Crosstalk in Balanced Interconnections Used for Differential Signal Transmission

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***Abstract*—This paper introduces a two-stage model for assessing crosstalk in balanced interconnections used for differential signal transmission, such as multipair cables. The first stage considers the interconnection as uniform and uses a change of variables based on the symmetries inherent to balancing, for the definition of a set of parameters to be measured. The second stage of the model takes into account the nonuniformity related to the fluctuations of the characteristics of the interconnection, using a first order perturbation expansion and a probabilistic approach. This model is compatible with published results on crosstalk in multipair cables.**

***Index Terms*—Balanced interconnection, crosstalk, interconnection, multipair cable, signal integrity, transmission.**

I. Introduction

EARLY telegraph and telephone transmission took place on open-wire lines, whose conductors were individually supported above ground by means of insulators on poles. At the beginning of the 20th century, engineers knew that the principal method of reducing crosstalk in such electrical interconnections is the use of a separate circuit for each signal to be transmitted, the circuits being substantially perfectly balanced to each other by means of very frequent transposition of the conductors of each circuit [1]. In transposition, the two conductors of each circuit exchange position at intervals along the line so as to balance out unwanted voltages and currents induced by adjacent circuits.

Even though transposed open-wire lines are not used anymore for signal transmission, transposition is a current technique used to obtain high-speed balanced interconnections, for instance in twisted-pair cables and in special trace structures in printed circuit boards [2]. A balanced interconnection is a propagation medium with p pairs, in which appropriate symmetries are used to obtain the cancellation of unwanted couplings between the pairs. Balanced interconnections are generally obtained using transposition, but not necessarily: for instance the star quad (for which $p = 2$) discussed later does not implement transposition. When balanced interconnections are used for differential transmission with suitable line transmitters, line receivers and terminations, crosstalk and echo will be low.

The authors have recently [3], [4] shown that crosstalk and echo can also be eliminated in uniform multiconductor interconnections if one uses modal transmission and matched terminations. This ZXtalk method provides n uncoupled transmission channels, using n transmission conductors and one reference conductor. It does not rely on a balanced interconnection and provides more channels per conductor than balanced interconnections used with differential signaling. However, both approaches implement modal transmission, and they would ideally provide uncoupled transmission channels.

As a consequence, the residual crosstalk showing up in real implementations of both approaches could be computed with a common theoretical framework. Instead of such a general and abstract development, this paper presents a correct derivation of crosstalk in balanced interconnections, in the case of multipair cable. Surprisingly, published material on the computation of crosstalk in multipair cables relies on the ad hoc "two-circuit model" outlined in Section II, which is not compatible with the accepted multiconductor transmission line (MTL) theory, and cannot be generalized to different transmission schemes. Our derivation, detailed in Section III to Section VIII, will be based on the MTL theory. It will therefore be more accurate than the two-circuit model and also adaptable to the ZXtalk method.

A major cause of the residual crosstalk in modal transmission schemes being small unwanted departures from the ideal properties of the propagation medium distributed along the length of the medium, a statistical approach will be used. The advantage of considering multipair cables in this paper is that it will be possible to compare our theory with abundant published experimental data, in Section IX.

II. Two-Circuit Model and MTL Theory

Crosstalk in multipair cables has mostly been studied with the following theoretical approach: two pairs are considered and the description of crosstalk is limited to electric and magnetic couplings between a *disturbing circuit* and a *disturbed circuit* [5, § 8.4 to § 8.8], [6, § 11.3], as shown in Fig. 1. This model uses only two coupling parameters distributed over the length of the cable: a mutual inductance M and an equivalent capacitance C_{eq}. This two-circuit model has been used to compute the crosstalk coupling loss at each end of the cable in three important cases: 1) the case of an electrical and/or a magnetic unbalance when the cable is short compared to wavelength; 2) the case of a nonuniform electrically long cable, using the assumption that the rms voltage at one end results from a summation of power caused by uncorrelated unbalance contributions along the cable; 3) the case of a uniform electrically long cable for which the voltage at one end results from a uniform unbalance along the cable. Case 3) is regarded as irrelevant in practical circuits, and the telecom industry has relied up to now on design rules based on Case 2), and on purely statistical models [7].

Independently, a standard method for computing crosstalk based on the MTL theory [8, § 6.2] has been the subject of many

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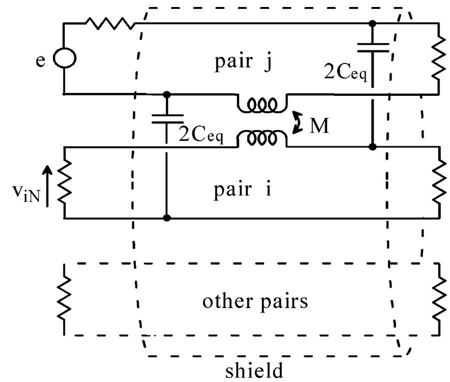

Fig. 1. Basic two-circuit model uses only the distributed parameters M and C_{eq} to compute the near-end crosstalk voltage v_{iN} and the far-end crosstalk voltage v_{iF}.

papers in the last 30 years. As is well known, MTL models appropriate for field coupling predictions (i.e., derivation of the emission of radiated fields or of the pick-up of external fields) must take into account the common-mode current on the cable, and therefore include an additional *reference conductor* [8, § 6.1.4 and § 7.1] running parallel to the cable. For instance, if the cable is inserted inside a metallic duct, it should be used as a reference conductor. For instance, if the cable is installed close to a flat metallic structure (ground plane), this structure should be the reference conductor. The two-circuit model can be viewed as a special case of the MTL theory implementing ad hoc initial assumptions to extract only 4 independent variables (two differential-mode currents and two differential-mode voltages) out of the MTL theory (which uses $4p$ independent variables in the case of an unshielded cable with p pairs or $4p + 2$ variables if the cable is shielded).

III. Symmetry Transforms

A. Natural Parameters

A cable contains p twisted pairs (with $p \geq 2$), eventually surrounded by an overall screen. Adhering to the vocabulary of the telecom industry, we will refer to a unshielded twisted-pair (UTP) cable or to a shielded twisted-pair (STP) cable, according to the case. The multipair cable is regarded as a multiconductor cable having n conductors, with $n = 2p$ in the case of a UTP cable or with $n = 2p + 1$ in the case of a STP cable. Let us number the n conductors of the cable in the following way:

$$\begin{aligned}
&\text{conductor } 1 = \text{1st wire of the first pair,}\\
&\text{conductor } 2 = \text{2nd wire of the first pair,}\\
&\dots\\
&\text{conductor } 2p-1 = \text{1st wire of the } p-\text{th pair,}\\
&\text{conductor } 2p = \text{2nd wire of the } p-\text{th pair,}\\
&\text{conductor } 2p+1 = \text{screen of the cable,}\\
&\quad \text{in the case of a STP cable only.}
\end{aligned}$$

The number attributed to each conductor will be used as index for the current flowing on the corresponding conductor (the positive direction is the one of current flowing into the cable at the near-end) and for the voltage of this conductor with respect to a reference conductor having any geometry. We will use $\mathbf{I}$ to denote the vector of these *natural currents* $i_1, \dots, i_n$ and $\mathbf{V}$ to denote the vector of these *natural voltages* $v_1, \dots, v_n$.

The standard cable parameters used in the MTL theory are the p.u.l. inductance, resistance, capacitance and conductance matrices (p.u.l. standing for "per-unit-length"), noted $\mathbf{L}$, $\mathbf{R}$, $\mathbf{C}$ and $\mathbf{G}$, respectively. We will call them the *natural matrices* of the problem. The four $n \times n$ symmetric natural matrices depend on the parameters of the reference conductor, i.e., its shape, size, position and distance to the cable. The MTL theory states that, at given radian frequency ω and abscissa z, all electrical phenomena taking place on the cable are described by the telegrapher's equations

$$\begin{cases} \frac{d\mathbf{V}}{dz} = -(\mathbf{R} + j\omega\mathbf{L})\mathbf{I} \\ \frac{d\mathbf{I}}{dz} = -(\mathbf{G} + j\omega\mathbf{C})\mathbf{V}. \end{cases} \tag{1}$$

$\mathbf{Z} = \mathbf{R} + \mathrm{j}\omega\mathbf{L}$ will be called the natural p.u.l. impedance matrix and $\mathbf{Y} = \mathbf{G} + \mathrm{j}\omega\mathbf{C}$ will be called the natural p.u.l. admittance matrix. As is well known, (1) is easily solved after a suitable diagonalization of the matrices $\mathbf{ZY}$ and $\mathbf{YZ}$ [8, § 6.2], [9]. The eigenvectors so obtained define the propagation modes and the eigenvalues correspond to the propagation constants.

B. Parameters After the Change of Variables

As will be shown in Section III-C, differential crosstalk voltages of well-balanced cables are very sensitive to small variations of some elements of the natural matrices, because they correspond to differences of natural voltages. The direct implementation of the usual MTL analysis approach is therefore not satisfactory for such interconnections.

In this paper, we will perform a linear change of variable (or linear transform) on the natural currents and voltages, in order to introduce in the MTL theory the convenient differential-mode currents and differential-mode voltages used in the two-circuit model, and the common-mode needed for field coupling assessments.

Such a transform must take into account the symmetries of the cable. It will therefore be called a *symmetry transform*. We will use $\mathbf{I}_S$ to denote the vector of the symmetry-transformed currents $i_{S1}, \dots, i_{Sn}$ and $\mathbf{V}_S$ to denote the vector of the symmetry-transformed voltages $v_{S1}, \dots, v_{Sn}$. In this paper, these currents and voltages will be respectively called *symmetrical currents* and *symmetrical voltages*. Our change of variables will be described with two invertible real matrices $\mathbf{A}$ and $\mathbf{B}$ such that

$$\begin{cases} \mathbf{V}_S = \mathbf{AV} \\ \mathbf{I}_S = \mathbf{BI}. \end{cases} \tag{2}$$

If we use (1) and (2), we obtain a symmetry-transformed telegrapher's equation as

$$\begin{cases} \frac{d\mathbf{V}_S}{dz} = -(j\omega\mathbf{L}_S + \mathbf{R}_S)\mathbf{I}_S \\ \frac{d\mathbf{I}_S}{dz} = -(j\omega\mathbf{C}_S + \mathbf{G}_S)\mathbf{V}_S \end{cases} \tag{3}$$

where $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are the symmetry-transformed p.u.l. inductance, resistance, capacitance and conductance matrices, respectively. These matrices are defined as

$$\begin{cases} \mathbf{L}_S = \mathbf{ALB}^{-1} \\ \mathbf{R}_S = \mathbf{ARB}^{-1} \end{cases} \quad \begin{cases} \mathbf{C}_S = \mathbf{BCA}^{-1} \\ \mathbf{G}_S = \mathbf{BGA}^{-1}. \end{cases} \tag{4}$$

If the symmetry transform is chosen in such a way that the transpose ${}^{\mathrm{t}}\mathbf{A}$ of $\mathbf{A}$ satisfies ${}^{\mathrm{t}}\mathbf{A} = \mathbf{B}^{-1}$, then the matrices $\mathbf{L}_S, \mathbf{R}_S, \mathbf{C}_S$ and $\mathbf{G}_S$ are symmetric. This property is of course very desirable, because it reduces the number of parameters to be measured. If $\mathbf{A}$ is real, this property also implies that the symmetry transform can be realized with a lossless linear network [10], [11, § III]. We will note $\mathbf{Z}_S = \mathbf{R}_S + \mathrm{j}\omega\mathbf{L}_S$ the symmetry-transformed p.u.l. impedance matrix and $\mathbf{Y}_S = \mathbf{G}_S + \mathrm{j}\omega\mathbf{C}_S$ the symmetry-transformed p.u.l. admittance matrix of the cable.

C. Example

For a STP cable with two pairs, we can consider the symmetry transform defined by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 1/2 & 1/2 & -1/2 & -1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1/2 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 \\ 1/2 & 1/2 & -1/2 & -1/2 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{5}$$

for which ${}^{\mathrm{t}}\mathbf{A} = \mathbf{B}^{-1}$. We see that, with this symmetry transform, v_{S1} and v_{S2} are the differential mode voltages, i_{S1} and i_{S2} are the differential mode currents, v_{S4} is the common mode voltage of the internal conductors with respect to the shield, i_{S4} is the common mode current on internal conductors, v_{S5} is the common mode voltage of the cable with respect to the reference conductor, and i_{S5} is the common mode current on the cable. The authors have designed and built an apparatus for the direct measurement of the symmetry-transformed matrices for such a symmetry transform [10], [11]. This instrument is intended to be connected between the cable and the two ports of a network analyzer. In order to illustrate the advantages of a direct measurement of the symmetry-transformed matrices, let us consider measurement results obtained at 1 MHz on a two-pair STP cable held at about 10 cm of a ground plane.

Using a conventional measurement technique [8, § 6.4 and § 6.5], the first four elements of the diagonal of the $\mathbf{L}$ matrix are found to be close to 1.3 μH/m, the 21 other elements ranging from 0.9 μH/m to 1 μH/m. Using our instrument for a direct measurement of $\mathbf{L}_S$ and $\mathbf{C}_S$, we found almost diagonal matrices, the diagonal elements for the differential mode being respectively close to 0.59 μH/m and 40 pF/m. The nondiagonal elements responsible for crosstalk could also be measured, their values being typically 60 dB below the diagonal elements. Using (4) and (5), we can check that such nondiagonal elements of $\mathbf{L}_S$ typically correspond to variations of the order of only 0,005 dB of the elements of $\mathbf{L}$. If we consider that such an accuracy is not achievable in practice, we can infer that direct measurements of the elements of $\mathbf{L}$ cannot lead to acceptable estimates of the nondiagonal elements of $\mathbf{L}_S$ using (4) and (5).

For well-balanced multipair cables, the nominal propagation parameters for the pairs are determined by the first p diagonal elements of the symmetry-transformed matrices defined by (4), and the nondiagonal elements of these matrices produce crosstalk. Consequently, for such cables, the direct measurement of the symmetry-transformed matrices is more effective for assessing crosstalk than conventional measurement techniques.

IV. Parameters for the UTP Cable

A. Ideally Balanced UTP Cable

We may now define a perfectly balanced UTP cable with p pairs as a cable for which the conductors of the same pair have the same p.u.l. impedances and admittances with respect to the reference conductor, and for which the excitation of any pair in differential mode induces no voltage and injects no current in any other conductor. If $\mathbf{X}$ is any one of the natural matrices $\mathbf{L}$, $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{G}$, we see that the elements $X_{\alpha,\beta}$ of $\mathbf{X}$ satisfy

$$\begin{aligned} &\forall\alpha \in \{1,\ldots,p\} \quad \forall\beta \in \{1,\ldots,p\} \\ &\qquad X_{2\alpha-1,2\alpha-1} = X_{2\alpha,2\alpha} \\ &(\alpha \neq \beta) \Rightarrow (X_{2\alpha-1,2\beta-1} = X_{2\alpha-1,2\beta} \\ &\qquad = X_{2\alpha,2\beta-1} = X_{2\alpha,2\beta}). \end{aligned} \tag{6}$$

Some averaging of the properties of a cable over its length being legitimate for low enough frequencies, it is clear that a suitable twisting of the pairs (i.e., transposition) may produce a cable almost perfectly balanced. We will also define a super-balanced UTP cable as a balanced cable in which any pair can be exchanged with any other pair without changing any natural matrix. For a super-balanced UTP cable there are three values X_A, X_B and X_M such that

$$\begin{aligned} &\forall\alpha \in \{1,\ldots,p\} \quad \forall\beta \in \{1,\ldots,p\} \\ &\qquad X_{2\alpha-1,2\alpha-1} = X_{2\alpha,2\alpha} = X_A \\ &\qquad X_{2\alpha-1,2\alpha} = X_{2\alpha,2\alpha-1} = X_B \\ &(\alpha \neq \beta) \Rightarrow (X_{2\alpha-1,2\beta-1} = X_{2\alpha-1,2\beta} \\ &\qquad = X_{2\alpha,2\beta-1} = X_{2\alpha,2\beta} = X_M). \end{aligned} \tag{7}$$

This corresponds to what Carson and Hoyt call "the ideal telephone transmission system" [12, eq. (19)]. For instance, with a proper numbering of the conductors, a perfect so-called star quad made of 4 wires twisted together will be super-balanced. However, the property (7) is much stronger than the property (6). We note that, for an ideal cable made of 4 twisted pairs, themselves twisted together in star quad configuration, we might (after averaging) get $\mathbf{X}$ matrices in the form

$$\mathbf{X} = \begin{pmatrix} X_A & X_B & X_M & X_M & X_M & X_M & X_N & X_N \\ X_B & X_A & X_M & X_M & X_M & X_M & X_N & X_N \\ X_M & X_M & X_A & X_B & X_N & X_N & X_M & X_M \\ X_M & X_M & X_B & X_A & X_N & X_N & X_M & X_M \\ X_M & X_M & X_N & X_N & X_A & X_B & X_M & X_M \\ X_M & X_M & X_N & X_N & X_B & X_A & X_M & X_M \\ X_N & X_N & X_M & X_M & X_M & X_M & X_A & X_B \\ X_N & X_N & X_M & X_M & X_M & X_M & X_B & X_A \end{pmatrix} \tag{8}$$

where X_A, X_B, X_M and X_N are four constants. This matrix has more symmetry than a perfectly balanced UTP cable, but less than a super-balanced one (with a super-balanced cable, we would have $X_M = X_N$).

B. Change of Variables for the UTP Cable

In this paper, we will consider the following symmetry transform for UTP cables:

$$\begin{cases} v_{s1} = v_1 - v_2 \quad \ldots \quad v_{sp} = v_{2p-1} - v_{2p} \\ v_{Sp+1} = \frac{1}{2}(v_1 + v_2) - \frac{1}{2}(v_3 + v_4) \\ \ldots \\ v_{Sj} = \frac{1}{2(j-p)}(v_1 + \ldots + v_{2j-2p}) \\ \quad -\frac{1}{2}(v_{2j-2p+1} + v_{2j-2p+2}) \\ \ldots \\ v_{S2p-1} = \frac{1}{2p-2}(v_1 + \ldots + v_{2p-2}) - \frac{1}{2}(v_{2p-1} + v_{2p}) \\ v_{S2p} = \frac{1}{2p}(v_1 + v_2 + \ldots + v_{2p}) \end{cases}$$

$$\begin{cases} i_{S1} = \frac{1}{2}(i_1 - i_2) \quad \ldots \quad i_{Sp} = \frac{1}{2}(i_{2p-1} - i_{2p}) \\ i_{Sp+1} = \frac{1}{2}(i_1 + i_2) - \frac{1}{2}(i_3 + i_4) \\ \ldots \\ i_{Sj} = \frac{1}{j-p+1}(i_1 + \ldots + i_{2j-2p}) \\ \quad -\frac{j-p}{j-p+1}(i_{2j-2p+1} + i_{2j-2p+2}) \\ \ldots \\ i_{S2p-1} = \frac{1}{p}(i_1 + \ldots + i_{2p-2}) - \frac{p-1}{p}(i_{2p-1} + i_{2p}) \\ i_{S2p} = i_1 + i_2 + \ldots + i_{2p}. \end{cases} \tag{9}$$

The symmetrical voltages $v_{S1}, \ldots, v_{Sp}$ (respectively, currents $i_{S1}, \ldots, i_{Sp}$) are called the differential mode voltages (respectively, currents). The symmetrical voltage v_{S2p} (respectively, current i_{S2p}) is the common-mode voltage (respectively, current) of the cable. Considering (2), we see that the matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by (9). We can easily establish that any two different rows of the matrices $\mathbf{A}$ and $\mathbf{B}$ are orthogonal and that they have the three following properties:

$$\begin{aligned} {}^t\mathbf{A} &= \mathbf{B}^{-1} \\ \det\, \mathbf{A} &= \det\, \mathbf{B} = \pm 1 \\ \mathbf{AB}^{-1} &= \text{diag}_n(\lambda_1, \ldots, \lambda_{2p}) \\ &\quad \text{with} \begin{cases} \lambda_1 = \ldots = \lambda_p = 2 \\ \lambda_{p+1} = 2/2 = 1 \\ \ldots \\ \lambda_j = (j-p+1)/(2j-2p) \\ \ldots \\ \lambda_{2p-1} = p/(2p-2) \\ \lambda_{2p} = 1/(2p). \end{cases} \end{aligned} \tag{10}$$

We note that thanks to the first property of (10), the matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are symmetric. The change of variable has been defined in such a way that, if the UTP cable is super-balanced, then the matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are diagonal. Hence, the matrices $\mathbf{Z}_S\mathbf{Y}_S = \mathbf{AZYA}^{-1}$ and $\mathbf{Y}_S\mathbf{Z}_S = \mathbf{BYZB}^{-1}$ are also diagonal and the symmetrical currents and voltages correspond to the modes for propagation (see the end of Section III-A above and the Section VI).

If the cable is only a perfectly balanced UTP cable, one can prove that the transformed matrix $\mathbf{X}_S$ of a natural matrix $\mathbf{X}$ can be divided in four square sub-matrices of order p according to

$$\mathbf{X}_S = \begin{pmatrix} \mathbf{X}_{SU} & \mathbf{0}_{p,p} \\ \mathbf{0}_{p,p} & \mathbf{X}_{SL} \end{pmatrix} \tag{11}$$

where $\mathbf{0}_{p,p}$ is the square null matrix of order p and where $\mathbf{X}_{SU}$ is a diagonal matrix of order p, with

$$\mathbf{X}_{SU} = q\, \text{diag}(X_{1,1} - X_{1,2}, X_{3,3} - X_{3,4}, \ldots, \\ X_{2p-1,2p-1} - X_{2p-1,2p}) \tag{12}$$

with $q = 2$ if $\mathbf{X} = \mathbf{L}$ or if $\mathbf{X} = \mathbf{R}$, or with $q = 1/2$ if $\mathbf{X} = \mathbf{C}$ or if $\mathbf{X} = \mathbf{G}$. The matrix $\mathbf{X}_{SL}$ is a symmetric square matrix of order p.

It is interesting to see how the transformed matrices look like in the case of a symmetry ranging between that of the perfectly balanced cable and of the super-balanced cable. In the case of a cable having the symmetry given by (8) for its natural matrices, if $\mathbf{X} = \mathbf{L}$ or if $\mathbf{X} = \mathbf{R}$, we get

$$\mathbf{X}_{SU} = 2 \begin{pmatrix} X_A - X_B & 0 & 0 & 0 \\ 0 & X_A - X_B & 0 & 0 \\ 0 & 0 & X_A - X_B & 0 \\ 0 & 0 & 0 & X_A - X_B \end{pmatrix} \tag{13}$$

and (14) shown at the bottom of the page. If $\mathbf{X} = \mathbf{C}$ or if $\mathbf{X} = \mathbf{G}$, we get

$$\mathbf{X}_{SU} = \frac{1}{2} \begin{pmatrix} X_A - X_B & 0 & 0 & 0 \\ 0 & X_A - X_B & 0 & 0 \\ 0 & 0 & X_A - X_B & 0 \\ 0 & 0 & 0 & X_A - X_B \end{pmatrix} \tag{15}$$

and (16) shown at the bottom of the next page. Of course, the matrices $\mathbf{X}_{SL}$ given by (14) and (16) become diagonal if $N = M$, because the cable is super-balanced.

V. Parameters for the STP Cable

A. Ideally Balanced STP Cable

If we were only investigating the crosstalk between the conductors of a STP cable, we could consider a problem without an external reference conductor, the cable screen being used as reference conductor. We would use the natural matrices of this

$$\mathbf{X}_{SL} = \begin{pmatrix} X_A + X_B - 2X_M & -X_M + X_N & \frac{4}{3}(X_M - X_N) & 0 \\ -X_M + X_N & \frac{3X_A + 3X_B - 2X_M - 4X_N}{4} & \frac{2}{3}(X_M - X_N) & 0 \\ \frac{4}{3}(X_M - X_N) & \frac{2}{3}(X_M - X_N) & \frac{6X_A + 6X_B - 8X_M - 4X_N}{9} & 0 \\ 0 & 0 & 0 & \frac{X_A + X_B + 4X_M + 2X_N}{8} \end{pmatrix} \tag{14}$$

problem: a p.u.l. inductance matrix $\mathbf{L}_I$, a p.u.l. resistance matrix $\mathbf{R}_I$, a p.u.l. capacitance matrix $\mathbf{C}_I$ and a p.u.l. conductance matrix $\mathbf{G}_I$. These "internal" natural matrices are square matrices of order $2p$ and they are defined using the numbering defined in the Section III for the conductors 1 to $2p$. The STP cable will be perfectly balanced if these natural matrices have the property (6). The STP cable will be super-balanced if these natural matrices have the property (7).

If we now consider an external conductor and define it as the new reference conductor, we can use the definitions of the Section III for the natural matrices $\mathbf{L}$, $\mathbf{R}$, $\mathbf{C}$ and $\mathbf{G}$ of this problem. They are square matrices of order $n = 2p + 1$. For the cable screen, we can also define a p.u.l inductance L_R, a p.u.l. resistance R_R, a p.u.l. capacitance C_R and a p.u.l. conductance G_R with respect to the reference conductor. Let us call "ideal magnetic screen" a conductive screen for which any current flowing on the screen and returning to the generator through the reference conductor produces no magnetic field and no electric field inside the cable screen. If we assume that the cable screen is an ideal magnetic screen, for $\mathbf{X} = \mathbf{L}$ and $\mathbf{X} = \mathbf{R}$, we can show that

$$\mathbf{X} = \begin{pmatrix} X_{I11} + X_R & \dots & X_{I2p} + X_R & X_R \\ \vdots & \ddots & \vdots & \vdots \\ X_{I2p1} + X_R & \dots & X_{I2p2p} + X_R & X_R \\ X_R & \dots & X_R & X_R \end{pmatrix} \quad (17)$$

where the X_{Iij} are the elements of the internal matrix $\mathbf{X}_I$. We note that if the screen has a nonnegligible resistivity at DC, the resistance matrix does not comply with (17), because a current flowing on the screen produces an electric field inside the screen. If we wish to consider that a screen is an ideal magnetic screen at low frequency, we might have to assume that its resistivity is negligible. We see that at frequencies so large that the thickness of an homogenous screen becomes much larger than its skin depth (in this case the p.u.l. transfer impedances $Z_{T1}, \dots Z_{T2p}$ of the internal conductors become very small), the screen might be regarded as an ideal magnetic screen.

Let us call "ideal electric screen" a screen which does not allow any capacitive or conductive coupling between the conductors of the pairs and the reference conductor. If we assume that the cable screen is an ideal electric screen, for $\mathbf{X} = \mathbf{C}$ and $\mathbf{X} = \mathbf{G}$, we can show that

$$\mathbf{X} = \begin{pmatrix} X_{I11} & \dots & X_{I12p} & -\sum_{i=1}^{2p} X_{I1i} \\ \vdots & \ddots & \vdots & \vdots \\ X_{I2p1} & \dots & X_{I2p2p} & -\sum_{i=1}^{2p} X_{I2pi} \\ -\sum_{i=1}^{2p} X_{Ii1} & \dots & -\sum_{i=1}^{2p} X_{Ii2p} & X_R + \sum_{i=1}^{2p}\sum_{j=1}^{2p} X_{Iij} \end{pmatrix} \quad (18)$$

where the X_{Iij} are the elements of the internal matrix $\mathbf{X}_I$. We know that cable screens (for instance those without significant apertures) can closely approximate ideal electric screens.

B. Change of Variables for the STP Cable

The symmetry transform used in this paper for STP cables is defined by (9) for the first $2p - 1$ symmetrical voltages and currents and by

$$\begin{cases} v_{S2p} = \frac{1}{2p}(v_1 + v_2 + \dots + v_{2p}) - v_{2p+1} \\ v_{S2p+1} = v_{2p+1} \end{cases}$$
$$\begin{cases} i_{S2p} = i_1 + i_2 + \dots + i_{2p} \\ i_{S2p+1} = i_1 + i_2 + \dots + i_{2p+1} \end{cases} \quad (19)$$

for the last two symmetrical voltages and currents. This and (2) define the matrices $\mathbf{A}$ and $\mathbf{B}$. Note that the example (5) corresponds to the case $p = 2$. The symmetrical voltages $v_{S1}, \dots, v_{Sp}$ (respectively, currents $i_{S1}, \dots, i_{Sp}$) are called the differential mode voltages (respectively, currents). The symmetrical voltage v_{S2p} (respectively, current i_{S2p}) can be called the common-mode voltage (respectively, current) of internal conductors. The symmetrical voltage v_{S2p+1} (respectively, current i_{S2p+1}') is the common-mode voltage (respectively, current) of the cable. We can easily establish the two following properties

$$\begin{aligned} {}^t\mathbf{A} &= \mathbf{B}^{-1} \\ \det\ \mathbf{A} &= \det\ \mathbf{B} = \pm 1. \end{aligned} \quad (20)$$

However, unlike the change of variable introduced for the UTP cable, we cannot say that any two different rows of the matrices $\mathbf{A}$ and $\mathbf{B}$ are necessarily orthogonal. Also, the matrix $\mathbf{AB}^{-1}$ is not diagonal, in general. We note that thanks to the first property of (20), the matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are symmetric. The change of variable has been defined in such a way that, if the STP cable is super-balanced and if its screen is an ideal electric screen and an ideal magnetic screen, then the matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are diagonal. Hence, the matrices $\mathbf{Z}_S\mathbf{Y}_S = \mathbf{AZYA}^{-1}$ and $\mathbf{Y}_S\mathbf{Z}_S = \mathbf{BYZB}^{-1}$ are also diagonal and the symmetrical currents and voltages correspond to the modes for propagation.

If the cable is only a perfectly balanced STP cable with an ideal magnetic and ideal electric screen, one can show that the symmetry-transformed matrix $\mathbf{X}_S$ of a natural matrix $\mathbf{X}$ can be written

$$\mathbf{X}_S = \begin{pmatrix} X_{IS11} & \dots & X_{IS12p} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ X_{IS2p1} & \dots & X_{IS2p2p} & 0 \\ 0 & \dots & 0 & X_{CM} \end{pmatrix} \quad (21)$$

$$\mathbf{X}_{SL} = \begin{pmatrix} X_A + X_B - 2X_M & \frac{4}{3}(-X_M + X_N) & 2(X_M - X_N) & 0 \\ \frac{4}{3}(-X_M + X_N) & \frac{12X_A + 12X_B - 8X_M - 16X_N}{9} & \frac{4}{3}(X_M - X_N) & 0 \\ 2(X_M - X_N) & \frac{4}{3}(X_M - X_N) & \frac{3X_A + 3X_B - 4X_M - 2X_N}{2} & 0 \\ 0 & 0 & 0 & 8X_A + 8X_B + 32X_M + 16X_N \end{pmatrix} \quad (16)$$

where the X_{ISij} are the elements of the symmetry-transformed matrix $\mathbf{X}_{IS}$ of the internal natural matrix $\mathbf{X}_I$ defined as if the internal conductor were a UTP cable, and where X_{CM} is equal to L_R or R_R or C_R or G_R according to the case. This property is very powerful, since a single element of $\mathbf{X}_S$ depend on the parameters of the reference conductor.

It should be noted that Kaden [13, chap. E] has established analytic formulas for the computation of the symmetry-transformed matrices $\mathbf{L}_{IS}$, $\mathbf{R}_{IS}$ and $\mathbf{C}_{IS}$ of a super-balanced star quad (these matrices are diagonal, and their diagonal elements correspond to the value computed by Kaden for the symmetric circuits, the phantom circuit and the asymmetric circuit).

VI. Crosstalk on a Uniform Multipair Cable

A uniform cable is a cable in which the symmetry-transformed matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ always take on the same value along the cable length. However, for many balanced cables of interest, this is true with a sufficient accuracy only after some averaging over the cable length (typically over a large number of twists of the twisted pairs).

Once the transformed matrices have been measured, they could be used to compute the natural matrices using (4). The usual computation of crosstalk [8, § 6.2], [9] starting with the diagonalization of $\mathbf{ZY}$ and $\mathbf{YZ}$ would then directly provide the natural voltages and currents, as said at the Section III-A.

Noting that (3) is formally identical to (1), we see that the computation of couplings may alternatively be performed using a diagonalization of $\mathbf{Z}_S\mathbf{Y}_S$ and $\mathbf{Y}_S\mathbf{Z}_S$. More precisely, we introduce two regular matrices $\mathbf{T}_S$ and $\mathbf{S}_S$ such that

$$\begin{cases} \mathbf{T}_S^{-1}\mathbf{Y}_S\mathbf{Z}_S\mathbf{T}_S = \mathbf{D} \\ \mathbf{S}_S^{-1}\mathbf{Z}_S\mathbf{Y}_S\mathbf{S}_S = \mathbf{D} \end{cases} \tag{22}$$

where

$$\mathbf{D} = \mathrm{diag}_n\left(\gamma_1^2, \ldots, \gamma_n^2\right) \tag{23}$$

is the diagonal matrix of order n of the eigenvalues. These eigenvalues are the squares of the propagation constants γ_i for waves traveling toward the far-end (they have a positive imaginary part). For the symmetry transforms defined at the Section IV-B and Section V-B, the matrices $\mathbf{Z}_S$ and $\mathbf{Y}_S$ are symmetric. Therefore, taking the transpose of the first line of (22) produces the second line for the particular choice of $\mathbf{S}_S$

$$\mathbf{S}_S = {}^t\mathbf{T}_S^{-1}. \tag{24}$$

This shows that $\mathbf{Z}_S\mathbf{Y}_S$ and $\mathbf{Y}_S\mathbf{Z}_S$ can be diagonalized into the same matrix $\mathbf{D}$. The matrices $\mathbf{T}_S$ and $\mathbf{S}_S$ define a "modal transform" for the symmetrical currents and voltages and the results of this transform are referred to as modal currents and modal voltages. If we use $\mathbf{I}_M$ to denote the vector of the modal currents $i_{M1}, \ldots, i_{Mn}$ and $\mathbf{V}_M$ to denote the vector of the modal voltages $v_{M1}, \ldots, v_{Mn}$, we have

$$\begin{cases} \mathbf{V}_S = \mathbf{S}_S\mathbf{V}_M \\ \mathbf{I}_S = \mathbf{T}_S\mathbf{I}_M. \end{cases} \tag{25}$$

We note that (22) implies that the column-vectors of $\mathbf{S}_S$ (respect. $\mathbf{T}_S$) are linearly independent eigenvectors of $\mathbf{Z}_S\mathbf{Y}_S$ (respect. $\mathbf{Y}_S\mathbf{Z}_S$). As a result, $\mathbf{S}_S$ and $\mathbf{T}_S$ are not uniquely defined by (22)–(24) alone because of: first the arbitrary ordering of the eigenvalues in (23), and second the arbitrary choice of eigenvector(s) corresponding to a given eigenvalue. However, for a perfectly balanced UTP cable and for a perfectly balanced STP cable with an ideal electric and ideal magnetic screen, (11), (12) and (21) show that the first p rows and columns of the symmetry-transformed matrices $\mathbf{L}_S$, $\mathbf{R}_S$, $\mathbf{C}_S$ and $\mathbf{G}_S$ are already diagonalized. Therefore, the first p rows and columns of the matrices $\mathbf{T}_S$ and $\mathbf{S}_S$ of such a cable could be chosen in such a way that they are the first p rows and columns of the identity matrix, and this choice is compatible with (24). If the cable is super-balanced, $\mathbf{Z}_S\mathbf{Y}_S$ and $\mathbf{Y}_S\mathbf{Z}_S$ are already diagonalized and $\mathbf{T}_S$ and $\mathbf{S}_S$ could be the identity matrix of order n. In the case of a well balanced UTP or STP cable, the matrices $\mathbf{T}_S$ and $\mathbf{S}_S$ could be chosen in such a way that they approach the first p rows and columns of the identity matrix. As a result, the differential voltages and currents on each pair would nearly correspond to propagation modes.

VII. Crosstalk on a Nonuniform Multipair Cable

The manufacturer of a balanced cable tries to produce an almost perfectly balanced cable, but the parameters of a real cable fluctuate around this ideal case. If the fluctuations were not present, the cable would be perfectly balanced, the differential voltages and currents would correspond to propagation modes as said above, and if the cable had balanced terminations, no crosstalk would degrade the signals transmitted on any pair.

In practice, some crosstalk occurs, because fluctuations are unavoidable. In this section, we will consider that the fluctuations of the cable parameters are known over the length of a section of cable. If we consider the nonuniform transformed telegrapher's equation, that is to say (3) where the parameters of the cable are z-dependent, we can derive a set of two independent equations

$$\begin{cases} \frac{d^2\mathbf{V}_S}{dz^2} - \frac{d\mathbf{Z}_S}{dz}\mathbf{Z}_S^{-1}\frac{d\mathbf{V}_S}{dz} - \mathbf{Z}_S\mathbf{Y}_S\mathbf{V}_S = 0 \\ \frac{d^2\mathbf{I}_S}{dz^2} - \frac{d\mathbf{Y}_S}{dz}\mathbf{Y}_S^{-1}\frac{d\mathbf{I}_S}{dz} - \mathbf{Y}_S\mathbf{Z}_S\mathbf{I}_S = 0. \end{cases} \tag{26}$$

It is then possible to split the symmetry-transformed matrix $\mathbf{Z}_S$ (respectively, $\mathbf{Y}_S$) into an homogenous (i.e., uniform) part $\mathbf{Z}_{hS}$ (respectively, $\mathbf{Y}_{hS}$) independent of z, and a (z-dependent) fluctuation part $\delta\mathbf{Z}_S$ (respectively, $\delta\mathbf{Y}_S$). If we show the dependence on the abscissa z, we obtain

$$\begin{cases} \mathbf{Z}_S(z) = \mathbf{Z}_{hS} + \delta\mathbf{Z}_S(z) \\ \mathbf{Y}_S(z) = \mathbf{Y}_{hS} + \delta\mathbf{Y}_S(z). \end{cases} \tag{27}$$

We will not assume that $\delta\mathbf{Z}_S$ and $\delta\mathbf{Y}_S$ have a zero average, but they are assumed to be small corrections. From now on, we will focus on the solution of the first line of (26) to compute $\mathbf{V}_S$, because all results for $\mathbf{I}_S$ can then be derived using a duality transform. $\mathbf{V}_S$ can be split according to

$$\mathbf{V}_S = \mathbf{V}_{hS} + \delta\mathbf{V}_S \tag{28}$$

where $\mathbf{V}_{hS}$ is a solution computed for $\delta\mathbf{Z}_S = 0$ and $\delta\mathbf{Y}_S = 0$. Following the derivation of [14] for the scalar case, we find that the equation for $\delta\mathbf{V}_S$ is

$$\frac{d^2\delta\mathbf{V}_S}{dz^2} - \mathbf{Z}_{hS}\mathbf{Y}_{hS}\delta\mathbf{V}_S = \Delta_{0S}(z)(\mathbf{V}_{hS} + \delta\mathbf{V}_S) + \Delta_{1S}(z)\left(\frac{d\mathbf{V}_{hS}}{dz} + \frac{d\delta\mathbf{V}_S}{dz}\right) \quad (29)$$

with

$$\Delta_{0S}(z) = \delta\mathbf{Z}_S(z)\mathbf{Y}_{hS} + \mathbf{Z}_{hS}\delta\mathbf{Y}_S(z) + \delta\mathbf{Z}_S(z)\delta\mathbf{Y}_S(z) \quad (30)$$

and

$$\Delta_{1S}(z) = \frac{d\delta\mathbf{Z}_S(z)}{dz}\left(\mathbf{Z}_{hS} + \delta\mathbf{Z}_S(z)\right)^{-1}. \quad (31)$$

We are looking for solutions of (29)–(31) for a section of cable of length L. We will assume that the section of cable is terminated at both ends by a network having an impedance matrix equal to the characteristic impedance matrix [8], [9] of the cable without the fluctuations, and that a suitable source at the near-end excites a single propagation mode j of the uniform cable (i.e., a mode computed without fluctuations).

Multiplying (29) on the left by the matrix $\mathbf{S}_S^{-1}$ obtained by applying the theory of the Section VI to the homogeneous problem defined by the matrices $\mathbf{Z}_{hS}$ and $\mathbf{Y}_{hS}$, and keeping only first order terms, we get

$$\frac{d^2\delta\mathbf{V}_M}{dz^2} - \mathbf{D}\delta\mathbf{V}_M \approx \Delta_{0M}(z)(\mathbf{V}_{hM} + \delta\mathbf{V}_M) + \Delta_{1M}(z)\frac{d\mathbf{V}_{hM}}{dz} \quad (32)$$

where

$$\Delta_{0M}(z) = \mathbf{S}_S^{-1}\left(\delta\mathbf{Z}_S(z)\mathbf{Y}_{hS} + \mathbf{Z}_{hS}\delta\mathbf{Y}_S(z)\right)\mathbf{S}_S \quad (33)$$

and

$$\Delta_{1M}(z) = \mathbf{S}_S^{-1}\frac{d\delta\mathbf{Z}_S(z)}{dz}\mathbf{Z}_{hS}^{-1}\mathbf{S}_S \quad (34)$$

where $\delta\mathbf{V}_M = \mathbf{S}_S^{-1}\delta\mathbf{V}_S$, where $\mathbf{V}_M = \mathbf{S}_S^{-1}\mathbf{V}_S$ and where $\mathbf{D}$ is given by (23) applied to the homogeneous problem. We see that the left-hand side of (32) contains no z-dependent coefficient. Using its Green's function

$$\mathbf{G}(z,z') = -\frac{1}{2}\mathrm{diag}_n\left(\frac{e^{-\gamma_1|z-z'|}}{\gamma_1}, \ldots, \frac{e^{-\gamma_n|z-z'|}}{\gamma_n}\right) \quad (35)$$

and noting that, because of our choice of termination, the section of cable behaves as if it was infinite in both directions, with $\delta\mathbf{Z}_S$ and $\delta\mathbf{Y}_S$ vanishing for $z < 0$ and $z > L$, we get

$$\delta\mathbf{V}_M \approx \int_0^L \mathbf{G}(z,z')\left[\Delta_{0M}(z')(\mathbf{V}_{hM} + \delta\mathbf{V}_M) + \Delta_{1M}(z')\frac{d\mathbf{V}_{hM}}{dz}\right]dz'. \quad (36)$$

In order to obtain the value of $\delta\mathbf{V}_M$ to the first order of the method of perturbation [14], [15, § 9.1], we only need to neglect the $\delta\mathbf{V}_M$ term in the right-hand side of (36). Since, according to our assumptions, $\mathbf{V}_{hM}$ is a vector having its element of the jth row equal to $V_{Mj}\exp(-\gamma_j z)$ and all other elements equal to 0, we get for the ith row of $\delta\mathbf{V}_M$ at the near-end

$$\left[\frac{\delta\mathbf{V}_M(0)}{V_{Mj}}\right]_i = \frac{j\omega}{2}\int_0^L F_{ij}^{\mathrm{NE}}(z)e^{-(\gamma_i+\gamma_j)z}dz \quad (37)$$

and for the ith row of $\delta\mathbf{V}_M$ at the far-end

$$\left[\frac{\delta\mathbf{V}_M(L)}{V_{Mj}}\right]_i = -\frac{j\omega e^{-\gamma_i L}}{2}\int_0^L F_{ij}^{\mathrm{FE}}(z)e^{(\gamma_i-\gamma_j)z}dz \quad (38)$$

where we have noted $[\mathbf{X}]_i$ the element of the row i of a vector $\mathbf{X}$, and with

$$F_{ij}^{\mathrm{NE}}(z) = \frac{1}{j\omega}\mathbf{S}_S^{-1}\left\{\gamma_j\left[\delta\mathbf{Z}_S(z)\mathbf{Z}_{hS}^{-1}\right]_{ij} - \gamma_i\left[\mathbf{Y}_{hS}^{-1}\delta\mathbf{Y}_S(z)\right]_{ij}\right\}\mathbf{S}_S \quad (39)$$

and

$$F_{ij}^{\mathrm{FE}}(z) = \frac{1}{j\omega}\mathbf{S}_S^{-1}\left\{\gamma_j\left[\delta\mathbf{Z}_S(z)\mathbf{Z}_{hS}^{-1}\right]_{ij} + \gamma_i\left[\mathbf{Y}_{hS}^{-1}\delta\mathbf{Y}_S(z)\right]_{ij}\right\}\mathbf{S}_S \quad (40)$$

where $[\mathbf{X}]_{ij}$ is the element of the row i and of the column j of a matrix $\mathbf{X}$. In the case of a lossless cable the propagation constants γ_i and γ_j are imaginary and proportional to ω, and F_{ij}^{NE} and F_{ij}^{FE} are therefore real and frequency-independent.

It is instructive to compare these results for the multipair cable according to a first-order perturbation theory, to the ones obtained by Wenger *et al.* [14] for a single transmission channel using a second-order perturbation theory. A single transmission channel corresponding to the case $i = j$, we see that the echo term (37) is identical to the single order formula (32) of [14], and that the channel term (38) looks different from the value 0 found by Wenger *et al.* to the first order. However, the discrepancy disappears if we consider that these authors assumed a zero mean for the fluctuations. We note that they found a nonvanishing channel term with the second order perturbation theory.

For $i \neq j$, (37) provides the near-end crosstalk (NEXT) coupling factor, and (38) the far-end crosstalk (FEXT) coupling factor, in the special case that we have considered for the terminations. These crosstalk terms are computed between two propagation modes of the uniform cable. In the case where the propagation constants γ_i and γ_j of two modes are equal, according to (38) the FEXT voltage is proportional to the length L of the section, multiplied by the average over the section of cable of the function defined by (40). It is interesting to consider the special case of an hypothetical cable made of perfectly conducting wires in a radially and axially homogenous and lossless dielectric. For such a cable, the product $\mathbf{Z}_S\mathbf{Y}_S$ is a constant, and all the propagation constants are equal to $j\omega/c$ where c is the velocity of e.m. waves in the dielectric. In this case, $\delta\mathbf{V}_M$ at the far-end is equal to zero, because the function defined by (40) is equal to zero. Even if these assumptions are not true for real cables, we expect the function defined by (40) for the FEXT to have a smaller modulus than the function defined by (39) for the NEXT.

VIII. Statistical Approach

A. Definitions and Analytical Results

In the Section VII we have investigated what we could derive from the knowledge of the fluctuations $\delta\mathbf{Z}_S$ and $\delta\mathbf{Y}_S$ of the cable parameters. In this section we will consider the more realistic case where these fluctuations are only known from some statistical properties. We can first compute the expectation of the coupling factors over many experiments using different samples of the same cable having the same length L. Assuming stationary processes for the fluctuation of the cable parameters, we get

$$\left\langle\left[\frac{\delta\mathbf{V}_M(0)}{V_{Mj}}\right]_i\right\rangle = \frac{j\omega\left\langle F_{ij}^{\mathrm{NE}}\right\rangle}{2(\gamma_i+\gamma_j)}\left(1-e^{-(\gamma_i+\gamma_j)L}\right) \quad (41)$$

and

$$\left\langle\left[\frac{\delta\mathbf{V}_M(L)}{V_{Mj}}\right]_i\right\rangle = \frac{j\omega\left\langle F_{ij}^{\mathrm{FE}}\right\rangle}{2(\gamma_j-\gamma_i)}\left(e^{-\gamma_j L}-e^{-\gamma_i L}\right). \quad (42)$$

The left-hand side of (41) and (42) are expectations of phasors. Assuming the fluctuations to be ergodic for the expectation [16, ch. 12], the average of the right-hand side of (41) and (42) can also be regarded as taken over a length of cable much larger than the distance necessary to obtain independent fluctuations. If $\gamma_i=\gamma_j$, the singularity in (42) should be removed using the limit of the right-hand side when $\gamma_j-\gamma_i$ tends to zero. In this case (41) and (42) are very similar to the classical results for the voltages induced at the ends of a transmission line made of two weakly coupled identical conductors above a ground plane, terminated with "matched" impedances to ground [17, equ. (23)]. We note that if the fluctuations $\delta\mathbf{Z}_S$ and $\delta\mathbf{Y}_S$ have a zero average, the expectations of the coupling factors (41) and (42) are equal to zero.

We can also compute the expectation of the square of the modulus of the coupling factors (thereafter called ESMCF), with the assumptions of the Section VII for the terminations. At the near-end, we get

$$\left\langle\left|\left[\frac{\delta\mathbf{V}_M(0)}{V_{Mj}}\right]_i\right|^2\right\rangle = \frac{\omega^2}{4}\int_0^L\int_0^L\left\langle F_{ij}^{\mathrm{NE}}(z_1)\overline{F_{ij}^{\mathrm{NE}}(z_2)}\right\rangle \times e^{-(\gamma_i+\gamma_j)z_1-\overline{(\gamma_i+\gamma_j)}z_2}dz_1dz_2 \quad (43)$$

The average inside the double integral is an autocorrelation and it should be viewed as a function of $\tau = z_2 - z_1$ only, because we assume the processes to be stationary and ergodic to the second order [16, ch. 12]. It therefore seems appropriate to use the variable τ and $s=z_2+z_1$ in the integral. For this change of variables, the new intervals of integration become $[-L,L]$ for τ, and $[|\tau|, 2L-|\tau|]$ for s. Noting α_i the real part and β_i the imaginary part of a propagation constant γ_i, we finally get the NEXT ESMCF

$$\left\langle\left|\left[\frac{\delta\mathbf{V}_M(0)}{V_{Mj}}\right]_i\right|^2\right\rangle = \frac{\omega^2}{4}\int_{-\infty}^{+\infty}C_{ij}^{\mathrm{NE}}(\tau)W_{ij}^{\mathrm{NE}}(\tau)e^{-j\tau(\beta_i+\beta_j)}d\tau \quad (44)$$

with the autocorrelation for the NEXT defined as

$$C_{ij}^{\mathrm{NE}}(\tau) = \left\langle\overline{F_{ij}^{\mathrm{NE}}(z)}F_{ij}^{\mathrm{NE}}(z+\tau)\right\rangle \quad (45)$$

and α_i and α_j being positive, with the window function for the NEXT defined as

$$\text{if } -L\le z\le L \quad W_{ij}^{\mathrm{NE}}(z) = \begin{cases} L-|z|, & \text{for } \alpha_i=\alpha_j=0 \\ e^{-L(\alpha_i+\alpha_j)}\frac{\sinh([L-|z|][\alpha_i+\alpha_j])}{\alpha_i+\alpha_j}, & \text{else}\end{cases}$$
$$\text{else } W_{ij}^{\mathrm{NE}}(z)=0. \quad (46)$$

In a similar way, the FEXT ESMCF can be derived as

$$\left\langle\left|\left[\frac{\delta\mathbf{V}_M(L)}{V_{Mj}}\right]_i\right|^2\right\rangle = \frac{\omega^2 e^{-2\alpha_i L}}{4} \times\int_{-\infty}^{+\infty}C_{ij}^{\mathrm{FE}}(\tau)W_{ij}^{\mathrm{FE}}(\tau)e^{-j\tau(\beta_j-\beta_i)}d\tau \quad (47)$$

with the autocorrelation for the FEXT defined as

$$C_{ij}^{\mathrm{FE}}(\tau) = \left\langle\overline{F_{ij}^{\mathrm{FE}}(z)}F_{ij}^{\mathrm{FE}}(z+\tau)\right\rangle \quad (48)$$

and the window function for the FEXT defined as

$$\text{if } -L\le z\le L \quad W_{ij}^{\mathrm{FE}}(z) = \begin{cases} e^{-L(\alpha_j-\alpha_i)}\frac{\sinh([L-|z|][\alpha_j-\alpha_i])}{\alpha_j-\alpha_i}, & \text{if } \alpha_i\ne\alpha_j \\ L-|z|, & \text{if } \alpha_i=\alpha_j\end{cases}$$
$$\text{else } W_{ij}^{\mathrm{FE}}(z)=0. \quad (49)$$

Four remarks can be made on these formula.

1) The integrations over $\mathbb{R}$ can of course be replaced by integrations on $[-L,L]$.
2) With a suitable modification of the integrands, the integrals over $\mathbb{R}$ can also be replaced with integrals on the interval $[0,L]$, using the fact that the value of an autocorrelation at $-\tau$ is the conjugate of its value at τ.
3) The autocorrelations defined by (45) and (48) do not depend on the length L of the section, but they are frequency-dependent through the ratios γ_i/ω and γ_j/ω appearing in (39) and (40). In the frequency range where the cable is not very lossy, C_{ij}^{NE} and C_{ij}^{FE} will only slightly depend on frequency.
4) The window functions defined by (46) and (49) depend on the length L of the section and on the frequency through the attenuation constants α_i and α_j.

We observe that the above theory, unlike the one presented in [14] for a single transmission channel, does not assume a short range for the coherence of the fluctuations of the cable parameters.

B. Meaning of Expectations in the Statistical Approach

In the sequel, we assume that $\mathbf{Z}_{hS}$ and $\mathbf{Y}_{hS}$ are chosen in such a way that the first p rows and columns of the matrices $\mathbf{T}_S$ and $\mathbf{S}_S$ are the first p rows and columns of the identity matrix (see the end of Section VI). For i and j ranging from 1 to p, $i \neq j$, H_{ij}^{NE} and H_{ij}^{FE} will be defined as the transfer functions between a differential-mode voltage at the near-end of the pair j and a differential-mode crosstalk voltage on the pair i, at the near-end and at the far-end, respectively (as shown in Fig. 1).

The expectations of the square of the modulus of the transfer functions H_{ij}^{NE} and H_{ij}^{FE} (thereafter called ESMTF) are equal to the corresponding ESMCF. They are frequency-dependent expectations, and can therefore be considered as ensemble averages of frequency-domain measurements of the square of the modulus NEXT and FEXT transfer functions over different realizations of the random variable, i.e., different samples of a section of cable of length L. Consequently, if a sufficient number of realizations is considered, the maximum values of the modulus of the NEXT and FEXT transfer functions are expected to be a few dB larger than the NEXT and FEXT ESMTF (at each frequency).

The expectation of the square of the noise voltage produced by a resistor at a given time t_0 happens to be directly measurable: the rms voltage across the resistor is a good estimate of this expectation, because the ensemble average can be replaced with a time average when the process is stationary and ergodic. We of course want to know if a similar phenomenon takes place for the NEXT and FEXT ESMTF. For investigating this question, we need a definition: we will say that the autocorrelations C_{ij}^{NE} and C_{ij}^{FE} have a short coherence length if $|C_{ij}^{\mathrm{NE}}(\tau)|$ and $|C_{ij}^{\mathrm{FE}}(\tau)|$ become very small when $|\tau|$ is larger than a coherence length τ_C, and if τ_C is much smaller than L, $1/\alpha_i$, $1/\beta_i$, $1/\alpha_j$ and $1/\beta_j$. In the case of a short coherence length, C_{ij}^{NE} and C_{ij}^{FE} behave in the integrals of (44) and (47) as a Dirac function $\delta(\tau)$ times a frequency-dependent constant, which will be noted K_{ij}^{NE} for the NEXT and K_{ij}^{FE} for the FEXT.

In the special case where the autocorrelations have short coherence length and losses are very small (i.e., L is much smaller than $1/\alpha_i$ and $1/\alpha_j$), we find

$$\frac{1}{L}\left\langle \left|H_{ij}^{\mathrm{NE}}\right|^2 \right\rangle \approx \frac{\omega^2}{4} K_{ij}^{\mathrm{NE}} \tag{50}$$

$$\frac{1}{L}\left\langle \left|H_{ij}^{\mathrm{FE}}\right|^2 \right\rangle \approx \frac{\omega^2}{4} K_{ij}^{\mathrm{FE}}. \tag{51}$$

These equations imply that averaging of the square of the modulus of the transfer functions takes place over the length L. Consequently, in this special case, the ensemble average can be replaced with an average over the length of the cable, and the NEXT and FEXT ESMTF for a length of cable L' such that $\tau_C \ll L' \ll L$ can be measured directly, with a single experiment.

IX. Compatibility With Known Experimental Results

A. Assumptions and Derivation

We have derived from the MTL theory a computation of crosstalk caused by fluctuations of the parameters of a balanced cable. We now want to check if our results are compatible with some published data on crosstalk in balanced cables.

If we consider cables for the telephone loop used at a high enough frequency (for instance above 100 kHz), we might assume the following.

a) Only the p differential mode currents and voltages are used for signal transmission (assuming that phantom circuits [18, § 3.5.7] are not used), and the cable being well balanced, the almost uncoupled differential mode voltages (and currents) correspond to propagation modes.
b) The impedance matrix of the termination at each cable end is equal to the characteristic impedance matrix.
c) The propagation constants are equal, so that we may write $\gamma_i = \gamma_j = \alpha + j\beta$.
d) The autocorrelations C_{ij}^{NE} and C_{ij}^{FE} have a short coherence length.

Let us compute the NEXT ESMTF. In practice we may here consider [14], [18, § 3.2.12] that

$$\alpha \approx \alpha_0 \sqrt{\omega} \tag{52}$$

where α_0 is a frequency-independent constant. We therefore get

$$W_{ij}^{\mathrm{NE}}(0) \approx \frac{1 - e^{-4L\alpha_0\sqrt{\omega}}}{4\alpha_0\sqrt{\omega}}. \tag{53}$$

For a long line (in the present case, "long" means that αL corresponds to a significant transmission loss), the exponential term in (53) becomes small, and using (44), we get

$$\left\langle \left|H_{ij}^{\mathrm{NE}}\right|^2 \right\rangle \approx \omega^{1.5} \frac{K_{ij}^{\mathrm{NE}}}{16\alpha_0}. \tag{54}$$

If we now consider the FEXT ESMTF predicted by (47), using the above assumptions, we get

$$\left\langle \left|H_{ij}^{\mathrm{FE}}\right|^2 \right\rangle e^{2\alpha L} \approx \omega^2 L \frac{K_{ij}^{\mathrm{FE}}}{4}. \tag{55}$$

We note that (54), (55) are different from (50)–(51). Consequently, performing ensemble averages is necessary to experimentally obtain the results given by (54) and (55).

B. Comparison With Known Results

Three kinds of experimental data and empirical models are available for cables used in the telephone loop: data on individual pair-to-pair NEXT and FEXT couplings, data on the power-sum NEXT and FEXT and data on worst case pair-to-pair NEXT and FEXT couplings. Pair-to-pair couplings vary in a complex manner with frequency, and these variations are different for each pair-to-pair combination. Power-sum quantities are formed from the sum of the pair-to-pair coupling powers of the other pairs in a binder group to a given pair [19]. The power summing process smoothes coupling variations in a given cable, in such a way that simple models provide a good description of the experimental data. Simple models are also available for the worst case NEXT loss (WNL) and the worst case FEXT loss (WFL) due to one pair-to-pair coupling. They roughly correspond to the envelope of many individual pair-to-pair couplings [20, Figs. 4 and 5], and they may be compared to (54) and (55), as explained in Section VIII-B above. Such models are

$$\mathrm{WNL}(f) = \mathrm{WNL}_1 f^{-1.5} \tag{56}$$

$$\mathrm{WFL}(f) = \mathrm{WFL}_1 f^{-2} L^{-1} e^{2\alpha L} \tag{57}$$

where f is the frequency, L is the loop length and where WNL_1 and WFL_1 are constants determined by measurements. These models may be applied to cables used in North America and Europe [7], [18, § 3.5.5], [21, Figs. 2 and 3], [22] in spite of the difference in their structure, but they are not very accurate.

The NEXT ESMTF given by (54) is independent from the line length, and increases with frequency at a rate close to 15 dB/decade if we assume that K_{ij}^{NE} is frequency-independent, in line with the third remark following (49). This agrees with (56). The product of the FEXT ESMTF by the $\exp(2\alpha L)$ factor is the inverse of the FEXT signal-to-crosstalk ratio. This product, given by (55) is increasing with the line length at a rate of 10 dB/decade, and with frequency at a rate of 20 dB/decade if we assume that K_{ij}^{FE} is frequency-independent, in line with the third remark following (49). This agrees with (57).

The agreement between (54), (55) and (56)–(57) has been obtained using several assumptions which would need to be investigated. For instance, it seems likely that manufacturing process and storage conditions or mechanical stress may introduce periodical imperfections which would invalidate the hypothesis of a short coherence length.

X. Conclusion

We have defined linear changes of variables for the voltages and currents, which cause differential-mode currents and differential-mode voltages to appear in the MTL theory. After this change of variables, a good balance of the cable corresponds to a small value for nondiagonal term of the symmetry-transformed matrices. Another benefit of using our symmetrical voltages and currents is that a connection to the reference conductor is not necessary for measuring them, except v_n and i_n. Consequently, we expect that the characteristics of the reference conductor have little influence on the elements of the symmetry-transformed matrices, except the elements of their last row and last column. We have proved this only when (21) is applicable. This question needs to be explored further.

The random departure from the ideally balanced case has been treated with a statistical approach applied to the MTL theory. As far as we know, this approach is new. Of course, the known limitations of the MTL theory apply, but they are much less limiting than the assumptions of the two-circuit model.

In Section IX, we have shown that our adaptation of the MTL theory is compatible with rough rules of thumb concerning the telephone loop, stating that the worst case NEXT loss or FEXT signal to crosstalk ratio decrease with x dB/decade of line length or frequency, x being equal to 0, 10, 15 or 20, according to the case. This result is not the purpose of this paper, since the two-circuit model already provides such a coarse agreement between theory and experience (at the cost of unjustified initial assumptions, though). The benefits of our MTL theory-based approach are as follows.

- All results are derived from the parameters of the cable, for instance K_{ij}^{NE} and K_{ij}^{FE} could be computed if the statistical properties of the fluctuations $\delta\mathbf{Z}_S$ and $\delta\mathbf{Y}_S$ are known.
- As shown in Section III, such fluctuations can be directly measured as symmetry-transformed matrices.
- All conductors of the cable are inherently considered in the MTL theory.
- The rough results of Section IX have been obtained with unnecessary assumptions which could be removed to improve the accuracy of predictions and expand their range of application.
- The statistical approach can clearly be adapted to the ZXtalk method, since it is also a modal transmission scheme for which the transmission channels are ideally uncoupled.

More work is needed to fully develop and validate the proposed analysis of crosstalk in balanced interconnections used for differential signal transmission. It seems that different models and experimental approaches will be necessary to cover balanced wiring on printed circuit boards (for frequencies up to ∼10 GHz), 2-pair and 4-pair cables used in local area networks (for frequencies up to ∼500 MHz) and multipair cables used in the telephone loop (for frequencies up to ∼20 MHz). It should then be possible to use such analysis for the design of better links, which could implement optimized interfaces or crosstalk cancellation schemes, or new designs of propagation medium.

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References

[1] "Dr. Campbell's Memoranda of 1907 and 1912," *Bell Syst. Tech. J.*, vol. 14, no. 4, pp. 553–572, Oct. 1935.
[2] D. G. Kam, H. Lee, J. Kim, and J. Kim, "A new twisted differential line structure on high-speed printed circuit boards to enhance immunity to crosstalk and external noise," *IEEE Microw. Wireless Compon. Lett.*, vol. 13, no. 9, pp. 411–413, Sep. 2003.
[3] F. Broydé, "Clear as bell—Controlling crosstalk in uniform interconnections," *IEEE Circuit Devices Mag.*, vol. 20, no. 6, pp. 29–37, Nov. 2004.
[4] F. Broydé and E. Clavelier, "A new method for the reduction of crosstalk and echo in multiconductor interconnections," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 2, pp. 405–416, Feb. 2005.
[5] R. Croze and L. Simon, *Transmission Téléphonique—Théorie des lignes*. Paris: Editions Eyrolles, 1952.
[6] *Transmission Systems for Communications Revised*, 4th ed. Murray Hill, NJ: Bell Telephone Laboratories, 1970.
[7] S. Galli and K. J. Kerpez, "Methods of summing crosstalk from mixed sources—Part I: Theoretical analysis," *IEEE Trans. Commun.*, vol. 50, no. 3, pp. 453–461, Mar. 2002.
[8] F. M. Tesche, M. V. Ianoz, and T. Karlsson, *EMC Analysis Methods and Computational Models*. New York: Wiley, 1997.

[9] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. New York: Wiley, 1994.
[10] F. Broydé and E. Clavelier, "Dispositif pour la caractérisation des câbles à paires," French patent FR0213629, Nov. 2002, publication number FR2846750, published on May 7, 2004.
[11] F. Broydé and E. Clavelier, "Déterminer la diaphonie dans les câbles multipaires," in *Proc. Actes du 12ème Colloq. Int. (CEM'04)*, Toulouse, France, Mar. 16–18, 2004, pp. 249–254 [Online]. Available: http://www.eurexcem.com/cluba/pu2toulou-a.htm
[12] J. R. Carson and R. S. Hoyt, "Propagation of periodic currents over a system of parallel wires," *Bell Syst. Tech. J.*, vol. 6, no. 3, pp. 495–545, Jul. 1927.
[13] H. Kaden, *Wirbelströme und Schirmung in der Nachrichtentechnik*. Berlin, Germany: Springer-Verlag, 1959.
[14] F. Wenger, T. Gustafsson, and L. J. Svensson, "Perturbation theory for inhomogeneous transmission lines," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 49, no. 3, pp. 289–297, Mar. 2002.
[15] P. M. Morse and H. M. Feshbach, *Methods of Theoretical Physics*. New York: McGraw-Hill, 1953.
[16] F. Roddier, *Distributions et transformation de Fourier*. New York: McGraw-Hill, 1981.
[17] D. B. Jarvis, "The effect of interconnections on high-speed logic circuits," *IEEE Trans. Electron. Comput.*, vol. EC-12, no. 5, pp. 476–487, Oct. 1963.
[18] P.-G. Fontolliet, *Systèmes de télécommunication*. Paris, France: Dunod, 1983.
[19] S. Galli, C. Valenti, and K. Kerpez, "A frequency-domain approach to crosstalk identification in xDSL systems," *IEEE J. Select. Areas Commun.*, vol. 19, no. 8, pp. 1497–1506, Aug. 2001.
[20] L. M. Garth, G. Huang, and J.-J. Werner, "Crosstalk mitigation for xDSL channels," *IEEE Circuits Syst. Soc. Newslett.*, vol. 10, no. 1, pp. 3–9, Mar./Apr. 1999.
[21] J. W. Cook *et al.*, "The noise and crosstalk environment for ADSL and VDSL systems," *IEEE Commun. Mag.*, pp. 73–78, May 1999.
[22] C. Valenti, "NEXT and FEXT models for twisted-pair north american loop plant," *IEEE J. Select. Areas Commun.*, vol. 20, no. 5, pp. 893–900, Jun. 2002.

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