



**CROSSHOLE RESISTIVITY AND ACOUSTIC VELOCITY
IMAGING: 2.5-D HELMHOLTZ EQUATION
MODELING AND INVERSION**

by

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Submitted in fulfillment of the requirements for
the degree of Doctor of Philosophy

September 1998

Contents

Abstract	V
Statement	VII
Acknowledgments	VIII
Chapter 1 Introduction	1
1.1 Crosshole Resistivity Imaging.....	1
1.2 Crosshole Seismic Imaging.....	3
1.3 2.5-D Approximation	5
1.4 Objectives of The Study and Outline of Thesis.....	7
Chapter 2 2.5-D Helmholtz Equation	9
2.1 2.5-D Helmholtz Equation.....	9
2.2 DC Electric Potential Equation	11
2.3 Acoustic Wave Equation.....	13
2.4 2.5-D Green's Functions	16
2.5 Principle of Reciprocity.....	19
2.6 The Finite Element Method (FEM).....	20
(1) Galerkin's Solution.....	21
(2) Variational Principle	22
Chapter 3 2.5-D DC Resistivity Modeling	25
3.1 Mixed-Boundary Condition	26
3.2 The FEM Formulation	28
(1) Inner Element Matrix.....	29
(2) Boundary Element Matrix	35
3.3 Inverse Fourier Transform.....	36
3.4 Implementation of The Method.....	39
3.5 Examples of Modeling.....	43
(1) Accuracy of The Method	43
(2) Resistivity Profiling.....	46
(3) Crosshole Applied Potential Surveying.....	46

(4) Monitoring of Underground Water Injection
 From Artificial Aquifer Storage..... 48

Chapter 4 2.5-D Acoustic Wave Modeling in

The Frequency-Domain 52

4.1 2.5-D Absorbing Boundary Condition 52

(1) Clayton-Engquist ABC..... 53

(2) General Form of The 2.5-D ABC..... 54

4.2 Boundary Condition for The Evanescent Field 57

4.3 Composite Boundary-Valued Solution 58

(1) The FEM Formulation 59

(2) Inner Element Matrix..... 60

(3) Boundary Element Matrix 62

(4) Numerical Modeling Results..... 64

4.4 A Damping Method 77

(1) The FEM Formulation 78

(2) Choice of The Damping Term 80

(3) Numerical Modeling Results 81

Chapter 5 Extension to 2.5-D Elastic Wave Modeling

in The Frequency-Domain 90

5.1 2.5-D Elastic Wave Equation 91

5.2 The FEM Formulation 95

5.3 Boundary Conditions 98

(1) A Viscous Boundary Condition..... 99

(2) Paraxial Approximation 100

(3) The Modified Higdon Formula..... 101

Chapter 6 Inversion Algorithms 104

6.1 Objective Functions 104

6.2 Tikhonov Regularization Solutions 108

(1) General Iterative Solution..... 109

(2) Iteratively Linearized Solution..... 110

(3) Conjugate Gradient Solution (CGS)..... 113

(4) Local-Search Quadratic Approximation CGS 114

6.3 The Smoothest Model Solution (Occam's Inversion)	120
6.4 Generalized Subspace Solution.....	122
Chapter 7 Computation of The Fréchet and Second Derivatives	128
7.1 Basic Definitions	129
(1) Bilinear Operator.....	129
(2) Fréchet and Second Derivatives.....	130
7.2 Common Algorithms.....	131
7.3 Explicit Expressions for DC Resistivity	134
7.4 Explicit Expressions for 2.5-D Acoustic Waves.....	139
7.5 Numerical Algorithms	142
7.6 Synthetic Examples.....	148
(1) Patterns in a Homogeneous Background.....	148
(2) Patterns for Imhomogeneous Models.....	152
Chapter 8 Synthetic Study on Crosshole DC Resistivity Imaging.....	157
8.1 Crosshole Surveying Configurations.....	157
8.2 Sensitivity Variation of Crosshole Measurements.....	162
8.3 Numerical Experiments of Crosshole Imaging.....	170
(1) Pole-Pole Array.....	172
(2) Pole-Bipole Array	173
(3) Bipole-Pole Array	176
(4) Bipole-Bipole Array.....	178
8.4 Numerical Experiments for Resistivity Profiling.....	183
8.5 Physical Model Imaging Experiments.....	188
Chapter 9 Numerical Simulations for Crosshole Acoustic	
Velocity Imaging with Spectral Data	191
9.1 Full-waveform Inversion.....	191
9.2 2.5-D Generalized Diffraction Tomography	195
9.3 Imaging with a Known Source Wavelet.....	199
(1) Using Real and Imaginary components.....	201
(2) Using Hartley-Spectral Data.....	204
(3) Using Amplitude and Phase Spectra.....	208
9.3 Imaging with an Unknown Source Wavelet.....	214

(1) Normalized Spectral Data.....	214
(2) Numerical Imaging Experiments.....	217
Chapter 10 Conclusions and Future Research	222
10.1 Forward Modeling	222
10.2 Imaging Algorithms	224
10.3 Numerical Imaging Results.....	225
10.4 Future Research.....	227
Appendix A The Hartley Transform	228
Appendix B Computation of The Boundary Integral.....	231
Appendix C Semi-Analytic Acoustic Solution for Two Semi-infinite Media in Contact.....	234
Appendix D A Constant-Block Approximation in The Computation of The Fréchet and Second Derivatives for Resistivity Imaging.....	237
Bibliography.....	239

Abstract

Crosshole resistivity and seismic velocity imaging (diffraction tomography or waveform inversion with the acoustic approximation) are two very useful techniques in geophysical exploration, because they may be employed to map conductivity or velocity inhomogeneities (geological anomalies) between boreholes. Such imaging techniques, when considered in mathematical terms, both reduce to 2.5-D Helmholtz equation modeling and inversion. This thesis presents (1) new finite element solutions for two specified forms of this kind of equation, which define 2.5-D resistivity and 2.5-D acoustic wave problems, as well as an extension to 2.5-D elastic waves solution; (2) iterative algorithms for tackling inversion problems, including Tikhonov regularisation, the smoothest model solution and the subspace solution, in which the generalised algorithms and their inter-relations are formulated, and new approximations for the Fréchet and second derivatives are developed; (3) numerical and physical model experiments for crosshole resistivity imaging with different electrode arrays (or configurations) and synthetic simulations for crosshole acoustic velocity imaging with different full-waveform spectral data (real and imaginary components, amplitude and phase, and the Hartley spectra).

The modeling examples for resistivity and acoustic wavefield show that the presented methods are effective and flexible to compute the 3-D physical response for 2-D arbitrary media. Furthermore, it is shown that the accuracy of the modeling mainly depends upon the wavenumber range and the number of wavenumber samples, both of which can be evaluated in terms of the typical shape of the transformed potential in the resistivity case and the critical values of the wavenumber of the media around the seismic source in the acoustic wave case, respectively. The constant-point approximation based on the explicit expressions for the Fréchet and second derivatives has been shown to be a robust and efficient algorithm for inversion. The numerical and physical imaging experiments show that some specified three- and four-electrode arrays are more suitable for crosshole resistivity imaging than the pole-pole array. For acoustic imaging with spectral data, the real and imaginary components, the Fourier amplitude

and the Hartley spectra are suitable for full-waveform inversion with a known source wavelet, but the phase data do not permit satisfactory object field reconstruction. It is shown in the thesis that the normalised Fourier/Hartley spectral data can be used for imaging when the seismic source wavelet is unknown.