logics as designate sufficiently many truth-values keep the principles of excluded middle and noncontradiction, as well as abduction (i.e. Clavius'). Among such logics, incidentally, are several systems studied by da Costa himself together with I. d'Ottaviano, and with Kotas. Diametrically opposed to alethic maximalism is the endorsement rule which I've shored up with a number of

arguments in several papers, to wit: It's, to some extent or other, true that p ⊢ p. For, what is more or less true is true (not necessarily wholly true). In other words; whatever is not altogether false is true (just true; I'm not saying entirely true). Owing to the endorsement rule, we can have both fuzzy sets and excluded middle as well as noncontradiction. Therefore, fuzziness neither is nor entails indeterminacy. (Going back to my sorites example: I see that the man yonder is bald and I also see that he is not bald — that follows from my former report due to involutivity of weak negation —, since I see that to some extent he's bald and I see that to some extent he's not bald. Thus, fuzziness entails, not failure of excluded middle, but both excluded middle, as well as noncontradiction, and some true contradictions.)

My third comment is that, consequently, a satisfactory logic underlying a fuzzy-set theory must be paraconsistent, i.e. such that it allows the presence of both a sentence and its negation without that presence leading to triviality (this is da Costa's term) or deliquescence (as I'd like to put it), i.e. to every wff being a theorem of the resulting theory. Da Costa says (on p. 222) that fuzzy-set theory may become a field of application of paraconsistent logics. I agree. But why? Should fuzziness be equated with indeterminacy, we should need a logic without excluded middle and nothing else.

My fourth comment is that paraconsistency or even acceptance of true contradictions nowise compels us to reject the principle of noncontradiction. Rejecting a sentence is different from negating it. To negate a sentence is to assert the sentence's negation (more accurately: asserting another sentence which results from prefixing a negation functor to the first sentence), whereas to reject a sentence is to refuse to accept it. However, da Costa suggests (on p. 131 and passim) that, whenever a contradiction is admitted as true, the principle of noncontradiction is to be given away. It is not: if we countenance degrees of truth — and hence also of falseness —, we can admit sentences which are both true (to some extent or other) and yet also false (in some degree at least, however small): such instances of the principle of noncontradiction as have true negations are false, but also true. Thus, all contradictions are false, even if at the same time some contradictions are somewhat true. (Incidentally, the same holds as regards the principle of identity; instead of giving up the very notion of identity for elementary particles, as da Costa suggests, it would be far better to admit that some things are both identical to themselves and, at the same time and at least to some extent, not identical to themselves.)

My fifth comment is that, if we are to account for comparative constructions — as da Costa suggests on p. 163 — through a fuzzy logical system, then the obvious course is to regard any difference of degree as a difference in degree of truth. Accordingly, acceptance of fuzziness leads us to accepting degrees of truth (not of proximity to truth or any other makeshift).

My last comment is that, once we accept both degrees of truth and the endorsement rule, we can have both a fuzzy negation (like Łukasiewicz negation), which is a weak or natural negation, and a strong negation or overnegation (classical

negation), to be read as 'not at all' or 'nowise': all classical theorems remain valid for strong negation, and therefore we ought to reject any overcontradiction, i.e any formula of the form 'p and it's not at all the case that p'.

Moreover, I don't think da Costa goes far enough in recognizing the range of fuzzy predicates, which in my opinion covers not just most of our everyday experience, but — as has been revealed by recent research in the field of fuzzy systems and applications thereof — many scientific are as: evolution can be satisfactorily accounted for only within a fuzzy-set theory; fuzzy approaches are by now thriving in are as such as medicine or geography, let alone in social sciences. (As for philosophy, the present writer has, in a number of papers, outlined a system of fuzzy ontology: existence itself as well as many other metaphysical notions can fruitfully be regarded as fuzzy properties.) and much more is in the offing. Interestingly, all those researches open the door to applications of paraconsistent logic, of which da Costa has been one of the pioneers.

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Conclusion

I, of course, have not aimed, is this brief commentary, at discussing all interesting topics in da Costa's book. There remain lots of insightful considerations and developments to be gone into. Many enlightening comments are devoted to issues such as incompleteness theorems, formal and informal semantics, paradoxes, alternative logics, Quine's latter view of logic, Aristotle's defence of the principle of noncontradiction and Łukasiewicz's interpretation thereof, and so on. Moreover, I'm afraid my criticisms may have unjustly overshadowed this engaging book's outstanding merits; the reader will find a great many thought-inspiring suggestions and remarks. I warmly recommend all researchers interested in philosophy of logic to read it. The book deserves an English translation in order for it to reach a larger audience.

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