

# Cryptanalysis of the N-Party Encrypted Diffie-Hellman Key Exchange Using Different Passwords

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**Abstract.** We consider the security of the n-party EKE-U and EKE-M protocols proposed by Byun and Lee at ACNS '05. We show that EKE-U is vulnerable to an impersonation attack, offline dictionary attack and undetectable online dictionary attack. Surprisingly, even the strengthened variant recently proposed by the same designers to counter an insider offline dictionary attack by Tang and Chen, is equally vulnerable. We also show that both the original and strengthened EKE-M variants do not provide key privacy, a criterion desired by truly contributory key exchange schemes and recently formalized by Abdalla *et al.* We discuss ways to protect EKE-U against our attacks and argue that the strengthened EKE-U scheme shows the most potential as a provably secure n-party PAKE.

**Keywords:** Password-authenticated key exchange, n-party, cryptanalysis, dictionary attack, collusion, key privacy.

## 1 Introduction

Password authenticated key exchange (PAKE) protocols [1, 5, 7, 8, 13, 16, 17, 20] enable two or more parties to share a common secret key for securing (via secret-key cryptography) subsequent communications among them. For systems that depend on human interactions, using a password is more practical than a high-entropy secret key since the former is easier for a human to memorize by heart rather than be tempted to write it down somewhere [13].

One of the first PAKEs was the Encrypted Key Exchange (EKE) due to Bellare and Merritt [5] for establishing a secret key between 2 parties. This was later extended to the 3-party case by Steiner *et al.* [20]. Further analysis and variants of the latter are found in [11, 16, 17, 1].

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In extending from a 2-party PAKE to a 3-party one, the basic question raised is how the parties will share the password. Consequently, we can classify group-based (involving more than 2 parties) PAKEs into two broad types [7], namely those that use a single shared password among all parties (SPWA) [6] and those where each party shares a distinct password with a trusted server (DPWA) [20, 16, 17, 1].

DPWA-type PAKEs allow trust to be partitioned among all clients such that in the event of any client being compromised or corrupted, it will not affect the security of the entire group; e.g. only secrets (session keys shared with him, and his password) known to the affected client need to be changed, but other innocent clients can continue using their existing passwords. This also means that less trust needs to be put on each individual client since the compromise of any client is less devastating to the security of the group. In contrast, a compromise of any client in an SPWA-type PAKE would require that the password shared by all clients be updated and re-communicated to each of them. Further, DPWA-type PAKEs are very much suited for mobile and distributed computing networks which are increasingly becoming prevalent, where the parties (clients) come from diverse environments thus are less understood. Under such circumstances, one would not want to put too much trust on any client.

Abdalla *et al.* [1] presented a formal security model for 3-party DPWA-type PAKEs by combining the Bellare *et al.* model [3] for 2-party PAKEs with the Bellare-Rogaway model [4] for 3-party key distribution schemes generalized to the password case. They also formally defined the notion of *key privacy* to differentiate truly contributory key exchange protocols from key distribution protocols. This notion, first mentioned in [20], roughly means that even though a third-party server's help is required to establish a session key between two clients, the server is not able to obtain any information on the value of that established session key. The goal of key privacy is to limit the amount of trust put into the server, where it is assumed that the server is honest but curious [1], thus clients prefer to have their established session key known only to themselves. This appropriately models real-life situations where privacy of secret information is well guarded by individuals. In fact, some other work in related information security fields are also moving in this direction, e.g. protocols proposed without the use of trusted third parties (TTP) in [9, 22], and research showing the subtlety of putting too much trust on TTPs [19, 12]. To achieve key privacy, it is necessary [1] to have a 2-party authenticated key exchange (AKE) between the two clients.

In this paper, we are concerned with DPWA schemes for the *n-party* case. More specifically, at ACNS '05 Byun and Lee [7] presented two variants of an *n-party* EKE protocol, respectively called *n-party* EKE-U and EKE-M for unicast and multicast networks. These appear to be the first known *n-party* EKE protocols with provable security. Tang and Chen [21] subsequently showed that EKE-U is vulnerable to an offline dictionary attack, and that EKE-M is vulnerable to an undetectable online dictionary attack [11]. Byun and Lee [8] promptly countered with strengthened variants, which we will also discuss in Sections 3 and 4.

Although PAKEs have been extensively studied especially in the last few years [1, 3, 5, 6, 7, 8, 13, 16, 17, 20], most of them consider either the 2-party or 3-party case. And it was only very recently that the first provably secure PAKEs for the 3-party and n-party cases were presented in [1] and [7] respectively. Thus this field (that of provably secure group PAKEs) has potentially unexplored areas of future work, e.g. how to extend the existing provably secure 2-party or 3-party PAKEs to the n-party case in an efficient yet secure manner, i.e. without involving too many inter-client communications that would cause a bottleneck to the network especially when  $n$  is large.

We show attacks on both the original and strengthened EKE-U that exploit the server as an oracle to generate messages supposedly from an innocent client. Meanwhile for both the original and strengthened EKE-M, we point out that they do not achieve the key privacy property that is desired of contributory key exchange protocols.

In our concluding section, we discuss how to improve the *strengthened* EKE-U to resist our attacks and argue that it is a worthwhile candidate for a provably secure n-party PAKE.

## 2 The N-Party EKE Protocols

The n-party EKE protocols due to Byun and Lee [7] involve  $n - 1$  clients and 1 server, and are specially designed to suit modern communication environments such as ad-hoc networks and ubiquitous computing, in particular EKE-U for unicast networks and EKE-M for multicast ones. *Unicast* networks allow for communication only between a single sender and a single receiver, while *multicast* networks allow for communication between a single sender and multiple receivers. For multicast networks, all messages from individual single senders can be sent in parallel during a single round to all receivers, thus more round-efficient group-based protocols can be designed in such networks.

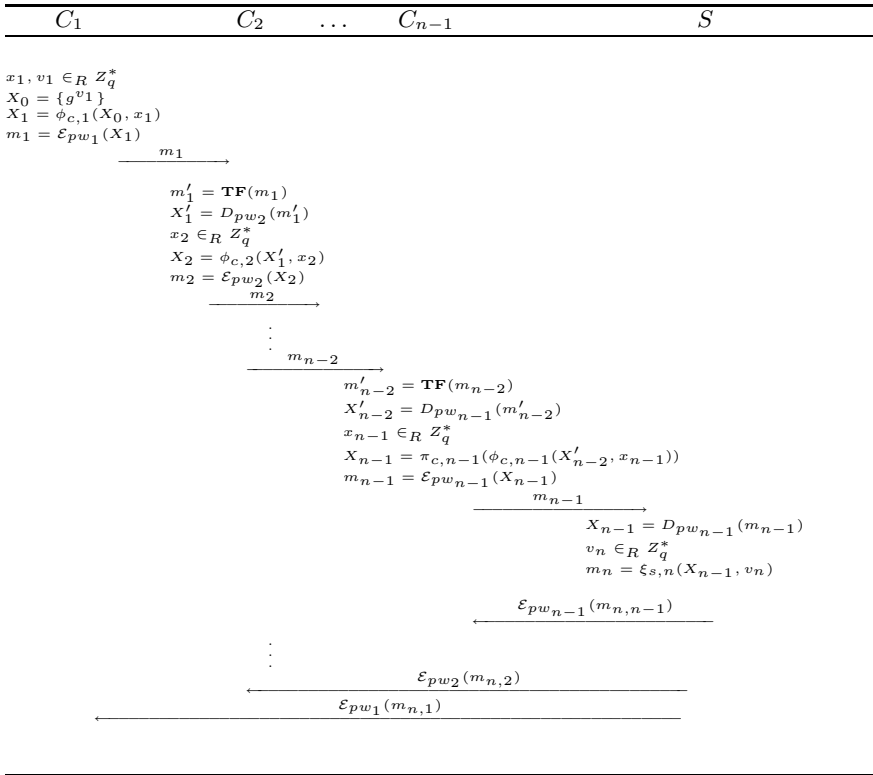
Note that all arithmetic operations in this paper are performed under cyclic group  $\mathbf{G} = \langle g \rangle$  of prime order.

### 2.1 N-Party EKE-U Protocol Variants

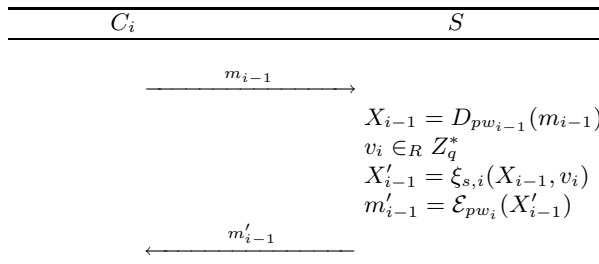
The EKE-U makes use of three types of functions which differ mainly in the number of elements produced at their respective outputs:

$$\begin{aligned}\pi(\alpha_1, \dots, \alpha_{i-1}, \alpha_i) &= \{\alpha_1, \dots, \alpha_{i-1}\}, \\ \phi(\{\alpha_1, \dots, \alpha_{i-1}, \alpha_i\}, x) &= \{\alpha_1^x, \dots, \alpha_{i-1}^x, \alpha_i, \alpha_i^x\}, \\ \xi(\{\alpha_1, \dots, \alpha_{i-1}, \alpha_i\}, x) &= \{\alpha_1^x, \dots, \alpha_{i-1}^x, \alpha_i^x\}.\end{aligned}$$

Note that  $\pi$  produces an output that is simply equal to its input less the last input element, and further  $\pi$  is only used by  $C_{n-1}$ . The functions  $\phi$  and  $\xi$  take in the same number of input elements, and their outputs are similar except that  $\phi$  has one more output element than  $\xi$ .



**Fig. 1.** Main protocol of n-party EKE-U



**Fig. 2.** TF protocol of n-party EKE-U

The main bulk of the EKE-U protocol is illustrated in Fig. 1, where clients  $C_1, \dots, C_{n-1}$  and the server  $S$  are arranged in a line. During the up-flow stage starting from  $C_1$ , each client  $C_i$  basically chooses its own secret  $x_i$  and calls the  $\phi$  function to raise the intermediate value  $X'_{i-1}$  to the power of this  $x_i$  in order to generate the value  $X_i$ . This is encrypted with client  $C_i$ 's password  $pw_i$  and sent to the next client  $C_{i+1}$  as the message  $m_i$ . Upon receipt of this,

	$S$	$C_1$	$C_2$	$\dots$	$C_{n-1}$
<b>Round 1</b>	$s_i \in_R Z_q^*$ $\mathcal{E}_{pw_i}(g^{s_i})$	$x_1 \in_R Z_q^*$ $\mathcal{E}_{pw_1}(g^{x_1})$	$x_2 \in_R Z_q^*$ $\mathcal{E}_{pw_2}(g^{x_2})$	$\dots$	$x_{n-1} \in_R Z_q^*$ $\mathcal{E}_{pw_{n-1}}(g^{x_{n-1}})$
<b>Round 2</b>	$N \in_R Z_q^*$ $sk_1 \oplus N \parallel \dots \parallel sk_{n-1} \oplus N$				

**Fig. 3.** N-party EKE-M

$C_{i+1}$  initiates an additional sub-protocol known as the **TF** protocol (see Fig. 2 and note that the function  $\xi$  is used here) with the server  $S$  so that the received message which was encrypted under client  $C_i$ 's password could be decrypted and re-encrypted under client  $C_{i+1}$ 's password to form  $m'_i$ . This then allows  $C_{i+1}$  to access the decrypted contents of  $m'_i$ , namely  $X'_i$  and the same process repeats until  $S$  receives the message  $m_{n-1}$  from  $C_{n-1}$ . The down-flow stage then starts by having  $S$  compute from  $m_{n-1}$  the keying material  $m_{n,i}$  meant for client  $C_i$  ( $i = 1, \dots, n - 1$ ), encrypt under  $pw_i$  and send these out to each corresponding client. Finally, each client with his own  $pw_i$  and  $x_i$  can perform the decryption and compute the session key  $sk = (m_{n,i})^{x_i} = g^{v_n \prod_{i=1}^{n-1} (v_i x_i)}$ .

Byun and Lee [7] also mention that an optional mutual authentication step based on key confirmation could be appended to the scheme if it is desired to ensure that all other clients have really computed the agreed session key  $sk$ . In this case, each client computes an authenticator  $\mathcal{H}(C_i \parallel sk)$ , which is the hash value of client index ( $C_i$ ) and new session key ( $sk$ ), and sends this to all other clients for verification. Note however that even if this step is made compulsory, it does not protect EKE-U against our attacks in Sections 3.3 and 3.4.

There is a strengthened version [8] of EKE-U and this will be explained in Section 3.4.

### 2.2 N-Party EKE-M Protocol Variants

EKE-M is much simpler than EKE-U and is shown in Fig. 3. It consists of two rounds. Round 1 is basically a simultaneous run of a 2-party PAKE between each client with the server to set up a secure channel (in the confidentiality sense) between them. In Round 2, the server distributes a common keying message to all clients via the secure channel. This will be used to form the common secret session key  $sk$  among all clients. More precisely, denote  $sk_i = \mathcal{H}_1(\mathcal{E}_{pw_1}(g^{x_1}) \parallel \dots \parallel \mathcal{E}_{pw_{n-1}}(g^{x_{n-1}}) \parallel g^{x_i s_i})$  and  $sk = \mathcal{H}_2(\mathcal{E}_{pw_1}(g^{x_1}) \parallel \dots \parallel \mathcal{E}_{pw_{n-1}}(g^{x_{n-1}}) \parallel sk_1 \oplus N \parallel \dots \parallel sk_{n-1} \oplus N \parallel N)$ . Note that  $\mathcal{H}_1$  and  $\mathcal{H}_2$  are standard hash functions.

There is also a strengthened version of EKE-M proposed by Byun and Lee [8] to prevent the undetectable online dictionary attack in [21]. The basic idea is to add an extra step after Round 1 where an authenticator  $\mathcal{H}(sk_i \parallel C_i)$  is broadcast by each client (or server) to be checked by all parties before Round 2 starts.

### 3 Cryptanalysis of the N-Party EKE-U Variants

In view of the low entropy password, the basic requirement for a PAKE is security against dictionary attacks on the password. Such attacks are typically online or offline, depending on whether or not the attacker needs to verify each guessed password by interacting online (being involved in a protocol run) with other parties. Another basic requirement of PAKEs is that they do not allow impersonation attacks where an attacker masquerades as any legitimate party because if this happens, there will be a non-achievement of mutual authentication.

#### 3.1 Tang-Chen Attack

Before describing our attacks, we first briefly discuss an insider offline dictionary attack on EKE-U given by Tang and Chen [21]. See [15] for a formal treatment of insider attacks on group AKEs.

The basic idea behind this attack is that a malicious client  $C_j$  modifies the first two components  $(g_1, g_2)$  in the message  $X_j$  of  $m_j = \mathcal{E}_{pw_j}(X_j)$  that it sends to  $C_{j+1}$  during the up-flow stage of the main protocol, such that they satisfy the relation  $g_1^\alpha = g_2$ . Then right at the end of the **TF** protocol when the server  $S$  returns  $m'_j = \mathcal{E}_{pw_{j+1}}(X'_j)$  to  $C_{j+1}$ , this is intercepted by the malicious  $C_j$  who then guesses the value of  $pw_{j+1}$  and verifies his guess by checking if the first two components that he had initially modified satisfy the given relation.

At first glance, it seems that this attack requires having to modify the message  $m_j = \mathcal{E}_{pw_j}(X_j)$ . However, as later pointed out in the same paper [21], this attack could work without this requirement. Instead, it suffices to decrypt  $m'_j$  with the guessed password  $pw_{j+1}$  and check if the last two components  $(\beta, \gamma)$  of  $X'_j$  satisfy the relation  $\beta^{x_j} = \gamma$ .

Note however that even with this relaxation, the latter attack still limits the malicious  $C_j$  to attack only his next neighbour  $C_{j+1}$  but not on the other clients because the components within in his possessed  $X'_{j-1}, X_j$  do not allow him to verify any two components of these other clients' messages without having to guess the secrets of the server  $v_i, i \in \{1, \dots, n\}$  or the secrets of other clients  $x_t$  ( $t \neq j$ ).

#### 3.2 By Any Outsider

Byun and Lee [7] have cleverly designed the EKE-U protocol such that the  $m_{n,i}$  within each keying material message  $\mathcal{E}_{pw_i}(m_{n,i})$  distributed by  $S$  to each client  $C_i$  in the down-flow stage does not have the random secret  $x_i$  chosen by client  $C_i$  in its exponent, thus only client  $C_i$  would be able to make use of its  $m_{n,i}$  ( $i^{\text{th}}$  component of the message  $m_n$ ) to generate the session key material  $K = (m_{n,i})^{x_i} = (g^{x_1 \dots x_{n-1}})^{v_1 \dots v_n}$ . Further, different functions  $(\phi, \pi, \xi)$  are used in the main and **TF** protocols, e.g. each of the three functions produces an output having different number of elements, and  $\phi$  is used in the main protocol while  $\xi$  is used in the **TF** protocol; thus it appears an attacker cannot exploit one protocol as an oracle for answering challenge-response-like queries in the other protocol.

However, note that this is only true for the communications during the up-flow stage of the main protocol from  $C_1$  through  $C_{n-1}$ , but not true from  $C_{n-1}$  to  $S$  because for the latter there is an extra function  $\pi$  (see Fig. 1 in addition to the function  $\phi$  that is used by  $C_{n-1}$ ). Thus the output of the composition of the functions  $\pi \circ \phi$  done by  $C_{n-1}$  during the main protocol results in the same number of elements as that of the output of the  $\xi$  function computed by  $S$  in the **TF** protocol; i.e.  $S$  can be exploited during the **TF** protocol as an oracle to generate messages supposedly generated by  $C_{n-1}$  during the main protocol when in fact  $C_{n-1}$  need not be present at all.

Our attack further exploits the fact that the messages transmitted during the **TF** protocol (Fig. 2) between a client and the server are similar in form to the messages transmitted during the up- or down-flow of the main unicast protocol (Fig. 1). In particular, message  $m_i$  and  $m'_{i-1}$  are both functions of  $\mathcal{E}_{pw_i}(\cdot)$ . Thus, the server  $S$  which is intended by the designers to act as an interpreter between two neighbouring clients,  $C_i$  and  $C_{i-1}$  could be used by the attacker as an oracle to generate messages  $m_i$  supposedly generated by the next neighbouring client  $C_i$  even when  $C_i$  is not present.

For ease of illustration, we take  $n = 4$  (as in Fig. 4) though it similarly applies for any  $n$ . Note that in this case,  $C_{n-1} = C_3$ .

1. The attacker captures the message  $m_2 = \mathcal{E}_{pw_2}(X_2)$  sent from  $C_2$  to  $C_3$  during the up-flow stage of the main protocol.
2. The attacker then initiates the **TF** protocol by forwarding this  $m_2$  to  $S$ .
3.  $S$  thinks<sup>1</sup> this is from  $C_3$  and decrypts it with  $pw_2$  to obtain  $X_2$ . It then computes

$$X'_2 = \xi(X_2, v_3) = \{g^{v_1v_2x_2v_3}, g^{v_1x_1v_2v_3}, g^{v_1x_1v_2x_2v_3}\} \tag{1}$$

and encrypts this with  $pw_3$  to get  $m'_2 = \mathcal{E}_{pw_3}(X'_2)$  and returns this  $m'_2$  thus completing the **TF** protocol.

4. The attacker now has  $m'_2$  which he simply reuses as  $m_3 = \mathcal{E}_{pw_3}(X_3) = \mathcal{E}_{pw_3}(X'_2)$  and then impersonates  $C_3$  by sending this to  $S$  in the main protocol. This completes the up-flow stage.
5. To start the down-flow stage,  $S$  decrypts  $m_3$  to obtain

$$X_3 = \{g^{v_1v_2x_2v_3}, g^{v_1x_1v_2v_3}, g^{v_1x_1v_2x_2v_3}\} \tag{2}$$

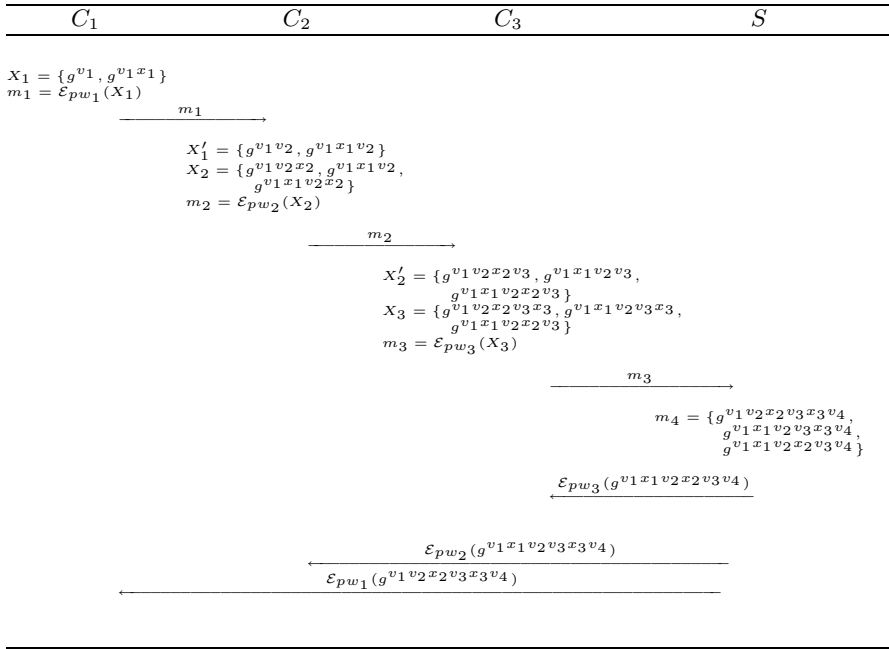
and then chooses  $v_4$  to compute

$$m_4 = \xi(X_3, v_4) = \{g^{v_1v_2x_2v_3v_4}, g^{v_1x_1v_2v_3v_4}, g^{v_1x_1v_2x_2v_3v_4}\}. \tag{3}$$

Each of these elements of  $m_4$ , denoted in turn as  $m_{4,1}, m_{4,2}, m_{4,3}$  are then encrypted with the respective passwords  $pw_i$  of client  $C_i$  ( $i = 1, \dots, 3$ ) and sent to each client respectively as  $\mathcal{E}_{pw_i}(m_{4,i})$  for ( $i = 1, \dots, 3$ ).

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<sup>1</sup> Note that there is no explicit authentication of a client by  $S$ . An apparent way for  $S$  to properly keep in sequence is to track the number of **TF** sessions that have been initiated with it. The  $i^{\text{th}}$  session would be taken to come from client  $C_{i+1}$  since  $C_1$  does not initiated any **TF** with  $S$ .



**Fig. 4.** An example of n-party EKE-U main protocol for  $n=4$

- 6. Each client  $C_i$  ( $i = 1, \dots, 3$ ) can then decrypt  $\mathcal{E}_{pw_i}(m_{4,i})$  and thus compute  $sk = (m_{4,i})^{x_i} = (g^{x_1 x_2})^{v_1 v_2 v_3 v_4}$ .

Note that though our attack can be used to attack only  $C_{n-1}$  and not any other client, the main plus is that it can be mounted by any outsider (in contrast to the attack in [21] which requires a malicious insider) and applies even without needing client  $C_{n-1}$  to be present. Having said that,  $C_{n-1}$ 's presence would pose no problem for the attacker either. Though the attacker is unable to recover the session key  $sk$  himself, he has successfully led all parties (except client  $C_{n-1}$  who is not present) to establish a totally new session key among them. This could also be viewed as a variant of the unknown key-share attack [10, 2, 14] in the n-party case since each client (except  $C_{n-1}$ ) believes it is sharing a session key with all other clients including  $C_{n-1}$  which is true, but  $C_{n-1}$  is not present and does not know that such a key has been established. In constrast, recall that an unknown key-share attack on a 2-party case is where one party  $A$  believes it is sharing a session key with  $B$  which is rightly so, but  $B$  instead believes it is sharing a session key with  $E \neq A$ .

To prevent this attack, the mutual authentication step (e.g. via key confirmation [14]) must be made compulsory. Nevertheless, when performed by a malicious insider, the mutual authentication step is no longer effective to prevent this attack, and it further becomes an offline dictionary attack allowing him to retrieve the password of  $C_{n-1}$ , as will be explained next.



### 3.3 By a Malicious Insider

A malicious client  $C_i$  could launch a more devastating variant of the previous attack since he could exploit it to further obtain the password of the innocent client  $C_{n-1}$ . This offline dictionary attack works as follows:

1. The attacker, client  $C_i$  ( $i \neq n-1$ ) performs steps 1 through 5 of Section 3.2.
2. Further, since the attacker is an insider, he could also decrypt the keying material intended for him  $\mathcal{E}_{pw_i}(m_{4,i})$ . We illustrate with an example. Consider  $C_1$  is the malicious client. It can be similarly shown for all other clients  $C_i$  for ( $i \neq n-1$ ). He can obtain  $g^{v_1 v_2 x_2 v_3 v_4}$  from  $\mathcal{E}_{pw_1}(m_{4,1}) = \mathcal{E}_{pw_1}(g^{v_1 v_2 x_2 v_3 v_4})$ .
3. With his value of  $x_1$ , he can compute  $y = (g^{v_1 v_2 x_2 v_3 v_4})^{x_1} = g^{v_1 x_1 v_2 x_2 v_3 v_4}$ .
4. He intercepts  $\mathcal{E}_{pw_3}(m_{4,3}) = \mathcal{E}_{pw_3}(g^{v_1 x_1 v_2 x_2 v_3 v_4})$  meant for client  $C_3$ , and makes guesses for all possible values of  $pw_3$ . For each guessed  $pw_3$ , he decrypts  $\mathcal{E}_{pw_3}(m_{4,3})$  and obtains  $z = g^{v_1 x_1 v_2 x_2 v_3 v_4}$ . He then checks if  $z$  equals  $y$ . The correct  $pw_3$  would satisfy this.

This attack can be mounted by any client  $C_i$  against  $C_{n-1}$ , thus it complements the attack in [21] where the attack is mounted by any client  $C_i$  against his neighbour  $C_{i+1}$ .

Note also that this attack works even with the mutual authentication step included since  $C_i$  has no problem in computing  $sk$ .

### 3.4 Attacking the Strengthened N-Party EKE-U

In [8], Byun and Lee suggested a strengthened n-party EKE-U protocol to counter the insider offline dictionary attack due to Tang and Chen [21].

Their basic idea to counter the attack is to use an ephemeral session key  $sk_i = \mathcal{H}(C_i \| S \| g^{a_i} \| g^{b_i} \| g^{a_i b_i})$  instead of the password  $pw_i$  to encrypt keying material during both the up- and down-flow of the main protocol, where  $a_i$  and  $b_i$  are the random number chosen by  $C_i$  and  $S$  respectively.

Nevertheless, we first remark that this strengthened variant also falls to our attacks in the Sections 3.2 and 3.3 since it inherits from the original version the same properties we exploited, i.e. (1) the composition of functions  $\pi \circ \phi$  produces an output with the same number of elements as that produced by  $\xi$ ; (2) messages transmitted during the **TF** protocol are the same in form to messages transmitted during the main protocol.

More interestingly, we have a further undetectable online dictionary attack [11] on this strengthened variant as follows, again assuming for the purpose of illustration that  $n = 4$  thus we have the parties  $C_1, C_2, C_3$  and  $S$ :

1. All malicious clients except  $C_1$  collude [18], meaning they share their secrets  $x_i$ .
2. They choose  $v$  and  $x$ , and for each guess of  $pw_1$ ,
  - (a) They compute  $m_1 = \mathcal{E}_{pw_1}(X_1)$  where  $X_1 = \{g^v, g^{vx}\}$ .
  - (b) Then  $C_2$  starts the **TF** protocol with  $S$ , etc., and the rest of the up-flow proceeds as normal.

- (c) Then during the down-flow, the keying material messages sent by  $S$  to  $C_1, C_2$  and  $C_3$  would be  $\mathcal{E}_{pw_1}(g^{vv_2v_3v_4x_2x_3})$ ,  $\mathcal{E}_{pw_2}(g^{vv_2v_3v_4xx_3})$  and  $\mathcal{E}_{pw_3}(g^{vv_2v_3v_4xx_2})$ .
- (d) Now the colluding clients  $C_2$  and  $C_3$  can easily obtain  $y = g^{vv_2v_3v_4}$  from  $\mathcal{E}_{pw_2}(g^{vv_2v_3v_4xx_3})$  or  $\mathcal{E}_{pw_3}(g^{vv_2v_3v_4xx_2})$ , and their knowledge of  $x, x_2$  and  $x_3$ .
- (e) They then use their current guess of  $pw_1$  to decrypt  $\mathcal{E}_{pw_1}(g^{vv_2v_3v_4x_2x_3})$  to get  $z$ . They compare this  $z$  with  $y^{x_2x_3}$ , where  $y$  was computed in the previous step. A match means the guess of  $pw_1$  is correct.

This is online because every time  $pw_1$  is guessed, the attackers have to initiate a protocol run with  $S$ , but this is undetectable because  $S$  would not notice anything wrong while  $C_1$  does not even have to be present.

The weakness exploited here is that the message from  $C_1$  to  $C_2$  is encrypted with a low-entropy password  $pw_1$  instead of  $sk_1$ . Thus a direct fix is to use  $sk_1$  in place of  $pw_1$  similar to how  $sk_i$  (for  $i \neq 1$ ) were used in place of  $pw_i$  for this strengthened EKE-U scheme.

## 4 N-Party EKE-M Does Not Provide Key Privacy

Byun and Lee [7] also proposed a multicast variant known as the n-party EKE-M protocol. It is illustrated in Fig. 3.

This variant does not exhibit the ‘ $S$ -oracle’ property of the U variant, i.e. the server  $S$  cannot be exploited as an oracle to generate messages that appear to be from a client, thus it does not appear to fall to our attacks on EKE-U. Nevertheless, there is one major problem with this M variant, namely that the server  $S$  is able to compute the session key  $sk$  established by the clients. This is quite unlike the U variant where even  $S$  is unable to know what  $sk$  is, and thus this M variant is undesirable in the sense that the privacy of the clients’ communications cannot be safeguarded against a third-party server.

This *key privacy* property is important because it would mean less trust [12, 19] needs to be put on a third-party server, who may not always be malicious but could sometimes be curious [1]. The first known n-party (for  $n=3$ ) EKE scheme to have this property is due to Steiner *et al.* [20] and this concept was later formally treated by Abdalla *et al.* [1]. Abdalla *et al.* argue that key privacy is the main difference between a key distribution protocol (for which the session key is known to the server) and a key exchange protocol (for which the session key remains unknown to the server). Thus, a true key exchange protocol where each party (in this case the client) contributes equal parts to the established session key, should have key privacy because the third-party server should not be able to listen in on future secret communications among the clients, and hence should not be able to know what this session key is.

Note that the strengthened EKE-M variant in [8] has the same problem even when mutual authentication via key confirmation is included, because the point here is that the server can compute  $sk$  even when  $C_i$  is not present, so mutual authentication is irrelevant.

We do not see any way to fix this with minor tweaks without destroying the basic structure of this M scheme, because essentially each client interacts only with the server, and never with each other, thus the keying material components that they contribute to the final establishment of the session key via a Diffie-Hellman way, can only be translated (decrypted with one password and re-encrypted with another) by the middleman  $S$ , thus  $S$  is able to view all communicated messages that it translates.

Alternatively, one could adopt the approach in [1] by appending one more phase where each client interacts directly with the other clients by contributing its secret part to jointly form the key but this would be infeasible for  $n > 3$  parties. Unless one resorts to using the method used for EKE-U where each client in turn adds his secret to the key material accumulatively while forwarding from one client to the next until it reaches the server. However, this is then essentially EKE-U and thus we end up destroying the original EKE-M structure.

If it is desired that this key privacy against the server be upheld, then this variant should not be used.

## 5 Conclusion

We have illustrated attacks (impersonation, dictionary or collusion attacks) on the n-party EKE-U variants proposed by Byun and Lee [7, 8].

EKE-U [7], even with strengthening [8], falls to our attacks in Sections 3.2 to 3.4, while EKE-M is not desirable as it does not provide key privacy. But to fix the key privacy problem requires clients to directly communicate with one another to contribute their secret key parts accumulatively, leading us therefore to EKE-U.

Thus it appears that *strengthened* EKE-U is the potential way to proceed for provably secure n-party PAKEs. Hence, to fix EKE-U, the mutual authentication step is compulsory in order to prevent the attack in Section 3.2, though attacks in Sections 3.3 and 3.4 still apply. A simple fix to prevent the attack in Section 3.3 is to require the server to check that  $x_{n-1} \neq 1$  before replying so that it is not exploited as an oracle. To prevent the attack in Section 3.4,  $C_1$  needs to also initiate the **TF** protocol to generate  $sk_1$  with the server and use  $sk_1$  instead of  $pw_1$  in constructing  $m_1$ .

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