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# Cross Dock Scheduling: Classification, Literature Review and Research Agenda

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## **Abstract**

At cross docking terminals incoming deliveries of inbound trucks are unloaded, sorted, moved across the dock and finally loaded onto outbound trucks, which immediately leave the terminal towards their next destination in the distribution chain. Accordingly, a cross dock is a consolidation point in a distribution network, where multiple smaller shipments can be merged to full truck loads in order to realize economies in transportation. In this context, the truck scheduling problem, which decides on the succession of truck processing at the dock doors, is especially important to ensure a rapid turnover and on-time deliveries. Due to its high real-world significance, several truck scheduling procedures have been introduced within the recent years, which all treat specific cross dock settings. In order to structure and promote scientific progress, the paper on hand introduces a classification of deterministic truck scheduling. With the help of this classification, existing literature is reviewed and future research needs are identified. Moreover, we represent a yet unexplored class of truck scheduling problems which is highly relevant in real-world distribution networks.

*Keywords:* Logistics; Cross Docking; Scheduling; Classification

## **1 Introduction**

A cross docking terminal is an intermediate node in a distribution network which is exclusively dedicated to the transshipment of truck loads. In contrast to traditional warehouses, a cross dock carries no or at least a considerably reduced amount of stock. Whenever an incoming truck arrives at the yard of a cross dock, it is assigned to a dock door, where inbound loads are unloaded and scanned to determine their intended destinations. The loads are then sorted, moved across the dock and loaded onto outbound

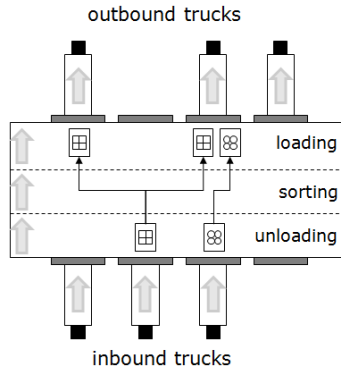


Figure 1: Schematic representation of a cross docking terminal

trucks for an immediate delivery elsewhere in the distribution system. Figure 1 gives a schematic representation of a cross docking terminal.

The primary purpose of a cross dock is to enable a consolidation of differently sized shipments with the same destination to full truck loads, so that economies in transportation costs can be realized (Apte and Viswanathan, 2000). This advantage makes cross docking an important logistics strategy receiving increased attention in today's globalized competition with its ever increasing volume of transported goods. Success stories on cross docking which resulted to considerable competitive advantages are reported for several industries with high proportions of distribution cost such as retail chains (Wal Mart; Stalk et al., 1992), mailing companies (UPS; Forger, 1995), automobile manufacturers (Toyota; Witt, 1998) and less-than-truckload logistics providers (Gue, 1999).

In contrast to traditional point-to-point deliveries, an additional transshipment of goods at the cross docking terminal slows down the distribution process and generates a significant amount of double handling. Consequently, efficient transshipment processes are required where inbound and outbound truckloads are synchronized, so that intermediate storage inside the terminal is kept low and on-time deliveries are ensured.

For this purpose, several scheduling procedures have been introduced within the recent years, which aim at solving the so called truck scheduling problem. This problem decides on the succession of inbound and outbound trucks at a given set of dock doors of the terminal. On the basis of the truck schedule, each inbound and outbound truck arriving at the yard is assigned to a specific dock door where shipments are processed. Obviously, this elementary problem consecutively arises during the daily cross dock operations and has vital influence on a rapid transshipment processes.

Different organizational and technical implementations lead to a large variety of possible truck scheduling problems in real-world settings. As cross docking is a comparatively new logistics strategy, there is not yet a massive body of academic literature on this subject. In fact, dedicated research on the short-term truck scheduling problem was published no earlier than 2005 (see McWilliams et al., 2005). Due to the immense practical importance, there has been a considerable amount of follow-up research in the meantime, however, and we strongly assume that this trend continues in the future. On

the one hand, this shows that we are in a formidable position to structure the field in an early stage of exploration, so that future research can be more easily coordinated. On the other hand, this means that the classification scheme needs to be easily adoptable and extendable, so that problem settings which have not yet been discussed can be readily considered. We therefore base our classification of deterministic truck scheduling problems on the very successful and widely accepted tuple notation for machine scheduling (Graham et al., 1976), provide a concise review on existing solution procedures and use the insights to identify important fields of interest for future research.

The remainder of the paper is organized as follows. Section 2 defines the scope of this review by characterizing the truck scheduling problem and establishing the relationship to interdependent decision problems. In Section 3 the truck scheduling classification is presented which is employed to review existing optimization models and solution procedures in Section 4. In Sections 5 and 6, future research needs are specified. In particular, a yet unexplored class of truck scheduling problems is introduced, which is highly relevant in real-world cross docking applications, before Section 7 concludes the paper.

## 2 Scope of review

In general, scheduling problems deal with the allocation of resources over time to perform a set of tasks being part of some process (e.g., Blazewicz et al., 2007, p. 1). In the special case of truck scheduling, the process of transshipment can be subdivided into the tasks unloading inbound trucks and loading outbound trucks, which are typically separated by a time lag for material handling inside the terminal, i.e., for scanning, sorting and moving shipments across the dock. These two tasks are to be processed by the resources “dock doors”, which can process one truck at a time and are assumed to be sufficiently equipped with loading equipment (e.g., hand stackers or fork lifts) and workers. Typically, truck scheduling uses a time related objective function in order to evaluate a given solution. As with other (operational) scheduling problems, cost consequences of task processing, e.g., delayed deliveries influencing customer satisfaction, are hard to quantify accurately, so that a time related surrogate objective often turns out to be the better (operational) choice. To conclude, a dispatcher of a cross docking terminal who seeks to solve a truck scheduling problem faces two interrelated decisions bound to some (time related) objective function: *where* and *when* the trucks should be processed at the dock doors of the terminal.

This (positive) definition of truck scheduling is now amended with a (negative) demarcation from related (and possibly interdependent) decision problems. The decision problems to be solved during the life cycle of a cross docking terminal ordered from strategic to operational are as follows:

- (i) Location of cross docking terminal(s)
- (ii) Layout of the terminal
- (iii) Assignment of destinations to dock doors

- (iv) Vehicle routing
- (v) Truck Scheduling
- (vi) Resource scheduling inside the terminal
- (vii) (Un-)Packing loads into (from) trucks

The strategic problem (i) of locating a single cross dock (or some other kind of intermediate warehouse) or a complete distribution network consisting of multiple cross docks is vividly discussed in scientific literature. A good starting point into location theory investigating intermediate nodes in a network of sites are the reviews on the hub location problem provided, e.g., by Campbell (1994) or Klose and Drexel (2005). In relation to the location problem, truck scheduling merely considers an isolated terminal with a given location.

The layout problem (ii) of a cross dock is investigated by Bartholdi and Gue (2004). Here, the number of dock doors and the shape of the terminal building (e.g., I, T or X-shaped) are to be determined. For truck scheduling, we presuppose a given terminal, so that the number of dock doors and their placement along the perimeter of the terminal are known. Consequently, the distance between any pair of doors is given, so that the time lag for material handling (at least the load-independent part) between any pair of doors can be anticipated accurately.

Typically, the truck scheduling problem presupposes that the assignment decision of trucks (and the destinations they serve) to dock doors is part of the short-term problem, so that each door may serve multiple destinations in varying succession per day depending on the actual truck schedule. However, the assignment of destinations to dock doors (problem (iii)) can also be executed on a mid-term horizon, with the result of each dock door exclusively serving a specific inbound or outbound destination for a longer period of time (e.g., a month). On the one hand, such a fixed assignment eases the allocation of shipments to trucks, since workers can “learn” the topology of the terminal and respective information systems become superfluous. On the other hand, a fixed assignment of doors to destinations restricts the degrees of freedom for short-term truck scheduling, because peak loads for single destinations cannot be absorbed by additional dock doors. Consequently, such a fixed assignment seems especially suited for steady commodity flows with a reliable distribution among inbound and outbound destinations. Tsui and Chang (1990, 1992) were the first to tackle the mid-term problem of assigning doors to destinations. On the basis of a representative distribution of shipments among related sites they solve the problem as a quadratic assignment problem, which minimizes the shipment flows between doors. Other contributions for this problem stem from Gue (1999), Bartholdi and Gue (2000), Bermudez and Cole (2001), Oh et al. (2006) as well as Bozer and Carlo (2008). If the decision of assigning doors to destinations is solved at an early (mid-term) stage, the short-term truck scheduling problem reduces to sequencing all trucks of equal destination at the respective dock door. However, in either case there remains a short-term truck scheduling problem. Typically, we assume that inbound and

outbound destinations are not previously fixed, so that door assignment is part of the truck scheduling problem.

Obviously, truck scheduling is also closely related to inbound and outbound vehicle routing (problem (iv)). On the inbound side, the vehicle routing schedule establishes the arrival times of trucks at the cross dock (see Lim et al., 2005; Chen et al., 2006; Lee et al., 2006). The estimated times can be directly taken up as inbound truck specific arrival times in truck scheduling. On the outbound side, succeeding vehicle routings possibly set boundaries on the earliest and latest departure time of outbound trucks. In spite of all interdependencies between (inbound and outbound) vehicle routing and truck scheduling it seems not meaningful to plan both decision problems simultaneously. Diverging time frames and the resulting complexity of a monolithic optimization model question such a simultaneous approach. Moreover, a cross dock is often operated by a third party logistics provider serving multiple forwarding companies, so that a centralized planning approach might be impossible.

For a given truck schedule, resource scheduling inside a terminal (problem (vi)), i.e., scanning, sorting and moving shipments across the dock, is a complex scheduling problem in itself, since multiple resources need to be coordinated. Li et al. (2004) as well as Álvarez-Pérez et al. (2008) model these tasks as a machine scheduling problem and present different meta-heuristics for its solution. Truck scheduling is heavily interdependent with this problem, because the actual time lag between each inbound and outbound task is the result of a detailed resource scheduling. Consequently, both planning tasks could be solved in a simultaneous manner. However, existing research abstains from such an advancement, because this would require to schedule each worker in detail, which would in turn necessitate a respective information system and limits workers' flexibility to react on unforeseen events. Thus, average handling times, e.g., determined from historical data, should capture this relation with sufficient preciseness, so that in the subsequent discussion, we assume given fixed time lags between inbound and outbound tasks, which only depend on the pair of doors between which the shipment is moved.

Finally, the packing of shipments inside trailers (problem (vii)) also influences task times for truck processing and handling times inside the dock. However, it seems not meaningful to interrelate packing decisions with truck scheduling, because the packing of inbound trucks is usually not known at the cross dock prior to opening the respective trailer. Furthermore, integrating packing aspects at the outbound side would also require to integrate vehicle routing, which determines the sequence of customer visits and, thus, the needed arrangement of shipments inside a trailer. This would, however, result in a very complex centralized planning approach. Consequently, we assume that the influence of packing times is negligible and already included in the transportation time lag.

Along with the (positive) definition of truck scheduling this (negative) separation from related decision problems defines truck scheduling and, thus, the scope of our review. The next section presents a classification which characterizes this scheduling task.

### 3 Classification

Classifications of complex and versatile optimization problems proved very successful to concisely identify and describe a specific optimization problem, so that the coordination of scientific efforts is eased considerably. The most successful and widely accepted classification schemes basing on a so-called tuple notation are dedicated to machine scheduling (Graham et al., 1979) and queueing systems (Kendall, 1953). Other tuple notations which successfully helped structuring complex research fields are, e.g., provided by Brucker et al. (1999) for project scheduling, Dyckhoff (1990) for cutting and packing, Boysen et al. (2006) and Boysen et al. (2009) for assembly line balancing and sequencing, respectively. In a tuple notation respective objectives and operational characteristics are referenced by a symbolic notation, so that in spite of the multitude of possible properties of a planning task, a particular model can be briefly described by a tuple.

Cross dock scheduling is closely related to traditional machine scheduling. Whenever possible we therefore take over the attributes of the Graham-notation. However, cross-dock scheduling bears some peculiarities, which cannot be directly denominated with the classical machine scheduling notation. For instance, in a cross docking terminal incoming shipments arriving on inbound trucks might deliver multiple product units, which are not preassigned to a specific truck but may satisfy the demand of multiple outbound trucks for the respective product, so that an assignment of product units to outbound trucks might be an additional decision task. The classical Graham-notation has no counterpart for these additional elements of cross docking. Thus, conventional machine scheduling attributes are to be augmented by special cross docking attributes which in combination form the truck scheduling classification. The classification scheme is structured as follows:

- Any truck scheduling problem will at least consist of three basic elements: door (processor) environment, operational characteristics and an objective to be followed. Accordingly, the presented classification will be based on those three elements which are noted as tuple  $[\alpha|\beta|\gamma]$ , where:

$\alpha$      door environment  
 $\beta$      operational characteristics  
 $\gamma$      objectives

- One major advantage of the tuple-notation is that any default value, represented by the symbol  $\circ$ , can be skipped when a tuple is actually specified. In the following notation, the symbol  $*$  always indicates that for the respective attribute the alternative values (except for  $\circ$ ) do not exclude each other and can be combined arbitrarily. As all attribute values are chosen such that they are unique, it is not necessary to specify the attribute designators within the tuples.

#### 3.1 Door environment

The jobs to be executed during cross docking are inbound (unloading) and outbound (loading) operations for which processors are required. These processors are the doors

of the dock. The door environment  $\alpha$  of a cross docking terminal can be represented by the two attributes  $\alpha_1$  and  $\alpha_2$ :

**Service mode**  $\alpha_1 \in \{E, M, EM, G\}$ : The service mode of a cross docking terminal influences the degrees of freedom in assigning inbound and outbound trucks to dock doors.

$\alpha_1 = E$  Each dock door is either exclusively dedicated to inbound or outbound operations. Such an *exclusive* mode of service is a widely spread guideline in real-world terminals. Typically, to ease product flows and supervision one side of the terminal is dedicated to inbound and the other to outbound operations.

$\alpha_1 = M$  On the other hand, also an intermixed sequence of inbound and outbound trucks to be processed per dock door can be allowed, because technical restrictions for a separation of inbound and outbound trucks do not exist. We label this service mode as the *mixed* mode.

$\alpha_1 = EM$  Additionally, both service modes can be applied in parallel, which means that a subset of doors is operated in exclusive service mode and the other in a mixed mode of service.

$\alpha_1 = G$  Finally, the assignment of trucks to dock doors can also be solved in a mid-term horizon, so that fixed assignments between doors and destinations exist (see Section 2). In this case, the door assignment of each truck is *given* by each trucks' destination. Consequently, truck scheduling reduces to a sequencing problem of a given truck set at each door.

Note that the case  $\alpha_1 = E$  is closely related to a flow shop system, where inbound and outbound doors build the first and second production stage, respectively. And further note that the case  $\alpha_1 = M$  is related to a processor environment with identical parallel processors. However, as was mentioned before the special cross dock setting that products arriving on the inbound side might be variably split among multiple outbound trucks cannot be directly covered by the Graham-notation. Thus, we prefer to highlight these peculiarities with novel attributes.

**Number of dock doors**  $\alpha_2 \in \{\circ, k\}$ : Typically, a cross dock consists of multiple dock doors. Gue (1999) reports on a terminal containing more than 500 doors, whereas the typical number ranges between 40 and 150. Consequently, it might be valuable to further specify the number of dock doors.

$\alpha_2 = \circ$  In the real-world, the number of dock doors varies from terminals to terminal. In the default case the number of doors may differ, too, so that algorithms dedicated to this case can solve truck scheduling problems having a facultative number of dock doors.



$\alpha_2 = k$  On the other hand, the number of dock doors can be restricted to a given number  $k$ , where  $k$  can be any positive integer. Especially valuable, i.e., for bound computation and complexity issues, might be the minimum number of dock doors. This minimum number amounts to  $\alpha_2 = 1$  and  $\alpha_2 = 2$  depending on whether a mixed mode of service ( $\alpha_1 = M$ ) or an exclusive ( $\alpha_1 = E$ ) one is employed, respectively.

### 3.2 Operational characteristics

The operational characteristics influencing the structure of truck scheduling can be classified by attribute set  $\beta$ , which contains 9 different attributes ( $\beta_1$  to  $\beta_9$ ).

**Preemption**  $\beta_1 \in \{\circ, pmtn\}$ : Preemption in the context of cross docking means that loading or unloading a truck is interrupted, the half-full trailer is removed from the dock, and replaced by another one. Later on, the unfinished trailer has to revisit the terminal to be finally processed.

$\beta_1 = \circ$  No preemption is allowed, so that a once docked trailer is completely processed.

$\beta_1 = pmtn$  Preemption of truck processing is allowed.

**Arrival times**  $\beta_2 \in \{\circ, r_j\}$ : Trucks are either already waiting on the yard and, thus, readily available to be called up or arrive after the start of the schedule, so that they may only be processed after their truck specific arrival time.

$\beta_2 = \circ$  All arrival times are zero.

$\beta_2 = r_j$  Arrival times differ per truck.

**Processing time**  $\beta_3 \in \{\circ, p_j = p, \underline{p} \leq p_j \leq \bar{p}\}$ : The service time (or processing time)  $p_j$  of a truck  $j$  comprises the whole time span to (un-)load its products. Note that we (and existing research) only deal with deterministic scheduling. Thus, we assume certainty about product loads arriving at the terminal, so that service times can be estimated upfront and are given parameters of a truck scheduling model. Furthermore, we also prescind from predetermined service intervals like they are proposed by Miao et al. (2007). Such an assumption seems somehow artificial, because it requires a rejection of trucks (lost shipments) whose given service windows can not be assured even if the violation amounts to just a few seconds. A real world justification for such strict time windows is not apparent. However, our classification can be easily extended at this point to account for both peculiarities. Hence, we distinguish processing time in analogy to machine scheduling as follows:

$\beta_3 = \circ$  Service times may vary from truck to truck, so that arbitrary processing times exist.

$\beta_3 = (p_j = p)$  All trucks have an processing times equal to  $p$ . The assumption of equidistant service slots can be seen as a reasonable approximation of reality, whenever vehicle capacities as well as the number and nature of products per vehicle do not strongly differ (see Boysen et al., 2007). As trailers are typically of a standardized size and cross docking aims at moving only full truck loads, this premise is fulfilled whenever all processed products are of comparable size (e.g., mail distribution centers) or all truck loads resemble a representative average truck load (e.g., rotational deliveries of special promotional offers to all stores of a retail chain).

$\beta_3 = (\underline{p} \leq p_j \leq \bar{p})$  No processing time  $p_j$  is less than  $\underline{p}$  or greater than  $\bar{p}$ .

**Deadlines**  $\beta_4 \in \{\circ, \tilde{d}_\lambda\}$ : Deadlines might restrict the departure time of trucks and shipments.

$\beta_4 = \circ$  No deadlines are assumed in the system (as a hard constraints). However, due dates may be defined as soft constraints which are taken up in the objective function.

$\beta_4 = \tilde{d}_\lambda$  Deadlines are defined which are to be met by trucks or shipments, where subscript  $\lambda \in \{i, o, j, s\}$  specifies for which element of truck scheduling due dates exist:

$\lambda = i$ : Only inbound trucks are bound to due dates, which means that these trucks need to be unloaded on-time to meet a later assignment.

$\lambda = o$ : Deadlines for the departure of outbound trucks need to be regarded to meet due dates negotiated with the customers.

$\lambda = j$ : Both inbound and outbound trucks might be bound by deadlines.

$\lambda = s$ : Finally, each single shipment might have a specific due date negotiated with the respective final recipient.

**Intermediate storage**  $\beta_5 \in \{\circ\lambda, \text{no-wait}\}$ : Although cross docking aims at minimizing inventory, at least intermediate storage inside a terminal is often inevitable.

$\beta_5 = \circ$  Typically, products are stored in front of the door the respective outbound truck is (to be) docked and, thus, remain in *intermediate stock* until loaded. The storage space inside the terminal might turn out as a bottleneck, which is covered by  $\lambda \in \{\circ, \text{limit}\}$ :

$\lambda = \circ$ : Unlimited storage space exists inside the terminal.

$\lambda = \text{limit}$ : Available stock space is *limited* by a given capacity.

$\beta_5 = \text{no-wait}$  Some products must not be intermediately stored at all, so that the *no-wait* property must hold. This is a common constraint for refrigerated products, e.g., frozen food, pharmaceuticals or flours, for which a defrost

threatens inside the uncooled terminal. Instead, these products must be instantaneously loaded on a cooled outbound truck once they are unloaded (see Boysen, 2007). Furthermore, it can also be an organizational guideline to instantaneously load products on their respective outbound trucks to avoid congestions inside a terminal. Note that the no-wait property corresponds to an intermediate buffer of zero capacity.

**Assignment restrictions**  $\beta_6 \in \{\circ, \text{doors}\}$ : Assignment restrictions confine the degrees of freedom in assigning trucks to doors.

$\beta_6 = \circ$  If *no* restrictions are to be considered any dock door is a possible choice for truck processing.

$\beta_6 = \text{doors}$  Some trucks might only be processed at a subset of *doors*, which fulfill specific requirements, e.g., a bus bar to cool freezer trailers or a wider dock for loading large products crosswise. Note that an exclusive mode of service ( $\alpha_1 = E$ ) is not considered as such an assignment restriction.

**Transshipment time**  $\beta_7 \in \{t_{io}, \circ, t_j = 0\}$ : We define the time lag between the arrival of shipments inside the terminal after having unloaded them from their respective inbound truck until their availability at an outbound door as the *transshipment time*. In the real-world, such a transshipment time depends on multiple factors, i.e., the disposability of resources (e.g., workers and fork lifts) and congestions inside the terminal. However, to reduce complexity of truck scheduling the transshipment time is approximated as being a given constant (see Section 2), which might differ as follows:

$\beta_7 = t_{io}$  In real-world terminals, distances to be covered by material handling and, thus, transshipment times  $t$  depend on the dock doors between which a shipment is moved. Typically, it takes considerably more transshipment time to move items between far distant doors than between neighboring ones. To model this relationship, an individual transshipment time for each pair of dock doors is to be considered as input data and the realized transshipment time for any shipment depends on the door assignment of the respective inbound truck  $i$  and outbound truck  $o$ .

$\beta_7 = \circ$  To reduce complexity of truck scheduling, it might be reasonable to assume transshipment times being a unique constant independent of the door assignment of trucks. In this case, a *single constant greater zero* covers the unique time lag for material handling of any shipment. This simplification is better suited for small terminals with only a few doors, where the differences in transshipment times caused by diverging distances are negligible compared to the service times of trucks.

$\beta_7 = (t_j = 0)$  The ultimate simplification of transshipment times reduces the constant to zero for each truck  $j$ . This might be a suited simplification to ease the

extraction of structural properties in mathematical models. In real-world terminals, this assumption can, for instance, be justified in the food industry, where only very small docks are utilized and products must be instantaneously stored in outbound trucks once they are unloaded. Here, short transshipment times are inevitable to ensure a continuous cooling chain (see Boysen, 2007).

**Outbound organization**  $\beta_8 \in \{\circ, \text{fix}\}$ : This tuple entry defines the organizational guidelines which decide on the points in time at which outbound trucks leave the terminal. The following two possibilities exist:

$\beta_8 = \circ$  An outbound truck leaves the terminal as soon as its predefined set of products is loaded.

$\beta_8 = \text{fix}$  An outbound truck departs at a predefined point in time, irrespective of the products loaded. Especially postal services depend on reliable departures in their multi-stage distribution networks and therefore often apply fixed schedules.

**Interchangeable products**  $\beta_9 \in \{\circ, \text{change}\}$ : The interchangeability of products mainly depends on whether or not value adding services (e.g., repacking) are fulfilled at a terminal.

$\beta_9 = \circ$  Any product arriving at the terminal is dedicated to a specific outbound truck. This might result from products being indeed individual, e.g., pre-commissioned shipments in retail industries, or is an organizational policy to ease allocation of products for the workers at the terminal.

$\beta_9 = \text{change}$  On the other hand, merely the number and types of products to be loaded per outbound truck might be defined, so that product units of a respective type can satisfy any outbound truck's demand for this product. Consequently, the assignment of product units to outbound trucks becomes part of the decision problem. Such a policy allows for a more flexible reaction on unforeseen events, like erring truck loads, and seems especially promising if a reduced number of standardized products is to be transshipped. On the other hand, a repacking of products inside the terminal is required, which slows down the transshipment process.

Note that these attributes might need to be extended if further organizational implementations become relevant. For instance, split deliveries might be allowed, so that a single shipment comprising several products can be divided among multiple outbound trucks which serve the same destination. Furthermore, the capacity of outbound trucks might be relevant. However, as these additional attributes have not been covered by truck scheduling research thus far, we abstain from including them to keep the classification as concise as possible.

### 3.3 Objectives

Finally, the optimization will be guided by some objective which evaluates solutions. In truck scheduling, the traditional machine scheduling objectives (see, e.g., Blazewicz et al., 2007), such as minimization of makespan or tardiness are also reasonable. However, in some cases an additional distinction can be made as to which element of a truck scheduling problem is subject to these objectives, i.e., shipments  $s$  or outbound trucks  $o$ . Furthermore, in the case of multi-objective optimization more than a single objective can be selected out of the following set:

$$\gamma \in \{\sum(w_\lambda)C_\lambda, C_{max}, L_{max}, \sum(w_\lambda)T_\lambda, \sum(w_\lambda)U_\lambda, \sum(w_p)S_p, S_{max}, -, \star\}^*.$$

$\gamma = \sum(w_\lambda)C_\lambda$  The completion time  $C_\lambda$  is the time an outbound truck  $o$  or a shipment  $s$  is finally processed and ready to leave the cross dock. Thus, to accelerate the turnover of goods and to reduce the probability of late shipments minimizing the sum of completion times ( $\gamma = \sum C_\lambda$ ) might be a reasonable objective. If shipments  $s$  (or the shipments contained in an outbound truck  $o$ ) are of diverging value also the weighted sum of completion times ( $\gamma = \sum w_\lambda C_\lambda$ ) can be considered.

$\gamma = C_{max}$  The schedule length or makespan is reached at the point in time the last shipment is finally loaded. Because shipments leave the terminal on outbound trucks, the makespan does not depend on the distinction between outbound trucks  $o$  and shipments  $s$ , so that  $C_{max} = \max_{o \in O}\{C_o\} = \max_{s \in S}\{C_s\}$ . This objective is especially suited to rapidly empty out the terminal, so that following trucks of adjacent planning runs can be processed.

$\gamma = L_{max}$  The maximum lateness can be minimized for each shipment  $s$  or outbound truck  $o$ . In the case of shipments as the reference point, maximum lateness is calculated as follows:  $L_{max} = \max_{s \in S}\{C_s - d_s\}$ , where  $d_s$  is the deadline of shipment  $s$ . This objective is especially suited whenever small delays are acceptable, e.g., lost time can be regained on the road, and only bigger delays notably derogate customer satisfaction.

$\gamma = \sum(w_\lambda)T_\lambda$  If already smaller delays influence customer satisfaction, it might be better to minimize the (weighted) tardiness, which can also be assigned to shipments and outbound trucks. For instance, each outbound trucks' tardiness amounts to:  $T_o = \max\{C_o - d_o; 0\} \forall o \in O$ .

$\gamma = \sum(w_\lambda)U_\lambda$  Furthermore, each delay can be harmful irrespective of its magnitude. In this case, the (weighted) number of tardy truck or shipments  $\sum_\lambda(w_\lambda)U_\lambda$  can be minimized, where  $U_\lambda = 1$ , if  $C_\lambda > d_\lambda$ .

$\gamma = \sum(w_p)S_p$  The cross docking concept relies on a rapid turnover of shipments. Thus, the minimization of stocked products  $p$  inside the terminal over the planning horizon possibly weighted (according to, e.g., size or value) with a product specific factor can be a valuable objective. At least in tendency, also the

danger of delayed shipments is reduced because inventory of once delivered products can only be decreased by loading them on outbound trucks to leave the terminal as early as possible. Moreover, a reduced stock size also minds congestions of vehicles for material handling inside the terminal.

- $\gamma = S_{max}$  Furthermore, it might be reasonable to minimize the maximum inventory level during the planning horizon, e.g., to not surmount available stock space or to reduce extraordinary congestions.
- $\gamma = -$  No objective function is applied whenever testing for feasibility, i.e., to meet deadlines, is considered.
- $\gamma = \star$  Some other (surrogate) objective is considered not specified in our classification.

## 4 Literature Review

In the following, we review existing truck scheduling research (in chronological order) on the basis of our classification scheme.

The first contribution to short-term truck scheduling stems from McWilliams et al. (2005). They investigate a real-world terminal of a postal service provider where delivered parcels are forwarded to outbound trucks by a system of interconnected conveyor belts. They tackle the resulting problem case  $[E|t_{io}|C_{max}]$  with a genetic algorithm, which is coupled with a simulation model (simulation based optimization). The simulation model is applied to evaluate the congestions on the conveyor belt system (and their impact on the makespan) caused by different inbound schedules.

Another real-world setting from the food industry is presented by Boysen (2007). The peculiarity of frozen foods and other refrigerated products, e.g., pharmaceuticals or flowers, is that the cooling chain must be intact. Consequently, a shipment once unloaded must instantaneously be stored in its respective cooled outbound trailer. No intermediate storage inside the uncooled terminal is allowed, so that the no-wait property ( $\beta_5 = \text{no-wait}$ ) must hold. For the case  $[E|p_j = p, \text{no-wait}, t_j = 0|\sum T_o]$  Boysen (2007) presents an exact Dynamic Programming approach extended with lower and upper bounds (so-called Bounded Dynamic Programming) and a heuristic simulated annealing approach.

A more stylized model with only a single inbound and a single outbound door operated in an exclusive mode of service ( $\alpha = E2$ ) is investigated by Boysen et al. (2007). For such a terminal setting, they aim at minimizing the makespan of the schedule ( $\gamma = C_{max}$ ) with the peculiarity of products being interchangeable between outbound trucks ( $\beta_9 = \text{change}$ ), so that the following constellation is considered:  $[E2|p_j = p, \text{change}|C_{max}]$ . Boysen et al. (2007) decompose the overall problem into an inbound and an outbound problem, which are proven to have identical structure. Iteratively, they solve inbound and outbound problems with different algorithms. This procedure is tested in a comprehensive computational study.

Chen and Lee (2007) extend the traditional two-machine flow-shop problem by precedence constraints between inbound tasks at the first stage and outbound tasks at the second stage. An outbound truck receives multiple dedicated shipments from diverging inbound trucks so that respective precedence constraints between inbound and outbound tasks enforce that an outbound truck on the second stage can not be processed before all its predecessor tasks have been completed on the first stage. Although, the resulting cross docking case with only one inbound and one outbound door is a stylized problem not directly applicable in the real-world, it is a good starting point for analyzing the structure of cross docking problems. Especially, the NP-hardness proof for the very basic case  $[E2|t_j = 0, p_j = p|C_{max}]$  is very useful since many real-world problems are generalizations of this problem. Furthermore, they present a branch and bound procedure for the case  $[E2|t_j = 0|C_{max}]$ , which is able to solve problems with up to 60 trucks to optimality.

The development of a decision support system called “LoadDock” for real-world cross docking in the less-than-truckload industry is documented by Chmielewski (2007). Such a decision support tool has to cover multiple real-world settings, so that many attributes of our classification are treated by Chmielewski:  $[EM|r_j, \tilde{d}_j, \text{limit}, \text{doors}, t_{io}|\star]$ . The solution approach bases on a network flow formulation where inbound and outbound doors along with different storage areas inside the terminal are defined as nodes with given capacity (deduced from processing speed per period). A heuristic solution approach for the extended network flow model basing on Column-Generation is presented and tested with real-world data.

Miao et al. (2007) investigate the following cross dock setting:  $[M|\text{limit}, t_{io}|\star]$ , which has important characteristics often relevant in the real-world, i.e., multiple doors ( $\alpha_2 = \circ$ ) operated in a mixed service mode ( $\alpha_1 = M$ ), a limited storage space inside the terminal ( $\beta_5 = \text{limit}$ ) and a transshipment time depending on the door assignment of the respective inbound and outbound trucks of a shipment ( $\beta_7 = t_{io}$ ). They also presuppose that each truck has a predefined and fixed service window during which a dock door is fully occupied. Whenever no door can be found to be reserved for the complete time span the truck cannot be processed at all and, thus, becomes a lost shipment. Consequently, one term of their objective function is to minimize penalty cost for lost shipments. An additional term covers operational costs which are mainly influenced by the (door assignment dependent  $\beta_7 = t_{io}$ ) distances to be covered by material handling devices. However, the model of Miao et al. (2007) suffers from the fact that trucks are counted as lost shipments even if their service window is only violated for a few seconds. Such a strict service window without the slightest variability seems hard to imagine in a real-world cross dock setting. The resulting optimization model is solved by a tabu search approach and a genetic algorithm. Especially, the tabu search approach shows very efficient in a computational study with differently sized test instances when compared with standard solver CPLEX.

Another stylized model with only a single door operated in a mixed service mode ( $\alpha = M1$ ) is considered by Boysen (2008). After proving NP-hardness for the investigated case:  $[M1|p_j = p, t_j = 0, \text{change}|\sum S_p]$  an exact Dynamic Programming approach and a Beam Search heuristic are presented and tested.

Yu and Egbelu (2008) treat the case  $[E2|change|C_{max}]$ , which also is a very elementary problem when products delivered by inbound trucks may serve product demands of multiple outbound trucks ( $\beta_9 = change$ ). However, with merely a single inbound and a single outbound door it is only a stylized model to investigate the structure of related (more complex) cross docking problems. For the solution of their problem Yu and Egbelu (2008) introduce a priority rule based heuristic. This heuristic is evaluated against a complete enumeration of all inbound and outbound sequences with test instances up to 12 trucks (6 inbound and 6 outbound) and 9 products.

Chen and Song (2009) extend the work of Chen and Lee (2007) by considering multiple parallel processors (multiple doors) per (inbound and outbound) stage. For the case  $[E|t_j = 0|C_{max}]$  heuristic procedures which are adoptions of the famous Johnson-rule for two machine flow-shop scheduling are presented and tested.

Table 1 summarizes the literature review, where the contributions of each paper are stated with the help of the following notation:

M	mathematical model	B	bound computation
HI	heuristic improvement procedure	HS	start heuristic for initial solution
HM	meta-heuristic	E	exact solution procedure
S	simulation approach	P	properties (e.g., complexity) of problem

Additionally, complexity of the problems is reported. If no complexity proof is provided by the authors, the label “open” highlights that the complexity status of the problem is unknown.

Publication	Notation	Complexity	Contribution
McWilliams et al. (2005)	$[E t_{io} C_{max}]$	open	HM, S
Boysen (2007)	$[E p_j = p, no-wait, t_j = 0 \sum T_o]$	open	M, HM, E
Boysen et al. (2007)	$[E2 p_j = p, change C_{max}]$	NP-hard	M, HS, HI, E, P
Chen and Lee (2007)	$[E2 t_j = 0 C_{max}]$	NP-hard	B, E, P
Chen and Lee (2007)	$[E2 t_j = 0, p_j = p C_{max}]$	NP-hard	P
Chmielewski (2007)	$[EM r_j, \tilde{d}_j, limit, doors, t_{io} \star]$	open	M, E
Miao et al. (2007)	$[M limit, t_{io} \star]$	NP-hard	M, HM, P
Boysen (2008)	$[M1 p_j = p, t_j = 0, change \sum S_p]$	NP-hard	M, HM, E, P
Yu and Egbelu (2008)	$[E2 change C_{max}]$	open	M, HS
Chen and Song (2009)	$[E t_j = 0 C_{max}]$	NP-hard	M, HS, B
this paper	$[E t_{io}, fix \sum w_s U_s]$	NP-hard	M, P
this paper	$[E t_i = 0, fix \sum w_s U_s]$	NP-hard	P

Table 1: Overview on truck scheduling research

## 5 Future research

Open research can be divided into three categories: (i) the unexplored cases of our classification, (ii) research needs in relation to interdependent planning problems, and (iii) implementation of real-world truck scheduling.



## 5.1 The unexplored cases

With the help of our classification scheme yet unexplored cases of truck scheduling can be easily identified. Up to now, any study on truck scheduling assumes the same outbound organization: An outbound trucks leaves the terminal not before all preassigned shipments are loaded. However, there exists another kind of outbound organization, which relies on fixed outbound schedules, i.e., an outbound truck serving a respective destination leaves the terminal at a given point in time. This alternative form of outbound organization ( $\beta_8 = fix$ ) has not been covered by truck scheduling research. To stimulate research on this important truck scheduling setting Section 6 further elaborates on fixed outbound schedules.

Another attribute not yet considered is preemption ( $\beta_1 = pmtn$ ). Although preemption is not a common policy in real-world truck processing it should be a fruitful field of research. As already simplified truck scheduling problems, e.g., with only one or two doors, turned out to be NP-hard preemption might be the additional characteristic, which allows the resulting problems to be solved in polynomial time. Then, these solution approaches could be applied, e.g., for bound computation, in more complex and realistic cases.

Only very few research papers consider a transshipment time, which depends on the door assignment of inbound and outbound trucks ( $\beta_7 = t_{io}$ ). This is somewhat astounding as this circumstance should be relevant in nearly any real-world cross dock. It takes much more time to transship products between far distant doors than between neighboring ones. Future research should further investigate the impact of this typical characteristic.

Finally, most truck scheduling problems considered in literature aim at minimizing the makespan ( $\gamma = C_{max}$ ). Research on other objectives should be intensified, as well. For this purpose, the classification scheme might be helpful, to systematically handle yet unexplored truck scheduling problems.

## 5.2 Research needs in relation to interdependent planning problems

The most important research question within this category refers to the problem whether or not destinations should be fixed over a mid-term horizon. Alternatively, the assignment of trucks to doors can also remain part of the truck scheduling problem. To answer this question the trade-off between a clear arrangement of material handling inside the terminal and the degrees of freedom for truck scheduling needs to be observed. On the one hand a durable assignment of destinations eases shipment allocation to doors for the workforce. On the other hand, it complicates finding good truck schedules, because peak loads for single destinations can not be absorbed by additional doors. It would be a valuable contribution if the disadvantage of a durable assignment with regard to efficient truck schedules could be evaluated with different data settings. This way, decision support could be retrieved under which real-world circumstances a durable assignment between doors and destinations is less disadvantageous for truck scheduling than in others.

Even if the assignment of doors to destinations is executed on a mid-term horizon there remains a truck scheduling problem, which is to sequence the trucks of a specific destinations at their respective door. For some cases of our classification the problem decomposes into single door problems. However, for other problems the relation of inbound trucks with regard to outbound departures hinders a decomposition. It would be a valuable contribution to investigate the structure of remaining truck scheduling problems. This would answer the question whether these problems are indeed easier (e.g., with regard to complexity) to solve compared to their counterpart truck scheduling problems including the door assignment of trucks.

Furthermore, the relationship to other planning problem should be investigated. In relation to material handling inside the terminal the following research question seems especially relevant: How to anticipate congestions of material handling devices resulting from different truck schedules? In relation to vehicle routing, the sensitivity of truck scheduling with regard to diverging arrival times of inbound trucks and departure times of outbound trucks seems worthwhile to investigate. A considerable sensitivity would be a hint that a simultaneous truck scheduling and vehicle routing could be a promising planning approach.

### 5.3 Implementation of real-world truck scheduling

Finally, implementing truck scheduling in real-world cross docks seems an especially challenging field. The most straightforward implementation would be do consider all trucks (i.e., all trucks already waiting on the yard plus all those which presumably arrive during the planing horizon) in a unique planning run. Then, the resulting schedule could fix truck processing over the complete planning horizon. However, arrival times of trucks are typically bound to heavy inaccuracies, because traffic congestions or engine failures delay inbound trucks with the utmost probability. Thus, the following research questions need to be answered in this context: Up to which “level of uncertainty” are expected arrival times of trucks useful information to be considered in truck scheduling? How to derive robust plans, i.e., plans which remain feasible in spite of (shorter) delays?

To further attenuate the impact of uncertain arrivals, truck scheduling is often applied in a rolling horizon setting. Scientific advice on how to dimension the planning horizon and how to link adjacent planning runs is still missing. In the extreme case, truck scheduling is executed once a door is released to merely determine the truck to be called-up taking over the empty door (see Boysen, 2007). Such an online procedure has the advantage, that uncertain truck arrivals become irrelevant because only trucks already waiting on the yard would need to be considered. In this context the question whether a complete planning run each time a door is released is actually better than a (simple) selection rule needs to be investigated.

Testing all planning scenarios (static planning vs. rolling horizon vs. online selection of the next truck) with real-world data seems a fruitful research task, as advice on the suitability of those alternatives under specific real-world circumstances could be gained.

Furthermore, organizational policies should be challenged. For instance, an exclusive mode of service ( $\alpha_1 = E$ ), which is often applied in many real-world terminals, eases

material handling inside the terminal. Nevertheless, a mixed mode of service ( $\alpha_1 = M$ ) leaves more degrees of freedom for truck scheduling and, thus, promises better plans. Quantifying the advantage of a mixed mode of service within different truck scheduling instances could provide valid information for the practitioner to reasonably decide on this organizational guideline. Another policy to question is the widespread exclusion of preemption ( $\beta_1 = pmtn$ ).

Finally, cross docks in different branches of industry should be investigated. Especially, material handling considerably deviates between branches. On the one hand, in retail or less-than-truckload industries material handling is mostly a manual task supported by fork lifts or pallet jacks (Gue, 1999). On the other hand, in postal services material handling is automated by conveyor belt systems (McWilliams et al., 2005). In automobile industry, even highly automated robots, which sort material into the sequence they are required at the final assembly (Just-in-Sequence), can be found. These and further peculiarities of different branches applying cross docking could be an important step towards learning the needs of different branches. At least, different cost structures (e.g., with regard to the products shipped and the penalty of delays) could be considered, so that the choice of an appropriate objective function for different cost structures would be enabled. This way existing research, which mainly deals with formulating and solving isolated truck scheduling models, could be enhanced to serviceable decision support in real-world cross docking.

## 6 Cross docking with fixed outbound schedules

As the literature review revealed, present research on truck scheduling is restricted to a single kind of outbound organization. Up to now, all studies presuppose that any outbound truck leaves the terminal not before all dedicated products or shipments are loaded ( $\beta_8 = \circ$ ). However, such an outbound organization is only possible if all shipments to arrive are actually known in advance at the respective cross docking terminal. Moreover, from an economic point of view it seems especially suited if few shipments of high value are transported. Consequently, this outbound pattern is, for instance, applied in cross docks of automobile industry, where large transport boxes of Just-in-Time materials are transshipped. Here, an early departure of an outbound truck ahead of an only slightly delayed inbound truck would jeopardize on time deliveries of Just-in-Time materials at the final assembly with the hazard of material stock outs and, thus, line stoppages.

Nevertheless there are several other industries where a multitude of smaller and low valued shipments are transported. In such a setting, delays of shipments are still undesired and, thus, to be reduced to a minimum; but they are by far not as harmful. Especially in larger hub-and-spoke networks where a multi-stage cross docking is applied, a reliable and steady flow of trucks seems much more essential. Consequently, especially postal services and less-than-truck load service providers typically rely on fixed outbound schedules ( $\beta_8 = \text{fix}$ ). Consequently, trucks are supposed to leave a terminal exactly at a predefined point in time over a fixed route to a specific destination (Chmielewski, 2007). All shipments for the respective destination which arrive before the truck's departure

are loaded and, thus, shipped the same day. Any other shipment is delayed up to the next day when the next truck serves the destination. In such a setting, truck scheduling should aim at minimizing the (weighted) number of shipments delayed up to the next day ( $\gamma = \sum (w_s)U_s$ ). Unfortunately, present research has not yet considered this highly relevant truck scheduling setting.

To stimulate this important field of truck scheduling research, we present an optimization model for the case:  $[E|t_{io}, \text{fix}|\sum w_s U_s]$  according to our classification. We presuppose that the outbound schedule is already planned over a mid-term horizon, so that all outbound trucks concerning the destination they serve, the point in time they leave the terminal and the dock doors they are served at are previously fixed. Thus, short-term truck scheduling has to determine the inbound schedule at a separated set of inbound doors (exclusive service mode:  $\alpha_1 = E$ ). Each inbound truck delivers shipments dedicated to multiple destinations any of which served by a specific outbound truck over a predetermined dock at a specific point in time ( $\beta_8 = \text{fix}$ ). Consequently, some shipments delivered by an inbound truck might reach their dedicated outbound trucks to be shipped the same day whereas others arrive late. Thus, in such a setting each inbound truck is bound to multiple and diverging dead lines. Note that, typically, in postal services the shipments arriving on an inbound truck are not previously announced to the cross dock, thus, average flows determined from historical data might need to be applied. A reasonable objective in such a setting is to minimize the weighted number of shipments delayed up to the next day ( $\gamma = \sum w_s U_s$ ). Furthermore, in real-world cross docks (especially larger ones) the diverging transshipment time for material handling between inbound and outbound door needs to be considered ( $\beta_7 = t_{io}$ ). In our model, we capture this context with a transshipment time  $t_{ko}$ , which measures the time lag between dock door  $k$  the respective inbound truck  $i$  is processed at and the dock door assigned to outbound truck  $o$ .

With the help of the notation summarized in Table 2 truck scheduling for the case  $[E|t_{io}, \text{fix}|\sum w_s U_s]$  consists of objective function (1) and constraints (2) to (8):

$$\text{Minimize } Z(C, X, Y) = \sum_{i \in I} \sum_{o \in O} w_{io} \cdot y_{io} \quad (1)$$

$I$	set of inbound trucks with $I = \{1, 2, \dots, n\}$
$O$	set of outbound trucks
$D$	set of inbound doors available for processing inbound trucks
$p_i$	processing time for unloading inbound truck $i$
$d_o$	departure time of outbound truck $o$
$t_{ko}$	transshipment time from inbound dock $k$ to the dock outbound truck $o$ is processed
$w_{io}$	weight, e.g., the number of products, of a shipment delivered by inbound truck $i$ dedicated to outbound truck $o$
$M$	big integer
$C_i$	continuous variable: completion time of inbound truck $i$
$x_{ij}^k$	binary variable: 1, if inbound truck $j$ is processed directly after inbound truck $i$ at dock door $k$ ; 0, otherwise
$x_{0i}^k$	binary variable: 1, if inbound truck $i$ is processed first at door $k$ ; 0, otherwise
$x_{i,n+1}^k$	binary variable: 1, if inbound truck $i$ is processed last at door $k$ ; 0, otherwise
$y_{io}$	binary variable: 1, if shipments delivered by inbound truck $i$ are too late to reach outbound truck $o$ ; 0, otherwise

Table 2: Notation

subject to

$$\sum_{k \in D} \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}^k = 1 \quad \forall j \in I \quad (2)$$

$$\sum_{i \in I} x_{0i}^k \leq 1 \quad \forall k \in D \quad (3)$$

$$\sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}^k = \sum_{\substack{i \in I \cup \{n+1\} \\ i \neq j}} x_{ji}^k \quad \forall j \in I; k \in D \quad (4)$$

$$C_i = \sum_{k \in D} \left( p_i \cdot x_{0i}^k + \sum_{j \in I} (C_j + p_i) \cdot x_{ji}^k \right) \quad \forall i \in I \quad (5)$$

$$y_{io} \cdot M \geq C_i - d_o - \sum_{k \in D} t_{ko} \cdot \left( \sum_{\substack{j \in I \cup \{0\} \\ i \neq j}} x_{ji}^k \right) \quad \forall i \in I; o \in O \quad (6)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i \in I \cup \{0\}; j \in I \cup \{n+1\}; k \in D \quad (7)$$

$$y_{io} \in \{0, 1\} \quad \forall i \in I; o \in O \quad (8)$$

The objective function (1) seeks to minimize the weighted number of delayed shipments, i.e., the number of shipments which remain in the terminal up to the next outbound truck (e.g., of the next day) serving the respective destination. Constraints (2) ensure that each inbound truck is processed exactly once. Inequalities (3) guarantee that

each dock door is utilized at most once by restricting the number of startup trucks to at most one per door. Constraints (4) ensure that the succession of inbound trucks at the dock doors is well-defined. These constraints play the same role as flow conservation constraints in many network flow problems. Constraints (5) define completion time  $C_i$  for each inbound truck  $i$ . Inequalities (6) ensure that shipments of late inbound trucks  $i$  can not reach a respective outbound truck  $o$  whenever completion time  $C_i$  exceeds departure time  $d_o$  plus movement time  $t_{ko}$  required to move a shipment processed at inbound door  $k$  to the outbound door truck  $o$  is processed at, where big integer  $M$  can be dimensioned as follows:  $M = \sum_{i \in I} p_i - \min_{o \in O} \{d_o + \min_{k \in D} \{t_{ko}\}\}$ . Finally, constraints (7) and (8) represent binary integrality requirement of 0-1 variables.

This model is NP-hard in the strong sense, which is proven in the appendix. Thus, future research should develop efficient exact and especially heuristic solution procedures for this and related optimization models. Especially, the close relationship of our model with parallel machines scheduling (e.g., Chen and Powell, 1999; M'Hallah and Bulfin, 2005) should be a good starting point in this direction.

## 7 Conclusions

This paper introduces a classification of truck scheduling problems basing on a tuple notation. With the help of this classification scheme existing research is briefly summarized and future research needs are identified. Especially, a yet unexplored truck scheduling problem for fixed outbound schedules is formalized by an optimization problem along with a complexity proof. In addition to the academic effort spent on describing the mathematical properties of alternative models and deriving suitable solutions procedures, there is an apparent lack of empirical research evaluating the goodness of fit of alternative truck scheduling approaches for real-world applications. Therefore, contributions which provide insights into this complex matter are to be seen as especially valuable.

## Appendix

We will prove NP-hardness for the case  $[E|t_j = 0, \text{fix}|\sum w_s U_s]$  by a transformation from the 3-Partition Problem, which is well known to be NP-hard in the strong sense (see Garey and Johnson, 1979). To ease representation, we will refer to the truck scheduling problem as CD. Note that the model for the case  $[E|t_{io}, \text{fix}|\sum w_s U_s]$  presented in Section 6 is a generalization of  $[E|t_j = 0, \text{fix}|\sum w_s U_s]$  and the reduction can be used to show NP-hardness for the former case as well.

**3-Partition Problem:** Given  $3q$  positive integers  $a_i$  ( $i = 1, \dots, 3q$ ) and a positive integer  $B$  with  $B/4 < a_i < B/2$  and  $\sum_{i=1}^{3q} a_i = qB$ , does there exist a partition of the set  $\{1, 2, \dots, 3q\}$  into  $q$  sets  $\{A_1, A_2, \dots, A_q\}$  such that  $\sum_{i \in A_j} a_i = B \quad \forall j = 1, \dots, q$ ?

**Transformation of 3-Partition to CD:** Consider an instance of CD with  $3q$  inbound trucks, whose processing times  $p_i$  equal the integer values  $a_i$  of 3-Partition. Furthermore, we have a single outbound truck  $o$  with a departure time  $d_o = B$  which receives shipments from all inbound trucks. Finally, we assume  $|D| = q$  dock doors for processing inbound trucks. As there is a one-to-one mapping between integer values of 3-Partition and inbound trucks of CD, this transformation is polynomial. The question we ask is whether we can find a solution for CD with objective value  $Z = 0$ , i.e., no delay of shipments.

A feasible solution for an instance of 3-Partition can be transformed to a feasible solution of the corresponding CD-instance in polynomial time by assigning each set of integers to a separate inbound door. As the sum of integer values of each set amounts to  $B$ , all three inbound trucks can be processed up to period  $B$  in facultative succession at their respective door, so that no delay at neither door occurs and the objective value of  $Z = 0$  is realized.

On the other hand, each feasible solution for any CD instance is also a feasible solution for 3-Partition. This holds true because any solution of CD with  $Z = 0$  must have exactly three inbound trucks assigned per door because of the restriction on the processing time values  $B/4 < p_i = a_i < B/2$ . Any solution with more or less than three trucks at a door must result in a makespan higher than  $B$ , so that a delay would be inevitable. Thus, any CD-solution with  $Z = 0$  must have a makespan of  $B$  at any door, so that a direct mapping between the trucks per door and the sets of integers exist.  $\square$

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