

# Cross-sectional Variation in Stock Returns: Liquidity and Idiosyncratic Risk

*Matthew Spiegel*<sup>†</sup> and *Xiaotong Wang*<sup>‡</sup>

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<sup>†</sup>Yale School of Management, P.O. Box 208200, New Haven, CT 06520-8200, phone: 203-432-6017, email: [matthew.spiegel@yale.edu](mailto:matthew.spiegel@yale.edu), www: <http://som.yale.edu/~spiegel>

<sup>‡</sup>Yale School of Management, P.O. Box 208200, New Haven, CT 06520-8200, email: [xiaotong.wang@yale.edu](mailto:xiaotong.wang@yale.edu).

## **Cross-sectional Variation in Stock Returns: Liquidity and Idiosyncratic Risk**

### **Abstract**

The roles played by idiosyncratic risk and liquidity in determining stock returns have recently received a great deal of attention. However, recent empirical tests have not examined the interaction between these two factors. As others have shown (and this paper confirms) stocks idiosyncratic risk and liquidity are negatively correlated. To what extent then is each variable responsible for the observed cross sectional patterns in stock returns? Overall, using monthly data, the paper finds that stock returns are increasing with the level of idiosyncratic risk and decreasing in a stock's liquidity. However, while both liquidity and idiosyncratic risk play a role in determining returns, the impact of idiosyncratic risk is much stronger and often eliminates liquidity's explanatory power. The point estimates indicate that a one standard deviation change in idiosyncratic risk has between 2.5 and 8 times the impact of a corresponding change in liquidity on cross sectional expected returns.

Questions regarding the factors that influence expected stock returns have long interested both academic and practitioner audiences. Some of the recent academic research on these issues has derived from two independent intellectual traditions. The first originates within the asset pricing literature and asks whether idiosyncratic risk plays a role in expected stock returns. The second derives from the market microstructure literature and looks at the relationship between liquidity and expected returns. To date there has not been an attempt to empirically connect these two lines of work, and yet there are good theoretical reasons to believe they are looking at related issues.<sup>1</sup> This paper addresses this gap in the literature by attempting to empirically disentangle the roles played by liquidity and idiosyncratic risk in stock returns.

Theoretical work by Merton (1987), and O'Hara (2003) indicates that liquidity should be priced by the market.<sup>2</sup> In contrast papers by Constantinides (1996), Heaton and Lucas (1996), and Vayanos (1998) imply that it should not be. In terms of idiosyncratic risk while the CAPM says it should not be priced other models like that of Merton (1987) indicate that it should be. A selective reading of the empirical literature (described in additional detail later on) can be used to support all of the above positions. Thus, it is of some academic interest to determine whether or not at least some of the contrasting results are due the interaction between liquidity and idiosyncratic risk.

When examining the paper's results it helps to divide liquidity measures into those that are "cost based" and those that are "reflective." Cost based measures attempt to quantify liquidity by examining the financial loss a trader incurs from a particular transaction. Examples include the bid-ask spread and Kyle's lambda. Reflective measures, such as volume, rely instead on the

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<sup>1</sup> For example, strategic inventory control models like Ho and Stoll (1980) or competitive models like Spiegel and Subrahmanyam (1995) predict that liquidity should be inversely related to idiosyncratic risk. Brunnermeier and Pedersen (2005) show that funding frictions also lead to this relationship. Section 2 contains a brief discussion of how this literature links idiosyncratic risk to liquidity.

<sup>2</sup> Merton's (1987) paper does not directly derive any results pertaining to liquidity. However, by differentiating the stock price with respect to its supply one can generate such results and the Appendix in this paper does so.

idea that liquidity should be associated with particular characteristics.<sup>3</sup> For example, high volume levels may indicate that a particular security is very liquid. But, volume does not tell us how costly it is to actually trade the security.

Overall, the analysis comes to four primary conclusions: First, univariate tests show that stock returns are decreasing in liquidity and increasing in idiosyncratic risk. Second, idiosyncratic risk and liquidity are strongly negatively correlated. Third, when both cost based liquidity measures and idiosyncratic risk are used to simultaneously explain returns cost based liquidity measures appear to play little or no role. That is controlling for idiosyncratic risk illiquid and liquid stocks have similar returns. Conversely, controlling for liquidity (cost based or otherwise) does not eliminate idiosyncratic risk's impact on returns. Fourth, the only liquidity measure that explains cross sectional returns beyond that found in other variables is dollar volume. However, even in this case the economic impact is small relative to that of idiosyncratic risk.

The economic importance of the above results can be seen in the returns various strategies produce. Portfolio sorts indicate that high-low strategies using idiosyncratic risk yield returns about eight times as large as those using volume. Point estimates from a regression analysis produce results nearly as large. On the NYSE or AMEX a one standard deviation change in a stock's idiosyncratic risk changes returns by an amount equal to 2.8 times a one standard deviation change in volume. A similar comparison with Nasdaq results in a ratio of four to one.

Understanding how idiosyncratic risk, liquidity and stock returns interact with each other can help us to understand the current set of findings in these areas and to sort out which market attributes are or are not priced. Numerous papers find that liquidity is negatively related to

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<sup>3</sup> Another example would be the number of investors holding a security.

expected stock returns.<sup>4</sup> At the same time another literature indicates that there exists a positive correlation between idiosyncratic risk and returns at the firm or market level.<sup>5</sup> But is the market pricing all of these factors as reported? If so it would appear that economically large returns can be obtained by combining the reported findings into a single portfolio loaded on those liquidity and idiosyncratic risk attributes with the highest returns. What this paper shows is that to a large degree the returns attributed to liquidity and idiosyncratic risk are in fact due mostly to idiosyncratic risk.

The numerous tests presented here, in which liquidity and idiosyncratic risk are allowed to compete as explanatory factors in cross sectional returns, corroborate each other. In one portfolios are created by first sorting on liquidity and then on idiosyncratic risk, in other cases the sorts are reversed. Sorting first on liquidity reduces the returns generated by going long the high idiosyncratic risk portfolio and short the low idiosyncratic risk portfolio by about .5% per month. Nevertheless, the strategy continues to generate statistically significant positive returns of about 1% per month. Reversing the procedure, so that the first sort is on idiosyncratic risk, largely eliminates all of the excess returns from going long the least liquid decile stocks and short the most liquid decile stocks. Another test uses a Fama-Macbeth regression that simultaneously allows liquidity, idiosyncratic risk, momentum, volume and firm size to influence returns. Once again, by itself cost based liquidity has explanatory power but not when idiosyncratic risk is simultaneously included. On the other hand idiosyncratic risk is statistically significant in every regression that it enters.

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<sup>4</sup>Among others see Amihud and Mendelson (1986), Amihud and Mendelson (1989), Amihud (2002), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Pastor and Stambaugh (2003), Acharya and Pedersen (2004), Baker and Stein (2004), Hasbrouck (2005).

<sup>5</sup>See, Lintner (1965), Douglas (1968), Lehmann (1990), Xu and Malkiel (2002), Goyal and Santa-Clara (2003), Ghysels, Santa-Clara and Valkanov (2004), and Fu (2005). However, there also exist studies that come to other conclusions. Bali, Cakici, Yang, and Zhang (2004) take issue with the results in Goyal and Santa-Clara (2003) while Guo and Savickas (2004) and Ang, Hodrick, Xing, and Zhang (2005) find a negative relationship between returns and idiosyncratic risk. Baker and Wurgler (2005) conclude that conditional on investor sentiment idiosyncratic risk can be positively or negatively correlated with the expected returns.

One caveat exists to the above conclusions regarding cost based liquidity measures. When dollar volume is excluded from the analysis Amihud's (2002) measure does provide out of sample explanatory power for cross sectional stock returns. However, with dollar volume it does not. Instead dollar volume itself becomes a statistically significant explanatory variable. In terms of explaining cross sectional returns, dollar volume always offers significant cross sectional explanatory power regardless of what other variables are included. This result parallels that in Brennan et al. (1998) who use a different set of control variables.

It is worth emphasizing the robustness of the results reported here. To avoid displaying one nearly identical table after another most of the discussion focuses on only a few liquidity measures and the results based on data from all three major exchanges (the NYSE, AMEX and Nasdaq). However, using any one of a long list of liquidity measures (see Section 1 and Footnote 7 for the list) yields essentially the same conclusions. The same holds true if the sample is restricted to NYSE and AMEX stocks; the qualitative patterns (including those involving volume) remain unchanged and typically the quantitative differences are economically small as well. Tests were also conducted on sub-period samples as well as samples based upon past returns. Neither made any qualitative difference to the reported results and thus the tables are not included.

Before proceeding to the empirical analysis, the results contained therein should not be read as implying that the set of cost based liquidity measures examined here are deficient in any way. These measures are important for any number of reasons (e.g. exploring the impact of institutions on investors). What this paper does indicate is that they are not priced once one allows for the impact of idiosyncratic risk. In terms of idiosyncratic risk itself, this paper leaves open the question of why Ang et al. (2005) find it is negatively correlated with returns in the daily data. While it is clearly of interest to understand why the daily and monthly data appear to produce contrasting results, it is a topic outside this paper's scope. Here the primary research

question is restricted to understanding the degree to which idiosyncratic risk and liquidity each contribute to observed cross sectional stock returns within the monthly data.

The paper is organized as follows: Section 1 describes the database used in this study. Section 2 examines the empirical relationships between idiosyncratic risk, liquidity and stock returns. Section 3 analyzes each factor's out of sample performance. Section 4 discusses the relationship between this paper and the extant literature. Section 5 concludes. Finally, the Appendix briefly reviews Merton (1987) and derives several empirical implications.

## 1. Data and Descriptive Statistics

Using the CRSP monthly stock return file data was obtained for the period covering January 1962 to December 2003. This was supplemented with four of the cost based liquidity measures discussed in Hasbrouck (2005). Professor Hasbrouck estimates them annually for each stock based upon that calendar year's data. Within any one month this paper includes a stock if its liquidity measures are available from Professor Hasbrouck's web for the *previous* calendar year.<sup>6</sup> In addition a stock is included in a particular month only if CRSP provides return, shares outstanding, price, and volume data for it in at least 24 of the previous 60 months. Due to the sample criteria this is smaller than the monthly average of 5,619 stocks in the CRSP database. Table 1 contains the sample summary statistics.

The four cost based liquidity estimators used in this paper are Gibbs, Gamma, Amihud, and Amivest. Each is described in detail by Hasbrouck (2005). For the most part this paper concentrates on the Gibbs estimator since it appears to have the most economic power (see Hasbrouck (2005)).<sup>7</sup> Unless otherwise noted, all references in this paper to liquidity refer to

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<sup>6</sup>This data requirement is designed to avoid a survivorship biased sample. A calendar year  $X$  liquidity measure only exists if there is sufficient data (the firm survived long enough) in a year to estimate it. These stocks produce monthly "alphas" of about 1%.

<sup>7</sup> Many of the tests in this paper were also conducted with the Probability of Informed Trading measure (PIN) from Easley, Kiefer, and O'Hara (1997) and Easley and O'Hara (2002) as well as Amihud's (2002) illiquidity measure, the Amivest liquidity ratio (see Hasbrouck (2005)), and the Pastor and Stambaugh

liquidity as measured by the Gibbs estimator. Since Hasbrouck (2005) provides an extensive description of each liquidity measure and how it is estimated only a brief overview of the Gibbs estimator is presented below along with a short description of the other three.

The Gibbs estimator is a Bayesian version of Roll's (1984) transactions cost measure

$$c = \begin{cases} \sqrt{-\text{cov}(r_t, r_{t-1})} & \text{if } \text{cov}(r_t, r_{t-1}) < 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

This measure derives from a model in which  $r_t = c\Delta q_t + u_t$  where  $q_t$  is a trade direction indicator (buyer or seller initiated),  $c$  the parameter to be estimated,  $\Delta q_t$  the change in the indicator from period  $t-1$  to  $t$ , and  $u_t$  an error term. Some simple algebra then leads to (1) under the assumption that buyer and seller initiated trades are equally likely.

The Gamma measure equals the Pastor and Stambaugh (2003) reversal parameter, the details for which can be found in either their paper or Hasbrouck (2005). The Amihud measure equals the log of the average daily absolute return over the daily dollar volume for the calendar year in question. This particular estimator conforms to the measure proposed in Amihud (2002) and a variant of it is also discussed in Hasbrouck's paper. Amivest equals the log of the average daily volume over the daily absolute return for the calendar year in question. The Amivest measure is not quite the negative of the Amihud measure since the ratios are averaged over the year prior to taking the logs.

Table 1 displays summary statistics for several of the variables examined here. Overall, the sub-sample statistics for the cost based liquidity measures used in this study are fairly close to those using the entire set of stocks available on Hasbrouck's web page. For example, the average value of the Gibbs sampler estimate is 0.013 for this paper's sample and for the whole sample it is 0.012. The greatest discrepancy across the means occurs with the Gamma measure. However,

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(2003) reversal measure. All produced qualitatively similar forecast results to those presented here and thus are not discussed in the text or included in every table for the sake of brevity. Tests were also conducted using real dollar liquidity measures (for those that are not unit free) rather than nominal measures. Again the results were qualitatively identical and are thus not reported here.



this appears to be driven by a few outliers since the sub-sample and full sample medians are reasonably close to each other while the full sample's standard deviation is much higher than that of the sub-sample. Thus, at least in terms of the first and second sample moments of the cost based liquidity parameter estimates, the firms included here correspond closely to the overall set of stocks for which data is available.

## 2. Idiosyncratic Risk, Liquidity and Size

### A. *A Very Brief Overview of the Theoretical Link between Idiosyncratic Risk and Liquidity*

This section contains only a cursory overview of why one expects idiosyncratic risk to be inversely related to a stock's overall liquidity. Readers interested in the substantive details are encouraged to consult the papers listed in footnote 1. Those familiar with these arguments may wish to skip this section.

In an inventory control model of market making there exist one or more "specialist firms" that agree to buy or sell securities on demand. These firms begin with  $x_0$  shares and seek to hold a target inventory of  $x$  shares by day's end. The firm's payoff typically equals its capital gains ( $G$ ) from trading during the day minus a cost related to the volatility of the security's final payoff ( $\sigma^2$ ) and a quadratic function of the difference between the final position ( $x_1$ ) and  $x$ . Typically, this leads to an optimization problem in which the objective function can be written as  $G - c(x_1 - x)^2 \sigma^2$ , where  $c$  is the cost of departing from the firm's optimal end of day holdings.

The specialist sets the slope of the supply function to trade off capital gains with both levels of trading and its willingness to hold an unbalanced position. For the firm, increasing the slope of the supply function provides a greater capital gain for a given trade size but also reduces the expected trade size itself. Anything that increases the specialist's sensitivity to missing its end of day target ( $c(x_1 - x)^2 \sigma^2$ ) leads it to become essentially more risk averse and thus increase the

slope of the supply function. That is, the specialist becomes less willing to provide liquidity.

One element that influences the specialist's holding cost is the final position's payoff variance  $\sigma^2$ .

For an individual security  $i$  its variance can be written as  $\sigma_i^2 = \beta_{i,MKT}^2 \sigma_{MKT}^2 + \beta_{i,F1}^2 \sigma_{F1}^2 + z_i^2$ ,

where  $\beta_{i,MKT}$  is the security's market beta,  $\sigma_{MKT}^2$  the market's variance,  $\beta_{i,F1}$  the security's beta for

some other risk factor labeled 1,  $\sigma_{F1}^2$  risk factor 1's variance, and  $z_i^2$  the stock's idiosyncratic

risk. If one now introduces a market index contract (like the S&P 500), and other similar

investment vehicles (like the Russell 2000 small capitalization stock index) the specialist can now

hedge out any factor risks associated with the stock. (For example, the specialist can use the

Wilshire 5000 to eliminate market risk, and the Russell 2000 and the S&P 500 to eliminate the

Fama-French small minus big factor.) This leaves his final cost from holding an unbalanced

position at day's end equal to  $c(x_1 - x)^2 z_i^2$ , which depends on the security's idiosyncratic risk

and not its total risk. Thus, if  $c$  does not vary systematically across securities, higher

idiosyncratic risk levels should be associated with lower levels of liquidity.

## **B. Estimating Idiosyncratic Risk**

While theoretical models provide a clear definition of idiosyncratic risk they do not offer

an obvious methodology for its estimation. Theoretically, it equals the return innovation's

standard deviation beyond what investors expected given that period's market return. But, the

models have nothing to say about how the market generates its expectations regarding the

innovation's variance and thus do not provide an empirical solution to this problem. Thus, this

paper accedes to recent practice and assumes that (unless otherwise noted) the Fama-French 3-

factor (FF3) model is the model used by the market. Given this, idiosyncratic risk equals the

standard deviation of the regression residual from:

$$R_t^i - R_{ft} = \alpha_i + \beta_{i,MKT} (R_{MKT,t} - R_{ft}) + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \varepsilon_{i,t} \quad (2)$$

Following standard notation,  $\beta_{i,x}$  equals the estimated loading on factor  $x$ ,  $R_{mkt,t}$  is the market return at time  $t$ ,  $SMB_t$  the return on small minus big capitalization stocks,  $HML_t$  the return on high minus low book to market stocks,  $R_t^i$  the time  $t$  return on stock  $i$ ,  $R_{ft}$  the time  $t$  risk free rate, and  $\varepsilon_{i,t}$  an error term.

In what follows idiosyncratic risk is always defined relative to the FF3 model. This was done since it seems likely that market makers tend to be employees of sophisticated trading firms that can employ vehicles to hedge out known risk factors. However, this is somewhat irrelevant to the primary issue explored in this paper; whether or not idiosyncratic risk explains some of the observed returns in the literature currently associated with liquidity. To the degree that the FF3 idiosyncratic risk measure is the “wrong” measure this will only favor liquidity as an explanatory variable. Presumably, if the tests should be done with the one factor CAPM model or some other model of the reader’s choice the results presented here would only be strengthened, that is idiosyncratic risk would explain even more of the return currently associated with liquidity.<sup>8</sup>

### a. OLS Estimates of Idiosyncratic Risk

At each month  $t$  the model uses the previous five year window to estimate the three factor model’s betas. Define the square root of  $(T - k)^{-1} \sum_{t=1}^T \hat{\varepsilon}_{i,t}^2$  as the OLS estimate of the idiosyncratic risk for the current month where  $T$  is the number of observations available over the time horizon and  $k$  is the number of estimated parameters (four in this case). A stock is included in the sample if 24 out of the 60 previous observations are available for estimation.

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<sup>8</sup> Naturally, the converse is true as well. If there exists a better liquidity measure than those tested here it may be possible to reduce the returns that this paper finds are attributable to idiosyncratic risk and instead attribute them to the new measure.

## b. EGARCH Estimation of Idiosyncratic Risk

While the static OLS model has seen extensive use in the idiosyncratic risk literature it cannot easily capture whatever time variation may exist in a stock's variance. For this a dynamic model like EGARCH is needed.<sup>9</sup> The EGARCH model estimates idiosyncratic risk via:

$$\begin{aligned}
 R_t^i - R_{ft} &= \alpha_i + \beta_{i,MKT} (R_{MKT,t} - R_{ft}) + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \varepsilon_{i,t} \\
 \varepsilon_{i,t} &= \sqrt{h_{i,t}} \times v_t \\
 \ln h_{i,t} &= \bar{\omega}_i + \sum_{m=1}^p \delta_{i,m} \ln h_{i,t-m} + \sum_{n=1}^q \eta_{i,n} (|v_{t-n}| - E|v_{t-n}| + \psi_i v_{t-n})
 \end{aligned} \tag{3}$$

The third equation describes the evolution of the conditional variance of  $\varepsilon_{i,t}$ . Here  $v_t$  is an i.i.d. error term with zero mean and unit variance,  $\bar{\omega}_i$  is the unconditional mean of  $\ln h_{i,t}$ . The  $\eta_{i,n}$ ,  $\psi_i$ ,  $\delta_{i,m}$  terms are estimated parameters. The  $h_{i,t}$  is the model's estimate of the  $\varepsilon_{i,t}$ 's conditional variance.<sup>10</sup>

At each month  $t$ , all available data prior to that date is used to estimate the EGARCH model (3). A stock must have 60 return observations available to be included in the sample. The previous period's EGARCH estimate of the conditional volatility (Eidio) is used as the estimate for this month's conditional idiosyncratic risk measure. Unless otherwise stated all forecasts using Eidio are out of sample.

<sup>9</sup> This model provides a natural alternative estimator for a firm's idiosyncratic risk. See Bollerslve (1986) and Nelson (1991) regarding the use and development of GARCH and EGARCH models.

<sup>10</sup> If the  $\eta_{i,n}$  are positive, the deviation of  $|v_{t-n}|$  from its expected value increases the variance of  $\varepsilon_{i,t}$ , and vice versa. The  $\psi_i$  parameter allows this effect to be asymmetric. If  $\psi_i = 0$  then a positive surprise ( $v_{t-n} > 0$ ) will have the same impact on conditional volatility ( $\ln h_{i,t}$ ) as a negative surprise ( $v_{t-n} < 0$ ). If  $-1 < \psi_i < 0$  then a positive surprise has a smaller impact on conditional volatility than a negative surprise. If  $\psi_i < -1$ , then a positive surprise reduces volatility while a negative surprise increases the conditional volatility.

A natural question to ask is which idiosyncratic risk measure is superior OLS (Idio) or EGARCH (Eidio)? Table 2 and Figure 1 report on a test developed here for comparing the relative accuracy of the OLS and EGARCH idiosyncratic risk estimators. For any date  $t$ , betas are estimated using data from periods  $t-60$  to  $t$  under the three factor model. Second, using these estimates the model's squared residual for period  $t$  is calculated. Call this the "true squared residual."<sup>11</sup> Third the OLS and EGARCH models are then estimated to produce period  $t$  forecasts. The OLS model uses data from  $t-61$  to  $t-1$ , while the EGARCH model uses all available date up to period  $t-1$ . From these estimates a forecast for the period  $t$  variance is produced. Fourth, the absolute difference between the true squared residual and each model's idiosyncratic risk forecast is recorded. Note that this four step procedure is designed to strongly favor the OLS model since it is used both in and out of sample and over nearly identical time periods. For the EGARCH model to win this "horse race" it must do a better job of predicting the out of sample realized OLS idiosyncratic risk estimates than the in sample OLS model itself.

Table 2 displays the results from the above procedure. The figures indicate that the EGARCH estimates of idiosyncratic risk are superior to those generated by the OLS model.<sup>12</sup> Overall, the OLS model's absolute prediction errors are on the order of 8% while those from the EGARCH model are about 4%. Comparing the medians produces similar conclusions. The Mann-Whitney rank-sum test and Kolmogorov-Smirnov test both indicate that these differences are statistically significant at any of the usual levels. A parametric t-test not reported here yields the same result. Figure 1 provides further verification. As it shows in 483 out of the 505 sample months the EGARCH model produced a lower average absolute prediction error across stocks

<sup>11</sup> The "true squared residual" is defined as

$$error_{i,t}^2 = \left\{ r_t - \left[ \hat{\alpha}_{i,t} + \hat{\beta}_{i,MKT} (r_{MKT,t} - r_{f,t}) + \hat{\beta}_{i,SMB} SMB_t + \hat{\beta}_{i,HML} HML_t \right] \right\}^2, \text{ where } \hat{\alpha}_t, \text{ and } \hat{\beta}_t$$

are OLS estimates using the previous 60 monthly observations from the Fama-French 3-factor model.

<sup>12</sup> Further tests, discussed later on in the paper, also confirm that Eidio does a superior forecasting job to Idio out of sample. Within sample, but unreported here, Table 3 to Table 5 are qualitatively the same regardless of which idiosyncratic risk measure is used.

than did the OLS model. Perhaps even more tellingly, in 261 months the EGARCH prediction error was less than half that produced by the OLS model.

The relative ranking of the OLS and EGARCH estimates in Table 2 and Figure 1 are consistent with those in Fu (2005) who finds that (in sample) the EGARCH model's estimates have a greater ability to explain cross sectional stock returns than do those from an OLS model. If the market in fact "knows" each security's true idiosyncratic risk then the above arguments indicate that the EGARCH model provides a better representation of those beliefs than does the OLS model. Because of this, except where otherwise indicated, the analysis that follows concentrates on the EGARCH measure's idiosyncratic risk estimates.

### **C. Correlations**

Many studies have shown that market liquidity and size are highly correlated with each other. But what about a security's idiosyncratic risk? Inventory control models such as Merton (1987), and Brunnermeier and Pedersen (2005) predict that there should exist a negative relationship between idiosyncratic risk and liquidity.<sup>13</sup> Empirically, Benston and Hagerman (1974) find that bid-ask spreads in the OTC market are positively correlated with the residual variance from the one factor market model.<sup>14</sup> Also, Stoll (1978) documents a relationship between a firm's return variance and the bid-ask spread on the Nasdaq. Thus, there is good reason to believe that liquidity may be more generally correlated with idiosyncratic risk.

Table 3 sorts stocks by idiosyncratic risk (Panel A), liquidity (Panel B) and size (Panel C) and examines whether or not this produces a similar sort on the other two variables. The results are quite strong. Panel A's sort by idiosyncratic risk produces perfect sorts on both size and liquidity. Since the rank correlations are perfect the p-values associated with these sorts are near

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<sup>13</sup> See the Appendix for a derivation of this result from Merton's (1987) model.

<sup>14</sup> Based upon a hand collected sample of 314 stocks using data from January 1963 through December 1967.

zero. Just as predicted by many theoretical models high idiosyncratic risk firms have low levels of liquidity. Also, it appears that small firms have more idiosyncratic risk than large firms. Thus, at this point at least, one cannot tell if the idiosyncratic risk leads to lower liquidity or if this is a spurious correlation caused by idiosyncratic risk's correlation with size.

Panel B sorts the data by liquidity (using the Gibbs sampler) while Panel C sorts the data by size. Both panels lead to the same conclusions reached by Panel A: size, liquidity, and idiosyncratic risk are highly correlated with each other.

#### ***D. Explaining Liquidity***

Table 4 regresses the Gibbs, Amihud, Amivest, and Gamma liquidity coefficients on a firm's idiosyncratic risk (Eidio), logged market capitalization (lmv), and dollar volume (nyamdvol for the AMEX and NYSE and nasdvol for the Nasdaq).<sup>15</sup> In every case idiosyncratic risk plays a very strong role in a stock's overall liquidity. For every measure other than Gamma the higher a stock's idiosyncratic risk the lower its liquidity (higher Gibbs, and Amihud and lower Amivest). These results are in line with the inventory control models of liquidity, Merton's (1987) limited participation model, and the Brunnermeier and Pedersen (2005) model of markets under funding constraints.

While the Table 4 result that idiosyncratic risk is positively correlated with Gamma seems anomalous it is consistent with Hasbrouck's (2005) findings. This may provide further evidence that stocks with larger Gamma values are in fact *less liquid* than those with lower values. As Hasbrouck (2005) notes (and as confirmed in this paper's Table 1) most of the Gamma estimates are positive. This is contrary to the supposition that Gamma measures liquidity via return reversals and should be negative. Also, Hasbrouck points out that larger Gamma values tend to be associated with stocks that other measures identify as less liquid. Table 4 adds

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<sup>15</sup> The table only reports results from a pooled OLS analysis. Tests were also done using Fama-Macbeth (1973). Since the results are qualitatively identical they are not reported here. Interested readers can contact the authors for copies of the relevant tables.

to these findings. It too shows that larger values of Gamma are associated with stock characteristics that are found in the less liquid stocks as identified by other measures.

The view that smaller values of Gamma imply less liquidity comes from the market microstructure inventory control literature. In such models less liquid stocks exhibit larger return reversals since orders push the price further from its fundamental value to compensate market makers for holding an imbalanced portfolio. However, other models such as Spiegel and Subrahmanyam (1995) (with competitive investors), and Vayanos (2001) (with strategic investors) conjecture that price dynamics are driven by those seeking to liquidate or accumulate large positions. Under this scenario less liquid stocks may produce larger and positive Gamma estimates. Consider, for example, a large investor in Vayanos' model that wishes to liquidate a position. For a liquid stock the entire position can be sold at once; leading to a price process without any serial correlation (a Gamma of zero). Conversely, for an illiquid stock the position will need to be "worked" over time leading to positively serially correlated returns (and a positive Gamma). Thus it is theoretically possible that Gamma is negatively correlated with liquidity.<sup>16</sup>

Returning to Table 4, based on each regression's  $R^2$  statistic, idiosyncratic risk accounts for between a third and a half of the model's explanatory power for both the Gibbs and Gamma measures. Size or volume have approximately as much explanatory power as idiosyncratic risk, but only separately. For example, under the Gibbs Sampler model the  $R^2$  statistic with only idiosyncratic risk in the regression equals .15. Adding size increases it to .27 while adding volume instead increases it to .29. However, adding both size and volume at the same time only brings the  $R^2$  to .31. Overall, size and volume apparently play similar and interchangeable roles when it comes to explaining these two liquidity measures.

For the Amihud and Amivest measures Table 4 indicates that firm size or volume explains most of each measure's value. As discussed earlier, these measures are functions of a

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<sup>16</sup> Determining whether models such as Spiegel and Subrahmanyam (1995) or Vayanos (2001) in fact explain the association between larger Gamma values and lower liquidity as determined by other measures is beyond the scope of this paper and thus will not be further examined here.



stock's absolute return and dollar volume. Thus, it is not surprising that log volume explains a significant fraction of each measure's value. However, the fact that volume explains over 80% of each measure's value is unexpected. Apparently the variability in the absolute return used by these measures plays only a secondary role.

The strong correlation between the Amihud and Amivest measures with volume and size make their interpretation difficult. Are they liquidity or volume measures or proxies for size? The counter claim would be that dollar volume reflects liquidity while size causes liquidity; thus a "good" measure should be highly correlated with these variables. These somewhat philosophical questions are beyond the scope of this paper. However, as later tables will show these measures do not appear to forecast cross sectional stock returns beyond what one can explain from dollar volume and idiosyncratic risk alone.<sup>17</sup>

Finally, Table 4 also shows that beyond size and dollar volume idiosyncratic risk appears to play its own role in explaining the ultimate value of the Amihud and Amivest measures. In every regression idiosyncratic risk comes in significant and of a sign that indicates that higher values lead to lower liquidity levels (positive coefficients for the Amihud and negative for the Amivest measure).

## ***E. Sorted Portfolio Returns***

Table 5 displays the returns from portfolios sorted two factors at a time (a simultaneous sort) using size, idiosyncratic risk, and the Gibbs sampler liquidity measure.<sup>18</sup> The reported returns in this table are from in sample sorts. (Liquidity in year  $t$  is based upon the liquidity

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<sup>17</sup> Because the Amihud and Amivest measures are so closely tied to volume and size subsequent tables generally do not display their corresponding results. Interested readers can obtain these results from the authors. Essentially, the results are qualitatively similar to those reported using dollar volume or size.

<sup>18</sup> An identical analysis was conducted using only NYSE and AMEX stocks. The results are qualitatively identical and any quantitative differences are minor as well. Table 5's results were also checked against the possibility that sparsely populated cells might account for the observed patterns. A table with the average number of stocks per cell per month shows that in fact all of the cells are well populated. Tables regarding all of the issues described in this footnote are available upon request.

parameter using year  $t$  data. The EGARCH idiosyncratic risk estimates use the entire available time series to estimate the model.) The purpose of this table is to see the degree to which particular parameters are associated with particularly high or low concurrent returns.

Table 5 Panel A examines the impact of size and idiosyncratic risk. Generally, studies find a negative monotonic relationship between firm size and return. However, as the panel shows this result is reversed once idiosyncratic risk is accounted for. In every case, holding idiosyncratic risk constant, the large capitalization stocks return more than the small capitalization stocks.<sup>19</sup> This result is consistent with Brennan et al. (1998) who reach a similar conclusion but instead control for volume.

In Table 5 the rank correlations between size and return are also dramatic; controlling for idiosyncratic risk larger firms have higher returns. How can all ten idiosyncratic risk deciles in Table 5 show higher returns for the largest firms relative to the smallest ones, when unconditionally small firms have higher returns? The answer lies in Table 3. Size and idiosyncratic risk are strongly negatively correlated. Thus, a sort on size is similar to a sort on idiosyncratic risk. What appears to be happening is that by sorting only on size the impact of idiosyncratic risk on returns dominates the results. Because Table 5 Panel A separates out these factors the size effect apparently reverses itself.

In terms of offering evidence on the extant theory perhaps the most telling pattern in Table 5 Panel A is the interaction between size and idiosyncratic risk on returns. In mathematical terms it appears that the cross derivative of returns on size and idiosyncratic risk is positive ( $\partial^2 r / \partial \text{Size} \partial \text{Eidio} > 0$ ). As shown in this paper's Appendix, this prediction can be derived from Merton's (1987) model even though his paper does not do so. This is telling since it may be the first time a prediction not actually stated in Merton's paper but still an implication of his model has been tested and verified.

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<sup>19</sup> Amihud and Mendelson's (1989) do not find a similar reversal of the size effect in their test of Merton's (1987) model. However, they used the standard deviation of daily returns as their proxy for idiosyncratic risk while this paper uses the EGARCH model's forecast.

Panel B does a double simultaneous sort on size and the Gibbs liquidity measure. Once again the size effect reverses itself.<sup>20</sup> In each case the Spearman rank correlations are significant at the 1% level for all but liquidity deciles 3, 9 and 10. For those the rank correlations are significant at the 5% level. While the rank correlations between decile and size vary from one liquidity decile to the next there is no real pattern (see the last column as an indicator of this). Reversing the analysis, there seems to be almost no relationship between liquidity and returns (the bottom row). Except for the two smallest size deciles the rank correlation between returns and liquidity are not significant at the usual levels.

Table 5 Panel B shows that the impact of liquidity on returns appears to vary across size classes but not in a particularly consistent manner. Consider what happens as one goes from small to large capitalization stocks. Initially, the illiquid stocks (decile 10) have lower returns than the more liquid stocks. Then as one moves across size deciles, the illiquid stocks have relatively higher returns and then lower returns compared to the more liquid stocks. Size however has the same impact across liquidity deciles; big firms have higher returns than small firms.

Table 5 Panel C finishes the two way comparisons this time between liquidity and idiosyncratic risk. High idiosyncratic risk firms produce higher returns in all ten liquidity deciles. Liquidity also produces very consistent results. Holding idiosyncratic risk constant in every case the illiquid stocks yield *lower* returns than the liquid stocks. A Spearman rank correlation test shows that this result comes not just from the extreme deciles. Rather holding idiosyncratic risk constant sorting on liquidity also tends to sort returns.<sup>21</sup>

What may be causing the inverse relationship between liquidity and returns is the fact that the Table 5 tests are all done in sample; a relationship that can also be found in the in sample

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<sup>20</sup> Brennan, Chordia and Subrahmanyam (1998) also find that controlling for dollar volume large capitalization stocks have higher returns than small capitalization stocks.

<sup>21</sup> A positive relationship between in sample liquidity and returns can also be found in the time series analysis of Amihud (2002), and Pastor and Stambaugh (2003).

tests of Amihud (2002), and Pástor and Stambaugh (2003). As noted in the introduction, conventional wisdom on Wall Street appears to be that when a security's value rises its liquidity increases. Conversely, right after a price drop liquidity falls. Table 5's results may simply reflect this. This is not the same as saying the *expected* return on a stock varies inversely with its liquidity. To test whether or not that is true out of sample tests are needed and the paper now turns to them.

### 3. Out of Sample Returns

#### A. Trading Strategies

To examine whether idiosyncratic risk indeed has predictive power in explaining cross sectional returns Table 6 examines a trading strategy based on both EGARCH and OLS out of sample idiosyncratic risk estimates. First, individual stocks are sorted into 10 value weighted portfolios based on the current month's forecasted idiosyncratic risk. This portfolio is then held for 1 month and rebalanced the next month. At each month  $t$ , the OLS estimates are based upon the previous 60 monthly return observation while the EGARCH model uses all data up to month  $t-1$ . Residuals, alphas, and betas are calculated via the Fama-French 3 factor model.

The last column in Table 6 displays the OLS idiosyncratic risk measure's ability to predict stock returns as measured by the three factor model's alpha. There is nearly no pattern. The Spearman rank correlation coefficient is near zero. Only the highest idiosyncratic risk decile indicates that the OLS model might have any out of sample predictive power. Overall though, out of sample, the OLS model's measure of idiosyncratic risk does little to help predict stock returns.<sup>22</sup>

The out of sample forecast results using the EGARCH model can be found in Table 6's columns six, seven and eight. Unlike the OLS case the sorts are now nearly perfect. Stocks with

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<sup>22</sup> Unlike the OLS results reported here, Ang et al. (2005) and Fu (2005) find that the linear model's forecast of idiosyncratic risk is correlated with returns. Both studies use daily data from month  $t$  to predict idiosyncratic risk in month  $t+1$ .

low levels of predicted idiosyncratic risk produce low returns while those with high levels produce high returns. The Spearman rank correlation coefficients are significant at the 5% level for all three columns and the 1% level for both the CAPM and Carhart-4 columns.<sup>23</sup>

Some intuition regarding the portfolios used to create Table 6's figures can be gleaned from their Sharpe ratios and their Goetzmann, Ingersoll, Spiegel and Welch (2004) manipulation proof measure's value.<sup>24</sup> The monthly Sharpe ratio for the market portfolio over the sample period equals .083. In contrast the high idiosyncratic risk and low idiosyncratic risk portfolios produce Sharpe ratios of .167 and .004 respectively. Using the manipulation proof measure yields an identical ranking. The market portfolio's value equals 1.0071 while the high and low idiosyncratic risk portfolios produce 1.0124 and 1.0003 respectively. These figures imply that compared to the overall market the high idiosyncratic risk portfolio is not that risky relative to the returns it produces. On the other hand, the low idiosyncratic risk portfolio produces a substantially lower score and certainly would make a poor stand alone investment.

Table 7 Panel A examines the out of sample relationship between the Gibbs sampler liquidity measure and stock returns. Unlike the in sample results in Table 5 liquidity is now weakly negatively correlated with returns. However, the relationship is modest at best. Yes, the least liquid stocks have higher alphas than the most liquid stocks under the one, three, and four factor models. However, the difference is not statistically significant under either the three or four factor model. Also the Spearman rank correlation between the deciles and returns is not significant at any reasonable level under any of the factor models and is even negative for the three factor model.

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<sup>23</sup> As noted earlier these results are consistent with those of Lehmann (1990) who, using monthly data, also found that returns increase in idiosyncratic risk. In comparison, using daily others have come to conflicting conclusions. Ang et al. (2005) find that idiosyncratic risk is negatively correlated with returns, while Fu (2005) finds that they are positively correlated.

<sup>24</sup> Goetzmann, et al. (2004) show that the manipulation proof measure eliminates the ability of a fund to "game" its score via the use of time varying volatilities or other mechanisms that might distort its return distribution. The measure has one free parameter and the paper recommends setting it to two, which is the value used to derive the results reported here.

Given the weak results in Table 7 Panel A, additional tests were run controlling for other factors to see if some type of interaction was preventing liquidity from influencing future returns. Panel B displays the returns from holding portfolios based on liquidity after first sorting on size. Once again the results are mixed. Controlling for size illiquid stocks have higher returns for the smallest firms but apparently lower returns for those in the fourth size quintile. The “Control for size” row in Panel B looks at the returns across liquidity deciles for a portfolio that is equally weighted across size quintiles. For these portfolios, while the low (decile ten) liquidity stocks have higher returns than high liquidity stocks (decile one) the t-statistic for their difference is only 1.84. Furthermore, a decrease in liquidity does not monotonically increase returns.

Controlling for idiosyncratic risk, Panel C in Table 7 further clouds the question of whether liquidity influences returns. For the high idiosyncratic risk quintile the low liquidity stocks (decile ten) appear to produce much higher returns than the high liquidity stocks (decile one). But this relationship is reversed in all of the other idiosyncratic risk quintiles. Furthermore, the return patterns as one goes from high to low liquidity stocks are not necessarily monotonic. The rank correlation coefficients are negative for the first four idiosyncratic risk quintiles. Except for the third quintile the Spearman rank correlation coefficients are statistically significant at the 5% level but as noted above are of different signs.<sup>25</sup> Overall, controlling for idiosyncratic risk it seems difficult to come to any general conclusions regarding liquidity’s ability to forecast stock returns.

Table 8 creates portfolios based upon dollar volume using data from the NYSE and AMEX only. This was done because Nasdaq volume may not be directly comparable.<sup>26</sup> In Panel A, the p1–p10 portfolios all exhibit positive returns indicating that high volume securities have lower future returns than low volume securities. For the CAPM alphas the rank correlation

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<sup>25</sup> For the third quintile the rank correlation coefficient is not significant at the usual levels.

<sup>26</sup> We thank Yakov Amihud for suggesting we include this table in the analysis. A similar table was also created using the entire data set (AMEX, NYSE, and Nasdaq). The results are qualitatively identical and are available upon request from the authors.

across deciles is significant at any reasonable level. However, for the three and four factor models the rank correlation p-values are only 9.8% and 10.8% respectively. Panel B repeats the analysis but now controls for several other variables. Note that even after controlling for idiosyncratic risk the p1–p10 portfolios yield positive and statistically significant returns in every row other than the lowest idiosyncratic risk quintile. In terms of the rank correlation coefficients controlling for idiosyncratic risk appears to help volume sort the out of sample alphas. For all five idiosyncratic risk quintiles and the “Control for Eidio” row the p-values are well below 1%.

Table 6 shows a strong relationship between future cross sectional returns and forecasted idiosyncratic risk. In contrast Table 7 shows almost no relationship between the Gibbs cost based liquidity measure and future stock returns. It is possible, however, that the difference in each variable’s predictive power comes from how frequently they are estimated relative to the holding periods. The idiosyncratic risk estimates in Table 6 are updated monthly and the holding periods are also limited to one month. However, Table 7 uses annual liquidity estimates and a one year holding period. To see if the different periodicities drive the results Table 9 repeats the analysis in Table 6 but this time with annual holding periods. The results are essentially unchanged. There remains a strong positive relationship between idiosyncratic risk and future returns.

Comparing Panel A in Table 8 with Table 9 indicates that while both dollar volume and idiosyncratic risk influence returns, the latter’s appears to be economically much more significant. In Table 8 the three and four factor returns from the p1–p10 portfolios return 3.81% and 3.06% respectively. Contrast this with the 25.63% and 25.90% returns in Table 9 under the same factor models. Also, note that while the three and four factor models considerably reduce the unexplained return from the p1–p10 strategy using volume (dropping it by about two-thirds) it has a negligible impact on the unexplained returns using from the same strategy using idiosyncratic risk.

Table 10 provides the results from a series of sequential sorts first on either size or the Gibbs liquidity measure and then idiosyncratic risk with monthly rebalancing. It provides a direct

contrast with the same exercise that was done with the sequential sorts in Table 7 Panel B where the second sort was on liquidity. While the sequential sorts in Table 7 Panel B fail to produce any consistent patterns, this is not true of Table 10. Under all controls, higher levels of idiosyncratic risk forecast higher returns. This relationship is independent of whether one first sorts on size, liquidity, or volume.<sup>27</sup> To the degree that the pattern changes cross sectionally it is that it is less pronounced for large capitalization stocks than for small capitalization stocks. Nevertheless, it remains statistically significant in all size groups. Liquidity, on the other hand, appears to have little or no impact on idiosyncratic risk's ability to forecast cross sectional returns.

## **B. Out of Sample Regression Analysis**

So far the sorted returns imply that while idiosyncratic risk influences expected stock returns liquidity does not. It is possible that the essentially univariate and bivariate sorts somehow hide liquidity's impact and overstate the impact of idiosyncratic risk. To determine the degree to which idiosyncratic risk, liquidity, size, lagged returns and dollar volume explain cross sectional variation in stock returns the approach proposed by Brennan, Chordia and Subrahmanyam (1998) (BCS) is used. Under the BCS method the first step estimates the risk adjusted return (alpha) relative to a multi-factor pricing model. Here, in each month, factor loadings are estimated using the previous 5-years of data via the three factor model. Thus, the out

of sample risk adjusted return ( $\hat{\alpha}_{i,t}$ ) equals:

$$\hat{\alpha}_{i,t} = (r_{i,t} - r_{f,t}) - \left( \hat{\beta}_{i,MKT} (r_{MKT,t} - r_{f,t}) + \hat{\beta}_{i,SMB} SMB_t + \hat{\beta}_{i,HML} HML_t \right). \quad (4)$$

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<sup>27</sup> Displayed are the results using either the Gibbs or Gamma liquidity measures but the same qualitative results were produced with Amihud's measure as well. The estimates controlling for volume were also conducted using only NYSE and AMEX stocks to remove the potential problems associated with differences in how they and the Nasdaq report volumes. The results are not only qualitatively identical but quantitatively very similar and thus for the sake of brevity are not displayed.



The second step of the algorithm runs a Fama-Macbeth type of regression. For each fixed  $t$ , the risk adjusted returns for each individual stock are regressed against the set of characteristics ( $Z_{i,t}$ ):

$$\hat{\alpha}_{i,t} = c_t Z_{i,t} + e_{i,t} \quad (5)$$

The vector  $\hat{\alpha}_t$  in (5) denotes the OLS estimates from the time series regression (4) while  $c_t$  is the coefficient to be estimated in (5). The average of the time series ( $T^{-1} \sum_{t=1}^T \hat{c}_t$  assuming there are  $T$  time periods) is then used to summarize the time series properties of each characteristic. As Brennan et al. (1998) argue the traditional errors-in-variable problem of the beta estimates will not affect the cross-sectional analysis here.

Table 11 reports separate in and out of sample tests following the BCS approach described above for the Gibbs, Gamma, and Amihud liquidity measures. This is done to see if the choice of measure does or does not influence the final conclusions. In Panels A, C, and E the cost based liquidity estimates are in sample values. Thus, the year  $t$  cost based liquidity estimate uses year  $t$  data. For Panels B, D, and F the cost based liquidity estimates are out of sample. In all six panels the idiosyncratic risk and volume estimates are out of sample.

Table 11's Panels A, C, and E show that in sample less liquid stocks (using cost based measures) exhibit lower returns than more liquid stocks.<sup>28</sup> This result is robust across specifications and is consistent with the earlier result using characteristic sorted portfolios. Also in line with the paper's earlier results higher idiosyncratic risk forecasts are associated with higher returns.

Panels B, D and F in Table 11 repeat the above analysis but now use the out of sample liquidity estimates. Out of sample, cost based liquidity only forecasts returns when idiosyncratic

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<sup>28</sup> This interpretation of Panel E's figures depends on whether larger values of Gamma reflect higher or lower levels of liquidity. This paper argued previously that on both empirical and theoretical grounds it appears that Gamma decreases with liquidity. It is under this interpretation of Gamma that Panel E confirms that findings in Panels A and C that less liquid stocks have lower in sample returns than more liquid stocks.

risk is excluded from the regression. The one partial exception is the Amihud measure so long as dollar volume is excluded. When both idiosyncratic risk and dollar volume are included the Amihud measure changes sign and becomes statistically insignificant. Idiosyncratic risk on the other hand remains significant no matter what other variables are included.

In all of the cases examined within Table 11 higher dollar volumes forecast lower future returns. However, like the portfolio strategies reported in Section 3.A the impact of dollar volume on returns is economically small compared to the impact of idiosyncratic risk. Consider the Table 11 results when the cost based liquidity estimators are not included but both idiosyncratic risk and volume are. Combining the estimated coefficients with Table 1's statistics implies that a one standard deviation change in idiosyncratic risk (0.0986) changes expected returns by approximately 26% per year (a monthly return of  $0.0986 \times 0.1876$ ). In contrast, a one standard deviation change in NYSE volume (2.42) changes expected returns by only about 9% per year (a monthly return of  $2.42 \times 0.0033$ ) and a one standard deviation change in Nasdaq volume (3.09) changes expected returns by about 6.5% per year (a monthly return of  $3.08 \times 0.0018$ ). Thus, a one standard deviation change in idiosyncratic risk has about 2.8 (NYSE-AMEX) or 4.0 (Nasdaq) times the impact of a similar change in volume.

## 4. Related Literature

The idea that idiosyncratic risk may influence stock returns has been generated within a variety of theoretical settings. Merton (1987) and Malkiel and Xu (2002) demonstrate idiosyncratic risk will impact expected returns in equilibrium if investors only trade in subsets of the available assets. Ou-Yang (2004) develops an integrated model of asset pricing and moral hazard and demonstrates that the equilibrium price is positively correlated with idiosyncratic risk. Jones and Rhodes-Kropf (2004) extend the standard principal-agent problem between investors and venture capitalists to show how and why diversifiable risk should be priced in venture capital deals even though investors are fully diversified. Barberis and Huang (2001) find that if agents

use rules of thumb related to mental accounting and loss aversion then idiosyncratic risk will be positively correlated with expected returns. Brunnermeier and Pedersen (2005) generate this relationship within a model where investors face margin requirements that limit their ability to maintain levered positions when stock prices turn downward.

Undoubtedly the above literature has both encouraged and been in response to the growing empirical literature relating idiosyncratic risk to stock returns. Lintner (1965) and Douglas (1968) find that idiosyncratic risk seems to explain some of the cross sectional variation in stock returns. However, Miller and Scholes (1972) note that the positive skewness within individual stock returns can lead, within the Lintner and Douglas empirical methodologies, to the spurious conclusion that higher idiosyncratic risk levels lead to higher returns. Lehmann (1990) then corrects for various econometric problems and once again finds that stock returns are increasing in idiosyncratic risk. More recently, Xu and Malkiel (2002) and Fu (2005) look cross sectionally at stocks and find (like Lehmann) that higher idiosyncratic risk stocks appear to have higher returns. Baker and Wurgler (2005) find that conditional on investor sentiment idiosyncratic risk can be positively or negatively correlated with the expected return. Switching to daily data Ang et al. (2005) come to the opposite conclusion of the studies using monthly data and conclude that stocks with the greatest levels of idiosyncratic risk produce the lowest returns.

At the macro level the evidence regarding the impact of idiosyncratic risk on cross sectional returns is also mixed. Goyal and Santa-Clara (2003) find that an increase in the average variance across stocks (largely idiosyncratic risk) leads to an increase in the expected return on the market index. Bali, Cakici, Yang, and Zhang (2004) argue that Goyal and Santa-Clara (2003)'s result is mostly driven by small Nasdaq stocks and is partly due to a liquidity premium. In response, Ghysels, Santa-Clara and Valkanov (2004) use a mixed data sampling approach and again find a significantly positive relationship between risk and return. Jiang and Lee (2005) show that after correcting for serial correlation in market level idiosyncratic volatility, there is a significant positive effect of idiosyncratic volatility on market return. However, Ang et al. (2005)

and Guo and Savickas (2004) find a significant negative relation between market level idiosyncratic risk and return.

The theoretical literature on whether or not liquidity should influence stock returns in an economically meaningful way has gone through several stages. Initially, papers such as Constantinides (1996), Heaton and Lucas (1996), and Vayanos (1998) argued that transactions costs are like a per trade tax and that investors can minimize its impact by just trading illiquid assets relatively infrequently. O'Hara (2003) however argues that because liquidity and asymmetric information are tied together, liquidity can have a large and economically significant impact on expected returns beyond what one might expect from viewing it as simply a "transactions cost" similar to a tax. This is somewhat along the lines of Subrahmanyam (1991). That paper shows that while a basket of securities may only be minimally impacted by asymmetric information in the market place the underlying securities may be strongly impacted by such issues.

Complementing the theoretical arguments arguing in favor of liquidity as a priced factor have been the empirical papers on the subject. Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Brennan et al. (1998), Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2004), Baker and Stein (2004), Hasbrouck (2005) all find returns decline with increased liquidity. These authors by and large agree that liquidity is either priced as a cross sectional characteristic, a risk factor over time, or maybe both. One conclusion that has arisen from this literature is that time varying liquidity changes tend to impact a large cross section of stocks simultaneously. This implies that liquidity may be priced because it will be unavailable when investors need it to convert any or all securities to cash. If in fact liquidity ebbs and flows across all securities simultaneously this may mitigate somewhat against the idea in Constantinides (1996), Heaton and Lucas (1996), and Vayanos (1998) that investors can avoid such costs by simply restricting their trades to the most liquid assets and rebalancing their portfolios less frequently.

## 5. Conclusion

This paper examines the relationship between stock returns, liquidity, and idiosyncratic risk. The findings strongly support the idea that liquidity and idiosyncratic risk are closely intertwined variables. High idiosyncratic risk firms tend to be those with the least liquidity. Merton's (1987) model predicts this should occur as do the inventory control models within the market microstructure literature.

Past research indicates that both idiosyncratic risk and liquidity influence stock returns. The goal of this paper has been to attempt to disentangle their individual affects. This paper confirms that either variable alone can explain some of the observed cross sectional variation in stock returns. However, when both are used simultaneously only idiosyncratic risk and liquidity as measured by dollar volume provides any out of sample explanatory power. In contrast, idiosyncratic risk appears to play a useful role regardless of what other variables one includes as controls.

In the end should one conclude from this paper that cost based liquidity measures play little or no role in expected stock returns? In the very unlikely event that this is true it is certainly too soon to reach such a conclusion. Liquidity is both difficult to define and measure. It is possible that there exist as yet untested or even undiscovered cost based liquidity measures that add explanatory power beyond that available from idiosyncratic risk forecasts. Furthermore, dollar volume does seem to influence cross sectional returns and it is if not itself liquidity then it is at least a partial reflection of it. Another possibility is that with better data even the current measures will produce superior out of sample forecasts. Looking forward, one can also view this paper as providing a way to test any new liquidity measure. By repeating the analysis presented here one can isolate those liquidity measures which impact expected returns from those that primarily result from market returns or reflect the influence of idiosyncratic risk.

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## 6. Appendix

What follows is drawn from Merton (1987) and follows his notation. There are  $n$  firms in the economy each of which produces an end of period return  $\tilde{R}_k$

$$\tilde{R}_k = \bar{R}_k + b_k \tilde{Y} + \sigma_k \tilde{\varepsilon}_k \quad (6)$$

where  $\tilde{Y}$  is the market factor's return,  $b_k$  the firm's loading on the market,  $\tilde{\varepsilon}_k$  the firm's idiosyncratic risk, and  $\sigma_k$  a constant. While many investors cannot trade every asset, they all have access to a risk free asset and a "market risk" asset (designated asset  $n+1$ ) that pays  $\tilde{Y}$ . Both the risk free and market risk assets are assumed to be in zero net supply.

There are a total of  $N$  investors, where  $N$  is large enough that each acts as a price taker. Investor  $j$  has a mean-variance utility function with identical risk aversion parameter  $\delta$ . Solving for each investor's optimal portfolio and the market clearing conditions yields expected equilibrium returns of  $\bar{R}_k$  for security  $k$  of

$$\bar{R}_k = R + b_k b \delta + \delta x_k \sigma_k^2 / q_k \quad (7)$$

where  $R$  is the risk free rate,  $b$  the market risk exposure in each investor's portfolio,  $q_k$  equals the fraction of the population that can invest in asset  $k$ , and  $x_k$  equals firm  $k$ 's size. The firm's size ( $x_k$ ) comes into the equilibrium equation via:

$$x_k = q_k \Delta_k / \delta \sigma_k^2. \quad (8)$$

where the  $\Delta_k$  term arises from the Lagrange multiplier in the investor's optimization problem that arises from the restriction that he cannot trade in all securities and equals

$$\Delta_k = \bar{R}_k - R - b_k (\bar{R}_{n+1} - R). \quad (9)$$

At this point equation (7) can be used to develop a number of comparative statics in order to generate the model's cross sectional predictions.

Define  $\psi(y)$  as the change in the log excess return over the risk free rate of an asset with respect to the log of some parameter  $y$  ( $d \log \left| \bar{R}_k - R \right| / d \log(y)$ ). Then (7) can be used to yield:

$$\psi(b_k) = q_k b_k b / (q_k b_k b + x_k \sigma_k^2) > 0, \quad (10)$$

$$\psi(x_k) = x_k \sigma_k^2 / (q_k b_k b + x_k \sigma_k^2) > 0, \quad (11)$$

$$\psi(\sigma_k^2) = \psi(x_k) > 0, \quad (12)$$

$$\psi(q_k) = -\psi(x_k) < 0. \quad (13)$$

These equations can be interpreted to produce the following empirical restrictions:

- (10) security returns are increasing in a firm's exposure to market risk,
- (11) security returns are increasing in a firm's size,
- (12) security returns are increasing in idiosyncratic risk, and
- (13) security returns decline in the number of investors that can trade it.

While the above implications can all be found in Merton (1987) other results can be drawn from both equation (11) and by differentiating (12) with respect to firm size ( $x_k$ ).

Typically, liquidity is interpreted as the change in return with respect to the change in supply. In this case, one can interpret the firm's "size" as supply (for example, the value held by market participants net of current noise trader purchases and sales) and then view (11) as the expected change in return with respect to supply. Thus, high values of (11) imply low levels of liquidity. Plugging this back into (7) then yields the result that expected returns decrease as the stock's liquidity increases.

Next, consider what happens to the equilibrium when one differentiates (12) with respect to firm size ( $x_k$ ). After performing this exercise one finds that:

- For large capitalization stocks the (positive) change in expected returns with respect to idiosyncratic risk is larger than for small capitalization stocks.
- Since the  $x_k$  and  $\sigma_k^2$  enter multiplicatively with each other the above also implies that the (positive) change in expected returns with respect to liquidity is larger for firms with more idiosyncratic risk.

Table 1: Summary Statistics

The summary statistics represent the time-series averages of the month by month cross-sectional means for each variable. There average 3493 stocks in each month over 504 months from January 1962 to December 2003. All variable estimates are taken from Joel Hasbrouck's web page at <http://pages.stern.nyu.edu/~jhasbrou/>.

**Alpha:** out of sample alpha for the current month based upon the FF-3 factor model with factor loadings estimated over the previous years.

**Eidio (s.d.):** conditional standard deviation estimates of the Fama-French 3-factor model by using EGARCH.

**Eidio (var.):** conditional variance estimates of the Fama-French 3-factor model by using EGARCH.

**Gibbs Sampler:** Hasbrouck (2005) Gibbs Sampler estimates of effective trading costs using the return under Roll's (1984) model ( $c^{\text{Gibbs}}$  in Hasbrouck's (2005) notation).

**Gamma:** Pastor and Stambaugh (2003) reversal measure.

**Amihud:** Log of the average daily absolute return over the dollar volume for one year.

**Amivest:** Log of the average daily dollar volume over the absolute return for one year.

**Firm Size:** natural logarithm of the stock price times the number of shares outstanding.

**Dvol:** natural logarithm of the dollar volume of trading in the security

**Retlag23:** natural logarithm of the cumulative return over the two months ending at the beginning of the previous month.

**Retlag46:** natural logarithm of the cumulative return over the three months ending three months prior to the current month.

**Retlag712:** natural logarithm of the cumulative return over the six month period ending six months prior to the current month.

Variables	Mean	Median	Std. Dev.
Sub-Sample Used in this Study			
Alpha	0.0015	0.0002	0.0144
Eidio (s.d.)	0.0906	0.1069	0.1562
Eidio (var.)	0.0291	0.0187	0.0986
Gibbs Sampler	0.0129	0.0108	0.0051
Gamma $\times 10^3$	0.0323	0.0211	0.0398
Amihud	5.8960	5.8302	0.7605
Amivest	-4.8491	-4.9459	1.0483
Firm Size	11.2078	11.0224	1.9908
nyamdvol	6.8507	5.9605	2.4201
nasdvol	3.0837	4.3166	3.0867
Retlag23	0.0050	0.0075	0.2121
Retlag46	0.0074	0.0140	0.2558
Retlag712	0.0150	0.0150	0.3638
All Available Data			
Gibbs Sampler	0.01183	0.01106	0.00482
Gamma $\times 10^3$	0.00068	0.01442	0.22735
Amihud	6.06993	6.09967	0.80847
Amivest	-5.08215	-5.21612	1.11807

Table 2: Comparison of Eidio and Idio

This table compares the absolute difference between the OLS and EGARCH estimates of the expected idiosyncratic risk and the out of sample idiosyncratic risk. Stocks are included in month  $t$  if CRSP provides 60 monthly returns over period  $t-61$  to  $t$ . Each model is run during training period and then used to forecast a stock's idiosyncratic risk in period  $t+1$ . The OLS model's training period data goes from  $t-60$  to  $t-1$  while the EGARCH model uses all available data up to  $t-1$ . The "true error" is defined as the period  $t$  residual from the three factor OLS model run with data from period  $t-60$  to  $t$ . The forecast error, defined as the difference between the squared forecast variance and the true error squared, is then recorded. This process is then repeated until the end of the sample period. The P-Values are for the hypothesis that the two forecast error distributions are the same.

	mean	median	Std Dev	P-Value	
				Mann-Whitney Test	Kolmogrov-Smirnov Test
Eidio_Difference	0.0425	0.0381	0.0284	0.00	0.00
Idio_Difference	0.0758	0.0763	0.0276		
Number of Obs.	1754327	1754327			

Table 3: The relationship between idiosyncratic risk, liquidity and size.

In each month value-weighted portfolios sorted by Eidio, Gibbs and Size are created. The time series cross sectional average and standard deviation of the other two characteristics for each portfolio are reported in the columns labeled Mean and Std. Dev. The sample period covers January 1962 to December 2003. Mean columns with + (-) and ++ (-) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Panel A: Portfolios sorted by Idiosyncratic Risk				
Rank	Size		Gibbs Sampler	
	Mean <sup>-</sup>	Std. Dev.	Mean <sup>++</sup>	Std. Dev.
1 (Low)	13.16	0.6492	0.0031	0.0009
2	12.52	1.6010	0.0035	0.0014
3	12.28	1.6505	0.0040	0.0016
4	12.31	1.5931	0.0047	0.0021
5	12.00	1.6088	0.0055	0.0024
6	11.73	1.6358	0.0066	0.0029
7	11.64	1.6732	0.0068	0.0032
8	11.70	1.7624	0.0069	0.0037
9	11.01	0.7498	0.0080	0.0045
10 (High)	9.96	0.7001	0.0124	0.0055

  

Panel B: Portfolios sorted by Hasbrouck's (2005) Gibbs Sampler				
Rank	Size		Eidio	
	Mean <sup>-</sup>	Std. Dev.	Mean <sup>++</sup>	Std. Dev.
1 (Low)	12.79	0.7540	0.07561	0.02438
2	11.32	1.2408	0.05883	0.00783
3	10.96	1.3003	0.06678	0.00771
4	10.84	1.3258	0.07337	0.01356
5	10.77	1.4735	0.08014	0.01730
6	10.64	1.3541	0.08465	0.02118
7	10.45	1.3780	0.09286	0.02634
8	9.88	1.3342	0.10590	0.03298
9	9.01	1.0155	0.11892	0.02752
10 (High)	8.20	0.6462	0.13467	0.02440

  

Panel C: Portfolios sorted by Size				
Rank	Gibbs		Eidio	
	Mean <sup>-</sup>	Std. Dev.	Mean <sup>-</sup>	Std. Dev.
1 (Low)	0.0260	0.0148	0.16970	0.04423
2	0.0165	0.0091	0.15162	0.03290
3	0.0125	0.0066	0.13683	0.03123
4	0.0100	0.0052	0.13271	0.02650
5	0.0079	0.0040	0.12429	0.03062
6	0.0064	0.0028	0.11523	0.02987
7	0.0052	0.0018	0.10662	0.02634
8	0.0041	0.0011	0.09371	0.02310
9	0.0033	0.0008	0.08312	0.01828
10 (High)	0.0026	0.0009	0.06212	0.01124

Table 4: Liquidity and Idiosyncratic Risk

Idiosyncratic risk is measured as the conditional volatility of the residual from the FF-3 factor model. Firms are included if they have at least 60 months of return data. Eidio represents the idiosyncratic risk, lmv represents the natural log of price times shares outstanding, nyamdvol equals the natural log of the price times the trading volume of the NYSE and AMEX stocks, it equals zero for all NASDAQ stocks, nasdvol equals the natural log of the price times the trading volume of the NASDAQ stocks, it equals zero for all NYSE and AMEX stocks. All of reported regression results use pooled OLS regressions. The sample starts from July 1962 and ends in December 2003. Robust Newey-West (1987) t-statistics are reported in square brackets for Pooled OLS regression.

Pooled OLS with Robust Newey-West T-statistics					
Measure	Eidio	lmv	nyamdvol	nasdvol	Adjusted R <sup>2</sup>
Gibbs Sampler	0.0058***	-0.0026***	-0.0017***	-0.0010***	0.3074
	[20.82]	[-37.79]	[-33.19]	[-20.47]	
	0.0066***		-0.0035***	-0.0026***	0.2874
	[20.93]		[-43.25]	[-39.41]	
	0.0059***	-0.0046***			0.2744
	[17.28]	[-46.47]			
	0.0087***				0.1479
	[46.20]				
Gamma*1000	0.0342**	-0.0113***	-0.0076***	-0.0083***	0.0026
	[2.48]	[-4.66]	[-3.01]	[-3.37]	
	0.0380**		-0.0154***	-0.0154***	0.0025
	[2.25]		[-7.76]	[-7.03]	
	0.0295**	-0.0200			0.0024
	[2.12]	[-8.36]			
	0.0411**				0.0012
	[2.32]				
Amihud	0.6948***	-0.2795***	-0.2964***	-0.2875***	0.8889
	[62.17]	[-92.81]	[-75.18]	[-83.92]	
	1.4152***		-0.4853***	-0.4606***	0.8633
	[21.41]		[-76.39]	[-84.71]	
	0.2156***	-0.0048***			0.8291
	[15.98]	[-45.63]			
	2.8440***				0.0538
	[96.34]				
Amivest	-1.0978***	0.5957***	0.6263***	0.6421***	0.8856
	[-45.63]	[90.87]	[70.10]	[94.44]	
	-2.6330***		1.0289***	1.0110***	0.8596
	[-104.89]		[63.36]	[78.53]	
	-1.2039***	1.3594***			0.8191
	[-42.20]	[37.25]			
	-5.3102***				0.0415
	[-84.07]				

Table 5: Average In Sample Returns For Portfolios Formed on Size, Eldio and Gibbs

Portfolios are formed monthly. Stocks are sorted into 10 portfolios based on Size, EGARCH estimated idiosyncratic risk (Eldio) and Gibbs Sampler estimates of effective cost respectively. Pair-wise portfolios are formed accordingly. The average return is the time-series average of the monthly value-weighted portfolio (simple) return. Column and row '10-1' represent to the difference in monthly returns between portfolio 10 and portfolio 1. Robust joint tests for the return difference between portfolio 10 and portfolio 1 equal to zero are all less than 1% for all cases. The sample period is January 1962 to December 2003. Columns and rows with + (-) and ++ (-- ) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Panel A: Average Monthly Returns for Portfolios formed on Eldio and Size											
	ME1 <sup>-</sup>	2	3	4	5	6	7	8	9	ME10 <sup>++</sup>	ME1-ME10 <sup>-</sup>
Eldio-Low <sup>++</sup>	-0.96%	-0.12%	0.05%	0.14%	0.37%	0.47%	0.65%	0.73%	0.77%	0.89%	-1.84%***
2 <sup>++</sup>	-1.46%	-0.69%	-0.21%	0.18%	0.32%	0.42%	0.66%	0.74%	0.79%	1.02%	-2.48%***
3 <sup>++</sup>	-1.75%	-0.96%	-0.51%	0.12%	0.26%	0.32%	0.49%	0.81%	0.77%	1.13%	-2.88%***
4 <sup>++</sup>	-2.19%	-1.15%	-0.41%	-0.21%	0.30%	0.42%	0.67%	0.66%	0.86%	1.14%	-3.34%***
5 <sup>++</sup>	-2.43%	-1.50%	-0.94%	-0.44%	-0.05%	0.41%	0.48%	0.72%	0.81%	1.33%	-3.75%***
6 <sup>++</sup>	-2.85%	-1.67%	-1.18%	-0.52%	-0.01%	0.07%	0.62%	0.67%	0.89%	1.76%	-4.61%***
7 <sup>++</sup>	-3.37%	-1.73%	-0.77%	-0.83%	0.16%	0.38%	0.44%	0.68%	1.27%	1.89%	-5.26%***
8 <sup>++</sup>	-4.01%	-2.16%	-1.47%	-0.54%	-0.34%	0.21%	0.64%	0.88%	1.51%	2.62%	-6.63%***
9 <sup>++</sup>	-4.60%	-2.22%	-0.96%	-0.79%	-0.11%	0.48%	0.75%	1.04%	1.94%	3.55%	-8.15%***
Eldio-High <sup>++</sup>	-3.39%	0.04%	0.74%	1.59%	2.20%	2.26%	2.87%	3.68%	4.60%	6.77%	-10.15%***
Eldio10-Eldio1 <sup>++</sup>	-2.43%***	0.16%	0.68%	1.45%*	1.83%**	1.79%**	2.22%***	2.95%***	3.83%***	5.88%***	
Panel B: Average Monthly Returns for Portfolios formed on Gibbs Sampler and Size											
	ME1 <sup>-</sup>	2	3	4	5	6	7	8	9	ME10	ME1-ME10
Gibbs-Low <sup>++</sup>	-1.73%	0.75%	0.46%	0.69%	0.63%	0.84%	0.83%	1.02%	0.92%	1.03%	-2.76%***
2 <sup>++</sup>	-0.26%	-0.28%	-0.15%	0.34%	0.12%	0.28%	0.81%	0.84%	0.76%	1.06%	-1.33%**
3 <sup>+</sup>	0.31%	-0.09%	0.11%	-0.42%	-0.28%	0.03%	0.30%	0.92%	0.87%	1.02%	-0.71%*
4 <sup>++</sup>	-0.36%	-1.49%	-1.14%	-0.65%	-0.08%	0.33%	0.49%	0.50%	0.93%	0.86%	-1.22%**
5 <sup>++</sup>	-1.33%	-1.68%	-1.13%	-0.57%	-0.40%	0.29%	0.48%	0.77%	0.74%	1.02%	-2.35%***
6 <sup>++</sup>	-2.54%	-1.90%	-1.07%	-0.21%	-0.20%	0.18%	0.19%	0.74%	0.78%	1.46%	-4.00%***
7 <sup>++</sup>	-3.72%	-1.82%	-1.13%	-0.51%	0.41%	0.19%	0.47%	0.72%	0.72%	1.05%	-4.77%***
8 <sup>++</sup>	-3.35%	-1.57%	-1.06%	-0.41%	0.40%	0.43%	0.44%	1.14%	1.64%	1.56%	-4.91%***
9 <sup>+</sup>	-3.99%	-1.46%	-1.16%	-0.35%	-0.03%	0.15%	1.09%	1.36%	2.76%	-0.96%	-3.03%***
Gibbs-High <sup>+</sup>	-4.03%	-1.42%	-0.45%	0.19%	0.83%	1.27%	1.01%	0.01%	0.12%	14.04%	-18.08%***
Gibbs10-Gibbs1 <sup>-</sup>	-2.30%***	-2.17%***	-0.91%*	-0.50%	0.20%	0.43%*	0.19%	-1.01%**	-0.80%*	13.02%***	
Panel C: Average Monthly Returns for Portfolios formed on Gibbs Sampler and Eldio											
	Eldio-Low <sup>-</sup>	2 <sup>-</sup>	3 <sup>-</sup>	4 <sup>-</sup>	5 <sup>-</sup>	6 <sup>-</sup>	7 <sup>-</sup>	8 <sup>-</sup>	9 <sup>-</sup>	Eldio-High <sup>-</sup>	Eldio10-Eldio1 <sup>-</sup>
Gibbs-Low <sup>++</sup>	1.00%	1.16%	1.08%	1.29%	1.15%	1.79%	2.26%	2.90%	2.76%	3.11%	2.11%***
2 <sup>++</sup>	0.83%	1.02%	1.14%	1.13%	1.14%	1.34%	1.30%	2.00%	2.68%	3.86%	3.04%***
3 <sup>++</sup>	0.79%	0.98%	1.06%	1.04%	1.04%	1.30%	1.45%	1.63%	2.41%	2.46%	1.67%***
4 <sup>++</sup>	0.88%	0.81%	0.96%	0.61%	1.02%	1.38%	1.28%	0.89%	1.54%	3.46%	2.58%***
5 <sup>+</sup>	0.58%	0.71%	0.69%	0.64%	0.64%	0.87%	0.62%	1.10%	1.34%	2.18%	1.60%***
6	0.60%	0.60%	0.69%	0.33%	0.59%	0.43%	1.04%	0.69%	0.67%	2.82%	2.22%***
7	0.53%	0.43%	0.77%	0.34%	0.62%	0.09%	0.04%	0.41%	0.69%	2.57%	2.04%***
8	0.31%	0.10%	0.42%	0.25%	0.17%	0.16%	0.11%	0.28%	-0.09%	1.97%	1.66%***
9	-0.02%	-0.17%	-0.14%	-0.36%	-0.69%	-0.74%	-0.60%	-0.75%	-1.13%	1.47%	1.49%***
Gibbs-High	-0.84%	-1.31%	-1.45%	-1.82%	-1.54%	-1.65%	-1.94%	-2.44%	-2.19%	0.38%	1.22%**
Gibbs10-Gibbs1 <sup>-</sup>	-1.85%***	-2.47%***	-2.53%***	-3.11%***	-2.70%***	-3.44%***	-4.20%***	-5.34%***	-4.95%***	-2.73%***	



Table 6: Portfolios Sorted by Idiosyncratic Risk

Individual stocks are sorted into ten portfolios each month based on their estimated conditional volatility (idiosyncratic risk) from an EGARCH model on Fama-French 3-factor model. Every month, conditional volatility is estimated by using the previous history to date. Estimates are only conducted if at least 60 return observations exist. Portfolios are rebalanced monthly. The columns labeled Mean and Std Dev are measured in monthly percentage terms and apply to the out of sample returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The column labeled OLS FF-3 Alpha replaces the EGARCH model with OLS residuals from a Fama-French 3-factor model to estimate each firm's idiosyncratic risk. Robust joint tests for the alphas equal to zero are all less than 1% for all cases. The sample period is January 1962 to December 2003. Columns with + (-) and ++ (-) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Rank	Mean	Std Dev	% mkt Share <sup>-</sup>	Size <sup>-</sup>	CAPM Alpha <sup>++</sup>	FF-3 Alpha <sup>+</sup>	Carhart-4 Alpha <sup>++</sup>	OLS FF-3 Alpha
Portfolios Sorted by Idiosyncratic Risk								
1Low	0.03%	0.0372	27.48%	12.33	-0.47%	-0.42%	-0.30%	-0.01%
					[-3.29]	[-3.75]	[-2.84]	[-1.65]
2	0.96%	0.0719	24.36%	12.19	0.02%	-0.02%	-0.01%	-0.05%
					[0.46]	[-0.50]	[-0.20]	[1.65]
3	1.19%	0.0470	16.04%	11.99	0.04%	0.14%	0.12%	0.01%
					[0.25]	[1.25]	[0.92]	[1.14]
4	1.17%	0.0757	10.98%	11.63	0.06%	0.15%	0.19%	0.10%
					[0.47]	[1.52]	[1.69]	[1.66]
5	0.98%	0.0694	7.34%	11.29	0.01%	-0.01%	0.07%	0.05%
					[0.27]	[-0.15]	[1.11]	[0.95]
6	1.00%	0.0511	5.08%	10.97	0.12%	0.07%	0.08%	0.15%
					[1.91]	[1.27]	[1.34]	[1.14]
7	0.98%	0.0425	3.52%	10.68	0.17%	0.11%	0.09%	0.14%
					[2.48]	[2.14]	[1.54]	[1.07]
8	1.09%	0.0568	2.42%	10.39	0.08%	0.11%	0.15%	0.09%
					[0.87]	[1.34]	[1.69]	[2.08]
9	0.96%	0.0659	1.71%	10.09	0.56%	0.60%	0.86%	-0.03%
					[1.94]	[1.89]	[3.17]	[-1.23]
10High	1.36%	0.0815	1.08%	9.69	0.96%	1.06%	1.27%	-0.77%
					[2.65]	[2.76]	[3.71]	[-3.50]
p10-p1	1.33%				1.43%	1.49%	1.58%	
	[3.21]				[3.72]	[3.76]	[4.33]	

Table 7: Portfolios Sorted by Gibbs Sampler (Annually-Rebalanced)

Individual stocks are sorted into ten portfolios at the end of year based on Hasbrouck's (2005) Gibbs Sampler estimates of effective cost averaged over the previous year. Value-weighted portfolios are rebalanced annually. Panel A reports on the out of sample returns from portfolio sorts using the Gibbs Sampler alone. Panel B reports on the out of sample returns using sequential sorts in which firms are first sorted on size or idiosyncratic risk and then on liquidity. Size is based upon the year end market capitalization prior to the period in which the portfolio is formed. Idiosyncratic risk is measured using the 3-factor EGARCH model's forecast with data up to the date the portfolio is created. The columns labeled Mean and Std Dev are measured in annual percentage terms and apply to the simple excess returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is January 1962 to December 2003. Columns (and in Panel B and C rows as well) with + (-) and ++ (-- ) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Rank	Mean <sup>+</sup>	Std Dev	% mkt Share <sup>++</sup>	Size <sup>-</sup>	Eidio <sup>++</sup>	CAPM Alpha	FF-3 Alpha	Carhart-4 Alpha
Panel A: Portfolios Sorted by Gibbs Sampler								
1Low	7.14%	0.1581	34.68%	13.57	0.0624	1.83% [1.62]	0.93% [1.39]	-0.81% [-1.50]
2	6.07%	0.1689	28.15%	13.02	0.0721	0.07% [0.27]	-0.19% [-0.35]	1.51% [1.91]
3	5.11%	0.2042	15.91%	12.50	0.0831	-2.12% [-1.87]	-2.52% [-2.34]	-1.47% [-0.98]
4	6.22%	0.1931	8.86%	12.02	0.0943	-0.46% [-1.10]	-0.34% [-0.48]	1.84% [3.27]
5	4.63%	0.2140	5.76%	11.61	0.1063	-2.71% [-4.59]	-3.40% [-5.21]	-1.16% [-1.61]
6	6.67%	0.2556	3.54%	11.20	0.1153	-1.22% [-1.11]	-1.52% [-1.91]	3.25% [2.76]
7	6.74%	0.2640	1.87%	10.76	0.1253	-1.62% [-0.58]	-4.18% [-1.68]	-2.79% [-1.23]
8	10.08%	0.2808	0.74%	10.29	0.1421	1.86% [1.32]	-2.04% [-3.07]	1.63% [1.06]
9	9.32%	0.3222	0.35%	9.72	0.1562	0.60% [0.36]	-2.74% [-3.30]	-2.51% [-1.56]
10High	16.47%	0.3932	0.13%	8.97	0.1654	7.99% [3.54]	2.24% [1.73]	1.77% [0.92]
p10-p1	9.33% [2.67]					6.16% [2.37]	1.30% [1.14]	2.58% [1.12]

Table 7: Portfolios Sorted by Gibbs Sampler (Annually-Rebalanced)

Individual stocks are sorted into ten portfolios at the end of year based on Hasbrouck's (2005) Gibbs Sampler estimates of effective cost averaged over the previous year. Value-weighted portfolios are rebalanced annually. Panel A reports on the out of sample returns from portfolio sorts using the Gibbs Sampler alone. Panel B reports on the out of sample returns using sequential sorts in which firms are first sorted on size or idiosyncratic risk and then on liquidity. Size is based upon the year end market capitalization prior to the period in which the portfolio is formed. Idiosyncratic risk is measured using the 3-factor EGARCH model's forecast with data up to the date the portfolio is created. The columns labeled Mean and Std Dev are measured in annual percentage terms and apply to the simple excess returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is January 1962 to December 2003. Columns (and in Panel B and C rows as well) with + (-) and ++ (-- ) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Panel B: Portfolios Sorted by Gibbs Sampler After Controlling for Size, FF 3 Factor Alphas											
	1 Low	2 <sup>+</sup>	3	4	5	6	7 <sup>-</sup>	8	9	10Big	10-1
Size Low <sup>++</sup>	-0.42%	-3.44%	3.23%	-1.34%	2.49%	7.48%	4.00%	8.74%	9.85%	20.46%	20.86%
	[-0.87]	[-7.89]	[1.39]	[-1.66]	[0.82]	[3.21]	[2.73]	[3.27]	[2.40]	[3.95]	[4.10]
2 <sup>+</sup>	-3.23%	-2.86%	-1.80%	-2.27%	2.80%	-0.96%	-1.36%	-1.73%	0.39%	0.60%	3.83%
	[-2.22]	[-2.75]	[-1.24]	[-2.07]	[0.19]	[-1.23]	[-1.20]	[-0.49]	[0.30]	[0.58]	[1.72]
3	-1.28%	-1.80%	-3.28%	-2.47%	-3.57%	-0.43%	-2.15%	-0.27%	0.75%	-3.04%	-1.76%
	[-1.63]	[-1.54]	[-4.78]	[-4.77]	[-7.45]	[-0.39]	[-1.37]	[-0.40]	[0.41]	[-3.05]	[-1.21]
4 <sup>-</sup>	2.36%	0.23%	-0.30%	-2.47%	-1.84%	-2.61%	-3.62%	-3.72%	-0.74%	-3.84%	-6.20%
	[4.77]	[0.43]	[-0.47]	[-3.77]	[-6.08]	[-5.44]	[-7.97]	[-9.46]	[-1.09]	[-6.60]	[-8.00]
Size Big <sup>-</sup>	-0.05%	0.14%	3.11%	0.85%	-1.76%	-2.08%	-2.41%	-0.43%	-1.82%	-1.16%	-1.11%
	[-0.07]	[0.21]	[6.05]	[1.81]	[-2.67]	[-2.59]	[-2.53]	[-0.56]	[-4.45]	[-1.54]	[-1.02]
Control for size <sup>+</sup>	-0.52%	-1.55%	0.19%	-1.54%	-0.38%	0.28%	-1.11%	0.52%	1.69%	2.60%	3.12%
	[-0.40]	[-1.21]	[0.87]	[-0.48]	[-1.17]	[0.73]	[-0.77]	[0.63]	[1.38]	[1.83]	[1.84]
Panel C: Portfolios Sorted by Gibbs Sampler After Controlling for Eidio, FF 3 Factor Alphas											
	1	2	3 <sup>+</sup>	4	5	6 <sup>+</sup>	7 <sup>++</sup>	8	9	10 <sup>+</sup>	10-1
Eidio Low <sup>-</sup>	-1.89%	-1.84%	-2.59%	-3.23%	-3.59%	-8.61%	-6.76%	-9.40%	-10.08%	-8.66%	-6.77%
	[-0.36]	[-0.99]	[-1.42]	[-2.32]	[-4.34]	[-3.55]	[-1.60]	[-1.84]	[-3.26]	[-2.56]	[-2.87]
2 <sup>-</sup>	0.17%	0.88%	-0.43%	0.79%	2.13%	-1.49%	-3.40%	0.51%	-3.91%	-8.87%	-9.05%
	[0.20]	[0.42]	[-0.72]	[0.93]	[0.96]	[-1.42]	[-5.17]	[0.61]	[-5.46]	[-6.08]	[-4.42]
3	0.77%	-4.00%	3.64%	-4.52%	-0.02%	0.80%	0.66%	-7.51%	-2.41%	-6.63%	-7.41%
	[0.53]	[-3.44]	[2.72]	[-4.59]	[-0.01]	[0.55]	[0.21]	[-2.81]	[-2.79]	[-4.99]	[-5.47]
4 <sup>+</sup>	-1.45%	1.72%	1.17%	-2.11%	0.08%	1.03%	-2.62%	-3.12%	-2.32%	-2.85%	-1.40%
	[-1.26]	[2.27]	[1.23]	[-2.95]	[0.13]	[0.45]	[-3.89]	[-4.13]	[-1.77]	[-2.19]	[-1.54]
Eidio High	5.87%	5.56%	18.40%	13.97%	5.77%	3.43%	11.87%	29.71%	26.14%	40.33%	34.46%
	[1.44]	[1.27]	[2.11]	[5.01]	[2.22]	[0.80]	[8.74]	[2.21]	[4.39]	[3.55]	[3.78]
Control for Eidio	0.69%	0.47%	4.04%	0.98%	0.87%	-0.97%	-0.05%	2.04%	1.48%	2.66%	1.96%
	[0.81]	[0.52]	[2.28]	[1.27]	[1.45]	[-1.08]	[-0.30]	[0.59]	[1.98]	[1.42]	[0.81]

Table 8: Portfolios Sorted by Dollar Volume NYSE and AMEX only (Annually-Rebalanced)

Individual stocks are sorted into ten portfolios at the end of year based on the monthly dollar volume averaged over the previous year. Value-weighted portfolios are rebalanced annually. Size is based upon the year end market capitalization prior to the period in which the portfolio is formed. Idiosyncratic risk is measured using the 3-factor EGARCH model's forecast with data up to the date the portfolio is created. The columns labeled Mean and Std Dev are measured in annual percentage terms and apply to the simple excess returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is January 1962 to December 2003. Columns (and in Panel B, C and D rows as well) with + (-) and ++ (--) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Rank	Mean <sup>-</sup>	Std Dev	% mkt Share <sup>++</sup>	Size <sup>++</sup>	Eidio <sup>-</sup>	CAPM Alpha <sup>-</sup>	FF-3 Alpha	Carhart-4 Alpha
Panel A: Portfolios Sorted by Dollar Volume								
1Low	15.37%	0.2367	0.09%	8.48	0.1795	9.75%	3.49%	2.99%
						[2.86]	[2.32]	[2.46]
2	12.72%	0.2602	0.21%	9.28	0.1590	6.27%	0.87%	-0.11%
						[1.69]	[0.61]	[-0.05]
3	12.94%	0.2345	0.44%	9.66	0.1496	6.70%	1.35%	1.65%
						[2.15]	[1.56]	[1.37]
4	11.69%	0.2125	0.80%	10.43	0.0920	6.06%	1.21%	1.82%
						[2.00]	[1.29]	[1.51]
5	9.75%	0.2145	1.35%	10.68	0.0865	3.45%	-0.99%	-0.47%
						[1.52]	[-1.18]	[-0.26]
6	8.50%	0.1897	2.27%	11.05	0.0808	2.79%	-0.87%	0.38%
						[1.37]	[-0.92]	[0.24]
7	8.69%	0.1885	3.78%	11.67	0.0776	3.03%	-0.64%	-0.41%
						[1.59]	[-0.61]	[-0.27]
8	8.12%	0.1794	6.76%	12.25	0.0732	2.59%	-0.63%	0.18%
						[1.42]	[-0.49]	[0.13]
9	7.27%	0.1664	14.04%	12.97	0.0705	1.80%	-0.29%	-0.57%
						[1.21]	[-0.31]	[-0.63]
10High	5.56%	0.1690	70.26%	14.38	0.0602	-0.50%	-0.32%	-0.06%
						[-0.65]	[-0.76]	[-0.10]
p1-p10	9.81%					10.24%	3.81%	3.06%
	[2.74]					[2.64]	[2.52]	[2.18]

Table 8: Portfolios Sorted by Dollar Volume NYSE and AMEX only (Annually-Rebalanced)

Individual stocks are sorted into ten portfolios at the end of year based on the monthly dollar volume averaged over the previous year. Value-weighted portfolios are rebalanced annually. Size is based upon the year end market capitalization prior to the period in which the portfolio is formed. Idiosyncratic risk is measured using the 3-factor EGARCH model's forecast with data up to the date the portfolio is created. The columns labeled Mean and Std Dev are measured in annual percentage terms and apply to the simple excess returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is January 1962 to December 2003. Columns (and in Panel B, C and D rows as well) with + (-) and ++ (-) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Panel B: Portfolios Sorted by Dollar Volume After Controlling for Size, FF 3 Factor Alphas											
	1 Low <sup>-</sup>	2 <sup>-</sup>	3	4	5	6	7	8	9	10Big	1-10 <sup>-</sup>
Size Low <sup>-</sup>	10.59%	4.40%	4.69%	4.24%	3.16%	-1.51%	-1.00%	0.41%	-3.62%	-10.12%	20.70%
	[3.64]	[2.03]	[2.92]	[2.30]	[1.50]	[-0.67]	[-0.66]	[0.18]	[-2.49]	[-4.33]	[4.10]
2 <sup>-</sup>	2.37%	2.10%	0.68%	2.16%	0.20%	-1.92%	-0.29%	-1.22%	-7.14%	-13.45%	15.83%
	[1.49]	[1.20]	[0.65]	[1.32]	[0.18]	[-2.01]	[-0.20]	[-0.54]	[-3.65]	[-7.03]	[6.17]
3 <sup>-</sup>	2.36%	3.95%	0.90%	1.24%	0.00%	-0.08%	-1.89%	-1.85%	-3.86%	-10.96%	13.32%
	[2.48]	[2.79]	[0.63]	[1.15]	[0.00]	[-0.05]	[-1.57]	[-1.88]	[-2.63]	[-6.87]	[5.99]
4 <sup>-</sup>	1.32%	0.33%	1.10%	-1.41%	0.46%	0.59%	0.77%	-1.32%	-2.46%	-4.77%	6.08%
	[0.94]	[0.23]	[0.85]	[-1.01]	[0.39]	[0.41]	[0.55]	[-0.97]	[-2.03]	[-3.00]	[3.70]
Size Big	-0.26%	0.25%	1.30%	1.29%	-0.38%	-0.95%	-0.28%	0.85%	0.19%	-0.33%	0.07%
	[-0.19]	[0.17]	[0.88]	[1.59]	[-0.36]	[-0.81]	[-0.28]	[0.72]	[0.22]	[-0.58]	[0.05]
Control for size <sup>-</sup>	3.27%	2.21%	1.74%	1.50%	0.69%	-0.77%	-0.53%	-0.63%	-3.38%	-7.93%	11.20%
	[4.11]	[2.32]	[1.99]	[1.71]	[0.94]	[-0.75]	[-0.69]	[-0.90]	[-4.03]	[-9.05]	[9.51]
Panel C: Portfolios Sorted by Dollar Volume After Controlling for Eidio, FF 3 Factor Alphas											
	1	2	3	4	5	6	7	8 <sup>-</sup>	9 <sup>-</sup>	10 <sup>-</sup>	1-10 <sup>++</sup>
Eidio Low <sup>-</sup>	2.61%	2.62%	2.14%	1.66%	1.59%	0.15%	0.77%	1.59%	0.72%	1.17%	1.45%
	[1.46]	[1.94]	[1.15]	[0.81]	[1.10]	[0.12]	[0.45]	[1.18]	[0.51]	[1.70]	[0.78]
2 <sup>-</sup>	1.75%	3.65%	2.17%	4.54%	2.28%	1.75%	0.97%	0.52%	-0.90%	-1.60%	3.35%
	[1.17]	[2.74]	[1.71]	[3.56]	[1.35]	[1.39]	[0.73]	[0.45]	[-0.83]	[-3.07]	[2.30]
3 <sup>-</sup>	5.41%	3.12%	0.42%	1.20%	-1.13%	-1.90%	-0.98%	-1.35%	-0.57%	-3.50%	8.91%
	[2.35]	[2.19]	[0.36]	[1.03]	[-1.04]	[-1.04]	[-0.49]	[-0.93]	[0.51]	[-2.63]	[3.57]
4 <sup>-</sup>	5.02%	-1.76%	-1.55%	3.77%	-0.04%	-1.76%	-0.61%	-4.39%	-3.92%	-4.07%	9.09%
	[2.22]	[-0.96]	[-1.45]	[1.81]	[-0.02]	[-1.03]	[-0.34]	[-2.73]	[-3.98]	[-2.46]	[3.10]
Eidio High <sup>-</sup>	4.82%	1.13%	0.75%	2.86%	-6.04%	-7.95%	-7.06%	-9.91%	-8.62%	-6.55%	11.37%
	[1.98]	[0.48]	[0.35]	[1.33]	[-2.74]	[-3.13]	[-2.98]	[-3.84]	[-3.20]	[-3.19]	[3.95]
Control for Eidio <sup>-</sup>	3.92%	1.75%	0.78%	2.81%	-0.67%	-1.94%	-1.38%	-2.71%	-2.66%	-2.91%	6.83%
	[3.66]	[1.65]	[1.12]	[2.39]	[-0.64]	[-2.33]	[-1.25]	[-2.74]	[-2.37]	[-4.03]	[5.15]
Panel D: Portfolios Sorted by Dollar Volume After Controlling for Amihud's Illiquidity measure, FF 3 Factor Alphas											
	1 <sup>+</sup>	2	3	4 <sup>+</sup>	5 <sup>+</sup>	6	7	8	9	10	1-10 <sup>++</sup>
Amihud Low	0.06%	1.55%	0.62%	-0.87%	-0.62%	-0.54%	-0.34%	0.34%	0.36%	-0.35%	0.41%
	[0.04]	[1.27]	[0.85]	[-0.77]	[-0.41]	[-0.46]	[-0.28]	[0.29]	[0.41]	[-0.61]	[0.30]
2 <sup>-</sup>	1.37%	0.54%	-0.66%	-0.51%	-0.20%	-1.25%	0.76%	-1.90%	-0.70%	-4.80%	6.17%
	[0.80]	[0.30]	[-0.54]	[-0.63]	[-0.13]	[-0.88]	[0.58]	[-1.33]	[-0.36]	[-2.97]	[2.55]
3 <sup>-</sup>	3.33%	0.38%	2.48%	-0.65%	-0.55%	-2.05%	-0.99%	-1.93%	-1.59%	-4.21%	7.54%
	[2.15]	[0.53]	[1.65]	[-0.41]	[-0.41]	[-1.56]	[-0.48]	[-1.30]	[-1.34]	[-1.82]	[2.83]
4 <sup>-</sup>	1.61%	3.72%	2.16%	1.66%	0.45%	-2.23%	1.45%	1.21%	-1.45%	-6.11%	7.72%
	[1.00]	[2.22]	[1.60]	[1.28]	[0.30]	[-1.88]	[1.18]	[0.81]	[-0.96]	[-3.38]	[2.75]
Amihud High <sup>-</sup>	6.20%	3.76%	5.65%	3.99%	2.16%	-0.92%	0.46%	1.24%	0.10%	-2.48%	8.68%
	[2.55]	[3.12]	[2.75]	[2.21]	[1.11]	[-0.57]	[0.30]	[0.56]	[0.05]	[-0.85]	[2.08]
Control for Amihud <sup>-</sup>	2.51%	1.99%	2.05%	0.72%	0.25%	-1.40%	0.27%	-0.21%	-0.66%	-3.59%	6.10%
	[2.25]	[2.35]	[2.12]	[0.75]	[0.24]	[-1.82]	[0.21]	[-0.25]	[-0.66]	[-2.68]	[3.03]

Table 9: Portfolios Sorted by Idiosyncratic Risk (Annually Rebalanced)

Individual stocks are sorted into ten portfolios at the end of year based on idiosyncratic risk. At the end of each year, all previous data is used to estimate the idiosyncratic risk relative to FF-3 factor model as the conditional volatility from EGARCH. A stock must have at least 60 monthly return observations for inclusion in the sample. A value-weighted portfolio is then held for 12 months before rebalancing. All returns are out of sample. The columns labeled Mean and Std Dev are measured in annual percentage terms and apply to the simple excess returns. Size reports the average log market capitalization for firms within the portfolio and % Mkt share reports the percentage of market share for each portfolio. Gibbs is Hasbrouck's (2005) Gibbs sampler estimate of the effective cost. The Alpha columns report Jensen's alpha with respect to CAPM, Fama-French 3-factor model and Carhart 4-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. Robust joint tests for the alphas equal to zero are all less than 1% for all cases. The sample period is January 1962 to December 2003. Columns with + (-) and ++ (--) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

Rank	Mean <sup>++</sup>	Std Dev	% mkt Share <sup>-</sup>	Size <sup>-</sup>	Gibbs <sup>++</sup>	CAPM Alpha <sup>++</sup>	FF-3 Alpha <sup>+</sup>	Carhart-4 Alpha <sup>++</sup>
Portfolios Sorted by Eidio								
1Low	5.15%	0.1557	27.68%	12.28	0.0026	-0.17% [-0.14]	-0.85% [-1.64]	-0.89% [-1.96]
2	6.96%	0.1686	24.32%	12.40	0.0028	-2.92% [-0.93]	-0.51% [-0.46]	-3.21% [-2.04]
3	7.55%	0.1756	16.98%	12.05	0.0030	-2.01% [-0.75]	-2.48% [-1.49]	-1.46% [-1.84]
4	6.49%	0.1819	9.64%	11.67	0.0033	-1.94% [-0.95]	-3.07% [-1.97]	-0.15% [-0.65]
5	6.47%	0.2266	7.50%	11.33	0.0038	-0.17% [-0.19]	-1.34% [-0.87]	-0.20% [-0.14]
6	6.28%	0.2158	5.62%	11.01	0.0044	1.26% [1.94]	-1.45% [-1.84]	-1.87% [-2.08]
7	8.73%	0.2776	3.44%	10.72	0.0050	3.79% [2.85]	1.12% [2.85]	1.57% [2.01]
8	12.44%	0.3018	2.27%	10.43	0.0058	4.98% [3.14]	6.74% [4.75]	6.96% [3.84]
9	20.44%	0.3320	1.62%	10.14	0.0067	18.57% [4.89]	10.75% [5.41]	9.87% [4.63]
10High	30.05%	0.6707	0.92%	9.73	0.0078	25.97% [6.81]	24.78% [6.81]	25.01% [6.81]
p10-p1	24.90% [3.96]					26.14% [7.89]	25.63% [8.11]	25.90% [7.89]

Table 10: Portfolios Sorted on Idiosyncratic Risk with Controls for Size, Liquidity, and Trading Volume

This table reports Fama and French (1993) alphas, with robust Newey-West (1987) t-statistics in square brackets. Each stock's idiosyncratic risk is estimated by regressing its return on the FF 3-factor model in a rolling five year historical window and idiosyncratic risk is measured by estimating EGARCH model. Each portfolio is rebalanced monthly. All returns are out of sample. The column "10-1" refers to the difference between portfolios 10 and 1. In the panel labeled "Size Quintiles", each month stocks are sorted into five quintiles base on their current market capitalization. Then within each size quintile, stocks are further sorted into 10 portfolios based on idiosyncratic risk. In the rows controlling for size, liquidity (Gibbs Sampler), liquidity (Gamma) and dollar volume "dependent" sequential sorts are performed. Each month, stocks are first sorted based on the first characteristic (size, liquidity (Gibbs Sampler), liquidity (Gamma) and dollar volume) and then, within each quintile stocks are then sorted on idiosyncratic risk. The ten idiosyncratic risk based portfolios are then averaged over each of the five characteristics portfolios. Hence, they represent idiosyncratic risk portfolio risk based portfolios controlling for characteristics. The Gibbs Sampler represents Hasbrouck (2005) Gibbs Sampler estimates of the effective cost, Gamma represents Pastor and Stambaugh (2003) reversal coefficient, volume represents the dollar volume. Columns and rows with + (-) and ++ (-) have positive (negative) Spearman rank correlations significant at the 5% and 1% levels respectively.

		Ranking on Idiosyncratic Risk											
		1 Low	2	3	4	5	6	7	8	9	10High	10-1	
		Size Controls											
Size Quintiles	1 <sup>++</sup>	-0.02%	0.05%	0.10%	0.14%	0.15%	0.53%	0.62%	0.19%	0.30%	2.29%	2.31%	
	Small	[-0.08]	[0.33]	[0.79]	[0.55]	[0.73]	[2.50]	[2.17]	[0.84]	[1.57]	[6.84]	[4.92]	
	2 <sup>++</sup>	-0.27%	-0.14%	-0.11%	-0.11%	0.02%	-0.10%	0.05%	0.28%	0.20%	1.80%	2.07%	
		[-2.78]	[0.87]	[-0.98]	[-0.79]	[0.87]	[-0.89]	[0.75]	[3.74]	[2.15]	[5.21]	[4.97]	
	3 <sup>++</sup>	-0.68%	0.04%	-0.10%	0.09%	0.52%	0.38%	-0.02%	0.45%	0.57%	0.94%	1.62%	
		[-3.75]	[0.47]	[-1.05]	[1.06]	[3.99]	[4.26]	[-1.10]	[4.75]	[6.06]	[2.43]	[3.26]	
	4 <sup>++</sup>	-0.03%	-0.13%	-0.07%	-0.00%	0.01%	0.10%	0.14%	0.09%	0.08%	0.78%	0.81%	
		[-1.21]	[-2.14]	[-0.74]	[-0.03]	[1.21]	[1.52]	[2.52]	[1.78]	[1.87]	[1.98]	[4.59]	
	Large	5 <sup>++</sup>	-0.17%	-0.06%	-0.14%	0.01%	0.03%	0.02%	0.04%	0.07%	0.13%	0.40%	0.57%
			[-1.84]	[-0.93]	[-1.37]	[0.98]	[0.84]	[0.74]	[1.14]	[1.58]	[1.96]	[1.83]	[2.18]
Control for Size <sup>++</sup>		-0.23%	-0.05%	-0.06%	0.03%	0.15%	0.18%	0.17%	0.22%	0.26%	1.24%	1.48%	
			[-2.12]	[-0.68]	[-0.71]	[0.14]	[1.87]	[1.96]	[1.74]	[1.93]	[4.21]	[6.10]	
		Liquidity Controls (Higher Values → Lower Liquidity)											
		1	2	3 <sup>++</sup>	4 <sup>+</sup>	5	6 <sup>++</sup>	7	8 <sup>++</sup>	9	10	10-1	
Gibbs Quintiles	1 <sup>++</sup>	-0.04%	-0.02%	-0.02%	-0.05%	0.07%	0.14%	0.15%	0.17%	0.22%	0.87%	0.91%	
	Small	[-0.87]	[-1.41]	[-0.13]	[-0.45]	[1.00]	[1.28]	[1.38]	[1.02]	[1.93]	[2.21]	[2.43]	
	2 <sup>++</sup>	-0.06%	-0.13%	0.13%	0.19%	0.31%	0.25%	0.53%	0.62%	0.88%	0.85%	0.91%	
		[-0.46]	[-1.31]	[1.21]	[1.51]	[2.44]	[1.41]	[3.37]	[3.17]	[4.85]	[1.98]	[2.25]	
	3 <sup>++</sup>	-0.30%	-0.20%	0.14%	0.33%	0.04%	0.27%	0.54%	0.69%	0.99%	0.89%	1.19%	
		[-1.01]	[-1.16]	[1.06]	[3.66]	[2.36]	[1.44]	[2.38]	[2.88]	[3.85]	[2.47]	[4.63]	
	4 <sup>++</sup>	-0.20%	-0.04%	0.17%	0.41%	0.45%	0.29%	0.37%	0.72%	0.95%	0.85%	1.05%	
		[-0.73]	[-0.57]	[1.53]	[3.89]	[3.47]	[2.45]	[2.15]	[2.87]	[4.03]	[6.13]	[4.87]	
	Large	5 <sup>++</sup>	-0.10%	-0.00%	0.19%	0.37%	0.38%	0.31%	0.47%	0.81%	0.94%	0.70%	
			[-0.82]	[-0.04]	[1.97]	[1.85]	[3.57]	[2.89]	[2.12]	[2.43]	[2.96]	[1.75]	[4.21]
Gibbs Sampler <sup>++</sup>		-0.14%	-0.08%	0.12%	0.25%	0.25%	0.25%	0.41%	0.60%	0.80%	0.81%	0.95%	
			[5.49]	[-1.41]	[1.23]	[1.94]	[1.84]	[1.72]	[2.03]	[1.86]	[2.05]	[9.38]	[6.78]
Gamma <sup>++</sup>		-0.06%	-0.09%	0.23%	0.14%	0.32%	0.30%	0.43%	0.71%	0.71%	0.83%	1.01%	
			[-1.24]	[-1.64]	[1.58]	[1.24]	[1.67]	[1.71]	[2.01]	[2.43]	[2.03]	[1.97]	[4.97]
Control for Volume <sup>++</sup>		-0.08%	-1.21%	-0.47%	-0.31%	-0.03%	0.22%	0.57%	0.85%	0.59%	0.86%	0.94%	
			[-1.94]	[-6.67]	[-6.44]	[-4.24]	[-0.39]	[2.75]	[5.79]	[7.35]	[3.69]	[4.22]	[3.96]





Table 11: Fama-Macbeth Regression of Risk Adjusted Return on Characteristics

This table reports the two-step Fama-Macbeth regression of the adjusted return on characteristics. Risk adjusted returns are estimated each month using the previous 5-year window if 24 out of 60 return observations are available. The Fama-French (1993) 3-factor model is used to create risk adjusted returns. Eidio represents the EGARCH estimates of the conditional idiosyncratic standard deviation based on Fama-French (1993) 3-factor model residuals. Sixty return observations must be available to run EGARCH regressions. Only data up to period  $t$  is used to by the EGARCH model when predicting a period  $t+1$  period's return based upon Eidio. Gibbs represents Hasbrouk's (2005) Gibbs sampler estimates of effective cost, lag Gibbs represents the lagged Gibbs sampler estimates based upon the prior calendar year's data. Amihud represents the liquidity measure in Amihud (2002) and lagAmihud is its value based upon the prior calendar year's data. Gamma represents the Pastor and Stambaugh (2003) reversal gamma and lag Gamma is its value based upon the prior calendar year's data. The symbol Lmv represents size lagged two months, measured as the natural log of price times shares outstanding. The symbol nyamdvol represents the natural log of the dollar volume for NYSE and Amex lagged two months. It equals zero for Nasdaq stocks. The nasdvol natural log of the dollar volume for Nasdaq stocks lagged two months. It is zero for NYSE and Amex stocks. The variable retlagxy represents the log of the stock's return from month  $-x$  to  $-y$ .

				Panel C: Risk Adjust Return Amihud Measure Current Year				Panel D: Risk Adjust Return Amihud Measure Lagged One Year				
	Coeff	T-stat		Coeff	T-stat		Coeff	T-stat		Coeff	T-stat	
Eidio (s.d.)	0.2720	12.01	***	0.1976	8.74	***	0.0014	2.61	***	0.2272	8.21	***
<b>Amihud</b>	<b>-0.0261</b>	<b>-7.32</b>	***	<b>-0.0235</b>	<b>-8.11</b>	***	<b>-0.0119</b>	<b>-9.28</b>	***	<b>-0.0107</b>	<b>-10.58</b>	***
Lmv	-0.0063	-3.59	***	-0.0135	-7.98	**	-0.0171	-3.45	***	-0.0122	-3.58	***
retlag23	0.0254	3.75	***	0.0176	5.63	***	0.0044	1.46	***	0.0068	2.37	***
retlag46	0.0102	4.30	***	0.0058	2.31	***	0.0052	2.10	**	0.0082	3.43	***
retlag712	0.0035	2.03	***	0.0004	1.65	*	0.0099	5.60	***	0.0074	4.17	***
Nyamdvoll	-0.0225	-7.25	***	-0.0168	-3.63	***	-0.0033	-8.70	***	-0.0025	-6.37	***
Nasdvol	-0.0122	-7.60	***	-0.0086	-1.69	*	-0.0018	-1.99	**	-0.0023	-4.39	***
Average Adjusted R <sup>2</sup>	0.0434			0.0196			0.0311			0.0245		0.0247
Panel D: Risk Adjust Return Amihud Measure Lagged One Year												
Eidio (s.d.)	0.2480	9.74	***	0.1976	8.74	***	0.0014	2.61	***	0.2272	8.21	***
<b>lag Amihud</b>	<b>-0.0022</b>	<b>-1.56</b>	***	<b>0.0010</b>	<b>2.57</b>	**	<b>0.0017</b>	<b>4.41</b>	***	<b>0.0023</b>	<b>4.53</b>	***
Lmv	0.0075	4.57	***	0.0014	2.68	***	-0.0007	-1.56	***	0.0049	10.99	***
retlag23	0.0192	7.12	***	0.0087	2.95	***	0.0044	1.50	***	0.0066	2.31	***
retlag46	0.0171	7.47	***	0.0095	3.82	***	0.0100	4.15	***	0.0094	3.94	***
retlag712	0.0126	7.27	***	0.0071	3.94	***	0.0074	4.17	***	0.0057	3.20	***
Nyamdvoll	-0.0074	-8.74	***	-0.0020	-4.45	***	-0.0025	-6.37	***	0.0081	4.60	***
Nasdvol	-0.0048	-4.58	***	-0.0009	-0.90		-0.0018	-1.99	**	0.0081	4.60	***
Average Adjusted R <sup>2</sup>	0.0306			0.0103			0.0311			0.0245		0.0281

Table 11: Fama-Macbeth Regression of Risk Adjusted Return on Characteristics

This table reports the two-step Fama-Macbeth regression of the adjusted return on characteristics. Risk adjusted returns are estimated each month using the previous 5-year window if 24 out of 60 return observations are available. The Fama-French (1993) 3-factor model is used to create risk adjusted returns. Eidio represents the EGARCH estimates of the conditional idiosyncratic standard deviation based on Fama-French (1993) 3-factor model residuals. Sixty return observations must be available to run EGARCH regressions. Only data up to period  $t$  is used to by the EGARCH model when predicting a period  $t+1$  period's return based upon Eidio. Gibbs represents Hasbrouk's (2005) Gibbs sampler estimates of effective cost, lag Gibbs represents the lagged Gibbs sampler estimates based upon the prior calendar year's data. Amihud represents the liquidity measure in Amihud (2002) and lagAmihud is its value based upon the prior calendar year's data. Gamma represents the Pastor and Stambaugh (2003) reversal gamma and lag Gamma is its value based upon the prior calendar year's data. The symbol Lmv represents size lagged two months, measured as the natural log of price times shares outstanding. The symbol nyamdvoll represents the natural log of the dollar volume for NYSE and Amex lagged two months. It equals zero for Nasdaq stocks. The nasdvoll natural log of the dollar volume for Nasdaq stocks lagged two months. It is zero for NYSE and Amex stocks. The variable retlagxy represents the log of the stock's return from month  $-x$  to  $-y$ .

Panel E: Risk Adjust Return Gamma Current Year																	
	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat					
Eidio (s.d.)	0.2055	9.27	***		0.1976	8.74	***				0.3478	8.52	***				
<b>Gamma</b>	<b>-2.5404</b>	<b>-3.25</b>	***		<b>-2.0947</b>	<b>-2.45</b>	**		0.0010	1.90	*	<b>-1.8321</b>	<b>-2.26</b>	**	<b>-2.5405</b>	<b>-3.25</b>	***
Lmv	0.0001	0.13		*	0.0010	1.90		*	0.0087	2.94	***	-0.0014	-4.67	***	0.0001	0.64	
retlag23	0.0079	2.75	***		0.0131	4.54	***		0.0087	2.94	***	0.0055	2.17	**	0.0077	2.76	***
retlag46	0.0097	4.05	***		0.0121	5.16	***		0.0097	4.04	***	0.0081	3.33	***	0.0110	3.61	***
retlag712	0.0068	3.83	***		0.0067	3.85	***		0.0068	3.89	***	0.0064	3.73	***	0.0088	3.68	***
Nyamdvoll	-0.0019	-5.34	***		-0.0023	-6.11	***		-0.0023	-6.00	***						
Nasdvoll	-0.0018	-4.02	***		-0.0021	-4.10	***		-0.0018	-1.72	*	-0.0021	-4.06	***			
Average Adjusted R <sup>2</sup>	0.0471				0.0271				0.0231			0.0201			0.0247		
Panel F: Risk Adjust Return Gamma Lagged One Year																	
	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat	Coeff	T-stat					
Eidio (s.d.)	0.1980	8.82	***		0.1976	8.74	***				0.1932	8.14	***				
<b>lag Gamma</b>	<b>1.2140</b>	<b>1.02</b>	***		<b>1.8798</b>	<b>2.11</b>	**		0.0010	1.90	*	<b>2.0671</b>	<b>2.40</b>	**	<b>1.3587</b>	<b>1.43</b>	***
Lmv	0.0033	6.94	***		0.0011	1.98	*		0.0087	2.94	***	-0.0014	-4.76	***	-0.0002	-0.76	***
retlag23	0.0130	4.49	***		0.0093	3.16	***		0.0087	2.94	***	0.0061	2.12	**	0.0091	3.00	***
retlag46	0.0122	5.18	***		0.0096	3.97	***		0.0097	4.04	***	0.0082	3.02	***	0.0109	3.57	***
retlag712	0.0099	5.63	***		0.0070	3.97	***		0.0068	3.89	***	0.0065	3.43	***	0.0009	5.85	***
Nyamdvoll	-0.0033	-8.66	***		-0.0023	-5.99	***		-0.0023	-6.00	***						
Nasdvoll	-0.0018	-1.75	*		-0.0011	-1.12			-0.0021	-4.06	***						
Average Adjusted R <sup>2</sup>	0.0374				0.0336				0.0231			0.0236			0.0251		

Figure 1: Per period average EGARCH (Eidio) and OLS (Idio) model idiosyncratic absolute forecast errors for individual stocks.

