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Criticism to the Twin's Paradox

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Abstract The so-called "twin's paradox" is considered an important issue in special relativity theory because it implies a profound understanding of space time structure. And yet, since its original formulation in 1911 by Paul Langevin, numerous alleged explanations for this disturbing paradox have been produced; as it seems, unsuccessfully. This remains a subject for heated debate. Why? Because in all those explanations one tries to reconcile the irreconcilable, this is, what seems to be a logical conclusion (based on the phenomenon of time dilation) with what is simply unacceptable: how can it be a difference in aging from twins without breaking the fundamental equivalency between frames of coordinates? The purpose of this research is, first, to point out the basic flaws in the premises of the usual "explanations" and then to provide a consistent answer to the problem. It is proven here that there is no twin's paradox and this despite the reality of time dilation. Proceeding without prejudice, simply following appropriate premises and mathematical equations, one finally discovers an astoundingly, wonderfully coherent resolution to the problem, and this in the frame of special relativity itself. The key to understand and finally resolve this puzzling issue is relativistic asynchrony, particularly past and future permutation. Finally, the implications of this understanding, as can be easily induced, go far beyond special relativity. If there is no different aging in inertial frames, regardless of their relative velocity, should this conclusion also apply to accelerated ones, this is, to general relativity?

Keywords Special Relativity, Twin's Paradox, Time Dilation, Relativistic Asynchrony

1. Introduction

To this day, I have not found in the literature or anywhere else a real satisfactory answer to the so-called "**twin's paradox**". I think it is still worthy to meditate on this uncomfortable paradox because its solution would bring a more profound comprehension of spacetime behavior and of how one measures coordinates in it; this is, of Relativity itself.

The basis for the paradox lies in a *"peculiar consequence"* of Special Relativity (SR), stated more than one century ago by Albert Einstein himself in his founding paper on the subject [1]:

> *"Consider in the points A and B of a coordinate frame K two clocks at rest, supposing they work in synchrony to whomever observe them in the frame at rest. Let us imagine now that we communicate to the clock in A a movement with velocity v along the straight line that lies both points, in the sense of B. When this clock arrives at B synchrony no longer exists: compared to the one which remained in B, the clock that has been moved presents a delay of $1/2\, tv^2/c^2$ (up to quantities of the fourth and higher orders), if we represent by t the duration of the displacement. We see at once that this result still holds if the clock moves from A to B along any arbitrary polygonal line, and this even when the points A and B coincide."*

Einstein admits that *"the deduced result for a polygonal line holds also for a continuously curved line"*, even in the case of closed curves. All these conclusions are then expected to be verified in the modern particle accelerators; in fact, they appear to be.

Translated in a more popular version, formulated by Paul Langevin in 1911 [2], these statements gave rise to the “twin’s paradox”. In short: one of two twins remains on Earth while the other gets on a rocket, travels with a velocity near to *c* and gets back; since the theory consistently sustains that a moving clock runs slower than a ‘stationary’ one, the voyager returns younger than his twin, when they meet again. The well-known paradox comes from the fact that, also consistently, SR takes all inertial frames to be equivalent. So, from the point of view of the travelling twin it is the other who is moving and the discrepancy on ages should be reverted. This is, of course, an absurd.

The French philosopher Henri Bergson caught the essence of this nonsense in his bold criticism to Einstein’s SR theory, particularly in the twin’s issue. He *“argued that there would be no such effect* [time passing at different rates for twins] *because the Lorentz transformations performed on the two reference frames were reciprocal”* [3]. He was right, though in fact he missed the point.

Since Langevin first attempt, many thinkers sought a way to explain or ‘undo’ this paradox. *“Max von Laue (1911, 1913) elaborated on Langevin's explanation. Using Hermann Minkowski's spacetime formalism, Laue went on to demonstrate that the world lines of the inertially moving bodies maximize the proper time elapsed between two events. He also wrote that the asymmetric aging is completely accounted for by the fact that the astronaut twin travels in two separate frames, while the Earth twin remains in one frame, and the time of acceleration can be made arbitrarily small compared with the time of inertial motion. Eventually, Lord Halsbury and others removed any acceleration by introducing the "three-brother" approach”* [4]. None of these arguments are fully convincing, so one may find numerous posterior “explanations” to the paradox, based whether on symmetry break concerning the situation of the twins due to accelerations (G-forces, etc.), whether on Doppler effect or on other factors outside uniform movement.

Is Einstein’s *“peculiar consequence”* paradoxical? In [5], Jean-Pierre Luminet sustains the answer is “no”. He writes:

> *“In scientific usage, a paradox refers to results which are contradictory, i.e. logically impossible. But the twin’s paradox is not a logical contradiction, and neither Einstein nor Langevin considered such a result to be paradoxical. Einstein only called it “peculiar”, while Langevin explained the different aging rates as follows: “Only the traveler has undergone an acceleration that changed the direction of his velocity”. He showed that, of all the wordlines joining two events (in this example the spaceship’s departure and return to Earth), the one that is not accelerated takes the longest proper time.*
>
> *The twin’s paradox, also called Langevin effect, underlines a limitation of the principle of relativity: points of view are symmetrical only for inertial reference systems. (…) Since there is no symmetry, Special Relativity is not contradicted by the realization that the twin who left Earth is younger that his sibling at the time of their reunion.”*

But this, in fact, does not settle the question. Further, the “peculiar consequence” became a quite serious issue. The debate on it may not be peaceful and even ignite passions. For instance, in the 1980s, a trio of authors argued with acrimony about whether Einstein changed or not his mind about it. In [6], Mendel Sachs sustains that *“Einstein abandoned his earlier view that there are material consequences, such as asymmetric aging, implied in the space-time transformations of relativity theory.”* In [7], four years later, the authors present a *“reply to a misleading paper by M. Sachs entitled (…)”* where *“he tried to convince the reader that Einstein changed his mind (…). Also, Sachs insinuates that he presented several years ago “convincing mathematical arguments” proving that the theory of relativity does not predict asymmetrical aging in the TP.”* The vised author responds briefly to this criticism in [8].

Now, as we saw above, an “alternative” to the nuisance is to dismiss the paradox as a problem of SR. For instance, Øyvind Grøn puts it this way [9]:

> *“One may get rid of the twin’s paradox at once by noting that in order to be able to meet, depart and meet again, at least one of the twins must accelerate. And within the special theory of relativity the principle of relativity is not valid for accelerated motion. Acceleration is absolute. Hence, at least one of the twins is not allowed to consider himself at rest. The twin with the greatest average velocity between the events P1 and P2 is youngest when the twins meet at P2”.*

The author uses the twin’s paradox as: *“a pedagogical entrance to the general theory of relativity. And the above resolution of the twin’s paradox is contrary to the spirit of to the general theory of relativity”*, he writes. This appears to be a bizarre assumption.

The author of [10] review, for his part, qualifies the paradox as a “naive interpretation”, affirming that:

> *“The really strange thing about time dilation is that it is symmetrical: if you and I have relative motion, then I see your clock to be running (with respect to our fames), and you see mine to be running slow. This is just one example of the weird logic of Einstein's theory of Special Relativity. (...) The naive interpretation – the reason why the situation is called a paradox – is to assume that the situation is completely symmetrical.”*

Quite recently, Gerrit Coddens, the nonconformist or disillusioned author of [11], reflects on the *“protocol which defines the journey (…) with respect to a given* *frame"*, stating that: *"It is this selection of a special reference frame which introduces the asymmetry. Hence, the reference frame wherein we define the protocol for the journey will act like an absolute frame and it is this unavoidable introduction which breaks symmetry between the twins."* In section 4.2, *"The true solution of the paradox"*, the author calls into question human *"physical intuition"* and finally concludes that *"there is thus not just one twin's paradox, but an uncountable infinity of them."* However, it seems to me that he does not really solve the problem, instead complicates it a lot.

Finally, one may find on YouTube several videos on the subject, such as these two, which justifiably restrict the analysis to SR: one essentially skeptical [12], understanding the importance of the "turn around" point, the other presenting a solution by a scientist quite confident in himself [13]. There are some fine analyses in YouTube videos, like these ones; but, along each explanation, something becomes obscure and unsatisfying. So, there is no explanation at all!

To resume, answers based on acceleration and deceleration are tempting ones, but incorrect because the problem directly arises from SR inertial frames; so, we should not look for an explanation outside these frames. Even if we do so, a supposed 'resolution' quickly shows itself as inconsistent. Besides, Special Relativity is, in my understanding, the most fundamental theory about space and time structure, energy, etc. It is one of the best verified theories in physics [5]; it is compatible with quantum theory, in Dirac's form – remark that, in his founding article, Einstein was careful to point out the coherence of his theory with Planck's equation: *"It is noteworthy that the energy and the frequency of a light complex vary with the observer state of motion according to the same law."* [1] §8 – and it allows inclusively to deal with accelerated frames and even tachyonic ones, according to the PtR theory I presented some years ago, which introduced the reality of negative time flux [14]. So, it is in SR theory that we must find an answer, but not "on demand". And there we find it, indeed!

It is quite reasonable to assume that there cannot exist paradoxes in Nature; in this case, that there cannot be a different aging from twins. But then, how is it, despite the reality of time dilation? The real answer to the paradox is to be found in the profound understanding of break of simultaneity and of spacetime behavior, this is, of our ways to measure phenomena in it. In the end, we discover that Special Relativity is yet a wonderful box full of surprises (as well as Nature, which the theory translates) and that our difficulties arise from approaches based, after all, on 'common sense'.

2. The Basics: Time Dilation and Length Contraction

Consider an observer **A** in a point $\mathbf{O_0}$, the origin of a supposed immobile coordinates frame S. Another observer, $\mathbf{A'}$, in the origin of an inertial frame S', is moving with velocity v along the x-axis, in the positive sense. According to Lorentz transformations,

$$\begin{cases} t = \gamma(t' + x'v/c^2) \\ x = \gamma(x' + vt') \\ y = y' \\ z = z', \end{cases} \quad \text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{1}$$

At the instant $t_0 = t'_0 = 0$ a light beam is emitted in the sense of the y-axis in both frames, S and S'; it is reflected in a mirror placed at a distance l above (event 1) and gets back to the observers (event 2). Naturally in both frames we have a similar own experience, which may be described by the events in S', z' being always null:

$$\begin{cases} t'_1 = l/c \\ x'_1 = 0 \\ y'_1 = l \end{cases} \quad \text{and} \quad \begin{cases} t'_2 = 2t'_1 = 2l/c \\ x'_2 = 0 \\ y'_2 = 0 \end{cases} \tag{2}$$

Now, seen from S, these coordinates become

$$\begin{cases} t_1 = \frac{t'_1}{\sqrt{1-\beta^2}} \\ x_1 = vt_1 \\ y_1 = y'_1 = l \end{cases} \quad \text{and} \quad \begin{cases} t_2 = \frac{t'_2}{\sqrt{1-\beta^2}} = 2t_1 \\ x_2 = vt_2 = 2x_1 \\ y_2 = y'_2 = 0. \end{cases} \tag{3}$$

The first equations on both sets express the *time dilation* phenomenon, from a moving frame S' towards the 'immobile' frame S – this is, concerning a moving clock towards an 'immobile' one:

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}} \tag{4}$$

This, along with *length contraction*,

$$\Delta x = \Delta x'\sqrt{1-\beta^2}, \tag{5}$$

can be easily deduced directly from the table (1). Both are fundamental and unattackable results of Special Relativity.

This "thought experiment" corresponds exactly to sending a light beam with an angle φ, given by $\sin\varphi = \sqrt{1-\beta^2}$ (Figure 1); the beam is reflected in a mirror at a distance l placed on the perpendicular to the x-axis from the point $\mathbf{O_1}$ with coordinate $x_1 = vt_1$, and then returned to a point $\mathbf{O_2}$ also in this axis, with coordinate $x_2 = 2x_1 = vt_2$ [see Figure 1]. It results that, in fact, the light beam takes a longer time to reach the point $\mathbf{O_2}$ than it does in the perpendicular way (equal to $t'_2 = 2l/c$): exactly $t_2 = t'_2/\sqrt{1-\beta^2}$.

On the other hand, the "thought experiment" concerning a beam of light emitted in the sense of the x-axis clearly shows the length contraction phenomenon, but it also displays another crucial feature of Special Relativity: the *break of simultaneity*.

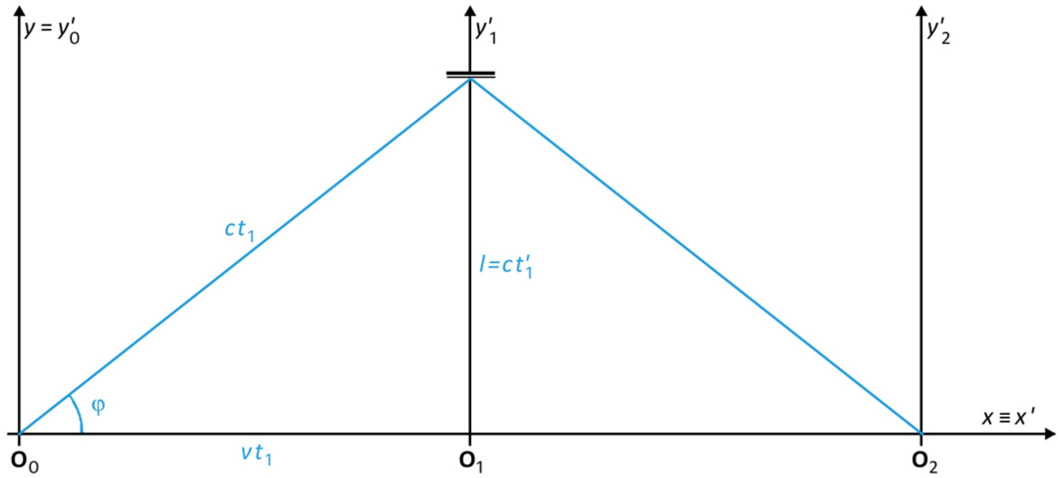

Figure 1. The experience of the light beam emitted in the perpendicular to a moving y'-axis.

3. The Twin's Paradox

3.1. The One-Way Voyage

We must get rid of rockets, accelerations and the idea of high velocities, because this has nothing to do with the reality of time dilation and Einstein's "peculiar consequence". Instead, we will extensively use the reflection experiment to measure time, this is, as a clock. But, instead of thinking the problem based on the symmetry of the situation for the movement of $\mathbf{A}'$ in S and of $\mathbf{A}$ in S', we will analyse it otherwise. Take a third observer, $\mathbf{B}$, immobile at the point $\mathbf{O_2}$, and, for our purposes, consider that they are all twins. Consider also [following the usual line of thought] that $\mathbf{B}$ sends a light beam at the same moment the observer $\mathbf{A}'$ does it; the mirror above $\mathbf{B}$ is again at the coordinate $y = l = l'$. The beam reaches the mirror, is reflected there and reaches the point $\mathbf{O_2}$ at the instant

$$t_{B2} = \frac{2l}{c} = \frac{2l'}{c} = t'_2,$$

that is, *before* the arrival of $\mathbf{A}'$ and of his own reflected beam, which occurs simultaneously at time $t_2 = t'_2/\sqrt{1-\beta^2}$. This is, of course, an obvious result.

But consider now the experiment from the point of view of $\mathbf{A}'$: the observer $\mathbf{B}$ is moving along the x-axis with velocity $-v$. Time t_{2B} is dilated to

$$t'_{B2} = \frac{t_{B2}}{\sqrt{1-\beta^2}} = \frac{t'_2}{\sqrt{1-\beta^2}}$$

and this means that — symmetrically, which is logical — $\mathbf{A}'$ receives his own beam *before* meeting $\mathbf{B}$! So, the laps of time between each observer receiving his own reflected beam and the other observer's doing so is opposite for each frame. This appears to be an insolvable contradiction and is equivalent to the traditional twin's paradox.

If we strongly believe in the consistency of Nature and of SR theory, we must search for an error somewhere in the premises. And, in fact, there is one, at least: it lies in the premise *"at the same moment (…)"*. The error comes from not taking the *break of simultaneity* into account. We must keep in mind that Lorentz transformations either for x coordinate, either for t coordinate, are functions of both the correspondent coordinates in the other frame. This is quite easy to understand in the case of the length coordinate (it is equivalent to the Galilean transformation) but harder in what comes to the time coordinate: though in the frame S the measure of time t_P does not depend on the position of a given point $\mathbf{P}$ in space, the correspondent time t'_P depends on t_P and on x_P; we must understand that, for instance, the aforementioned coordinates t_2 and t'_2 are the coordinates of the observer $\mathbf{A}'$ in each frame, S and S' respectively:

$$\begin{cases} t_2 = t_{A'2} \\ x_2 = x_{A'2} = vt_2 \end{cases} \text{and} \quad \begin{matrix} t'_2 = t'_{A'2} \\ \text{and} \quad x'_{2A'} = 0 \end{matrix} \Rightarrow t'_{2A'} = t_2\sqrt{1-\beta^2}.$$

This is quite different from the coordinates in S' of $\mathbf{A}$ at the instant t_2:

$$\begin{cases} t'_{A2} = \dfrac{t_2}{\sqrt{1-\beta^2}} \\ x'_{A2} = -\dfrac{x_2}{\sqrt{1-\beta^2}}. \end{cases}$$

We will return to this issue in a while. For now, using suitable notations, we will write the inverse Lorentz transformation, for *the same time coordinate in* S, $t = t_B = t_A$, as:

$$\begin{cases} t'_A = \frac{t}{\sqrt{1-\beta^2}} \\ x'_A = -\frac{vt}{\sqrt{1-\beta^2}} \end{cases} \text{and} \quad \begin{cases} t'_B = \frac{t - x_B v/c^2}{\sqrt{1-\beta^2}} = t'_A - \frac{x_B v/c^2}{\sqrt{1-\beta^2}} \\ x'_B = \frac{x_B - vt}{\sqrt{1-\beta^2}} = x'_A + \frac{x_B}{\sqrt{1-\beta^2}}. \end{cases} \tag{6}$$

This clearly shows that the synchronous clocks in S, 'owned' by $\mathbf{A}$ and $\mathbf{B}$, appear as asynchronous in the frame S' of the observer $\mathbf{A}'$.

Now, if we look for the coordinates in S' of the observer **B** corresponding to the instant $t'_0 = 0$ where *the light beam is emitted by the observer* $\mathbf{A}'$, we get from the transformation table above, considering that $x_B = x_2 = vt_2$ for every instant t,

$$\begin{cases} t'_{B0} = -\frac{\beta^2 t_2}{\sqrt{1-\beta^2}} \\ x'_{B0} = \frac{x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{7}$$

Remark that, at this moment, for $\mathbf{A}'$ *the observer* $\boldsymbol{B}$ *is in the past.* For the instant t_2, when $\mathbf{A}'$ receives its beam and meets **B**, we get

$$\begin{cases} t'_{B2} = t_2 \frac{1-\beta^2}{\sqrt{1-\beta^2}} = t_2\sqrt{1-\beta^2} = t'_2 \\ x'_{B2} = \frac{x_2 - vt_2}{\sqrt{1-\beta^2}} = 0. \end{cases} \tag{8}$$

The first result represents the time dilation of t_2; but, above all, it means that, from the point of view of $\mathbf{A}'$, when he reaches **B**, *both have aged exactly the same*: t'_2.

So, as *transcribed in* $\boldsymbol{B}$ *coordinates*, the difference between final and initial length and time coordinates for the experience "$\mathbf{A}'$ sends and receives a reflected light beam" yields:

$$\begin{cases} \Delta t'_{B2} = t'_{B2} - t'_{0B} = t_2\left(\frac{1-\beta^2}{\sqrt{1-\beta^2}} + \frac{\beta^2}{\sqrt{1-\beta^2}}\right) = \frac{t_2}{\sqrt{1-\beta^2}} \\ \Delta x'_{B2} = -x'_{B0} = -\frac{x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{9}$$

Both differences express a dilation (in particular, $\Delta t'_{2B} > t_2$); but in the end, coherently, their quotient gives the velocity of **B** relatively to $\mathbf{A}'$:

$$v'_B = \frac{\Delta x'_{B2}}{\Delta t'_{B2}} = -\frac{x_2}{t_2} = -v.$$

Further,

$$\begin{cases} \Delta t'_{B2} = \Delta t'_{A2} = t'_{A2} \\ \Delta x'_{B2} = \Delta x'_{A2} = x'_{A2}, \end{cases}$$

and this makes all sense. The mutual dilation may seem strange and contradict Relativity. But it does not. First, it is to be expected regarding time; second, concerning $\Delta x'$, it comes from the fact that the phenomenon of length contraction requires the measure of $\Delta x'$ to be taken at the same instant t'; this does not happen here (where $\Delta x'$ simply expresses a difference of x' coordinates).

Also remark that all the precedent reasoning may be applied as well to the movement of $\mathbf{A}'$ relatively to **B**, for this one emitting the light beam; it is enough to consider the frame S centred in $\mathbf{O_2}$ and the x-axis oriented in the opposite sense.

Finally, from the strict point of view of the observer $\mathbf{A}'$, the path between **A** and **B** may be seen as a rigid rule, immobile in the frame S, with length $\Delta x = x_2$. Therefore, it appears in S' with a contracted length $|\Delta x'| = x_2\sqrt{1-\beta^2}$, which $\mathbf{A}'$ covers in the same time $\Delta t' = t'_2 = t_2\sqrt{1-\beta^2}$ it takes the light to go and return from the mirror above him; so, also coherently, we get for the movement of **B** in S' the same velocity

$$v'_B = -\frac{|\Delta x'|}{\Delta t'} = -\frac{\Delta x}{\Delta t} = -v.$$

3.2. The Simultaneous Reception

If we wish to evaluate the relative aging of the twins, we may do it in an alternative way at the very moment they meet, because only then both clocks are in mutual proximity. In the frame S, the observer $\mathbf{A}'$ and his reflected light beam meet **B** in time $t_2 = t'_2/\sqrt{1-\beta^2}$; on the other hand, the beam experience made by **B** takes a time t'_2. So, for both light beams (the one sent by $\mathbf{A}'$ and the one sent by **B**) to arrive at the same time to the point $\mathbf{O_2}$, **B** must send his own beam at the instant

$$t_3 = t_2 - t'_2 = t_2\left(1 - \sqrt{1-\beta^2}\right).$$

In the frame S', the inverse transformation gives (making $x_3 = x_2$):

$$\begin{cases} t'_3 = \dfrac{t_3 - \frac{vx_3}{c^2}}{\sqrt{1-\beta^2}} = \dfrac{t_2\left(1-\sqrt{1-\beta^2}\right) - \frac{vx_2}{c^2}}{\sqrt{1-\beta^2}} = \dfrac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1-\beta^2}} - t_2 = t'_2 - t_2 \\ x'_3 = \dfrac{x_3 - vt_3}{\sqrt{1-\beta^2}} = \dfrac{x_2 - vt_2 + vt_2\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}} = \dfrac{x_2 - vt_2}{\sqrt{1-\beta^2}} + vt_2 = x'_2 + vt_2. \end{cases}$$

But, since $x_2 = vt_2$, it results $x'_2 = 0$ and, therefore,

$$\begin{cases} t'_3 = -t_3 = t'_2 - t_2 \\ x'_3 = vt_2 = x_2. \end{cases}$$

Remark that a *negative time* appears once again, precisely the symmetrical of t_3. This is the key to understand and resolve the problem. It means that the emission of the beam by **B**, which in the frame S is *posterior* to the emission of the beam by $\mathbf{A}'$, is in this one's proper frame S' *anterior* to it. So, one concludes that, in S', **B** is *younger* when he sends his light beam than $\mathbf{A}'$ is when he sends his own beam. Further: when the moving observer **B** reaches $\mathbf{A}'$ and both receive their reflected beams, a time laps t'_2 has passed in S'. Therefore, in this frame, a total time laps $\Delta t'$ exist *since the emission of the beam by* **B**, which is given by

$$\Delta t' = t'_2 - t'_3 = t_2,$$

this being exactly the time required in S for the beam emitted by $\mathbf{A}'$ to return to him and to reach **B**! Remember that this describes how $\mathbf{A}'$ observes, in his proper frame S', the experience "emission and reception of a light beam by his twin **B**". So, in a way, this result was to be expected: it is the symmetrical process and corresponds to time dilation; but it becomes fully understandable by the fact that in the frame S' the observer **B** emits his beam *before* his twin $\mathbf{A}'$ does it.

The point here is that this reasoning is interchangeable between frames, simply permuting $\mathbf{A}'$ and **B**: *for each twin, the other sends his light beam before he does it!* This is a quite amazing conclusion but is the basis for the understanding of *how* Nature resolves the imbroglio. The same delay compensates the longer time laps $\Delta t' = t_2$ for the moving twin's experience compared with the time laps t'_2 for the immobile one's experience, in sort that, at the end, **both twins have aged exactly the same**: their proper time laps t'_2.

We see then that there is indeed no real conflict between frames evaluations. The mutual time dilation (t_2) is a real relativistic phenomenon but does not affect the *effective aging* of $\mathbf{A'}$ and $\mathbf{B}$.

3.3. The Round Trip

This seems to be true (and to settle the question) concerning $\mathbf{A'}$ and $\mathbf{B}$. But what about $\mathbf{A}$?

Well, in the frame S, the age of the twins $\mathbf{A}$ and $\mathbf{B}$ is supposed to be always the same. So, a reasonable hypothesis is that, when reaching the starting point, after reverting his movement in $\mathbf{O_2}$, $\mathbf{A'}$ and $\mathbf{A}$ should meet with the same age. But how is this possible? The answer is to be found in a subtle and intriguing interference that comes out in the *turning point* $\mathbf{O_2}$.

Let us re-examine the issue from the point of view of the observer $\mathbf{A'}$. As we have seen, when he reaches $\mathbf{O_2}$, the coordinates of $\mathbf{A}$ are

$$\begin{cases} t'_{A2} = \frac{t_2}{\sqrt{1-\beta^2}} \\ x'_{A2} = -\frac{x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{10}$$

Take a close look at these formulae. One must remind that they concern *the beam experience made by* $\mathbf{A'}$, not by his twin $\mathbf{A}$ (where the coordinate t_2 would be equal to t'_2 and t'_{2A} to t_2). What the formulae say is essentially that $\mathbf{A}$ is *in the future* for $\mathbf{A'}$ when this one, after a time laps t'_2, reaches $\mathbf{B}$; the temporal difference is given by

$$\Delta t' = t'_{A2} - t'_2 = \frac{\beta^2}{1-\beta^2} t'_2 \ ;$$

[for instance, for $\beta = 0.5$, this gives $\Delta t' = 1/3\, t'$]. In this future moment, $\mathbf{A}$ is at a distance $|x'_{A2}|$ behind $\mathbf{A'}$. This is compatible with $x'_{A2}/t'_{A2} = -v$ but is strange! However, stranger things are yet to come.

From a theoretical point of view, when $\mathbf{A'}$ instantly reverts his movement, it is advisable to consider a new immobile frame $\boldsymbol{S}$, the former S reset and centered in $\mathbf{O_2}$. Then, $\mathbf{A'}$ and his also new frame $\boldsymbol{S'}$ will move in the opposite direction of the x-axis, with velocity $-v$, $\boldsymbol{S'}$ being coincident with $\boldsymbol{S}$ at the instant $\mathbf{t'} = \mathbf{t} = 0$. In these circumstances, naturally, Lorentz transformations apply by simply reversing the sign of v.

As we did before, for $\mathbf{B}$, we will write

$$\begin{cases} \mathbf{t'}_A = \frac{\mathbf{t} + \mathbf{x}_A v/c^2}{\sqrt{1-\beta^2}} = \frac{\mathbf{t} - x_2 v/c^2}{\sqrt{1-\beta^2}} \\ \mathbf{x'}_A = \frac{\mathbf{x}_A + v\mathbf{t}}{\sqrt{1-\beta^2}} = \frac{v\mathbf{t} - x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{11}$$

So, we will have (almost) similar equations to (7) and (8) for $\mathbf{t'} = \mathbf{t} = 0$ and for the final $\mathbf{t}_4 = t_2$:

$$\begin{cases} \mathbf{t'}_{A0} = -\frac{\beta^2 t_2}{\sqrt{1-\beta^2}} \\ \mathbf{x'}_{A0} = -\frac{x_2}{\sqrt{1-\beta^2}} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{t'}_{A4} = t_2\sqrt{1-\beta^2} = t'_2 \\ \mathbf{x'}_{A4} = 0. \end{cases} \tag{12}$$

In each case, we are interested in the **summative time and length** coordinates, from the beginning (the departure of $\mathbf{A'}$ from $\mathbf{O_0}$), symbolized by the bold capital letters $\mathbf{T'}_A$ and $\mathbf{X'}_A$. To obtain them we must add to $\mathbf{t'}_A$ and $\mathbf{x'}_A$ the respective new initial coordinates given by (10). The result is:

$$\begin{cases} \mathbf{T'}_{A0} = t_2\sqrt{1-\beta^2} = t'_2 \\ \mathbf{X'}_{A0} = -\frac{2x_2}{\sqrt{1-\beta^2}} \end{cases} \tag{13}$$

and

$$\begin{cases} \mathbf{T'}_{A4} = t_2\left(\sqrt{1-\beta^2} + \frac{1}{\sqrt{1-\beta^2}}\right) \\ \mathbf{X'}_{A4} = -\frac{x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{14}$$

First, we see from $\mathbf{T'}_{A0} = t'_2$, that, for $\mathbf{A'}$ simply reversing its course, at that moment the advance in time of $\mathbf{A}$ is overturned: both 'find' themselves with the same age. This is the most astounding phenomenon!

Then, at the end of the double light beam experience – this is, the round trip –, $\mathbf{A'}$ meets $\mathbf{A}$ after a time lapse $\mathbf{T'}_{A4}$, which is equal for both. Remark that, in fact, the equation includes both contraction and dilation positive components and that all the reasoning leads to the same result if it is applied to the movement of $\mathbf{A'}$ in relation to $\mathbf{A}$. So, **there is no longer a contradiction regarding the measure of time in the two reference frames; both twins have aged the same**: $\mathbf{T}_4 = \mathbf{T}_{A'4} = \mathbf{T'}_{A4}$.

One may also express $\mathbf{T}_4$ as

$$\mathbf{T}_4 = \frac{2-\beta^2}{\sqrt{1-\beta^2}}\ t_2 \quad \text{or} \quad \mathbf{T}_4 = \frac{2-\beta^2}{1-\beta^2}\ t'_2.$$

If $\beta = 0$, this conduces to $t_2 = t'_2$ and the consistent result $\mathbf{T}_4 = 2\ t'_2$. For $\beta > 0$, $\mathbf{T}_4 > 2\ t'_2$ (for instance, making $\beta = 0.5$, it comes $\mathbf{T}_4 = 7/3\, t'_2$). In fact, $\lim \mathrm{T}_4 = \infty$ for $\beta \to 1$. Besides, the difference between the summative time for moving and immobile frames is given by

$$\mathbf{T}_4 - 2\ t'_2 = \frac{\beta^2}{1-\beta^2}\ t'_2 = \frac{\beta^2\ t_2}{\sqrt{1-\beta^2}} = -\mathbf{t'}_{A0}\,,$$

this is, in modulus, the non-summative time coordinate of $\mathbf{A}$ relatively to $\mathbf{A'}$ at the turning point. One should say that this increment in time is provided by the reversion of the movement of $\mathbf{A'}$. In a way, it is what is needed *to bring* $\boldsymbol{A}$ *to the present* of $\mathbf{A'}$.

To end this paper in beauty, one just needs to deduce the summative coordinates for the observer $\mathbf{B}$. His initial coordinates are those expressed in (8):

$$\begin{cases} t'_{B2} = t_2 \dfrac{1-\beta^2}{\sqrt{1-\beta^2}} = t_2\sqrt{1-\beta^2} \\ x'_{B2} = \dfrac{x_2 - vt_2}{\sqrt{1-\beta^2}} = 0. \end{cases}$$

Regarding the new frames $\boldsymbol{S}$ and $\boldsymbol{S}'$, since $\mathbf{x}_B = 0$ at every moment, one gets:

$$\begin{cases} \mathbf{t}'_B = \frac{\mathbf{t}}{\sqrt{1-\beta^2}} \\ \mathbf{x}'_B = \frac{v\mathbf{t}}{\sqrt{1-\beta^2}}; \end{cases} \tag{15}$$

and this gives, for the turning point,

$$\begin{cases} \mathbf{t}'_{B0} = 0 \\ \mathbf{x}'_{B0} = 0\,, \end{cases}$$

which is quite natural, meaning that $\mathbf{B}$ is not affected by the reversion of the movement of $\mathbf{A}'$; and also

$$\begin{cases} \mathbf{t}'_{B4} = \dfrac{t_2}{\sqrt{1-\beta^2}} \\ \mathbf{x}'_{B4} = \dfrac{x_2}{\sqrt{1-\beta^2}}. \end{cases}$$

Therefore, one obtains the following summative coordinates:

$$\begin{cases} \mathbf{T}'_{B0} = t'_2 \\ \mathbf{X}'_{B0} = 0, \end{cases} \tag{16}$$

and

$$\begin{cases} \mathbf{T}'_{B4} = t_2\left(\sqrt{1-\beta^2} + \frac{1}{\sqrt{1-\beta^2}}\right) \\ \mathbf{X}'_{B4} = \frac{x_2}{\sqrt{1-\beta^2}}. \end{cases} \tag{17}$$

This finally means that $\mathbf{B}$ ages exactly the same from his twins $\mathbf{A}'$ and $\mathbf{A}$, when these ones meet again, which is $\mathbf{T}_4$, thus confirming our hypothesis.

4. Conclusions

Nature creates the effect of temporal dilation; but it also provides ways to overcome it, under certain circumstances, avoiding contradictions and ensuring that physical events are independent of coordinate frames. The main element in this 'magic' is synchronicity, including the fact that the distinction between past and future is not an objective feature: it depends on coordinate frames.

Long ago, Einstein demonstrated, in his seminal paper on Relativity, that synchronicity is broken from one coordinate frame to another. This is a crucial fact. Once we deeply understand it, which is quite difficult because it often goes against the common sense that – for cause – still rules our mind, we begin to be able to find solutions.

The final conclusion here is that, despite the reality of time dilation, it is not true that the "traveling twin" returns younger than the one who "stayed at home": they meet again at exactly the same age.

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