Cross-Sectional Asset Pricing with Individual Stocks: Betas versus Characteristics

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Main question

- Are expected returns related to
  - Risk/betas, OR
  - Characteristics

- If both, which is more important?

How to answer?

- Use portfolios
  - Helps mitigate EIV problem (Fama and MacBeth, 1973)
  - But,
    - Less efficient (Ang, Liu, and Schwarz, 2010)
    - Method of grouping is important (Lewellen, Nagel, and Shanken, 2010)

- Use individual securities
  - But, EIV problem

What we do

- Use individual securities
- Correct for EIV bias (Litzenberger and Ramaswamy, 1979; Shanken, 1992; Kim, 1995)
    - And a few other biases
- Allow betas to change over time (Rosenberg and Guy, 1976; Shanken, 1990; Fama and French, 1997; Avramov and Chordia, 2006)
- Quantify relative contribution of risk loadings versus characteristics in explaining the cross-section of returns

Two-pass procedure

- Time-series regression (TSR)
  - Condition betas on
    - Firm characteristics (Size, B/M, six-month return), or
    - Firm characteristics (Size, B/M, six-month return) + Macro variables (Term spread, Default spread)
  - One-, three-, and four-factor models
- Cross-sectional regression (CSR)

- For now, we do not allow for time-variation in risk premia but relax this later on in the third-stage

TSR

- Unconditional TSR

$$R_{it} = B_{0i} + B_i F_t + \varepsilon_{it}$$

- Conditional TSR
  - Define zts_{it} to be a p×1 vector of conditioning variables
  - Redefine scaled intercept and factors as

$$F_{it}^{*} = [zts'_{(p-1)it-1}, zts_{i1t-1}F_t', \ \ldots, zts_{ipt-1}F_t']'$$

  - Then, the TSR is

$$R_{it} = B_{0i}^{*} + B_i^{*} F_{it}^{*} + \varepsilon_{it}$$

TSR …

- Example of a one-factor model with firm characteristics conditioning variables

$$R_{it} = (B_{i0}^{*} + B_{i1}^{*} Sz_{it-1} + B_{i2}^{*} BM_{it-1} + B_{i3}^{*} \mathrm{Re}\,t6_{it-2})$$
$$+ (B_{i0}^{Mkt*} + B_{i1}^{Mkt*} Sz_{it-1} + B_{i2}^{Mkt*} BM_{it-1} + B_{i3}^{Mkt*} \mathrm{Re}\,t6_{it-2}) \mathrm{Mkt}_{t}$$
$$+ \varepsilon_{it}$$

Characteristics are cross-sectionally demeaned

- Then the implied market beta is given by

$$B_{it-1}^{Mkt} = (B_{i0}^{Mkt*} + B_{i1}^{Mkt*} Sz_{it-1}^{*} + B_{i2}^{Mkt*} BM_{it-1}^{*} + B_{i3}^{Mkt*} \mathrm{Re}\,t6_{it-2})$$

CSR

- Cross-sectional regression (CSR) using OLS

$$R_t = \gamma_{0t} + \hat{B}_{t-1}\gamma_{1t} + Zcs_{t-1}\gamma_{2t} + \varepsilon_t$$

$$\hat{\Gamma}_t \equiv \left(\hat{\gamma}_{0t}, \hat{\gamma}'_{1t}, \hat{\gamma}'_{2t}\right)'$$

$$\hat{\Gamma}_t = \left(\hat{X}'_t \hat{X}_t\right)^{-1} \hat{X}'_t R_t, \text{ where } \hat{X}_t \equiv \left[1_{N_t} : \hat{B}_{t-1} : Zcs_{t-1}\right]$$

CSR biases

$$\hat{X}_t = \left[1_{N_t} : \hat{B}_{t-1} : Zcs_{t-1} \right]$$

$$= \left[1_{N_t} : B_{t-1} : Zcs_{t-1} \right] + \left[0 : \hat{B}_{t-1} - B_{t-1} : 0 \right]$$

$$= X_t + \left[0 : U_t : 0 \right]$$

- Estimation error in betas, *U*, is the cause of all the trouble

CSR biases – EIV

- $\hat{X}_t'\hat{X}_t$ contains a term $U_t'U_t$, which is the (cross-sectional sum of the) of the estimation error variance in betas
- This increases the “denominator” and causes the classic EIV problem
- Fortunately, we know the estimation error in betas from TSR

$$\hat{\Gamma}_t^{\mathrm{EIV}} = \left(\hat{X}_t'\hat{X}_t - \sum_{i=1}^{N_t} M'\hat{\Sigma}_{\hat{B}_{it-1}} M \right)^{-1} \hat{X}_t' R_t$$

$$\hat{\Sigma}_{\hat{B}_{it-1}} = Zts_{it-1}'\hat{\Sigma}_{\hat{B}_i^*}^{\mathrm{White}} Zts_{it-1}, \text{ and } M = \begin{bmatrix} 0_{k\times 1} & I_{k\times k} & 0_{k\times k_2} \end{bmatrix}$$

Intuition …

- For a one-factor model with no additional characteristics

$$\gamma^{OLS} = \frac{\text{cov}_{cs}(\hat{\beta}_{it}, R_{it})}{\text{var}_{cs}(\hat{\beta}_{it})} = \frac{\text{cov}_{cs}(\beta_{it}, R_{it})}{\text{var}_{cs}(\hat{\beta}_{it})}$$

- The denominator is equal to the cross-sectional variation of betas (this is what we want) plus the cross-sectional variation of beta estimation error (this is what we want to remove). Fortunately

$$\begin{aligned}\text{var}_{cs}(\hat{\beta}_{it}) &= \text{var}_{cs}(\beta_{it}) + \text{var}_{cs}(\hat{\beta}_{it} - \beta_{it}) \\ &= \text{var}_{cs}(\beta_{it}) + \text{var}_{cs}(\sigma_{ei}^2 \Sigma_F^{-1})\end{aligned}$$

- This means

$$\gamma^{EIV} = \frac{\text{cov}_{cs}(\beta_{it}, R_{it})}{\text{var}_{cs}(\hat{\beta}_{it}) - \text{var}_{cs}(\sigma_{ei}^2 \Sigma_F^{-1})}$$

CSR biases – 1-month bias

- The "numerator" contains $U_t'\varepsilon_t$. This should cancel out in balanced panels and homosekadastic case
  - In one-factor case, this term is equal to the sum of

$$\left(Rm_t - \overline{Rm}\right)\varepsilon_{it}^2$$

- In heteroskedastic case (as observed in the data), this term is not zero and leads to the second correction

$$bias_t = \begin{pmatrix} 0 \\ \sum_{i=1}^{N_t} Zts'_{it-1}(F_{di}^{*\prime}F_{di}^{*})^{-1}F_{dit}^{*}{}'e_{it}^2 \\ 0_{k_2 \times 1} \end{pmatrix}$$

CSR biases – another bias

- Both the denominator and the numerator contain term such as $U_t' Zcs_{t-1}$
- Since Zcs contains price-related variables (size, B/M, six-month return), this term is also not zero
- This necessitates a third correction
  - We assume for simplicity an AR(1) process for size and B/M to implement this correction

Final formula

$$\hat{\Gamma}_t^{\text{final}} = \left(\hat{X}_t' \hat{X}_t - \sum_{i=1}^{N_t} M' \hat{\Sigma}_{B_{it-1}} M - \text{anotherbias}_t \right)^{-1} \left(\hat{X}_t' R_t - bias_t \right)$$

Contribution measures

- Using average CSR estimates

$$E_{t-1}\left[R_t\right] = \overline{\hat{\gamma}_0} + E_{t-1}^{\text{beta}}\left[R_t\right] + E_{t-1}^{\text{char}}\left[R_t\right], \text{where}$$

$$E_{t-1}^{\text{beta}}\left[R_t\right] = \hat{B}_{t-1}\overline{\hat{\gamma}_1}, \text{and } E_{t-1}^{\text{char}}\left[R_t\right] = Zcs_{t-1}\overline{\hat{\gamma}_2}$$

- Using cross-sectional variation at time *t*

$$\%(\text{Betas}) = \text{var}_{cs}\left(E_{t-1}^{\text{beta}}\left[R_t\right]\right) \Big/ \text{var}_{cs}\left(E_{t-1}\left[R_t\right]\right)$$

$$\%(\text{Chars}) = \text{var}_{cs}\left(E_{t-1}^{\text{char}}\left[R_t\right]\right) \Big/ \text{var}_{cs}\left(E_{t-1}\left[R_t\right]\right)$$

Contribution measures …

$$\text{var}_{cs}\left(E_{t-1}^{\text{beta}}\left[R_t\right]\right) = \overline{\hat{\gamma}'}_1 \text{var}_{cs}\left(\hat{B}_{t-1}\right)\overline{\hat{\gamma}}_1$$

- Have to again correct the cross-sectional variance of estimated betas using the same trick as in the regular EIV correction

- Standard errors for contributions
  - Resample gammas using their standard errors
  - Recalculate contribution numbers
  - Repeat 1,000 times to obtain empirical distribution
    - Use Efron's procedure to account for non-linearity

Data

- All common stocks on NYSE, AMEX, and NASDAQ
- Sample: 1951 to 2011
- Price greater than $1 (for CSR)

- At least five years of data
  - Could lead to survivorship bias
  - We do not include stocks in CSR in the first five years of their life

One-factor model

	Panel A: All Stocks			
	Bias uncorrected		Bias corrected	
zts→	Firm	Firm + Macro	Firm	Firm + Macro
Cnst	0.686	0.825	0.838	1.060
	(4.02)	(4.68)	(4.96)	(6.11)
B_{Mkt}	0.331	0.235	0.158	−0.053
	(2.13)	(1.56)	(0.91)	(−0.31)
Sz	−0.043	−0.048	−0.045	−0.050
	(−1.56)	(−1.72)	(−1.67)	(−1.82)
B/M	0.282	0.267	0.274	0.269
	(6.04)	(5.79)	(5.92)	(5.68)
Ret6	1.248	1.294	1.097	1.065
	(8.93)	(9.06)	(6.86)	(6.64)
Nstocks	2,025	2,025	2,025	2,025
% Betas	20.7	14.0	3.6	0.5
% Chars	82.7	88.4	97.6	99.5
% Diff	62.0	74.4	93.9	99.0
	(6.5,98.6)	(10.7,99.9)	(49.3,100.0)	(88.3,100.0)

Three-factor model

	Panel A: All Stocks			
	Bias uncorrected		Bias corrected	
zts→	Firm	Firm + Macro	Firm	Firm + Macro
Cnst	0.406	0.537	0.532	0.677
	(3.39)	(4.70)	(3.52)	(4.79)
B_{Mkt}	0.254	0.186	0.072	0.000
	(1.64)	(1.24)	(0.43)	(−0.00)
B_{SMB}	0.241	0.206	0.233	0.191
	(2.30)	(2.03)	(1.96)	(1.73)
B_{HML}	−0.006	0.020	0.018	0.029
	(−0.06)	(0.21)	(0.17)	(0.30)
Sz	0.003	−0.008	0.001	−0.007
	(0.15)	(−0.48)	(0.04)	(−0.36)
B/M	0.269	0.242	0.273	0.276
	(9.40)	(8.65)	(6.93)	(6.98)
Ret6	1.257	1.288	1.115	1.103
	(10.40)	(10.91)	(7.68)	(7.90)
Nstocks	2,025	2,025	2,025	2,025
% Betas	38.8	36.3	23.5	19.9
% Chars	61.4	61.8	71.4	72.0
% Diff	22.6	25.6	47.8	52.1
	(−28.4,86.1)	(−22.0,89.7)	(−10.2,99.4)	(2.9,99.5)

Four-factor model

	Panel A: All Stocks			
	Bias uncorrected		Bias corrected	
zts→	Firm	Firm + Macro	Firm	Firm + Macro
Cnst	0.398	0.510	0.566	0.663
	(3.47)	(4.67)	(3.79)	(4.81)
B_{Mkt}	0.257	0.198	0.095	0.084
	(1.65)	(1.31)	(0.46)	(0.53)
B_{SMB}	0.239	0.213	0.197	0.171
	(2.26)	(2.08)	(1.67)	(1.54)
B_{HML}	0.008	0.034	0.054	0.045
	(0.08)	(0.36)	(0.52)	(0.45)
B_{MOM}	0.186	0.202	0.428	0.404
	(1.28)	(1.43)	(2.48)	(2.59)
Sz	0.005	−0.005	−0.003	−0.014
	(0.28)	(−0.30)	(−0.14)	(−0.73)
B/M	0.276	0.251	0.272	0.291
	(10.33)	(9.66)	(6.96)	(7.48)
Ret6	1.265	1.307	1.062	0.999
	(12.72)	(13.40)	(8.21)	(7.69)
Nstocks	2,025	2,025	2,025	2,025
% Betas	40.9	45.0	31.9	38.5
% Chars	54.4	49.3	55.4	50.8
% Diff	13.5	4.3	23.5	12.3
	(−23.7,75.6)	(−30.8,68.6)	(−15.5,74.3)	(−22.9,73.1)

Time variation in risk premia

- Allow predictability in risk premia with dividend-price ratio, term spread, and default spread as predictive variables x

$$\hat{\gamma}_t = c_0 + c_1' x_{t-1} + v_t$$

- Calculate fitted values

$$\hat{\gamma}_{t-1}^{fit} = \hat{c}_0 + \hat{c}_1' x_{t-1}$$

- Recalculate the contribution numbers as

$$E_{t-1}\left[R_t\right] = \hat{\gamma}_0 + E_{t-1}^{\text{beta}}\left[R_t\right] + E_{t-1}^{\text{char}}\left[R_t\right], \text{where}$$

$$E_{t-1}^{\text{beta}}\left[R_t\right] = \hat{B}_{t-1}\hat{\gamma}_{1t-1}^{fit}, \text{and } E_{t-1}^{\text{char}}\left[R_t\right] = Zcs_{t-1}\hat{\gamma}_{2t-1}^{fit}$$

Four-factor

	B_{Mkt}	B_{SMB}	B_{HML}	B_{MOM}	Sz	B/M	Ret6
Panel C.1: Conditional betas (*zts* = Firm)							
Cnst	−1.245	0.555	0.718	1.569	−0.063	0.323	1.907
	(−1.45)	(1.16)	(1.49)	(1.62)	(−0.70)	(2.24)	(3.20)
Payout	21.717	−21.518	−11.963	−18.608	4.558	−10.237	0.961
	(1.18)	(−2.02)	(−1.11)	(−1.13)	(2.13)	(−3.09)	(0.07)
Def	0.324	0.696	−0.094	−0.479	−0.110	0.304	−1.029
	(0.58)	(2.31)	(−0.27)	(−0.80)	(−1.84)	(2.83)	(−2.73)
Term	4.425	−5.374	−2.838	7.374	−1.739	5.547	5.520
	(0.35)	(−0.69)	(−0.35)	(0.67)	(−1.12)	(2.25)	(0.66)
Adj-R^2	0.12	0.33	0.07	0.32	0.54	1.94	1.21
% Betas = 42.3, % Chars = 47.4, % Diff = 5.1 (−9.6, 49.7)							

Panel C.2: Conditional betas (*zts* = Firm + Macro)							
Cnst	−1.128	0.373	0.308	1.625	−0.158	0.533	1.473
	(−1.62)	(0.82)	(0.68)	(1.93)	(−2.08)	(3.77)	(2.59)
Payout	20.282	−19.560	−10.058	−15.151	5.671	−8.887	7.233
	(1.24)	(−1.96)	(−0.97)	(−0.95)	(3.08)	(−2.61)	(0.55)
Def	0.209	0.813	0.224	−0.660	−0.062	0.020	−1.022
	(0.44)	(2.94)	(0.74)	(−1.32)	(−1.29)	(0.20)	(−2.54)
Term	7.086	−7.554	−2.270	3.793	−2.303	6.831	11.201
	(0.60)	(−1.02)	(−0.30)	(0.33)	(−1.70)	(2.75)	(1.23)
Adj-R^2	0.29	0.59	−0.23	0.65	1.37	1.44	0.92
% Betas = 50.2, % Chars = 43.4, % Diff = −6.8 (−23.8, 35.7)							

Conclusion

- Risk premium on SMB strong but that on HML weak
- Reject all factor models
  - Rejection not news
- Both betas and characteristics matter
- Characteristics often more important than betas
  - However, four-factor model conditional bias-corrected betas explain more of returns than characteristics with time-varying risk premia

Next steps

Potential problems

- Use of full-sample betas
- Data restriction of minimum 5 years

Limitations

- Conclusions valid only for the factor models and the characteristics analyzed here