IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.Doi Number

Crystal Structure Algorithm (CryStAl): A Metaheuristic Optimization Method

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ABSTRACT Metaheuristics are computational procedures that intelligently lead the search process through the efficient exploration of the search space associated with an optimization problem. With the progressive outburst of problems with large data sets in various fields, there is an ongoing quest for enhancing existing metaheuristic algorithms as well as developing new ones with greater accuracy and efficiency. In general, a powerful and efficient metaheuristic algorithm is based on a rich inspiration source, implemented effectively through a precise mathematical model. Aiming to develop a highly efficient, nature-inspired optimization algorithm, here we propose a novel metaheuristic called Crystal Structure Algorithm (CryStAl). This method is chiefly inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points, which is a natural phenomenon that can be seen in the symmetric arrangement of constituents (i.e. atoms, molecules, or ions) in crystalline minerals such as quartz. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of this novel algorithm, 12 different classical and modern metaheuristic algorithms are selected from the literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly. The obtained results, further supported by a complete statistical analysis, demonstrated that the proposed algorithm is capable of providing very competitive results, outperforming the other metaheuristics in most cases.

INDEX TERMS Crystal Structure Algorithm (CryStAl), lattice, function, metaheuristic, optimization, statistical analysis.

I. INTRODUCTION

Many design problems in nature can be considered as optimization problems that demand appropriate optimization techniques and methods to be dealt with. Nowadays, design problems have become extremely complex for which classical optimization methods based on mathematical principles are incapable of providing satisfactory results in a reasonable period of time. Gradient-based methods, which utilize the gradient of the objective function for the configuration of the optimization problem, are a type of these mathematical methods. Over the past few decades, exploring the deficiencies of classical optimization methods and introducing new efficient optimization algorithms have been of great interest. Based on recent technological advances, there is a growing interest in introducing new optimization methods with enhanced efficiency, accuracy, and increased speed rate for tackling difficult optimization problems. Besides, some other concerns in dealing with some specific issues such as the local optima issues alongside the smoothness and convexity of the search spaces have been of great importance for a long period of time.

The presented concerns about the classical optimization algorithms have led optimization experts to a new methodology in solving different optimization problems called “Metaheuristic”. Glover [1] firstly proposed this term in 1986 which is comprised of the main word, i.e. heuristics, and a prefix, i.e. meta, which both have Greek origins. The term “heuristic” comes from *heuriskein* which is an old Greek word meaning “to discover”, while “meta” means “beyond the normal or natural limits of something”. Metaheuristics are solution techniques that implement higher-level strategies into search processes in order to guide an optimization process to perform a powerful search into the search space with some special capabilities such as avoiding local optima.

As presented by Sörensen [2], the history of utilizing metaheuristics as the solution methods for dealing with real-world problems can generally be categorized into five distinct periods. In the first period which is named the “pre-theoretical” period (until 1940), there was not any formal presentation of heuristics and metaheuristics methods. Despite that, these methods had been used for solving some simple optimization problems in this period. In the second period which is from

1940 to 1980 and known as the "early" period, some studies were conducted on heuristics which was the first formal introduction and discussion in this field. In the third period which is called the "method-centric" period (1980 to 2000), multiple metaheuristics were proposed and developed for specific applications which extended the field of heuristics and metaheuristics. In the fourth period, which is from 2000 until now and known as the "framework-centric" period, the methodology of utilizing metaheuristics as frameworks alongside methods has been successfully presented with considerable growth of intuition in this field. In the fifth or last period which is named the "scientific" or "future" period, the design and introduction of new metaheuristics will turn into a matter of science rather than art. A summary of the abovementioned historical periods is presented in Table 1.

TABLE 1. Summary of the historical periods of metaheuristics evolution.

No.	Name	Duration	Details
1	Pre-theoretical	Until 1940	No formal presentation with limited applications.
2	Early	1940 to 1980	Heuristics were formally introduced and discussed.
3	Method-centric	1980 to 2000	Multiple metaheuristics were proposed and developed for specific applications.
4	Framework-centric	2000 to now	The methodologies of utilizing metaheuristics as frameworks alongside various methods have been successfully presented.
5	Scientific or future	Future	The design and introduction of new metaheuristics will turn into a matter of science rather than art.

Considering the development of various metaheuristic algorithms, four classifications can be made in terms of their inspiration. The first category is entitled "evolutionary algorithms" including the Memetic Algorithm (MA) [3], Genetic Algorithm (GA) [4], Differential Evolution (DE) [5], and the Evolution Strategies (ES) [6], which were developed based on the biological evolution and reproduction. The second category contains swarm intelligence-based algorithms that were formed based on the cooperative behavior of decentralized and self-organized natural or artificial systems. The Particle Swarm Optimization (PSO) [7], Ant Colony Optimization (ACO) [8], Artificial Bee Colony (ABC) [9], Cat Swarm Optimization (CSA) [10], Firefly Algorithm (FA) [11], Krill Herd (KH) algorithm [12], and Slap Swarm Algorithm (SSA) [13] are some of the well-known methods in this category. The third category consists of algorithms motivated by physical laws. The Simulated Annealing (SA) [14], Magnetic Optimization Algorithm (MOA) [15], Gravitational Search Algorithm (GSA) [16], Charged System Search (CSS) algorithm [17], Ray Optimization Algorithm (ROA) [18], Colliding Bodies Optimization (CBO) [19], Multiverse Algorithm (MVO) [20], and the Sine Cosine Algorithm (SCA) [21] are some methods belonging to this category. Beyond these methods, some other metaheuristic algorithms were presented based on the lifestyle of humans and animals (the fourth category) such as the Harmony Search (HS) [22], Teaching–learning-based Optimization (TLBO) [23], Creativity-Oriented Optimization Algorithm (COOA) [24], Human Behavior-Based Optimization (HBBO) [25], and the Gaining Sharing Knowledge-based algorithm (GSK) [26]. In addition to these standard algorithms, some other challenges in developing, upgrading, or hybridizing standard algorithms have also been achieved [27-38]. A summary of these metaheuristic algorithms is presented in Table 2.

In this paper, a novel metaheuristic optimization method called Crystal Structure Algorithm (CryStAl) is proposed which is inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of CryStAl, 12 different classical and modern metaheuristic algorithms are selected from the literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly.

TABLE 2. Summary of the classification of the metaheuristic algorithms.

Classification	Algorithm	Year of Proposal
Evolution	Memetic Algorithm (MA) [3]	1989
	Genetic Algorithm (GA) [4]	1992
	Differential Evolution (DE) [5]	1997
	Evolution Strategies (ES) [6]	2002
Swarm intelligence	Particle Swarm Optimization (PSO) [7]	1995
	Ant Colony Optimization (ACO) [8]	1996
	Artificial Bee Colony (ABC) [9]	2006
	Cat swarm Optimization (CSA) [10]	2006
	Firefly Algorithm (FA) [11]	2010
	Krill Herd (KH) algorithm [12]	2012
	Slap Swarm Algorithm (SSA) [13]	2017
Physical laws	Simulated Annealing (SA) [14]	1983
	Magnetic Optimization Algorithm (MOA) [15]	2008
	Gravitational Search Algorithm (GSA) [16]	2009
	Charged System Search (CSS) algorithm [17]	2010
	Ray Optimization Algorithm (ROA) [18]	2012
	Colliding bodies Optimization (CBO) [19]	2014
	Multi-verse Algorithm (MVO) [20]	2016
	Sine Cosine Algorithm (SCA) [21]	2016
Lifestyle	Harmony Search (HS) [22]	2001
	Teaching–learning-based optimization (TLBO) [23]	2011
	Creativity-Oriented Optimization Algorithm (COOA) [24]	2015
	Human Behavior-Based Optimization (HBBO) [25]	2017
	Gaining Sharing Knowledge-based algorithm (GSK) [26]	2019

In general, the efficiency of novel metaheuristic algorithms in producing improved solutions to well-known optimization problems has been a significant research challenge for algorithm developers in recent decades. Considering the source of inspiration and the mathematical model as the two foundations of metaheuristic algorithms, this mission can generally be accomplished by utilizing solid mathematical models developed based on suitable inspirational concepts.

In this regard, this paper proposes CryStAl as a metaheuristic algorithm conceptualized based on the principles underlying the formation of crystal structures as a well-known physical paradigm in nature. This method is implemented using a fully-detailed mathematical model comprised of the details of crystalline configurations which have been established by crystallographers over the past few centuries. By developing a metaheuristic based on such a rich inspiration source followed by a precise mathematical model, we have shown that excellent results in dealing with different optimization problems can be achieved.

It should be also noted that the proposed approach, i.e. CryStAl, is a parameter-free metaheuristic algorithm in which there is no internal parameter to be determined throughout the optimization procedure. In other words, a notable feature of this algorithm is its parameter-free framework in which the exploitation and exploration phases of optimization are adjusted through the main loop of the algorithm. Besides, the position updating process of candidate solutions in this method is conducted in four separate phases in which the local and global searches of the entire search space are satisfied in a more precise way that results in excellent responses.

A summary of this paper is as follows. In section 2, the inspirational background of the proposed algorithm alongside the mathematical model of the new optimization algorithm is presented. In section 3, some mathematical functions with different characteristics are presented for further utilization in evaluating the proposed metaheuristic algorithm along with some other alternative approaches. In section 4, the selected alternative metaheuristic algorithms for comparative purposes are presented in detail. In section 5, the results of CryStAl alongside the other metaheuristics in dealing with mathematical test functions are presented. In section 6, a comprehensive statistical analysis is conducted to compare the results of the new algorithm with the other metaheuristic approaches. In section 7, the main findings of this paper including the conclusions alongside some suggestions for future challenges are presented accordingly.

II. CRYSTAL STRUCTURE ALGORITHM (CRYSTAL)

A. INSPIRATION

Solid minerals the constituent components (molecules, atoms, or ions) of which are regularly and repeatedly arranged in three spatial directions or have a crystallographic order are called crystals. Crystalline solids are highly diverse and can have isotropic or anisotropic properties. The word crystal has Greek roots and means "frozen by cold". They believed that if water was kept at very low temperatures for some time, it would become stable at high temperatures. "Crystal" is also an Arabic word derived from the Greek word "berlis" meaning emerald [39]. A representative example of a typical crystal is depicted in Fig. 1a.

The earliest references to the regular arrangement of particles that make up crystals can be found in the works of Johannes Kepler in 1619 and Robert Hooke in 1665. Sometime later in 1690, Christine Hogens studied the optical properties of calcite crystals and hypothesized that the crystals were made of very small particles with a definite shape. Since then, different physical and chemical formulations for crystals have been proposed and investigated experimentally [39]. Furthermore, crystals and their rich symmetries have inspired the conception and design of many man-made structures, mechanisms, and artworks [40-80].

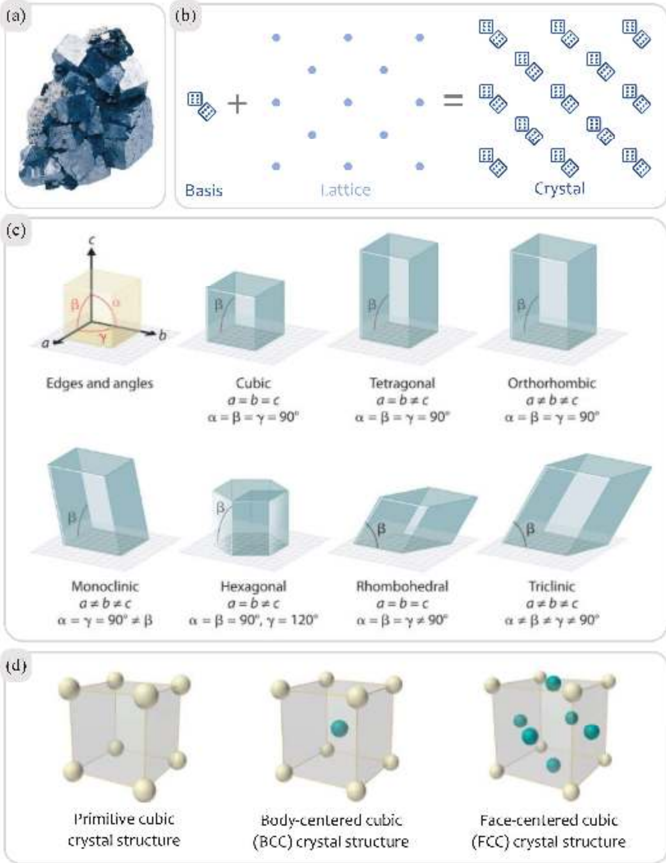

FIGURE 1. **(a) An example of a natural crystal called Galena. (b) Definition of a crystal as a basis added to a lattice. (c) Various lattice configuration options. (d) Three common varieties of the cubic crystal system (Parts a, c, and d are adapted from [39]).**

The underlying component of a crystal is a "lattice" which represents a periodic array of points in predefined spaces, though it is not capable of defining the specific locations of atoms in the material. On the other hand, the location of atoms in the structure of crystals is determined by the "basis" associated with each lattice point. Hence, crystals are determined by the combination of these two elements, i.e. the basis and the lattice, as illustrated in Fig 1b.

Since the lattice determines only the overall shape of the crystal, different geometrical shapes can be composed considering the fact that infinite geometrical shapes are found in nature; however, here we consider some of the most well-known regular shapes, as represented in Fig. 1c.

For the basis, different configurations of atoms in the lattice can be considered in which the location of atoms can be in the corner points alongside other irregular patterns. In Fig. 1d, this aspect is represented in a simple cubic crystal system.

As a mathematical representation of these aspects should be defined for numerical investigations, the Bravais model [39] is considered in this paper for defining crystal configurations. In this model, a periodic crystal structure is defined by considering infinite lattice shape in which any lattice pint is described by the location of their lattice point with a vector as follows:

$$r = \sum n_i a_i , \quad (1)$$

where n_i is an integer, a_i is the shortest vector along the principal crystallographic directions, and i is the number of crystal corners.

B. MATHEMATICAL MODEL

In this section, the mathematical model of CryStAl is presented in which the basic concepts of crystals are utilized with necessary modifications. In this model, each candidate solution of the optimization algorithm is considered as a single crystal in the space. For iterative purposes, a number of crystals are randomly determined for initialization.

$$Cr = \begin{bmatrix} Cr_1 \\ Cr_2 \\ \vdots \\ Cr_i \\ \vdots \\ Cr_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^d \\ x_2^1 & x_2^2 & \dots & x_2^j & \dots & x_2^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^1 & x_n^2 & \dots & x_n^j & \dots & x_n^d \end{bmatrix}, \begin{cases} i = 1,2,\dots,n \\ j = 1,2,\dots,d \end{cases} \quad (2)$$

where n is the number of crystals (i.e., candidate solutions) and d is the dimension of the problem. The initial positions of these crystals are randomly determined in the search space as follows:

$$x_i^j(0) = x_{i,\min}^j + \xi(x_{i,\max}^j - x_{i,\min}^j), \quad \begin{cases} i = 1,2,\dots,n \\ j = 1,2,\dots,d \end{cases} \quad (3)$$

where $x_i^j(0)$ determines the initial position of the crystals; $x_{i,\min}^j$ and $x_{i,\max}^j$ are the minimum and maximum allowable values, respectively, for the j^{th} decision variable of the i^{th} candidate solution; and ξ is a random number in the interval [0,1].

Based on the concept of 'basis' in crystallography, all the crystals at the corners are considered as the *main crystals*, Cr_{main}, determined randomly by considering the initially-created crystals (candidate solutions). It should be noted that the random selection process for each step is determined by omitting the current Cr. The crystal with the *best* configuration is determined as Cr_b while the mean values of randomly-selected crystals are denoted by F_c.

To update the positions of the candidate solutions in the search space, basic lattice principles are considered in which four kinds of updating process are determined as follows:

(i) Simple cubicle:

$$Cr_{new} = Cr_{old} + rCr_{main}, \quad (4)$$

(ii) Cubicle with the best crystals:

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 Cr_b, \quad (5)$$

(iii) Cubicle with the mean crystals:

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 F_c, \quad (6)$$

(iv) Cubicle with the best and mean crystals:

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 Cr_b + r_3 F_c, \quad (7)$$

where, in the four equations above, Cr_{new} is the new position, Cr_{old} is the old position, and r, r_1, r_2 and r_3 are random numbers.

It should be mentioned that exploration and exploitation, as two critical features of metaheuristics, have been considered in this algorithm through (4) to (7) in which local and global searches are conducted simultaneously. In order to deal with the solution variables x_i^j violating the boundary conditions of the variables, a mathematical flag is defined in which for the x_i^j outside the variables range, the flag orders a boundary change for the violating variables. The terminating criterion is considered based on the maximum number of iterations in which the optimization process is terminated after a fixed number of iterations. The pseudo-code of the algorithm is presented in Fig. 2.

```
procedure Crystal Structure Algorithm (CryStAl)
        Create random values for initial positions (x_i^j) of initial
crystals (Cr_i)
        Evaluate fitness values for each crystal
        while (t < maximum number of iterations)
            for i=1: number of initial crystals
                Create Cr_main
                Create new crystals by Eq. 4
                Create Cr_b
                Create new crystals by Eq. 5
                Create F_c
                Create new crystals by Eq. 6
                Create new crystals by Eq. 7
                    if new crystals violate boundary conditions
                        Control the position constraints for new crystals and
amend them
                    end if
                Evaluate the fitness values for new crystals
                Update Global Best (GB) if a better solution is found
            end for
            t=t+1
        end while
        Return GB
end procedure
```

FIGURE 2. The pseudo-code of the Crystal Structure Algorithm (CryStAl).

III. MATHEMATICAL TEST FUNCTIONS

In this section, a number of mathematical functions are selected to be utilized as test functions for the performance evaluation of the proposed algorithm. A total number of 239 mathematical functions are tested which are categorized into four different groups based on their specific characteristics. These functions have been derived from various references [41-45] in which different mathematical functions with different characteristics had been reviewed and presented for utilization in the validation of novel metaheuristic algorithms.

In the first group, 117 mathematical functions are presented which have minimum and maximum dimensions of 2 and 10, respectively. Among these functions, which are named F_1 to F_{117}, the first 90 functions have 2 dimensions whereas the other 27 functions have dimensions of 3 to 10. In this paper, these functions are called the 'two-dimensional (2D)' test functions and are presented in Table 3. The second group of mathematical functions consists of 58 test functions in which the dimensions of functions are variable due to their specific formulations and are called the '*N*-dimensional (*N*D)' test functions. A maximum number of dimensions of 50 is considered in dealing with the functions of this group, called the 50-dimensional (50D) test functions, which are named F_{118} to F_{175} and presented in Table 4. For the third group, the mathematical functions of the second group are considered with the maximum dimension of 100 and are called the 100-dimensional (100D) test functions; these functions, named F_{175} to F_{233}, are presented in Table 5. For the fourth group, three composite and three hybrid mathematical functions are considered which are named F_{233} to F_{239}, presented in Table 6. In these tables, C, NC, D, ND, S, NS, Sc, NSc, U, and M denote Continuous, Non-Continuous, Differentiable, Non-Differentiable, Separable, Non-Separable, Scalable, Non-Scalable, Unimodal, and Multi-modal, respectively. Furthermore, *R*, *D*, and *Min*. represent the variables range, variables dimension, and the global minimum of the functions.

Based on the fact that a larger number of mathematical functions (239 functions) are considered in this paper, the 3D plots for some of these functions are presented in the following. The 3D plots for some of the 2D functions are shown in Fig. 3, while those of the 50D and 100D functions are depicted in Figs. 4 and 5, respectively. The complete mathematical formulations of these test functions are presented in Refs. [81-85].

IV. ALTERNATIVE METAHEURISTICS FOR COMPARISON

In order to evaluate the overall performance of the proposed algorithm, some different optimization algorithms are utilized as alternative approaches to provide a valid comparative study. The utilized metaheuristics for this purpose are the ABC, ACO, BA, FA, GA, HS, MFO, MVO, PSO, SA, SCA, and SSA. Based on the fact that some of the selected optimization algorithms are recently proposed or developed for special purposes, the most recent and improved versions of these algorithms are used in this paper. Knowing that the internal parameters of the optimization algorithms have the most vital role in their convergence performance, a parameter summary of the selected algorithms is presented in Table 7. The values of these parameters have been determined using the reference-based parameter identification process in which the internal parameters of these algorithms are selected based on relevant previously published research papers.

In many metaheuristic algorithms, some specific parameters are utilized for tuning the exploration and exploitation rates during the optimization process which are often problem-dependent parameters and so they should be tuned for each specific optimization problem. The mentioned parameters for the alternative algorithms in Table 7 were derived from the latest and most successful configurations of these algorithms available in the literature which resulted in acceptable optimum results in most of the previously considered optimization problems.

Knowing that such algorithms are potentially vulnerable to entrapment in local optima or even having convergence problems, we have proposed CryStAl as a simple algorithm without any internal or external parameters to be tuned. This characteristic can be considered as the major advantage of this algorithm over competing algorithms. In fact, as mentioned earlier in this section, CryStAl considers exploration and exploitation through (4) to (7) where local and global searches are performed simultaneously.

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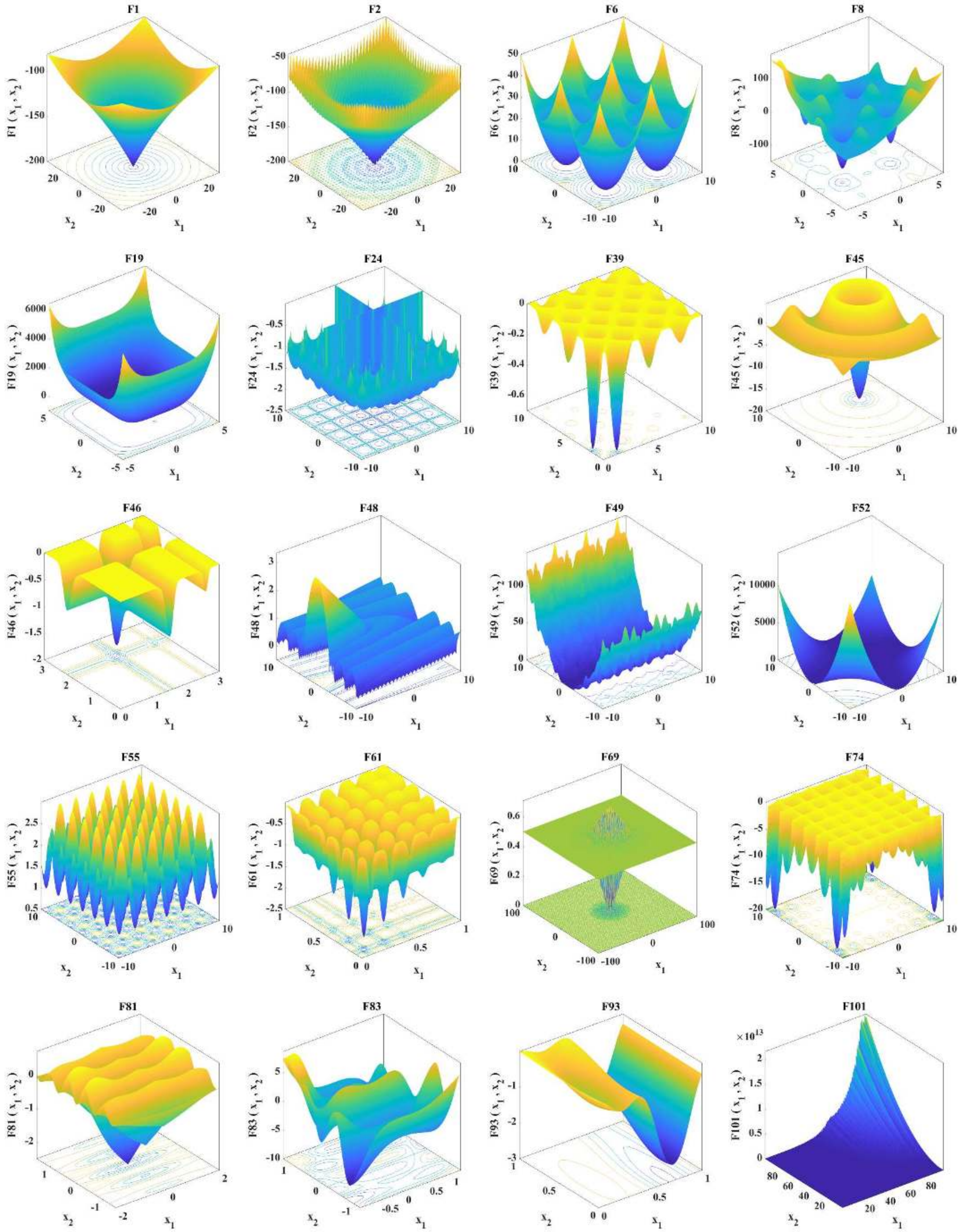

FIGURE 3. The 3D plots of the 2D mathematical functions (Continues on the next page →)

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

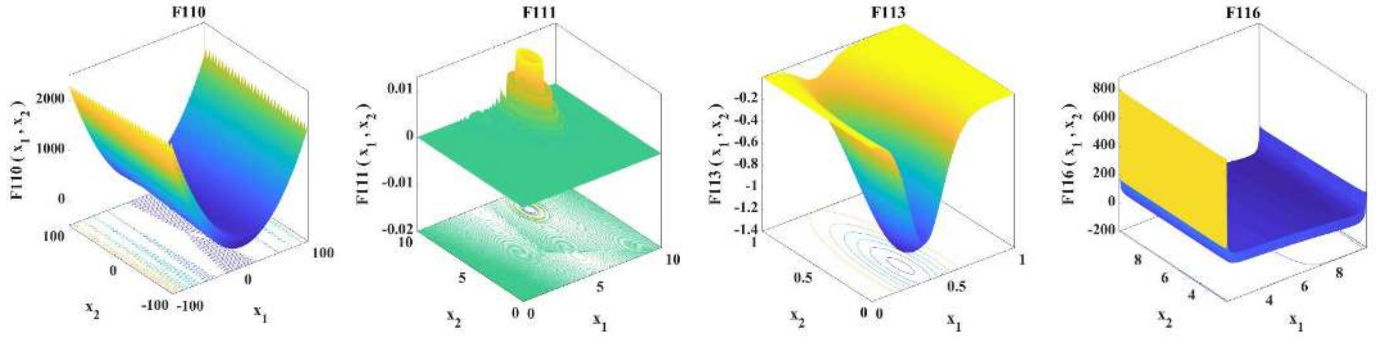

FIGURE 3. (Continued). The 3D plots of the 2D mathematical functions.

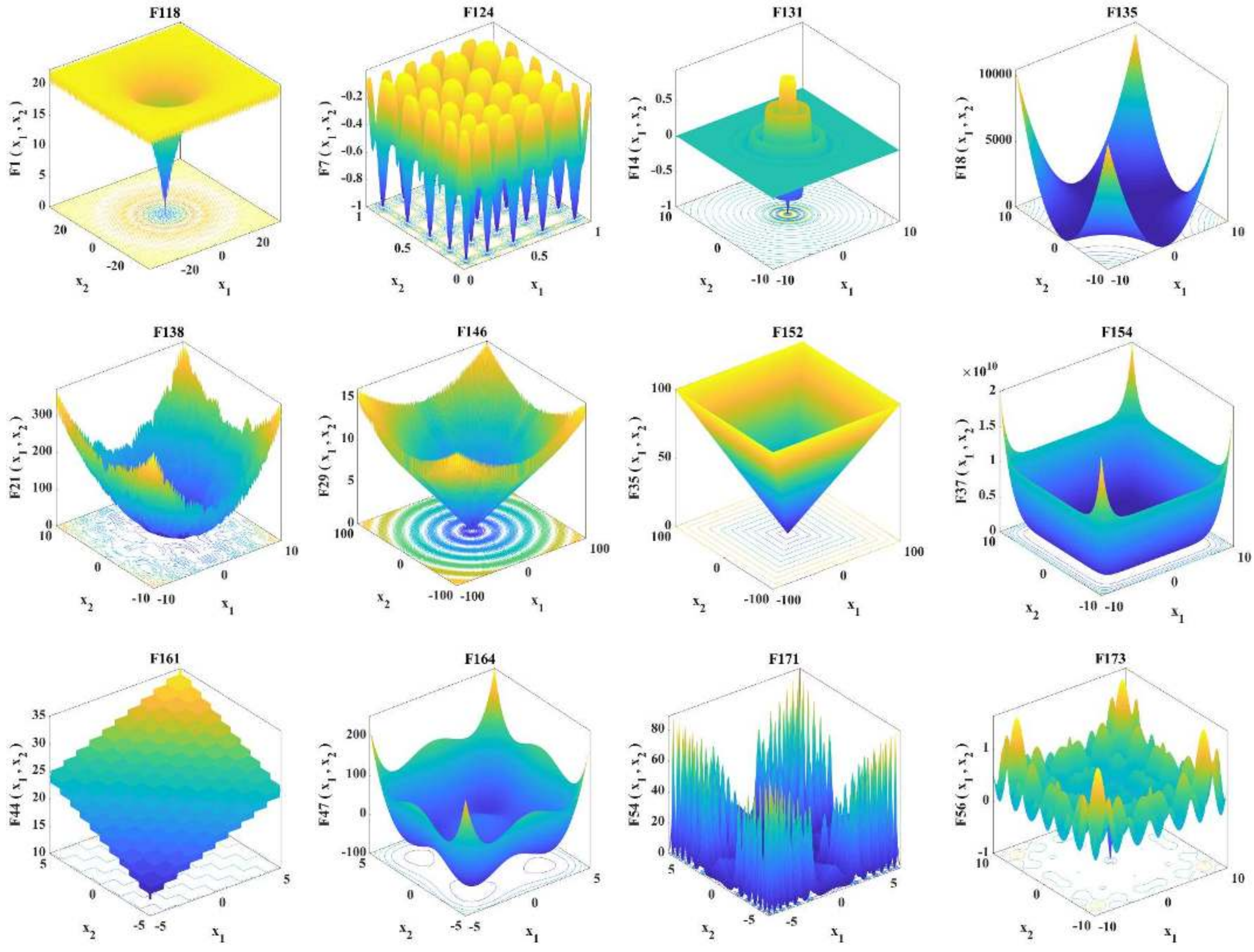

FIGURE 4. The 3D plots of the 50D mathematical functions.

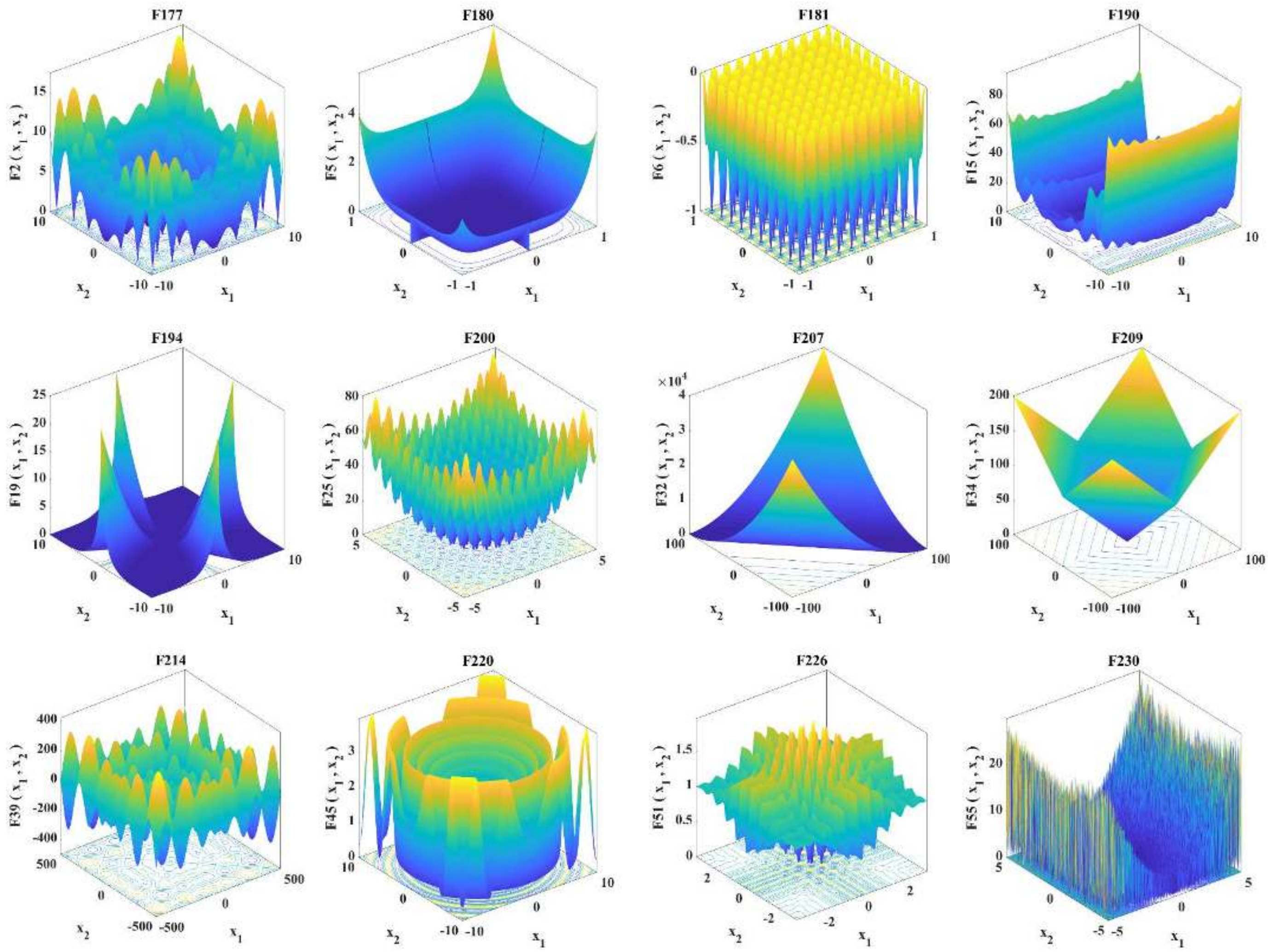

FIGURE 5. The 3D plots of the 100D mathematical functions.

TABLE 3. Details of the 2D to 10D mathematical functions (First group)

No.	Name	Type	R	D	Min.
F_1	*Ackley 2*	*C, D, NS, Sc,M*	[-35, 35]	2	-200
F_2	*Ackley 3*	*C, D, NS, NSc,U*	[-32, 32]	2	-195.629
F_3	*Adjiman*	*C, D, NS, NSc, M*	[-1, 2] & [-1, 1]	2	-2.02181
F_4	*Bartels Conn*	*C, ND, NS, NSc, M*	[-500, 500]	2	1
F_5	*Beale*	*C, D, NS, NSc, U*	[-4.5, 4.5]	2	0
F_6	*Becker-Lago*	*S*	[-10, 10]	2	0
F_7	*Biggs EXP2*	*C, D, NS, NSc, M*	[0, 20]	2	0
F_8	*Bird*	*C, D, NS, NSc, M*	[-2π, π]	2	-106.765
F_9	*Bohachevsky 1*	*C, D, S, NSc, M*	[-100, 100]	2	0
F_{10}	*Bohachevsky 2*	*C, D, NS, NSc, M*	[-100, 100]	2	0
F_{11}	*Bohachevsky 3*	*C, D, NS, NSc, M*	[-100, 100]	2	0
F_{12}	*Booth*	*C, D, NS, NSc, U*	[-10, 10]	2	0
F_{13}	*Branin RCOS*	*C, D, NS, NSc M*	[-5, 10] & [0, 15]	2	0.397887
F_{14}	*Branin RCOS 2*	*C, D, NS, NSc, M*	[-5, 15]	2	5.559037
F_{15}	*Brent*	*C, D, NS, NSc, U*	[-10, 10]	2	0
F_{16}	*Bukin 4*	*C, ND, S, NSc, M*	[-15, -5] & [-3, 3]	2	0
F_{17}	*Bukin 6*	*C, ND, NS, NSc, M*	[-15, -5] & [-3, 3]	2	0
F_{18}	*Camel - 3 Hump*	*C, D, NS, NSc, M*	[-5, 5]	2	0
F_{19}	*Camel - 6 Hump*	*C, D, NS, NSc, M*	[-5, 5]	2	-1.0316
F_{20}	*Carrom table*	*NS*	[-10, 10]	2	-24.1568
F_{21}	*Chen Bird*	*C, D, NS, NSc, M*	[-500, 500]	2	-2000
F_{22}	*Chen V*	*C, D, NS, NSc, M*	[-500, 500]	2	-2000
F_{23}	*Chichinadze*	*C, D, S, NSc, M*	[-30, 30]	2	-42.9444
F_{24}	*Cross-in-Tray*	*C, NS, NSc, M*	[-10, 10]	2	-2.06261
F_{25}	*Cube*	*C, D, NS, NSc, U*	[-10, 10]	2	0
F_{26}	*Damavandi*	*C, D, NS, NSc, M*	[0, 14]	2	0
F_{27}	*Deckkers-Aarts*	*C, D, NS, NSc, M*	[-20, 20]	2	-24771.1
F_{28}	*Easom*	*C, D, S, NSc, M*	[-100, 100]	2	-1
F_{29}	*El-Attar-Vidyasagar-Dutta*	*C, D, NS, NSc, M*	[-500, 500]	2	1.7128
F_{30}	*Egg Crate*	*C, D, NS, Sc, M*	[-5, 5]	2	0
F_{31}	*Exp 2*	*S*	[0, 20]	2	0
F_{32}	*Freudenstein Roth*	*C, D, NS, NSc, M*	[-10, 10]	2	0
F_{33}	*Giunta*	*C, D, S, Sc, M*	[-1, 1]	2	0.060447
F_{34}	*Goldstein Price*	*C, D, NS, NSc, M*	[-2, 2]	2	3
F_{35}	*Hansen*	*C, D, S, NSc, M*	[-10, 10]	2	-165.953
F_{36}	*Himmelblau*	*C, D, NS, NSc, M*	[-5, 5]	2	0
F_{37}	*Hosaki*	*C, D, NS, NSc, M*	[0, 5] & [0, 6]	2	-2.3458
F_{38}	*Jennrich-Sampson*	*C, D, NS, NSc, M*	[-1, 1]	2	124.3612
F_{39}	*Keane*	*C, D, NS, NSc, M*	[0, 10]	2	-0.67367
F_{40}	*Leon*	*C, D, NS, NSc, U*	[-1.2, 1.2]	2	0
F_{41}	*Levy 3*	*S*	[-10, 10]	2	-176.542
F_{42}	*Levy 5*	*NS*	[-10, 10]	2	-176.138
F_{43}	*Matyas*	*C, D, NS, NSc, U*	[-10, 10]	2	0
F_{44}	*McCormick*	*C, D, NS, NSc, M*	[-1.5, 4] & [-3, 3]	2	-1.9133
F_{45}	*Mexican hat*	*NS*	[-10, 10]	2	-19.6683
F_{46}	*Michaelewicz 2*	*S*	[0, π]	2	-1.8013
F_{47}	*Mishra 3*	*C, D, NS, NSc, M*	[-10, 10]	2	-0.18465
F_{48}	*Mishra 4*	*C, D, NS, NSc, M*	[-10, 10]	2	-0.19941
F_{49}	*Mishra 5*	*C, D, NS, NSc, M*	[-10, 10]	2	-1.01983
F_{50}	*Mishra 6*	*C, D, NS, NSc, M*	[-10, 10]	2	-2.28395
F_{51}	*Mishra 8*	*C, D, NS, NSc, M*	[-10, 10]	2	0
F_{52}	*Mishra 10*	*C, D, NS, NSc, M*	[-10, 10]	2	0
F_{53}	*Parsopoulos*	*C, D, S, Sc, M*	[-5, 5]	2	0
F_{54}	*Pen Holder*	*C, D, NS, NSc, M*	[-11, 11]	2	-0.96354
F_{55}	*Periodic*	*S*	[-10, 10]	2	0.9
F_{56}	*Price 1*	*C, ND, S, NSc, M*	[-500, 500]	2	0
F_{57}	*Price 2*	*C, D, NS, NSc, M*	[-10, 10]	2	0.9
F_{58}	*Price 3*	*C, D, NS, NSc, M*	[-500, 500]	2	0
F_{59}	*Price 4*	*C, D, NS, NSc, M*	[-500, 500]	2	0
F_{60}	*Quadratic*	*C, D, NS, NSc*	[-10, 10]	2	-3873.72
F_{61}	*Ripple 1*	*NS*	[0, 1]	2	-2.2
F_{62}	*Ripple 25*	*NS*	[0, 1]	2	-2
F_{63}	*Rosenbrock Modified*	*C, D, NS, NSc, M*	[-2, 2]	2	34.3712
F_{64}	*Rotated Ellipse*	*C, D, NS, NSc, U*	[-500, 500]	2	0
F_{65}	*Rotated Ellipse 2*	*C, D, NS, NSc, U*	[-500, 500]	2	0
F_{66}	*Rump*	*C, D, NS, NSc, U*	[-500, 500]	2	0

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

F_{67}	Scahffer 1	C, D, NS, NSc, U	[-100, 100]	2	0
F_{68}	Scahffer 2	C, D, NS, NSc, U	[-100, 100]	2	0
F_{69}	Scahffer 3	C, D, NS, NSc, U	[-100, 100]	2	0.001567
F_{70}	Scahffer 4	C, D, NS, NSc, U	[-100, 100]	2	0.292579
F_{71}	Schwefel 2.6	C, D, NS, NSc, U	[-100, 100]	2	0
F_{72}	Schwefel 2.36	C, D, S, Sc, M	[0, 500]	2	-3456
F_{73}	Table 1 / Holder Table 1	C, D, S, NSc, M	[-10, 10]	2	-26.9203
F_{74}	Table 2 / Holder Table 2	C, D, S, NSc, M	[-10, 10]	2	-19.2085
F_{75}	Table 3 / Carrom Table	C, D, NS, NSc, M	[-10, 10]	2	-24.1568
F_{76}	Testtube Holder	C, D, S, NSc, M	[-10, 10]	2	-10.8723
F_{77}	Trecanni	C, D, S, NSc, U	[-5, 5]	2	0
F_{78}	Trefethen	C, D, NS, NSc, M	[-10, 10]	2	-3.30687
F_{79}	Tripod	C, D, NS, NSc, M	[-100, 100]	2	0
F_{80}	Ursem 1	S	[-2.5, 3] & [-2, 2]	2	-4.81681
F_{81}	Ursem 3	NS	[-2, 2] & [-1.5, 1.5]	2	-2.5
F_{82}	Ursem 4	NS	[-2, 2]	2	-1.5
F_{83}	Ursem Waves	NS	[-0.9, 1.2] & [-1.2, 1.2]	2	-7.307
F_{84}	Venter Sobiezcczanski-Sobieski	C, D, S, NSc	[-50, 50]	2	-400
F_{85}	Wayburn Seader 1	C, D, NS, Sc, U	[-500, 500]	2	0
F_{86}	Wayburn Seader 2	C, D, NS, Sc, U	[-500, 500]	2	0
F_{87}	Wayburn Seader 3	C, D, NS, Sc, U	[-500, 500]	2	21.35
F_{88}	Zettl	C, D, NS, NSc, U	[-5, 10]	2	-0.00379
F_{89}	Zirilli or Aluffi-Pentini	C, D, S, NSc, U	[-10, 10]	2	-0.3523
F_{90}	Zirilli 2	C, D, S, S, M	[-500, 500]	2	0
F_{91}	Biggs EXP3	C, D, NS, NSc, M	[0, 20]	3	0
F_{92}	Gulf Research Problem	C, D, NS, NSc, M	[0.1, 100] & [0, 25.6] & [0, 6.5]	3	0
F_{93}	Hartman 3	C, D, NS, NSc, M	[0, 1]	3	-3.86278
F_{94}	Helical Valley	C, D, NS, Sc, M	[-10, 10]	3	0
F_{95}	Meyer-Roth	NS	[0, 1]	3	4.00E-05
F_{96}	Mishra 9	C, D, NS, NSc, M	[-10, 10]	3	0
F_{97}	Wolfe	C, D, S, Sc, M	[0, 2]	3	0
F_{98}	Biggs EXP4	C, D, NS, NSc, M	[0, 20]	4	0
F_{99}	Colville	C, D, NS, NSc, M	[-10, 10]	4	0
F_{100}	Corana	DC, ND, S, Sc, M	[-500, 500]	4	0
F_{101}	DeVilliers Glasser 1	C, D, NS, NSc, M	[1, 100]	4	0
F_{102}	Gear	NS	[12, 60]	4	2.70E-12
F_{103}	Kowalik	NS	[-5, 5]	4	0.000308
F_{104}	Miele Cantrell	C, D, NS, NSc, M	[-1, 1]	4	0
F_{105}	Shekel 5	C, D, NS, Sc, M	[0, 10]	4	-10.1532
F_{106}	Shekel 7	C, D, NS, Sc, M	[0, 10]	4	-10.4029
F_{107}	Shekel 10	C, D, NS, Sc, M	[0, 10]	4	-10.5364
F_{108}	Biggs EXP5	C, D, NS, NSc, M	[0, 20]	5	0
F_{109}	DeVilliers Glasser 2	C, D, NS, NSc, M	[1, 60]	5	0
F_{110}	Dolan	C, D, NS, NSc, M	[-100, 100]	5	-529.871
F_{111}	Langerman-5	C, D, NS, Sc, M	[0, 10]	5	-0.965
F_{112}	Biggs EXP6	C, D, NS, NSc, M	[-20, 20]	6	0
F_{113}	Hartman 6	C, D, NS, NSc, M	[0, 1]	6	-3.32236
F_{114}	Trid 6	C, D, NS, NSc, M	[-36, 36]	6	-50
F_{115}	Ann-XOR	NS	[-1, 1]	9	0.95979
F_{116}	Paviani	C, D, NS, Sc, M	[2.0001, 10]	10	-45.778
F_{117}	Trid 10	C, D, NS, NSc, M	[-100,100]	10	-210

TABLE 4. Details of the 50D mathematical functions (Second group).

No.	Name	Type	R	D	Min.
F_{118}	Ackley 1	C, D, NS, Sc,M	[-35, 35]	50	0
F_{119}	Alpine 1	C, ND, S, NSc,U	[-10, 10]	50	0
F_{120}	Brown	C, D, NS, Sc, U	[-1, 4]	50	0
F_{121}	Chung Reynolds	C, D, PS, Sc, U	[-100, 100]	50	0
F_{122}	Csendes	C, D, S, Sc, M	[-1, 1]	50	0
F_{123}	Deb 1	C, D, S, Sc, M	[-1, 1]	50	-1
F_{124}	Deb 3	C, D, S, Sc, M	[0, 1]	50	-1
F_{125}	Dixon & Price	C, D, NS, Sc, U	[-10, 10]	50	0
F_{126}	Extended Easom	C, D, S, NSc, M	[-2π, 2π]	50	-1
F_{127}	Exponential	C, D, NS, Sc, M	[-1, 1]	50	-1
F_{128}	Griewank	C, D, NS, Sc, M	[-100,100]	50	0
F_{129}	Holzman 2	S	[-10, 10]	50	0
F_{130}	Hyper-ellipsoid	C, U	[-500, 500]	50	0
F_{131}	Inverted cosine wave	NS	[-10, 10]	50	-49
F_{132}	Levy 8	NS	[-10, 10]	50	0
F_{133}	Mishra 1	C, D, NS, Sc, M	[0, 1]	50	2
F_{134}	Mishra 2	C, D, NS, Sc, M	[0, 1]	50	2
F_{135}	Mishra 7	C, D, NS, NSc, M	[-10, 10]	50	0
F_{136}	Mishra 11	C, D, NS, NSc, M	[-10, 10]	50	0
F_{137}	Pathological	C, D, NS, NSc, M	[-100, 100]	50	0
F_{138}	Pint´er	C, D, NS, Sc, M	[-10, 10]	50	0
F_{139}	Powell Singular	C, D, NS, Sc, U	[-4, 5]	50	0
F_{140}	Powell Singular 2	C, D, NS, Sc, U	[-4, 5]	50	0
F_{141}	Powell Sum	C, D, S, Sc, U	[-1, 1]	50	0
F_{142}	Rastrigin	C, D, S, M	[-5.12, 5.12]	50	0
F_{143}	Qing	C, D, S, Sc, M	[-500, 500]	50	0
F_{144}	Quintic	C, D, S, NSc, M	[-10, 10]	50	0
F_{145}	Rosenbrock	C, D, NS, Sc, U	[-30, 30]	50	0
F_{146}	Salomon	C, D, NS, Sc, M	[-100, 100]	50	0
F_{147}	Schumer Steiglitz	C, D, S, Sc, U	[-100, 100]	50	0
F_{148}	Schwefel	C, D, PS, Sc, U	[-100, 100]	50	0
F_{149}	Schwefel 1.2	C, D, NS, Sc, U	[-100, 100]	50	0
F_{150}	Schwefel 2.4	C, D, S, NSc, M	[0, 10]	50	0
F_{151}	Schwefel 2.20	C, ND, S, Sc, U	[-100, 100]	50	0
F_{152}	Schwefel 2.21	C, ND, S, Sc, U	[-100, 100]	50	0
F_{153}	Schwefel 2.22	C, D, NS, Sc, U	[-100, 100]	50	0
F_{154}	Schwefel 2.23	C, D, NS, Sc, U	[-10, 10]	50	0
F_{155}	Schwefel 2.25	C, D, S, NSc, M	[0, 10]	50	0
F_{156}	Schwefel 2.26	C, D, S, Sc, M	[-500, 500]	50	-418.98
F_{157}	Sphere	C, D, S, Sc, M	[0, 10]	50	0
F_{158}	Step	DC, ND, S, Sc, U	[-100, 100]	50	0
F_{159}	Step 2	DC, ND, S, Sc, U	[-100, 100]	50	0
F_{160}	Step 3	DC, ND, S, Sc, U	[-100, 100]	50	0
F_{161}	Stepint	DC, ND, S, Sc, U	[-5.12, 5.12]	50	-275
F_{162}	Stretched V Sine Wave	C, D, NS, Sc, U	[-10, 10]	50	0
F_{163}	Sum Squares	C, D, S, Sc, U	[-10, 10]	50	0
F_{164}	Styblinski-Tang	C, D, NS, NSc, M	[-5, 5]	50	-1958.3
F_{165}	Trid	C, D, NS, NSc, U	[-D^2, D^2]	50	-22050
F_{166}	Trigonometric 1	C, D, NS, Sc, M	[0, π]	50	0
F_{167}	Trigonometric 2	C, D, NS, Sc, M	[-500, 500]	50	1
F_{168}	W / Wavy	C, D, S, Sc, M	[-π, π]	50	0
F_{169}	Xin-She Yang (1)	DC, ND, NS, Sc, M	[-20, 20]	50	-1
F_{170}	Xin-She Yang (2)	DC, ND, NS, Sc, M	[-10, 10]	50	0
F_{171}	Xin-She Yang (3)	DC, ND, NS, Sc, M	[-2π, 2π]	50	0
F_{172}	Xin-She Yang (4)	DC, ND, NS, Sc, M	[-5, 5]	50	0
F_{173}	Xin-She Yang (5)	DC, ND, NS, Sc, M	[-10, 10]	50	-1
F_{174}	Xin-She Yang (6)	DC, ND, NS, Sc, M	[-5, 5]	50	0
F_{175}	Zakharov	C, D, NS, Sc, M	[-5, 10]	50	0

TABLE 5. Details of the 100D mathematical functions (Third group).

No.	Name	Type	R	D	Min.
F_{176}	Ackley 1	C, D, NS, Sc,M	[-35, 35]	100	0
F_{177}	Alpine 1	C, ND, S, NSc,U	[-10, 10]	100	0
F_{178}	Brown	C, D, NS, Sc, U	[-1, 4]	100	0
F_{179}	Chung Reynolds	C, D, PS, Sc, U	[-100, 100]	100	0
F_{180}	Csendes	C, D, S, Sc, M	[-1, 1]	100	0
F_{181}	Deb 1	C, D, S, Sc, M	[-1, 1]	100	-1
F_{182}	Deb 3	C, D, S, Sc, M	[0, 1]	100	-1
F_{183}	Dixon & Price	C, D, NS, Sc, U	[-10, 10]	100	0
F_{184}	Extended Easom	C, D, S, NSc, M	[-2π, 2π]	100	-1
F_{185}	Exponential	C, D, NS, Sc, M	[-1, 1]	100	-1
F_{186}	Griewank	C, D, NS, Sc, M	[-100,100]	100	0
F_{187}	Holzman 2	S	[-10, 10]	100	0
F_{188}	Hyper-ellipsoid	C, U	[-500, 500]	100	0
F_{189}	Inverted cosine wave	NS	[-10, 10]	100	-99
F_{190}	Levy 8	NS	[-10, 10]	100	0
F_{191}	Mishra 1	C, D, NS, Sc, M	[0, 1]	100	2
F_{192}	Mishra 2	C, D, NS, Sc, M	[0, 1]	100	2
F_{193}	Mishra 7	C, D, NS, NSc, M	[-10, 10]	100	0
F_{194}	Mishra 11	C, D, NS, NSc, M	[-10, 10]	100	0
F_{195}	Pathological	C, D, NS, NSc, M	[-100, 100]	100	0
F_{196}	Pint´er	C, D, NS, Sc, M	[-10, 10]	100	0
F_{197}	Powell Singular	C, D, NS, Sc, U	[-4, 5]	100	0
F_{198}	Powell Singular 2	C, D, NS, Sc, U	[-4, 5]	100	0
F_{199}	Powell Sum	C, D, S, Sc, U	[-1, 1]	100	0
F_{200}	Rastrigin	C, D, S, M	[-5.12, 5.12]	100	0
F_{201}	Qing	C, D, S, Sc, M	[-500, 500]	100	0
F_{202}	Quintic	C, D, S, NSc, M	[-10, 10]	100	0
F_{203}	Rosenbrock	C, D, NS, Sc, U	[-30, 30]	100	0
F_{204}	Salomon	C, D, NS, Sc, M	[-100, 100]	100	0
F_{205}	Schumer Steiglitz	C, D, S, Sc, U	[-100, 100]	100	0
F_{206}	Schwefel	C, D, PS, Sc, U	[-100, 100]	100	0
F_{207}	Schwefel 1.2	C, D, NS, Sc, U	[-100, 100]	100	0
F_{208}	Schwefel 2.4	C, D, S, NSc, M	[0, 10]	100	0
F_{209}	Schwefel 2.20	C, ND, S, Sc, U	[-100, 100]	100	0
F_{210}	Schwefel 2.21	C, ND, S, Sc, U	[-100, 100]	100	0
F_{211}	Schwefel 2.22	C, D, NS, Sc, U	[-100, 100]	100	0
F_{212}	Schwefel 2.23	C, D, NS, Sc, U	[-10, 10]	100	0
F_{213}	Schwefel 2.25	C, D, S, NSc, M	[0, 10]	100	0
F_{214}	Schwefel 2.26	C, D, S, Sc, M	[-500, 500]	100	-418.98
F_{215}	Sphere	C, D, S, Sc, M	[0, 10]	100	0
F_{216}	Step	DC, ND, S, Sc, U	[-100, 100]	100	0
F_{217}	Step 2	DC, ND, S, Sc, U	[-100, 100]	100	0
F_{218}	Step 3	DC, ND, S, Sc, U	[-100, 100]	100	0
F_{219}	Stepint	DC, ND, S, Sc, U	[-5.12, 5.12]	100	-575
F_{220}	Stretched V Sine Wave	C, D, NS, Sc, U	[-10, 10]	100	0
F_{221}	Sum Squares	C, D, S, Sc, U	[-10, 10]	100	0
F_{222}	Styblinski-Tang	C, D, NS, NSc, M	[-5, 5]	100	-3916.6
F_{223}	Trid	C, D, NS, NSc, U	[-D^2, D^2]	100	-171600
F_{224}	Trigonometric 1	C, D, NS, Sc, M	[0, π]	100	0
F_{225}	Trigonometric 2	C, D, NS, Sc, M	[-500, 500]	100	1
F_{226}	W / Wavy	C, D, S, Sc, M	[-π, π]	100	0
F_{227}	Xin-She Yang (1)	DC, ND, NS, Sc, M	[-20, 20]	100	-1
F_{228}	Xin-She Yang (2)	DC, ND, NS, Sc, M	[-10, 10]	100	0
F_{229}	Xin-She Yang (3)	DC, ND, NS, Sc, M	[-2π, 2π]	100	0
F_{230}	Xin-She Yang (4)	DC, ND, NS, Sc, M	[-5, 5]	100	0
F_{231}	Xin-She Yang (5)	DC, ND, NS, Sc, M	[-10, 10]	100	-1
F_{232}	Xin-She Yang (6)	DC, ND, NS, Sc, M	[-5, 5]	100	0
F_{233}	Zakharov	C, D, NS, Sc, M	[-5, 10]	100	0

TABLE 6. Details of the composite and hybrid mathematical functions (Fourth group).

No.	Descriptions	R	D	Min.
F_{234}	*Basic Functions: Sphere Function* $f_1, f_2, f_3, \ldots, f_{10}$ = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}]$ = [5/100, 5/100, 5/100, …, 5/100]	[-5, 5]	10	0
F_{235}	*Basic Functions: Griewank Function* $f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}]$ = [5/100, 5/100, 5/100, …, 5/100]	[-5, 5]	10	0
F_{236}	*Basic Functions: Griewank Function* $f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [1, 1, 1, \ldots, 1]$	[-5, 5]	10	0
F_{237}	*Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions* f_1, f_2 = Ackley Function f_3, f_4 = Rastrigin Function f_5, f_6 = Weierstrass Function f_7, f_8 = Griewank Function f_9, f_{10} = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}]$ = [5/32, 5.32, 1, 1, 5/0.5, 5/0.5, 5/100, 5/100, 5/100, 5/100]	[-5, 5]	10	0
F_{238}	*Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions* f_1, f_2 = Rastrigin Function f_3, f_4 = Weierstrass Function f_5, f_6 = Griewank Function f_7, f_8 = Ackley Function f_9, f_{10} = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}]$ = [1/5, 1/5, 5/0.5, 5/0.5, 5/100, 5/100, 5/32, 5/32, 5/100, 5/100]	[-5, 5]	10	0
F_{239}	*Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions* f_1, f_2 = Rastrigin Function f_3, f_4 = Weierstrass Function f_5, f_6 = Griewank Function f_7, f_8 = Ackley Function f_9, f_{10} = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}]$ = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1] $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}]$ = [0.1×1/5, 0.2×1/5, 0.3×5/0.5, 0.4×5/0.5, 0.5×5/100, 0.6×5/100, 0.7×5/32, 0.8×5/32, 0.9×5/100, 1×5/100]	[-5, 5]	10	0

TABLE 7. Parameter summary of the alternative metaheuristic algorithms.

Metaheuristic	Parameter	Description	Value
ABC	N_{pop}	Colony Size	50
	N_o	Number of Onlooker Bees	50
	L	Abandonment Limit Parameter	60
	a	Acceleration Coefficient Upper Bound	1
ACO	N_{pop}	Archive Size	50
	N_s	Sample Size	50
	q	Intensification Factor	0.5
	ζ	Deviation-Distance Ratio	1
BA	N_{pop}	Number of Scout Bees	50
	N_{ss}	Number of Selected Sites	25
	N_{se}	Number of Selected Elite Sites	10
	N_{rs}	Number of Recruited Bees for Selected Sites	25
	N_{re}	Number of Recruited Bees for Elite Sites	50
	r	Neighborhood Radius	0.1
	r_{damp}	Neighborhood Radius Damp Rate	0.95
FA	N_{pop}	Number of Fireflies (Swarm Size)	50
	γ	Light Absorption Coefficient	1
	β	Attraction Coefficient Base Value	2
	α	Mutation Coefficient	0.2
	α_{damp}	Mutation Coefficient Damping Ratio	0.98
	δ	Uniform Mutation Range	±0.05
GA	p_c	Crossover Percentage	0.8
	p_m	Mutation Percentage	0.3
	μ	Mutation Rate	0.02
	β	Roulette wheel selection pressure	1
HS	HMS	Harmony Memory Size	50
	N_{new}	Number of New Harmonies	20
	$HMCR$	Harmony Memory Consideration Rate	0.9
	PAR	Pitch Adjustment Rate	0.1
	FW	Fret Width (Bandwidth)	±0.02
	FW_{damp}	Fret Width Damp Ratio	0.995
PSO	N_{pop}	Swarm Size	50
	w	Inertia Weight	1
	w_d	Inertia Weight Damping Ratio	0.99
	c_1	Personal Learning Coefficient	2
	c_2	Global Learning Coefficient	2
SA	N_{pop}	Population Size	50
	M_{subit}	Maximum Number of Sub-iterations	15
	T_0	Initial Temperature	0.025
	α	Temperature Reduction Rate	0.99
	N_m	Number of Neighbors per Individual	5
	μ	Mutation Rate	0.5
	σ	Mutation Range (Standard Deviation)	0.1

V. NUMERICAL RESULTS

In this section, the obtained results of the optimization run for CryStAl alongside the alternative metaheuristic approaches in dealing with the mathematical test functions are presented. The optimization problem is formulated with the maximum population size taken as 50 and the maximum number of Function Evaluations (FEs) selected to be 150000 for all of the metaheuristics. The maximum number of iterations in each algorithm is adjusted based on the selected maximum number of FEs. As collecting quantitative results are of great importance in dealing with different optimization problems, CryStAl and the other algorithms are utilized 100 times with different initializations and the mean and standard deviation (std) of the best approximated solutions in the last iteration are reported. A tolerance of 1×10^{-12} is also considered for the convergence results of the algorithms in which the optimization runs are stopped at this tolerance of the Global Best (GB). It is assumed that the GB results are achieved by these optimization runs within this tolerance and the results of the GB are utilized instead of the final results of the optimization runs. The number of FEs are also calculated based on the selected tolerance. It should be noted that the above-mentioned is utilized as the stopping criterion in order to save time from a computational complexity perspective. In other words, if the algorithm reaches to this tolerance of the global best for the considered problem, the global best is reported as the final solution of the algorithm which requires less computational time. Therefore, the computational time for the considered 100 optimization runs will be reasonable. Besides, the initial random state of each optimization run for each alternative algorithm has been selected equally in order to form a fair judgment about the performance of the proposed and alternative algorithms.

The detailed results of CryStAl and the other selected methods are presented in the Supplementary Materials which includes the convergence history of the proposed algorithm. It turned out that CryStAl can find the exact global results of 156 functions (65%); moreover, its result is very close to the global best result for 83 problems. Further investigations into the results of CryStAl compared to those of the other methods are performed in the next sections using some advanced statistical approaches. Moreover, the convergence curves of the proposed algorithm in dealing with some of the considered mathematical test functions are provided in the Supplementary Materials.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

VI. STATISTICAL ANALYSIS

In this section, the maximum error values of the optimization convergence data have been calculated and utilized for statistical analysis. To this end, the difference between the Global Best (GB) of the functions and the obtained optimal values resulted from the optimization runs are considered as the error values. For statistical analysis purposes, four statistical tests have been conducted in which the Kolmogorov-Smirnov (K-S) test is utilized for normality issues, the Mann-Whitney U (M-W) test is implemented for comparing the summation of the ranks of different metaheuristics in a two-by-two comparing manner, the Kruskal-Wallis (K-W) test is conducted for comparing the overall rankings of the metaheuristics by considering the mean of their rankings, and the Post-Hoc (P-H) analysis is conducted based on the results of the K-W tests for further investigations.

A. Kolmogorov-Smirnov Test

There are two kinds of statistical tests which are applicable to all of the obtained statistical data from multiple applications, known as the parametric and non-parametric statistical tests. One of the most important criteria which demonstrates the possibility of utilizing each method in a specific situation is the Kolmogorov-Smirnov test. This test shows that the distribution of data is either normal or non-normal in which the distribution of each sample among the statistical data are considered and checked accordingly. If the K-S test is rejected, the data are normally distributed, and there is the possibility of using parametric statistical tests for the research. Conversely, if the K-S test is confirmed, the data do not have a normal distribution, so the nonparametric tests should be used in the study.

The results of the K-S test for the error values of the minimum, mean, standard deviation, and maximum function evaluations of the optimization runs for the 2D, 50D, and 100D functions are presented in Table 8. This test is conducted as a two-sample test in which the distributions of the CryStAl data are compared with the data obtained from other metaheuristics. It should be noted that if the Asymptotic Significance (Asymp. Sig.) value is less than 0.05, the presented data are not distributed normally, so the non-parametric statistical tests should be conducted for further investigations. The obtained results of the K-S test demonstrate that the Asymp. Sig. values in most of the investigated cases are less than 0.05, so the non-parametric statistical tests should be utilized for further considerations.

In Table 9, the maximum difference between the statistical data of CryStAl and the other metaheuristics are also presented in order to have an initial judgment about the obtained results of the new algorithm. The maximum and minimum differences of CryStAl with the alternative algorithms are represented by bold font-weight and underlined font, respectively. The bolded values designate those algorithms which have the maximum difference with CryStAl among other metaheuristics, while the underlined values show the algorithms which have the minimum difference with CryStAl among other metaheuristics.

B. Mann-Whitney U Test

The Mann-Whitney U (M-W) test is a non-parametric test that allows two groups of data to be compared in which the null hypothesis denotes that it is equally likely that a randomly-selected value from one sample will be less than or greater than a randomly-selected value from a second sample. This test can be used to investigate whether two independent samples were selected from populations having the same distribution. This test provides the summation of the ranks for two sets of statistical data considered for comparative analysis. As an essential criterion, if the summation of the ranks for one sample has lower values than the other one, the one with a smaller sum of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the M-W test for different mathematical functions based on the obtained results of the optimization runs are presented in Tables 10 to 12. In these tables, the upper and lower values are the summation of the ranks related to the alternative metaheuristics and CryStAl, respectively. Based on the statistical results, the related values of CryStAl for the summation of the ranks in most cases are lower than those of the other metaheuristics (bolded values in the table) which demonstrates the superiority of CryStAl to its competitors in dealing with optimization functions.

C. Kruskal-Wallis Test

The Kruskal-Wallis (K-W) test is a non-parametric method for testing whether or not different statistical samples are originated from the same distribution. It is used for comparing two or more independent samples of equal or different sample sizes. It extends the Mann-Whitney U test, which is used for comparing only two groups. A significant K-W test indicates that at least one sample stochastically dominates another sample. This test provides the mean of the ranks for multiple sets of statistical data which are considered for comparative analysis. As an important criterion, if the mean of the ranks for one sample has lower values than the other ones, the one with a smaller mean of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the K-W test for different studied functions based on the obtained results of the optimization runs have been presented in Tables 13 to 15. Based on the results, the CryStAl related values for the mean of the ranks in most of the cases are lower than the related values for the other metaheuristics which represents the superiority of CryStAl. In these tables, the bolded values are related to the metaheuristic which is superior to the other ones while the values related to CryStAl are all underlined.

D. Post-Hoc Analysis

Post-hoc is a Latin phrase, meaning "after this" or "after the event". In a scientific study, a Post-Hoc (P-H) analysis consists of statistical analyses that were not specified before the data was seen. A P-H analysis involves looking at the data after a study has been concluded, and trying to find patterns that were not the primary objectives of the study.

In this section, the P-H analysis is conducted in order to derive the overall rankings of the metaheuristic algorithms for all of the 2D, 50D, and 100D functions based on the achieved results of the K-W test. The overall rankings of the metaheuristics obtained by the P-H analysis are presented in Table 16. It should be noted that CryStAl provides a success estimation of 100 percent in outranking the other metaheuristics, which demonstrates the superiority of this proposed novel optimization algorithm.

TABLE 8. The K-S test results (Asymp. Sig.) for different algorithms.

Main Algorithm	*Function Type*	*Data Type*	*Alternative Metaheuristic Algorithms*											
			ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*
CryStAl	**2D**	**Min.**	6.92E-07	8.66E-01	8.09E-11	1.79E-21	2.07E-01	3.51E-01	9.97E-01	2.35E-13	1E+00	3.61E-16	4.92E-10	2.72E-01
		Mean	1.30E-04	8.66E-01	1.11E-15	1.15E-16	2.91E-14	5.04E-03	8.19E-02	4.73E-12	4.04E-02	8.35E-14	1.56E-07	7.92E-03
		Std.	2.01E-10	2.07E-01	6.59E-21	5.04E-25	4.80E-22	6.95E-04	1.14E-01	8.43E-20	4.04E-02	1.26E-22	8.35E-14	2.09E-05
		Fun. Evl.	6.47E-09	1.14E-01	9.99E-15	4.80E-22	4.73E-12	8.35E-14	2.07E-01	1.79E-21	2.76E-02	1.00E-18	3.20E-11	1.79E-21
	50D	**Min.**	2.48E-14	1.09E-15	4.17E-17	3.23E-11	1.40E-03	1.12E-13	6.67E-04	4.69E-07	1.05E-02	2.17E-16	1.40E-03	3.06E-04
		Mean	9.37E-06	4.69E-07	1.69E-08	6.67E-04	6.07E-01	3.06E-04	5.76E-05	3.24E-01	7.65E-01	9.37E-06	2.22E-01	3.24E-01
		Std.	3.06E-04	3.06E-04	1.32E-06	1.46E-01	9.99E-01	9.30E-02	6.67E-04	7.65E-01	7.65E-01	1.05E-02	4.54E-01	6.07E-01
		Fun. Evl.	1.40E-03	6.67E-04	1.35E-04	1.35E-04	2.22E-01	1.35E-04	8.99E-01	6.67E-04	7.65E-01	5.76E-05	1.46E-01	3.37E-02
	100D	**Min.**	2.04E-12	4.87E-13	2.48E-14	1.21E-10	5.76E-05	8.27E-12	5.31E-08	1.61E-07	1.40E-03	1.12E-13	4.69E-07	3.06E-04
		Mean	3.58E-06	3.58E-06	1.61E-07	6.67E-04	6.07E-01	1.35E-04	2.37E-05	2.22E-01	3.24E-01	5.76E-05	5.58E-03	8.99E-01
		Std.	3.06E-04	3.06E-04	5.76E-05	3.37E-02	9.99E-01	5.58E-03	1.35E-04	8.99E-01	6.07E-01	2.85E-03	1.05E-02	6.07E-01
		Fun. Evl.	1.92E-02	1.05E-02	6.67E-04	6.67E-04	5.70E-02	6.67E-04	9.30E-02	6.67E-04	3.24E-01	6.67E-04	5.58E-03	5.70E-02

TABLE 9. The K-S test results (the overall difference between data) for different algorithms.

Main Algorithm	*Function Type*	*Data Type*	*Alternative Metaheuristic Algorithms*											
			ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*
CryStAl	**2D**	**Min.**	0.3504	0.0769	0.4444	**0.6325**	0.1368	0.1197	0.0513	0.4957	0.0427	0.5470	0.4274	0.1282
		Mean	0.2821	0.0769	0.5385	**0.5556**	0.5128	0.2222	0.1624	0.4701	0.1795	0.5043	0.3675	0.2137
		Std.	0.4359	0.1368	0.6239	**0.6838**	0.6410	0.2564	0.1538	0.6068	0.1795	0.6496	0.5043	0.3077
		Fun. Evl.	0.4017	0.1538	0.5214	**0.6410**	0.4701	0.5043	0.1368	0.6325	0.1880	0.5897	0.4530	0.6325
	50D	**Min.**	0.7241	0.7586	**0.7931**	0.6379	0.3448	0.7069	0.3621	0.5000	0.2931	0.7759	0.3448	0.3793
		Mean	0.4483	0.5000	**0.5517**	0.3621	0.1379	0.3793	0.4138	0.1724	0.1207	0.4483	0.1897	0.1724
		Std.	0.3793	0.3793	**0.4828**	0.2069	0.0690	0.2241	0.3621	0.1207	0.1207	0.2931	0.1552	0.1379
		Fun. Evl.	0.3448	0.3621	0.3966	0.3966	0.1897	0.3966	0.1034	0.3621	0.1207	**0.4138**	0.2069	0.2586
	100D	**Min.**	0.6724	0.6897	**0.7241**	0.6207	0.4138	0.6552	0.5345	0.5172	0.3448	0.7069	0.5000	0.3793
		Mean	0.4655	0.4655	**0.5172**	0.3621	0.1379	0.3966	0.4310	0.1897	0.1724	0.4138	0.3103	0.1034
		Std.	0.3793	0.3793	**0.4138**	0.2586	0.0690	0.3103	0.3966	0.1034	0.1379	0.3276	0.2931	0.1379
		Fun. Evl.	0.2759	0.2931	0.3621	0.3621	0.2414	0.3621	0.2241	**0.3621**	0.1724	0.3621	0.3103	0.2414

TABLE 10. The M-W test results (summation of the ranks) for 2D mathematical functions.

Main Algorithm	*Function Type*	*Data Type*	*Alternative Metaheuristic Algorithms*											
			ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*
CryStAl	**2D**	**Min.**	16014.00	14047.50	16845.50	18294.50	14779.00	14616.50	14146.50	16896.50	14064.00	17406.50	16912.50	14595.00
			11481.00	**13447.50**	**10649.50**	**9200.50**	**12716.00**	**12878.50**	**13348.50**	**10598.50**	**13431.00**	**10088.50**	**10582.50**	**12900.00**
		Mean	15181.00	14235.50	17494.00	17703.00	17137.00	15344.00	14945.50	16680.00	14752.00	16954.00	16323.00	14893.00
			12314.00	**13259.50**	**10001.00**	**9792.00**	**10358.00**	**12151.00**	**12549.50**	**10815.00**	**12743.00**	**10541.00**	**11172.00**	**12602.00**
		Std.	16016.50	14528.00	17779.00	18090.50	17774.00	15501.00	14778.50	17393.00	14751.00	17583.50	16729.00	15552.50
			11478.50	**12967.00**	**9716.00**	**9404.50**	**9721.00**	**11994.00**	**12716.50**	**10102.00**	**12744.00**	**9911.50**	**10766.00**	**11942.50**
		Fun. Evl.	15657.00	**13306.00**	17180.00	18550.00	16372.00	16576.00	14609.00	18323.00	14689.00	18335.00	17293.00	17633.00
			11838.00	14189.00	**10315.00**	**8945.00**	**11123.00**	**10919.00**	**12886.00**	**9172.00**	**12806.00**	**9160.00**	**10202.00**	**9862.00**

TABLE 11. The M-W test results (summation of the ranks) for 50D mathematical functions.

Main Algorithm	*Function Type*	*Data Type*	*Alternative Metaheuristic Algorithms*											
			ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*
CryStAl	**50D**	**Min.**	4570.00	4632.00	4753.00	4563.00	3995.00	4610.00	3765.00	4308.00	3625.00	4702.00	3870.00	4003.00
			2216.00	**2154.00**	**2033.00**	**2223.00**	**2791.00**	**2176.00**	**3021.00**	**2478.00**	**3161.00**	**2084.00**	**2916.00**	**2783.00**
		Mean	4210.00	4292.50	4468.50	4089.50	3545.00	4060.00	4080.50	3681.00	**3390.50**	4268.00	3710.00	3539.00
			2576.00	**2493.50**	**2317.50**	**2696.50**	**3241.00**	**2726.00**	**2705.50**	**3105.00**	3395.50	**2518.00**	**3076.00**	**3247.00**
		Std.	3943.00	4002.00	4209.00	3795.00	**3384.50**	3760.50	3953.00	3530.50	**3289.00**	3938.00	3562.50	**3387.50**
			2843.00	**2784.00**	**2577.00**	**2991.00**	3401.50	**3025.50**	**2833.00**	**3255.50**	3497.00	**2848.00**	**3223.50**	3398.50
		Fun. Evl.	3937.00	3968.00	4069.00	4063.00	3697.00	4048.00	3536.00	3983.00	**3355.00**	4079.00	3748.00	3816.00
			2849.00	**2818.00**	**2717.00**	**2723.00**	**3089.00**	**2738.00**	**3250.00**	**2803.00**	3431.00	**2707.00**	**3038.00**	**2970.00**

TABLE 12. The M-W test results (summation of the ranks) for 100D mathematical functions.

Main Algorithm	*Function Type*	*Data Type*	*Alternative Metaheuristic Algorithms*											
			ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*
CryStAl	**100D**	**Min.**	4516.00	4555.00	4712.00	4571.00	4113.00	4610.00	4251.00	4381.00	3943.00	4650.00	4303.00	4047.00
			2270.00	**2231.00**	**2074.00**	**2215.00**	**2673.00**	**2176.00**	**2535.00**	**2405.00**	**2843.00**	**2136.00**	**2483.00**	**2739.00**
		Mean	4163.00	4193.00	4360.00	4069.00	3517.00	4145.00	4171.00	3688.00	3587.00	4208.00	3987.00	3469.00
			2623.00	**2593.00**	**2426.00**	**2717.00**	**3269.00**	**2641.00**	**2615.00**	**3098.00**	**3199.00**	**2578.00**	**2799.00**	**3317.00**
		Std.	3889.00	3918.00	4071.00	3756.00	**3347.00**	3865.00	4042.00	3485.00	3461.00	3916.00	3824.00	**3325.00**
			2897.00	**2868.00**	**2715.00**	**3030.00**	3439.00	**2921.00**	**2744.00**	**3301.00**	**3325.00**	**2870.00**	**2962.00**	3461.00
		Fun. Evl.	3825.50	3858.00	3995.00	3995.00	3766.50	3989.00	3739.00	3989.00	3655.50	3997.00	3888.50	3775.50
			2960.50	**2928.00**	**2791.00**	**2791.00**	**3019.50**	**2797.00**	**3047.00**	**2797.00**	**3130.50**	**2789.00**	**2897.50**	**3010.50**

TABLE 13. The K-W test results (mean of the ranks) for 2D mathematical functions.

Ranking	2D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	**CryStAl**	**558.8803**	**CryStAl**	**533.0342**	**CryStAl**	**491.7137**	**CryStAl**	**474.2222**
2	PSO	595.5940	ACO	592.4744	ACO	580.1197	ACO	484.2607
3	ACO	601.7350	SSA	631.2607	SSA	622.9487	PSO	545.1838
4	MFO	608.3205	ABC	649.4615	MFO	629.0043	MFO	555.2906
5	SSA	648.7650	PSO	668.5684	PSO	630.4573	HS	698.1624
6	HS	659.7521	MFO	687.2735	ABC	685.3034	GA	700.3162
7	GA	682.7180	HS	727.0556	HS	701.8889	ABC	768.0342
8	ABC	793.7863	SCA	829.2863	SCA	843.2094	SSA	817.4274
9	MVO	875.8120	MVO	830.9573	MVO	860.1026	SCA	884.8205
10	BA	905.4060	SA	870.3333	SA	899.4744	BA	903.0769
11	SCA	927.8162	GA	897.7094	GA	931.0171	MVO	1000.7521
12	SA	950.4103	FA	983.9444	FA	992.2222	SA	1004.3590
13	FA	1084.0043	BA	991.6410	BA	1025.5385	FA	1057.0940
Chi-sq.	253.8161		168.4093		225.9612		332.7130	
Prob>Chi-sq.	2.1883E-47		1.0096E-29		1.3755E-41		6.1870E-64	

TABLE 14. The K-W test results (mean of the ranks) for 50D mathematical functions.

Ranking	50D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	**CryStAl**	**193.3621**	**CryStAl**	**263.3879**	**PSO**	**290.0776**	**PSO**	**280.4397**
2	PSO	244.8793	PSO	269.4224	CryStAl	307.8707	CryStAl	281.2414
3	MFO	297.6034	GA	296.2586	GA	308.6638	MFO	313.6552
4	SCA	299.5000	SSA	298.9310	SSA	312.3190	GA	351.0603
5	GA	300.1638	MVO	321.7931	MVO	337.1121	SCA	359.1983
6	SSA	309.4655	SCA	332.7845	SCA	348.0776	SSA	377.9138
7	MVO	348.3276	HS	389.2241	HS	380.3190	ABC	404.4828
8	HS	429.8621	FA	413.8190	FA	396.2500	MVO	406.4828
9	FA	449.7500	MFO	423.7672	SA	420.7500	ACO	411.5000
10	SA	489.8362	SA	450.6207	MFO	433.0259	HS	427.7328
11	ABC	490.7586	ABC	456.6638	ABC	434.9483	FA	429.3707
12	ACO	508.1121	ACO	473.9224	ACO	447.7500	BA	429.9569
13	BA	545.8793	BA	516.9052	BA	490.3362	SA	434.4655
Chi-sq.	189.8568		105.8629		61.5589		99.1510	
Prob>Chi-sq.	4.0193E-34		3.9241E-17		1.1716E-08		8.1644E-16	

TABLE 15. The K-W test results (mean of the ranks) for 100D mathematical functions.

Ranking	100D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	**CryStAl**	**171.7069**	**CryStAl**	**259.5517**	**SSA**	**298.1638**	**CryStAl**	**278.2328**
2	PSO	274.6810	SSA	283.4741	GA	299.2241	PSO	340.7586
3	SSA	291.4397	GA	285.7414	CryStAl	305.3793	MFO	358.7500
4	GA	307.5172	PSO	304.1983	PSO	323.3362	GA	364.9741
5	MVO	342.7672	MVO	314.6724	MVO	327.7759	SSA	366.7328
6	SCA	349.0948	SCA	389.4483	FA	387.4310	ABC	375.9569
7	MFO	350.9914	FA	405.6379	SCA	400.7069	ACO	382.9741
8	FA	438.8103	HS	417.7241	HS	403.2586	SCA	389.6810
9	HS	451.3190	MFO	427.6121	SA	413.5862	HS	409.3276
10	SA	466.2414	SA	435.2241	ABC	420.7759	MVO	409.4569
11	ABC	472.2845	ABC	445.9655	ACO	426.6810	BA	410.1379
12	ACO	479.4483	ACO	452.9741	MFO	441.4052	FA	410.1897
13	BA	511.1983	BA	485.2759	BA	459.7759	SA	410.3276
Chi-sq.	149.9211		86.7540		50.0012		65.2723	
Prob>Chi-sq.	5.8829E-26		2.0918E-13		1.3965E-06		2.4278E-09	

TABLE 16. The P-H analysis results for all of the mathematical functions.

Ranking	2D & 50D & 100D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	**CryStAl**	**1011.2554**	**CryStAl**	**1167.9185**	**CryStAl**	**1213.4700**	**CryStAl**	**1077.0343**
2	PSO	1181.8820	SSA	1262.1524	SSA	1271.1373	PSO	1198.1524
3	MFO	1301.1180	PSO	1296.0794	PSO	1304.7704	MFO	1253.2403
4	SSA	1322.7361	GA	1463.3519	ACO	1467.9335	ACO	1306.1953
5	GA	1358.6159	MVO	1492.2876	ABC	1507.1803	GA	1403.3948
6	SCA	1552.6202	ACO	1520.1352	GA	1510.8391	SSA	1506.7189
7	HS	1575.1717	ABC	1523.9678	HS	1516.0665	HS	1507.9700
8	ACO	1577.6159	SCA	1542.6180	MFO	1517.6009	ABC	1555.5579
9	MVO	1591.6459	MFO	1543.5536	MVO	1526.1524	SCA	1632.6674
10	ABC	1679.0107	HS	1553.2382	SCA	1566.9571	BA	1739.8498
11	SA	1818.0408	SA	1697.1931	SA	1679.9893	MVO	1792.3734
12	BA	1847.7639	FA	1759.9850	FA	1738.1974	SA	1834.5622
13	FA	1877.5236	BA	1872.5193	BA	1874.7060	FA	1887.2833
Chi-sq.	270.5922		141.3699		125.5517		334.3474	
Prob>Chi-sq.	6.8410E-51		3.1677E-24		4.8084E-21		2.7999E-64	

VII. CEC 2017 COMPETITION RESULTS

In order to evaluate the overall performance of the proposed algorithm, CryStAl, it is necessary to consider state-of-the-art mathematical test functions alongside state-of-the-art algorithms. To this end, a recent competition on single-objective real-parameter numerical optimization named "CEC 2017" [86] is considered in this section. In this regard, a list of 30 mathematical functions are studied and presented in Table 17; the mathematical details of these functions have been presented by the CEC 2017 competition committee [86].

TABLE 17. Summary of the CEC 2017 test functions [46].

Function type	*Func. No.*	*Function details*	*Func. Min.*
Unimodal functions	G_1	*Shifted and Rotated Bent Cigar Function*	100
	G_2	*Shifted and Rotated Sum of Different Power Function*	200
	G_3	*Shifted and Rotated Zakharov Function*	300
Simple multimodal functions	G_4	*Shifted and Rotated Rosenbrock's Function*	400
	G_5	*Shifted and Rotated Rastrigin's Function*	500
	G_6	*Shifted and Rotated Expanded Schaffer's F6 Function*	600
	G_7	*Shifted and Rotated Lunacek Bi_Rastrigin Function*	700
	G_8	*Shifted and Rotated Non-Continuous Rastrigin's Function*	800
	G_9	*Shifted and Rotated Levy Function*	900
	G_{10}	*Shifted and Rotated Schwefel's Function*	1000
Hybrid functions	G_{11}	*Hybrid Function* 1 ($N = 3$)	1100
	G_{12}	*Hybrid Function* 2 ($N = 3$)	1200
	G_{13}	*Hybrid Function* 3 ($N = 3$)	1300
	G_{14}	*Hybrid Function* 4 ($N = 4$)	1400
	G_{15}	*Hybrid Function* 5 ($N = 4$)	1500
	G_{16}	*Hybrid Function* 6 ($N = 4$)	1600
	G_{17}	*Hybrid Function* 6 ($N = 5$)	1700
	G_{18}	*Hybrid Function* 6 ($N = 5$)	1800
	G_{19}	*Hybrid Function* 6 ($N = 5$)	1900
	G_{20}	*Hybrid Function* 6 ($N = 6$)	2000
Composition functions	G_{21}	*Composition Function* 1 ($N = 3$)	2100
	G_{22}	*Composition Function* 2 ($N = 3$)	2200
	G_{23}	*Composition Function* 3 ($N = 4$)	2300
	G_{24}	*Composition Function* 4 ($N = 4$)	2400
	G_{25}	*Composition Function* 5 ($N = 5$)	2500
	G_{26}	*Composition Function* 6 ($N = 5$)	2600
	G_{27}	*Composition Function* 7 ($N = 6$)	2700
	G_{28}	*Composition Function* 8 ($N = 6$)	2800
	G_{29}	*Composition Function* 9 ($N = 3$)	2900
	G_{30}	*Composition Function* 10 ($N = 3$)	3000
Search range: $[-100,100]^D$			

The statistical results of the CryStAl algorithm in dealing with these test functions (CEC 2017) with 10 dimensions are presented in the Supplementary Materials where the results of three other successful algorithms are also presented. It should be noted that the error values, rather than the global best values, of each run are considered in this competition and the statistical results are based on the best error values of 51 independent runs. The results show that the proposed CryStAl algorithm is capable of providing eminently acceptable results in dealing with these test functions of different dimensions.

VIII. COMPUTATIONAL COST AND COMPLEXITY ANALYSIS

In this section, the computational cost and complexity of the proposed CryStAl method are examined and analyzed where three different approaches are considered to acquire a better understanding of these properties. In the first approach, the computational cost procedure of the CEC 2017 benchmark suite is determined while the results of three other state-of-the-art algorithms are also considered to form a fair judgment. In the CEC 2017 computational scenario, four different computational times, namely T_0, T_1, T_2 and $\widehat{T}_2$, are considered based on four specific mathematical procedures. T_0 refers to the running time of a predefined mathematical procedure [46], T_1 denotes the computational time for evaluation of the G_{18} test function considering 200000 function evaluations, T_2 represents the computational time of the considered metaheuristic algorithm (CryStAl in this paper) for evaluation of the G_{18} test function considering 200000 function evaluations, and $\widehat{T}_2$ refers to the mean values of five different assessments of T_2. The results of this scenario for the proposed and alternative algorithms are presented in Table 18 which demonstrates the capability of the proposed CryStAl algorithm in producing competitive results.

TABLE 18. Computational complexity results of CryStAl compared to other approaches.

Metaheuristics	*Properties*	*Results (sec)*
EBO with CMAR [47]	T_0	0.0413
	T_1	0.8218
	$\widehat{T}_2$	7.5794
	$(\widehat{T}_2 - T_1)/T_0$	163.6223
LSHADE-cnEpSin [48]	T_0	0.1093
	T_1	0.8391
	$\widehat{T}_2$	2.1835
	$(\widehat{T}_2 - T_1)/T_0$	12.30009
MM-OED [49]	T_0	2.157784
	T_1	0.146416
	$\widehat{T}_2$	6.704923
	$(\widehat{T}_2 - T_1)/T_0$	3.039417
CryStAl (the present study)	T_0	0.027387
	T_1	0.144345
	$\widehat{T}_2$	5.378017
	$(\widehat{T}_2 - T_1)/T_0$	191.10059

In computer science, "Big O notation" is a mathematical notation that determines the required running time and memory space of an algorithm by considering its growth rate in dealing with different inputs. In the following, the computational cost of the proposed CryStAl method is presented using this notation which is the second approach for testing the complexity of the proposed algorithm. For CryStAl, the random selection process in the initialization phase of the algorithm has a computational complexity of $O(NP\times D)$ where NP is the initial population size and D is the dimension of the problem. The computational complexity of the objective function evaluation in the initialization phase of the algorithm is calculated as $O(NP)\times O(F(x))$ where $F(x)$ demonstrates the objective function value. After the initialization phase, the main loop of the algorithm is started based on the previously determined maximum number of iterations (MaxIter). By the consideration of the worst-case scenario, each line has a computational complexity of MaxIter in the main loop of the algorithm. In this loop, four new position vectors are created for each of the current vectors so the position updating process of the problem will have a computational complexity of $O(\text{MaxIter}\times NP\times D\times 4)$. In addition, the objective function evaluation in the main loop has a computational complexity of $O(\text{MaxIter}\times NP\times 4)\times O(F(x))$.

In general, the overall capacity of a metaheuristic algorithm depends on the balance between exploration and exploitation while the convergence speed is also an important factor in its evaluation. In order to demonstrate these properties for the proposed CryStAl algorithm, as the third complexity approach, the diversity graphs of CryStAl are plotted for functions F_1, F_{61}, and F_{83} in the Supplementary Materials. As can be seen from these results, the population in the optimization process by CryStAl tends to localize the search for achieving better results.

IX. REAL-WORLD OPTIMIZATION PROBLEMS

In this section, the applicability of the proposed algorithm, CryStAl, is investigated by considering some real-world optimization problems which can be a great challenge for the proposed method. In this regard, we have considered six difficult power electronics problems on synchronous optimal pulse-width modulation (SOPWM) which is used to regulate medium-voltage (MV) drives. This approach provides a significant decrease of switching frequency without raising the distortion, which leads to the reduction of switching losses that enhances the performance of the inverter. Generally, switching angles are calculated by reducing the distortion of current. In this study, this problem is considered as a constrained optimization problem which is benchmarked by CEC 2020 [90] regarding real-world constrained optimization. In this paper, six configurations of this problem are determined and solved by the proposed CryStAl with a simple penalty approach for constrained handling purposes. A brief explanation of these problems is presented in Table 19 while the comparative results are provided in the Supplementary Materials. The findings of this study demonstrated that the proposed method is capable of producing eminently acceptable and even better results in dealing with these challenging problems.

Based on the presented results in this and previous sections, it can be concluded that the proposed algorithm produces excellent results in most of the considered cases. One of the key aspects of this study is the conducted statistical analysis to evaluate the capability of this algorithm in dealing with an extensive set of test problems. The employed benchmark test problems of CEC and the competitive results of CryStAl in dealing with these problems demonstrate that this algorithm can be considered as a successful metaheuristic approach.

TABLE 19. Description of the investigated real-world design problems.

No. (*CEC No.*)	*Name*	*D*	*g*	*h*
M_1 (RC 45)	*SOPWM for 3-level Inverters*	25	24	1
M_2 (RC 46)	*SOPWM for 5-level Inverters*	25	24	1
M_3 (RC 47)	*SOPWM for 7-level Inverters*	25	24	1
M_4 (RC 48)	*SOPWM for 9-level Inverters*	30	29	1
M_5 (RC 49)	*SOPWM for 11-level Inverters*	30	29	1
M_5 (RC 50)	*SOPWM for 13-level Inverters*	30	29	1

X. CONCLUSIONS

This paper proposed a novel metaheuristic method called Crystal Structure Algorithm (CryStAl), inspired by the underlying principles of the formation of crystal structures from the addition of the basis to the lattice points. Four groups of mathematical test functions were selected in order to efficiently evaluate the performance of CryStAl with a total number of 12 different metaheuristic algorithms. A complete statistical analysis was conducted to provide a valid judgment about the performance of this method. The most important findings of this paper are as follows:

(i) CryStAl is superior to the other metaheuristics in converging to the global bests of the mathematical functions based on the selected tolerance.

(ii) The results of the K-S test demonstrated that the maximum difference between CryStAl and the other metaheuristics is about FA and BA in most of the cases.

(iii) The results of the M-W test showed that the summation of the ranks for CryStAl in most of the cases is lower than those of the other metaheuristics.

(iv) The results of the K-W test manifested that CryStAl is 100% successful in outranking the other metaheuristics for the 2D functions in all of the cases such as the minimum, mean, and standard deviation values alongside the number of function evaluations.

(v) The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values of the 50D test functions while the PSO outranks CryStAl in the standard deviation and function evaluation.

(vi) The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values alongside the number of function evaluations of the 100D test functions while the SSA and GA outrank CryStAl in the standard deviation values.

(vii) The overall comparison of CryStAl and the alternative metaheuristics considering all of the 2D, 50D, and 100D test functions demonstrated that CryStAl is 100 percent successful in outranking the other metaheuristics in all of the cases.

As future challenges, different applications of CryStAl can be explored and its capabilities in dealing with difficult test problems can be examined. Besides, new configurations of this algorithm can be considered as other researchers may have different viewpoints on the presented methodology.

ACKNOWLEDGMENTS

This research was supported by a research grant from the University of Tabriz (Number: 1615). Funding for Open Access was provided by the University of Liverpool.

CODE AVAILABILITY

The MATLAB implementation of CryStAl is accessible at: https://www.mathworks.com/matlabcentral/fileexchange/91850-crystal-structure-algorithm-crystal

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

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XI. SUPPLEMENTARY MATERIALS

The detailed outputs of 100 optimization runs for different algorithms in dealing with different mathematical functions are presented, here. Tables S1-S4 present the (*i*) minimum values, (*ii*) mean values, (*iii*) standard deviations, and (*iv*) mean values of the maximum number of function evaluations, respectively, of the optimization runs for CryStAl and the alternative metaheuristics in dealing with the 2D mathematical functions. Similarly, Tables S5-S8 present the abovementioned outputs (i.e., *i*, *ii*, *iii,* and *iv*) associated with the 50D mathematical functions; and Tables S9-S12 present these outputs associated with the 100D mathematical functions. For the composite mathematical functions, the mean values and standard deviations of the optimization runs are shown in Tables S13 and S14, respectively.

TABLE S1. The minimum values of the 2D mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_1	-200	-200	-199.999542	-199.9902	-200	-200	-200	-199.999918	-200	-199.99977	-200	-200	-200
F_2	-195.629	-195.629	-195.629	-195.628953	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629
F_3	-2.02181	-2.02181	-2.02181	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678	-2.02180678
F_4	1.00000025	1	1.00048649	1.52589967	1	1	1	1.00001485	1	1.00001161	1	1	1
F_5	1.62E-10	0	5.27E-10	7.68E-09	0	0	0	8.72E-11	0	1.53E-10	3.76E-07	0	0
F_6	6.46E-10	0	2.72E-09	6.55E-08	0	0	0	2.20E-11	0	2.45E-11	2.00E-06	0	0
F_7	0	0	1.87E-09	1.05E-09	0	0	0	1.09E-10	0	2.40E-12	1.06E-08	0	0
F_8	-106.764537	-106.764537	-106.764537	-106.764536	-106.764537	-106.764537	-106.764537	-106.764537	-106.764537	-106.764537	-106.764529	-106.764537	-106.764537
F_9	1.28E-10	0	1.57E-05	0.00145075	0	0	0	3.06E-08	0	3.91E-08	0	0	0
F_{10}	7.49E-12	0	4.01E-05	0.00408756	0	0	0	1.31E-07	0	1.60E-07	0	0	0
F_{11}	8.10E-10	0	9.26E-06	0.00294394	0	0	0	1.07E-08	0	1.46E-06	0	0	0
F_{12}	1.42E-12	0	2.37E-09	5.01E-09	0	0	0	2.41E-11	0	5.03E-09	1.36E-06	0	0
F_{13}	0.39788736	0.39788736	0.39788738	0.39788736	0.39788736	0.39788736	0.39788736	0.39788736	0.39788736	0.39788736	0.39789071	0.39788736	0.39788736
F_{14}	5.559037	5.56323577	5.559037	5.55898736	5.559037	5.559037	5.559037	5.559037	5.559037	5.559037	5.559037	5.559037	5.559037
F_{15}	0	0	6.31E-10	0	0	0	0	0	0	0	0	0	0
F_{16}	1.01E-11	0	4.00E-06	2.27E-07	0	0	0	3.15E-08	0	2.63E-08	6.54E-08	0	0
F_{17}	0.04191438	0.01908949	0.02996532	0.03570534	0.0003309	0.00017205	0.00042588	0.01698081	0.00011688	0.031259	0.06017514	0.00072075	0.00010196
F_{18}	0	0	4.15E-11	5.14E-09	0	0	0	3.20E-11	0	1.05E-11	0	0	0
F_{19}	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
F_{20}	-24.1568155	-24.1568155	-24.1568155	-24.1568153	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568047	-24.1568155	-24.1568155
F_{21}	-1001.63819	-999.429109	-999.998329	-1000	-1000	-1000.00858	-1000	-1000	-2000.004	-1000	-1000	-1000	-2000.004
F_{22}	-999.98605	-999.988219	-999.998206	-999.979757	-1000	-1000.00001	-1000	-999.999043	-1000.00707	-999.982027	-1014.92465	-1000.00013	-2000
F_{23}	-42.9443866	-42.944387	-42.9443841	-42.9443486	-42.944387	-42.944387	-42.944387	-42.944387	-42.944387	-42.944387	-42.944385	-42.944387	-42.944387
F_{24}	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261187	-2.06261183	-2.06261187	-2.06261187
F_{25}	1.33E-07	0	1.66E-06	1.57E-07	1.57E-05	2.39E-08	0	5.38E-10	0	2.94E-07	2.74E-08	0	0
F_{26}	0.0485563	2	5.71E-07	0.00101176	2	0	2	9.70E-07	0	2.63E-06	0.00026857	0	0
F_{27}	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938	-24771.0938
F_{28}	-0.99768942	-1	-0.99999989	-0.99759744	-1	-1	-1	-1	-1	-0.99999999	-0.99999178	-1	-1
F_{29}	1.7128	1.7128	1.71485298	16.694644	1.7128	1.7128	1.7128	1.71279045	1.7128	1.7128	1.71296286	1.7128	1.7128
F_{30}	0	0	6.85E-09	1.34E-07	0	0	0	1.14E-10	0	7.31E-11	0	0	0
F_{31}	0	0	7.89E-10	2.36E-08	0	0	0	3.07E-12	0	2.13E-12	8.11E-08	0	0
F_{32}	2.59E-08	0	1.66E-06	1.47E-06	0	0	0	3.78E-09	0	1.56E-09	2.24E-05	0	0
F_{33}	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447042	0.06447044	0.06447042	0.06447042
F_{34}	3	3	3.00000005	3	3	3	3	3	3	3	3	3	3
F_{35}	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532	-165.9532

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F_{36}	4.26E-10	0	5.87E-09	3.61E-08	0	0	0	8.64E-10	0	5.33E-10	3.35E-05	0	0
F_{37}	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458
F_{38}	124.362182	124.362182	124.3622	124.362182	124.362182	124.362182	124.362182	124.362182	124.362182	124.362182	124.362223	124.362182	124.362182
F_{39}	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752	-0.67366752
F_{40}	1.02E-06	0	6.13E-09	1.63E-10	1.26E-09	4.25E-10	0	8.34E-12	0	2.09E-09	4.89E-07	0	0
F_{41}	-176.540166	-176.541001	-176.541793	-176.541712	-176.541793	-176.541793	-176.541793	-176.541793	-176.541793	-176.541793	-176.541485	-176.541793	-176.541793
F_{42}	-176.1375	-176.1375	-176.1375	-176.137367	-176.1375	-176.1375	-176.1375	-176.137569	-176.1375	-176.137567	-176.137212	-176.1375	-176.1375
F_{43}	1.72E-11	0	4.25E-09	8.29E-09	0	0	0	3.51E-12	0	1.15E-09	0	0	0
F_{44}	-1.9133	-1.9133	-1.9133	-1.91322295	-1.91322295	-1.91322295	-1.91322295	-1.91322295	-1.91322295	-1.91322295	-1.91322289	-1.91322295	-1.91322295
F_{45}	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683
F_{46}	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013
F_{47}	-0.18465	-0.18465	-0.18465	-0.18424729	-0.18465	-0.18465	-0.18465	-0.18438254	-0.184621	-0.18425697	-0.1830757	-0.18465	-0.18465
F_{48}	-0.199407	-0.199407	-0.199407	-0.19912339	-0.199407	-0.19722262	-0.199407	-0.19916811	-0.199407	-0.19897075	-0.19878456	-0.19939706	-0.199407
F_{49}	-1.01983	-1.01983	-1.01983	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952	-1.01982952
F_{50}	-2.28394984	-2.28394984	-2.28394984	-2.28394973	-2.28394984	-2.28394984	-2.28394984	-2.28394984	-2.28394984	-2.28394984	-2.28394921	-2.28394984	-2.28394984
F_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{52}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{53}	1.14E-07	9.74E-08	1.01E-10	6.89E-09	0	0	0	3.04E-11	0	2.84E-12	0	0	0
F_{54}	-0.96354	-0.96354	-0.96354	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483	-0.96353483
F_{55}	0.90000304	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
F_{56}	9.27E-09	0	6.58E-07	0.00683902	0	0	0	9.73E-08	0	4.21E-08	4.64E-07	0	0
F_{57}	0.90000304	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
F_{58}	2.75E-07	0	3.08E-05	0.65478977	0	0	0	2.39E-06	0	0.00013503	1.45E-07	0	0
F_{59}	1.05E-12	0	2.99E-07	0.00088534	0	0	0	1.01E-11	0	2.21E-09	0	0	0
F_{60}	-3873.72418	-3873.72418	-3873.72418	-3873.72417	-3873.72418	-3873.72418	-3873.72418	-3873.72418	-3873.72418	-3873.72418	-3873.72418	-3873.72418	-3873.72418
F_{61}	-2.19998648	-2.2	-2.19970474	-2.19999998	-2.2	-2.2	-2.2	-2.19999995	-2.2	-2.19999997	-2.19980986	-2.2	-2.2
F_{62}	-2	-2	-1.99999999	-2	-2	-2	-2	-2	-2	-2	-1.99999999	-2	-2
F_{63}	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712	34.3712
F_{64}	4.49E-09	0	6.86E-05	5.30688414	0	0	0	4.89E-07	0	4.00E-06	0	0	0
F_{65}	1.69E-10	0	2.18E-06	0.08010113	0	0	0	2.73E-08	0	5.21E-10	0	0	0
F_{66}	3.44E-09	0	0.00011014	0.54931796	0	0	0	2.62E-07	0	4.71E-07	0	6.76E-12	0
F_{67}	0	0	0	4.86E-11	0	0	0	0	0	0	0	0	0
F_{68}	0	0	0	8.72E-09	0	0	0	0	0	0	0	0	0
F_{69}	0.00156696	0.00156685	0.00156789	0.00158439	0.00156685	0.00156685	0.00156685	0.00156686	0.00156685	0.00156686	0.00156685	0.00156685	0.00156685
F_{70}	0.292579	0.292579	0.292579	0.29249201	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579
F_{71}	6.79E-05	0	0.00172823	0.00407868	0	0	0	0.00014271	0	0.00104623	0.000318	1.43E-08	0
F_{72}	-3456	-3456	-3456	-3455.47539	-3456	-3456	-3456	-3456	-3456	-3455.99999	-3455.99934	-3456	-3456
F_{73}	-26.920336	-26.920336	-26.920336	-26.9203354	-26.9203356	-26.9203356	-26.9203356	-26.9203356	-26.9203356	-26.9203356	-26.9203295	-26.9203356	-26.9203356
F_{74}	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2084623	-19.2085	-19.2085
F_{75}	-24.1568155	-24.1568155	-24.1568155	-24.1568153	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568155	-24.1568047	-24.1568155	-24.1568155
F_{76}	-10.8722962	-10.8723	-10.8723	-10.8722996	-10.8723	-10.8723	-10.8723	-10.8723001	-10.8723	-10.8723001	-10.8723	-10.8723	-10.8723
F_{77}	0	0	1.41E-08	1.80E-08	0	0	0	2.27E-11	0	8.97E-12	0	0	0
F_{78}	-3.30680638	-3.30686865	-3.3063708	-3.30678858	-3.30686865	-3.30686865	-3.30686865	-3.30686845	-3.30686865	-3.30686851	-3.3068559	-3.30686865	-3.30686865
F_{79}	0.00030939	0	0.00028705	0.02441967	0	0	0	1.00E-04	0	4.34E-05	0.00012689	4.78E-09	0
F_{80}	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406	-4.81681406
F_{81}	-2.5	-2.5	-2.5	-2.49998689	-2.5	-2.5	-2.5	-2.49999836	-2.5	-2.49999922	-2.5	-2.5	-2.5
F_{82}	-1.5	-1.5	-1.5	-1.49999235	-1.5	-1.5	-1.5	-1.49999981	-1.5	-1.49999807	-1.5	-1.5	-1.5
F_{83}	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30699873	-7.30697316	-7.30699873	-7.30699873
F_{84}	-400	-400	-400	-399.99998	-400	-400	-400	-400	-400	-400	-400	-400	-400
F_{85}	1.66E-06	0	0.00684184	3.22391612	0	0	0	1.53E-05	0	5.88E-05	2.63E-06	0	0
F_{86}	5.85E-09	0	2.31E-05	18.4887501	0	0	0	9.56E-07	0	1.80E-06	2.99E-09	0	0
F_{87}	21.35	21.35	21.35	21.6385138	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35

F_{88}	-0.003791	-0.003791	-0.003791	-0.00379123	-0.003791	-0.003791	-0.003791	-0.003791	-0.003791	-0.003791	-0.003791	-0.003791	-0.003791
F_{89}	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523
F_{90}	2.20E-11	0	6.72E-05	0.10612488	0	0	0	4.04E-07	0	2.47E-08	0	0	0
F_{91}	3.18E-08	0	1.34E-06	8.22E-07	6.35E-10	0	0	6.67E-10	0	5.47E-09	1.24E-05	0	0
F_{92}	0	1.58E-07	0	2.10E-05	1.15E-07	1.52E-07	0	4.41E-10	0	3.01E-06	5.91E-05	0	0
F_{93}	-3.86277979	-3.86277979	-3.86277787	-3.86277979	-3.86277979	-3.86277979	-3.86277979	-3.86277979	-3.86277979	-3.8627797	-3.86276606	-3.86277979	-3.86277979
F_{94}	8.97E-08	0	2.04E-08	1.26E-07	0	0	0	4.85E-10	0	9.93E-09	3.96E-10	0	0
F_{95}	4.36E-05	4.36E-05	0.01367838	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592	0.02453592
F_{96}	3.98E-12	0	4.65E-08	1.02E-11	0	0	0	0	0	0	1.55E-11	0	0
F_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{98}	0.00771286	0	0.00086166	8.45E-06	3.65E-06	1.49E-06	0	4.11E-09	0	5.73E-06	0.00113997	3.07E-12	0
F_{99}	0.00290505	0	0.35667569	0.00081715	0.00355628	3.79E-05	4.23E-11	1.54E-06	0	0.00035321	0.01745587	0	0
F_{100}	0	0	24.9246777	383413.322	0	0	0	0	0	0	0	0	0
F_{101}	0	0	0	735.44057	7.40247274	1.32E-06	0	1.81042873	0	1.11616512	47.9087956	155.684788	0
F_{102}	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12
F_{103}	0.00080613	0.00090043	0.00052484	0.00034588	0.00032396	0.00055839	0.00030784	0.00030784	0.00030784	0.00036295	0.00031517	0.00030784	0.00030784
F_{104}	1.52E-10	0	1.42E-12	0	0	2.57E-09	0	0	0	0	0	0	0
F_{105}	-10.1531997	-10.1531997	-10.1520702	-10.1524423	-10.1531997	-10.1531997	-10.1531997	-10.1531991	-10.1531997	-10.1455371	-9.44429044	-10.1531997	-10.1531997
F_{106}	-10.4029	-10.4029	-10.4029	-10.4027552	-10.4029	-10.4029	-10.4029	-10.4029374	-10.4029	-10.3992164	-9.86524684	-10.4029	-10.4029
F_{107}	-10.5364	-10.5364	-10.5308349	-10.5357467	-10.5364	-10.5364	-10.5364	-10.5364078	-10.5364	-10.5293091	-9.80039745	-10.5364	-10.5364
F_{108}	0.00637568	0.00264999	0.00049407	3.29E-05	0.0001421	0.00011144	4.33E-05	1.12E-06	2.37E-06	9.77E-05	0.00216789	5.04E-06	0
F_{109}	0	0	0	12.5022596	0.80545718	0.09224651	0.00328598	1.1302493	0.02501146	0.41369325	16.2876424	3.24049383	0
F_{110}	-529.8714	-529.8714	-529.8714	-529.484937	-529.870857	-529.8714	-529.8714	-529.8714	-529.8714	-529.852125	-529.738453	-529.8714	-529.8714
F_{111}	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999	-0.9649999
F_{112}	0.00980343	0.00565566	0.00730794	0.00366127	0.00228053	0.0013744	1.84E-05	2.79E-05	2.79E-06	0.00305842	0.00422035	4.50E-07	0
F_{113}	-3.32236	-3.32236	-3.32199185	-3.32236	-3.32236	-3.32236	-3.32236	-3.32236	-3.32236	-3.32208876	-3.26058965	-3.32236	-3.32236
F_{114}	-49.9999999	-50	-49.9451732	-49.771456	-49.9999226	-50	-50	-49.999999	-50	-49.8691333	-49.3201588	-50	-50
F_{115}	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979	0.95979
F_{116}	-45.778	-0.07016289	-38.3053001	-45.7759857	-45.778	-45.778	-45.778	-45.778	-45.778	-45.66838	-39.4532271	-45.778	-45.778
F_{117}	-209.989509	-210	-134.058401	-36.1946776	-209.940989	-209.992732	-210	-209.99994	-210	-195.394318	-167.587261	-210	-210

TABLE S2. The mean values of the 2D mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*												
	ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*	*CryStAl*
F_{1}	-200	-200	-199.91	-199.923	-200	-200	-200	-199.999	-200	-199.998	-200	-200	-200
F_{2}	-195.629	-195.629	-195.62	-195.627	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629	-195.629
F_{3}	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181	-2.02181
F_{4}	1.000003	1	1.574072	30.37011	1.000066	1	1	1.000563	1	1.002087	1	1	1
F_{5}	6.79E-08	0	0.000106	3.87E-07	0.00764	0.000105	0	0.099069	0.060966	3.67E-06	2.42E-05	0	0
F_{6}	5.27E-07	0	2.27E-05	5.35E-06	2.00E-10	0	0	4.11E-09	0	9.04E-09	0.000206	0	0
F_{7}	9.07E-12	0	3.46E-05	1.05E-06	1.15E-09	0	0	4.31E-09	0	4.29E-09	1.75E-05	0	0
F_{8}	-106.765	-106.765	-106.763	-106.764	-106.765	-106.765	-106.765	-104.625	-106.375	-106.765	-106.755	-106.765	-106.765
F_{9}	5.27E-09	0	0.215317	0.416159	1.78E-08	0	0	9.84E-06	0	2.00E-05	0	9.87E-13	0
F_{10}	2.63E-08	0	0.14211	0.285438	0.010916	0.017465	0	8.47E-06	0	0.010918	0	9.98E-13	0
F_{11}	3.42E-07	0	0.082213	0.224658	0.001024	0	0	3.21E-06	0	0.002657	0	2.29E-13	0
F_{12}	6.52E-10	0	0.000678	1.29E-05	1.36E-06	0	0	1.53E-08	0	5.15E-07	0.000111	0	0
F_{13}	0.397888	0.397887	0.397951	0.397892	0.397888	0.397887	0.397887	0.397887	0.397887	0.397888	0.398102	0.397887	0.397887
F_{14}	5.564912	6.706896	5.561705	5.634131	6.473536	5.610723	6.106183	6.426086	6.618413	5.947018	5.723289	5.571553	6.530843
F_{15}	7.49E-12	0	0.00042	0	0	0	0	0	0	0	0	0	0

***F*₁₆**	2.94E-10	0	0.001084	1.43E-05	5.54E-09	0	0	1.06E-06	0	6.00E-07	1.53E-06	2.48E-11	0
***F*₁₇**	0.173053	0.178046	0.172048	0.186095	0.044596	0.018926	0.035232	0.09046	0.0176	0.228093	0.274983	0.025628	0.011866
***F*₁₈**	6.49E-13	0	2.76E-05	4.19E-07	9.91E-10	0	0	1.36E-09	0	5.82E-09	0	0	0
***F*₁₉**	-1.0316	-1.0316	-1.03153	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
***F*₂₀**	-24.1568	-24.1568	-24.1568	-24.1567	-24.1568	-24.1568	-24.1568	-24.1568	-24.1568	-24.1568	-24.1503	-24.1568	-24.1568
***F*₂₁**	-695.72	-605.028	-844.147	-1000	-994.452	-1000	-1000	-997.962	-1010	-991.486	-1000	-1000	-2000
***F*₂₂**	-732.249	-740.56	-816.092	-91.357	-971.375	-1000	-1000	-986.052	-1000	-797.894	-674.479	-1000	-2000
***F*₂₃**	-42.9231	-42.9398	-42.856	-42.9406	-42.9444	-42.9444	-42.9444	-42.9265	-42.8505	-42.9444	-42.9441	-42.9131	-42.9444
***F*₂₄**	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261	-2.06261
***F*₂₅**	9.24E-05	0.002826	0.007084	4.73E-05	0.136287	0.072765	0.002105	1.76E-07	0	0.00033	0.000134	0	0
***F*₂₆**	1.681503	2	1.407737	1.876688	2	1.880148	2	1.660001	1.98	1.978996	0.170617	0.94	1.98
***F*₂₇**	-24771.1	-24771.1	-24764.9	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1	-24771.1
***F*₂₈**	-0.10623	-1	-0.97808	-0.74896	-0.95	-1	-1	-0.98	-1	-1	-0.99977	-1	-1
***F*₂₉**	1.712806	1.7128	21.04069	9813.009	1.788259	1.7128	1.7128	1.714105	1.7128	1.73127	1.723573	1.7128	2.218053
***F*₃₀**	4.33E-10	0	0.000375	8.29E-06	1.32E-08	0	0	2.26E-08	0	5.32E-08	0	0	0
***F*₃₁**	3.87E-11	0	3.13E-05	1.02E-06	1.88E-09	0	0	3.62E-09	0	5.39E-09	1.70E-05	0	0
***F*₃₂**	7.22E-06	8.281526	0.033996	0.000271	5.23E-06	0	1.077181	5.65E-07	2.987749	5.80E-06	0.010323	0	0.497958
***F*₃₃**	0.06447	0.06447	0.064472	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.064473	0.06447	0.06447
***F*₃₄**	3	3	3.001124	3.000002	3	3	3	3	3	3	3.000001	3	3
***F*₃₅**	-165.901	-165.908	-165.936	-165.953	-162.753	-164.673	-164.246	-165.953	-165.953	-165.953	-165.923	-165.953	-165.953
***F*₃₆**	5.16E-07	0	0.000305	7.81E-06	4.48E-08	0	0	3.91E-08	0	8.65E-08	0.001648	0	0
***F*₃₇**	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.3458	-2.34576	-2.3458	-2.3458
***F*₃₈**	124.3622	124.3622	124.3836	124.3622	124.3679	124.3622	124.3622	124.3622	124.3622	124.3635	124.3709	124.3622	124.3622
***F*₃₉**	-0.67367	-0.67367	-0.67364	-0.67367	-0.67367	-0.67367	-0.67367	-0.67367	-0.5518	-0.67367	-0.67367	-0.67367	-0.67367
***F*₄₀**	9.10E-05	0.000816	0.000108	9.51E-09	0.030091	0.00439	0.000851	1.20E-09	0	5.16E-06	5.55E-05	0	0
***F*₄₁**	-176.445	-176.448	-176.513	-176.53	-174.367	-176.23	-175.921	-176.542	-176.542	-176.542	-176.476	-176.542	-176.542
***F*₄₂**	-176.138	-176.138	-174.863	-176.132	-173.274	-169.774	-174.549	-166.538	-170.137	-175.821	-176.103	-176.138	-176.138
***F*₄₃**	1.40E-09	0	2.36E-05	4.30E-07	1.87E-06	0	0	5.01E-10	0	4.26E-07	0	0	0
***F*₄₄**	-1.9133	-1.91329	-1.9133	-1.91322	-1.91322	-1.91322	-1.91322	-1.91322	-1.91322	-1.91322	-1.91321	-1.91322	-1.91322
***F*₄₅**	-19.6683	-19.6683	-19.6683	-19.6683	-19.4973	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683	-19.6683
***F*₄₆**	-1.8013	-1.8013	-1.80127	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.8013	-1.7526	-1.8013	-1.8013
***F*₄₇**	-0.18465	-0.18465	-0.18465	-0.18119	-0.13471	-0.13202	-0.16046	-0.14622	-0.09802	-0.17434	-0.17323	-0.17962	-0.18408
***F*₄₈**	-0.19941	-0.19941	-0.19941	-0.19628	-0.16725	-0.14778	-0.19254	-0.16752	-0.11332	-0.19157	-0.18803	-0.19358	-0.19941
***F*₄₉**	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983	-1.01983
***F*₅₀**	-2.28395	-2.28395	-2.28367	-2.28394	-2.25877	-2.27974	-2.28395	-2.28395	-2.28395	-2.28395	-2.28377	-2.28395	-2.28395
***F*₅₁**	0	0	0	0	0	0	0	0	0	0	0	0	0
***F*₅₂**	1.84E-10	7.33E-10	2.00E-09	8.71E-10	4.57E-11	0	0	4.09E-11	0	1.65E-10	1.19E-09	0	0
***F*₅₃**	1.34E-05	2.13E-05	2.45E-06	3.77E-07	1.03E-10	0	0	9.60E-10	0	2.08E-09	3.71E-06	0	0
***F*₅₄**	-0.96354	-0.96354	-0.96354	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353	-0.96353
***F*₅₅**	0.902001	0.962548	0.901658	0.900005	0.933	0.902043	0.944	0.946	0.916	0.906469	0.9	0.9	0.9
***F*₅₆**	3.82E-06	0	0.04855	7.265038	1.04E-07	0	0	1.00E-05	0	5.80E-06	0.000322	9.24E-13	0
***F*₅₇**	0.902001	0.962548	0.901658	0.900005	0.933	0.902043	0.944	0.946	0.916	0.906469	0.9	0.9	0.9
***F*₅₈**	6.00E-05	0.002329	53.58117	75037.07	0.016974	0.003093	0.003485	0.003609	0.003634	0.055932	0.000122	0.003263	0.000331
***F*₅₉**	1.08E-09	4.83E-10	3.217993	47660.31	0.000202	0.000226	5.40E-07	1.67E-06	2.00E-10	0.006514	3.17E-11	1.25E-09	3.88E-12
***F*₆₀**	-3873.72	-3873.72	-3873.7	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72	-3873.72
***F*₆₁**	-2.19779	-2.2	-2.18412	-2.19999	-2.2	-2.2	-2.19562	-2.19999	-2.09512	-2.19999	-2.19396	-2.2	-2.2
***F*₆₂**	-2	-2	-1.99958	-2	-2	-2	-1.99842	-2	-1.89374	-2	-1.99992	-2	-2
***F*₆₃**	34.63038	63.88666	34.39487	35.95635	64.11007	35.16388	55.37452	64.48909	60.52621	52.99172	34.3712	34.3712	39.91923
***F*₆₄**	1.29E-07	0	2.15377	212.2401	0.000517	0	0	8.38E-05	0	0.00057	0	1.04E-11	0
***F*₆₅**	1.07E-08	0	0.25049	24.566	1.66E-05	0	0	8.82E-06	0	3.76E-05	0	9.35E-13	0
***F*₆₆**	1.01E-06	4.64E-08	3.177045	81320.38	5.82E-06	0	0	0.000194	0	0.000915	2.14E-08	2.89E-07	0
***F*₆₇**	3.17E-11	0	0.000197	6.23E-05	0	3.13E-05	0	0	0	0	0	0	0

F_{68}	2.16E-08	0	0.000578	0.000364	9.39E-05	4.70E-05	0	9.34E-16	0	1.37E-06	0	0	0
F_{69}	0.001645	0.001567	0.003091	0.002703	0.001931	0.002116	0.001711	0.001567	0.001567	0.001671	0.001567	0.001567	0.001567
F_{70}	0.292579	0.292579	0.293035	0.293369	0.292682	0.292817	0.292579	0.292579	0.292579	0.292592	0.292579	0.292579	0.292579
F_{71}	0.000655	0	0.176494	0.286967	0.230016	0.160978	0	0.00098	0	0.038452	0.008734	2.99E-07	0
F_{72}	-3456	-3456	-3456	-703.783	-3456	-3456	-3421.44	-2799.36	-1797.12	-3456	-1555.19	-3456	-3456
F_{73}	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9203	-26.9167	-26.9203	-26.9203
F_{74}	-19.2085	-19.2085	-19.2085	-19.2084	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2085	-19.2058	-19.2085	-19.2085
F_{75}	-24.1568	-24.1568	-24.1568	-24.1567	-24.1568	-24.1568	-24.1568	-24.1568	-24.1568	-24.1568	-24.1503	-24.1568	-24.1568
F_{76}	-10.8713	-10.8667	-10.8677	-10.8723	-10.8618	-10.8596	-10.8452	-10.8681	-10.8533	-10.8656	-10.8722	-10.8715	-10.8606
F_{77}	2.19E-11	0	3.74E-05	6.06E-07	9.37E-10	0	0	2.13E-09	0	5.97E-09	0	0	0
F_{78}	-3.21521	-3.30517	-2.93195	-3.28198	-3.16981	-3.18557	-3.03354	-3.30389	-3.27734	-3.26257	-3.30053	-3.2675	-3.30588
F_{79}	0.009538	0.86	0.155776	0.276014	0.690017	0.733009	0.38	0.710733	0.72	0.030843	0.014375	0.2	0.86
F_{80}	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681	-4.81681
F_{81}	-2.5	-2.5	-2.49974	-2.49991	-2.5	-2.5	-2.5	-2.49998	-2.5	-2.49997	-2.5	-2.5	-2.5
F_{82}	-1.5	-1.5	-1.5	-1.49994	-1.5	-1.5	-1.5	-1.49999	-1.5	-1.49998	-1.5	-1.5	-1.5
F_{83}	-7.307	-7.307	-7.307	-7.307	-7.2184	-7.30681	-7.30448	-7.30593	-6.79353	-7.30531	-7.30469	-7.30565	-7.30428
F_{84}	-400	-400	-399.971	-399.997	-400	-400	-400	-400	-400	-400	-400	-400	-400
F_{85}	0.000124	0	17.13242	3.5E+08	0.043665	0.035885	0	0.00137	0	0.136084	0.000884	5.58E-11	0
F_{86}	3.56E-07	0	22.16767	23150.16	0.004687	7.82E-06	0	1.68E-05	0	0.018548	2.53E-06	6.66E-13	0
F_{87}	21.35	21.35	22.41224	1416.965	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35
F_{88}	-0.00379	-0.00379	-0.00371	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379	-0.00379
F_{89}	-0.3523	-0.3523	-0.35222	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523	-0.3523
F_{90}	2.30E-09	0	0.351821	20.70758	5.44E-07	0	0	1.24E-05	0	2.18E-05	0	1.76E-12	0
F_{91}	1.30E-06	0	0.003473	1.91E-05	1.88E-05	3.61E-05	0	3.88E-08	0	1.50E-06	0.010875	0	0
F_{92}	0	5.91E-05	0.001564	0.000346	0.002274	0.001185	0.000288	1.22E-06	0	0.000913	0.001276	8.94E-06	0
F_{93}	-3.86278	-3.86278	-3.86201	-3.86278	-3.86278	-3.86278	-3.86278	-3.86278	-3.86278	-3.86278	-3.85578	-3.86278	-3.86278
F_{94}	1.91E-05	2.58E-07	0.09117	7.91E-05	0.03659	0.125796	8.76E-09	4.55E-07	0	3.47E-05	2.33E-07	0	0
F_{95}	4.38E-05	4.36E-05	0.017801	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536
F_{96}	1.23E-07	4.26E-10	0.024673	3.44E-07	4.28E-06	0	0	0.000771	0	2.16E-07	0.004265	1.54E-12	0
F_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{98}	0.080279	0.046843	0.030226	0.000171	0.023366	0.008451	0.010558	0.00373	0.006743	0.000341	0.045975	0.005773	0
F_{99}	0.038426	0.073525	6.551541	0.065175	1.960562	2.321403	0.833085	0.000126	0.184534	0.072768	0.545461	0.295544	0
F_{100}	0	0	23862.17	1501111	0.012237	0	0	0.114507	0	19.93036	0	0.304799	0
F_{101}	0	15.07788	3549.441	3914.083	2291.35	182.4623	7315.347	1694.244	1041.104	109.0851	580.2653	3045.064	0
F_{102}	3.52E-11	1.06E-11	6.45E-10	2.24E-11	1.13E-08	2.59E-09	6.57E-09	3.18E-10	1.40E-09	1.75E-10	4.50E-10	3.13E-10	1.09E-11
F_{103}	0.00113	0.001049	0.001421	0.000584	0.002501	0.001589	0.001091	0.003468	0.000871	0.000725	0.00075	0.000726	0.000345
F_{104}	4.54E-07	1.10E-09	1.30E-06	0	1.20E-09	1.48E-08	0	0	0	0	0	0	0
F_{105}	-10.135	-5.4187	-7.25559	-9.96915	-6.05559	-5.17998	-6.84528	-8.02409	-5.89825	-9.48143	-3.26644	-9.11489	-7.78635
F_{106}	-10.4029	-9.88825	-8.29251	-10.394	-7.73703	-6.76033	-8.78852	-8.73896	-7.5522	-10.3356	-4.78972	-10.1154	-9.86789
F_{107}	-10.5364	-10.029	-7.84355	-10.5275	-7.09249	-8.02427	-8.44896	-9.24558	-7.51599	-10.3427	-5.63423	-10.0067	-10.3787
F_{108}	0.014842	0.013436	0.01972	0.005536	0.016654	0.016391	0.013246	0.011797	0.010575	0.010778	0.065421	0.013656	0.012721
F_{109}	0	0	1002.409	199.6639	129.8825	21.81037	2485.693	16.74176	916.8418	12.35025	85.59185	231.0146	779.9745
F_{110}	-529.871	-529.871	-529.871	-524.122	-529.637	-529.866	-529.36	-510.04	-504.734	-529.71	-526.975	-527.358	-529.808
F_{111}	-0.965	-0.96386	-0.83246	-0.965	-0.82785	-0.90131	-0.81102	-0.94808	-0.94601	-0.95366	-0.5473	-0.94717	-0.96196
F_{112}	0.03517	0.013086	0.071682	0.008535	0.068606	0.037912	0.01202	0.004663	0.004478	0.005668	0.011768	0.007685	0.002925
F_{113}	-3.32236	-3.27706	-3.27363	-3.29177	-3.27587	-3.30567	-3.2355	-3.24963	-3.28064	-3.26971	-2.97084	-3.22462	-3.28422
F_{114}	-50	-50	-39.0147	-48.7025	-49.9963	-49.999	-50	-50	-50	-49.5316	-47.3083	-50	-50
F_{115}	0.95979	0.95979	0.95979	0.964521	0.967206	0.965784	0.96446	0.96661	0.967831	0.965206	0.961667	0.965358	0.964105
F_{116}	-45.778	0.001066	-27.4893	-45.7725	-45.778	-45.778	-45.778	-45.778	-45.778	-45.4809	-32.4647	-45.778	-45.778
F_{117}	-209.849	-210	413.6937	468.218	-208.358	-204.938	-51.2971	-209.999	-210	-174.486	-85.7053	-210	-210

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

TABLE S3. **The standard deviation values of the 2D mathematical functions for different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{1}	1.46E-06	0	0.11697	0.043876	0.000343	0	0	0.000415	0	0.000902	0	1.72E-07	0
F_{2}	4.57E-13	4.57E-13	0.026056	0.001822	4.57E-13	4.57E-13	4.57E-13	4.57E-13	4.57E-13	4.57E-13	2.23E-05	4.57E-13	4.57E-13
F_{3}	3.57E-15	3.57E-15	3.57E-15	2.32E-13	6.07E-13	2.12E-15	2.23E-15	6.05E-13	2.23E-15	1.50E-10	1.01E-10	1.86E-15	2.23E-15
F_{4}	2.02E-06	0	1.855692	27.92913	0.00055	0	0	0.00047	0	0.002118	0	2.07E-07	0
F_{5}	8.95E-08	0	0.000174	3.75E-07	0.076205	0.001045	0	0.257578	0.207786	3.73E-06	2.25E-05	0	0
F_{6}	5.71E-07	0	6.16E-05	6.04E-06	1.11E-09	0	0	3.95E-09	0	1.41E-08	0.000218	0	0
F_{7}	1.32E-11	0	8.19E-05	1.25E-06	3.28E-09	0	0	3.18E-09	0	6.28E-09	2.23E-05	0	0
F_{8}	1.51E-08	5.76E-14	0.00364	9.37E-05	1.25E-07	5.81E-14	5.58E-14	7.768992	2.737232	1.59E-06	0.01179	4.88E-14	5.64E-14
F_{9}	6.76E-09	0	0.241608	0.213791	1.01E-07	0	0	1.04E-05	0	2.44E-05	0	2.08E-12	0
F_{10}	3.42E-08	0	0.193934	0.131701	0.04782	0.059525	0	7.74E-06	0	0.047598	0	2.00E-12	0
F_{11}	3.55E-07	0	0.1418	0.147322	0.002541	0	0	3.22E-06	0	0.002872	0	5.74E-13	0
F_{12}	7.75E-10	0	0.00262	1.25E-05	2.59E-06	0	0	1.28E-08	0	6.11E-07	0.00011	0	0
F_{13}	1.98E-07	1.06E-15	0.000173	5.30E-06	8.42E-07	1.06E-15	1.06E-15	1.27E-08	1.06E-15	1.32E-06	0.000285	3.33E-15	1.06E-15
F_{14}	0.008209	0.33967	0.013576	0.298728	0.765016	0.246441	0.639373	0.800394	0.664052	0.581751	0.42245	0.125155	0.693471
F_{15}	8.00E-12	0	0.000974	0	0	0	0	0	0	0	0	0	0
F_{16}	2.29E-10	0	0.001046	9.02E-06	3.64E-08	0	0	7.64E-07	0	5.32E-07	1.40E-06	3.77E-11	0
F_{17}	0.079052	0.081999	0.120939	0.088395	0.086186	0.01268	0.016531	0.038717	0.013479	0.11426	0.139176	0.015809	0.008066
F_{18}	1.12E-12	0	8.85E-05	3.92E-07	4.32E-09	0	0	1.39E-09	0	9.90E-09	0	0	0
F_{19}	1.56E-15	1.56E-15	0.000214	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15
F_{20}	3.57E-15	3.57E-15	3.57E-15	0.000132	5.50E-09	3.57E-15	3.57E-15	1.06E-07	3.57E-15	6.31E-07	0.006701	3.57E-15	3.57E-15
F_{21}	294.5935	321.8987	252.0488	0	45.8701	0.000858	0	9.204521	100.0004	57.71089	0	0	9.14E-13
F_{22}	257.4091	283.2728	269.4483	199.916	138.8434	1.66E-06	3.63E-07	24.13377	0.000707	255.1144	355.7277	1.28E-05	0
F_{23}	0.063962	0.044719	0.11658	0.003496	1.82E-06	5.01E-14	5.00E-14	0.088077	0.183071	2.34E-05	0.000276	0.11468	5.00E-14
F_{24}	3.91E-09	2.68E-15	7.61E-06	9.12E-07	4.06E-10	2.66E-15	2.68E-15	6.12E-10	2.68E-15	2.82E-09	2.31E-06	2.26E-15	2.68E-15
F_{25}	9.06E-05	0.002336	0.012222	4.00E-05	0.44976	0.112	0.006962	2.96E-07	0	0.000503	0.000126	0	0
F_{26}	0.591209	0	0.808781	0.466089	1.02E-09	0.47678	0	0.755048	0.2	0.20015	0.506176	1.003227	0.2
F_{27}	0	0	16.56524	0.022115	0	0	0	0	0	0	0	0	0
F_{28}	0.239433	0	0.051027	0.368956	0.219025	0	0	0.140705	0	3.73E-06	0.000256	1.03E-13	0
F_{29}	1.59E-05	2.45E-15	25.97304	21626.32	0.142321	2.45E-15	2.45E-15	0.001054	2.45E-15	0.024848	0.011757	2.45E-15	3.554586
F_{30}	4.41E-10	0	0.000963	7.85E-06	1.20E-07	0	0	2.26E-08	0	9.13E-08	0	0	0
F_{31}	2.65E-10	0	8.73E-05	1.07E-06	6.76E-09	0	0	4.09E-09	0	1.16E-08	1.80E-05	0	0
F_{32}	1.16E-05	18.19306	0.06375	0.000288	1.62E-05	0	6.155968	1.29E-06	11.88543	1.03E-05	0.011351	0	4.979582
F_{33}	2.24E-13	1.15E-16	3.63E-06	4.60E-10	4.95E-12	1.01E-16	1.10E-16	4.54E-11	9.95E-17	2.04E-10	2.33E-06	9.72E-17	1.06E-16
F_{34}	8.60E-10	0	0.002821	1.99E-06	7.10E-08	0	0	5.97E-08	0	5.63E-07	1.31E-06	0	0
F_{35}	0.094071	0.08846	0.063975	0.000267	7.656492	5.092304	5.817196	5.71E-14	5.71E-14	5.71E-14	0.07045	5.71E-14	5.71E-14
F_{36}	8.80E-07	0	0.001149	8.13E-06	1.32E-07	0	0	3.86E-08	0	1.52E-07	0.0019	0	0
F_{37}	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	2.68E-15	5.67E-05	2.68E-15	2.68E-15
F_{38}	1.63E-05	1.46E-13	0.057363	5.44E-06	0.006417	1.58E-13	1.41E-13	5.59E-07	1.46E-13	0.001862	0.007678	1.24E-13	1.46E-13
F_{39}	4.46E-08	8.41E-16	5.47E-05	2.92E-11	2.79E-13	8.54E-16	8.93E-16	3.40E-11	0.183365	7.74E-10	4.04E-10	6.94E-16	8.93E-16
F_{40}	9.56E-05	0.000622	0.000159	1.02E-08	0.033576	0.007434	0.002606	1.31E-09	0	9.74E-06	5.52E-05	0	0
F_{41}	0.10269	0.095281	0.079009	0.010656	7.96583	3.106311	4.370877	5.63E-06	2.44E-13	3.54E-05	0.08097	1.03E-12	2.39E-13
F_{42}	3.43E-13	3.43E-13	4.061857	0.006086	9.150015	12.78853	6.959556	16.5717	14.38812	3.161242	0.03696	3.43E-13	3.43E-13
F_{43}	1.39E-09	0	5.83E-05	4.18E-07	2.64E-06	0	0	5.23E-10	0	5.49E-07	0	0	0
F_{44}	4.46E-16	2.69E-05	4.46E-16	6.47E-08	4.00E-09	4.02E-15	4.02E-15	6.39E-10	4.02E-15	1.97E-08	1.49E-05	3.93E-15	4.02E-15
F_{45}	3.21E-14	3.21E-14	3.21E-14	3.21E-14	1.710081	3.21E-14	3.21E-14	3.21E-14	3.21E-14	3.21E-14	3.21E-14	3.21E-14	3.21E-14
F_{46}	1.56E-15	1.56E-15	7.68E-05	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	1.56E-15	0.1911	1.56E-15	1.56E-15

F_{47}	5.58E-17	5.58E-17	5.58E-17	0.001612	0.044453	0.035126	0.038199	0.053157	0.048742	0.020678	0.006164	0.005853	0.004009
F_{48}	4.18E-16	4.18E-16	4.18E-16	0.001653	0.045516	0.031678	0.025054	0.051891	0.056908	0.011659	0.005478	0.008297	4.18E-16
F_{49}	8.93E-16	8.93E-16	8.93E-16	3.35E-10	1.78E-15	1.79E-15	1.79E-15	1.96E-11	1.79E-15	1.77E-09	2.58E-08	1.97E-15	1.79E-15
F_{50}	3.97E-11	2.68E-15	0.000674	1.30E-05	0.100155	0.041961	2.68E-15	1.29E-08	2.68E-15	1.55E-07	0.000174	3.29E-15	2.68E-15
F_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{52}	3.77E-10	2.04E-09	1.18E-08	1.91E-09	2.93E-10	0	0	8.55E-11	0	3.21E-10	2.73E-09	0	0
F_{53}	1.37E-05	1.94E-05	5.86E-06	4.42E-07	7.68E-10	0	0	9.03E-10	0	2.75E-09	1.21E-05	0	0
F_{54}	1.34E-15	1.34E-15	1.34E-15	1.93E-07	7.73E-11	2.34E-15	2.34E-15	1.36E-10	2.34E-15	3.41E-10	5.67E-06	2.34E-15	2.34E-15
F_{55}	0.002835	0.038217	0.006525	4.64E-06	0.047258	0.01407	0.049889	0.050091	0.036845	0.024207	7.81E-16	7.81E-16	7.81E-16
F_{56}	4.42E-06	0	0.119418	6.217954	8.76E-07	0	0	9.59E-06	0	1.09E-05	0.000346	1.82E-12	0
F_{57}	0.002835	0.038217	0.006525	4.64E-06	0.047258	0.01407	0.049889	0.050091	0.036845	0.024207	7.81E-16	7.81E-16	7.81E-16
F_{58}	7.03E-05	0.003357	181.8071	129654.2	0.042136	0.004139	0.00372	0.003527	0.003726	0.076535	0.000774	0.003699	0.001473
F_{59}	1.88E-09	5.64E-10	12.96031	174515.5	0.000969	0.000523	2.44E-06	4.36E-06	3.86E-10	0.017011	8.55E-11	2.53E-09	1.65E-11
F_{60}	3.70E-11	8.23E-12	0.043967	0.000557	6.25E-06	8.27E-12	8.23E-12	4.47E-07	8.23E-12	9.79E-06	0.000198	8.81E-12	8.23E-12
F_{61}	0.001372	4.02E-15	0.016154	8.34E-06	1.17E-06	4.02E-15	0.024493	1.11E-05	0.106125	2.65E-05	0.002968	2.23E-12	4.02E-15
F_{62}	1.32E-06	0	0.001635	2.05E-08	4.00E-09	0	0.015826	4.14E-08	0.108826	3.19E-08	8.82E-05	0	0
F_{63}	0.466797	14.92037	0.119471	7.804749	17.25627	5.576686	19.87839	17.01006	18.86708	19.87305	3.57E-14	3.57E-14	13.81995
F_{64}	1.48E-07	0	4.754616	227.1377	0.001235	0	0	9.16E-05	0	0.000701	0	1.64E-11	0
F_{65}	1.20E-08	0	0.596604	21.71972	6.15E-05	0	0	9.36E-06	0	5.06E-05	0	1.65E-12	0
F_{66}	9.75E-07	1.02E-07	11.90008	262168.6	4.76E-05	0	0	0.000809	0	0.001919	1.94E-07	2.71E-06	0
F_{67}	8.98E-11	0	0.000831	0.000206	0	0.00022	0	0	0	0	0	0	0
F_{68}	5.06E-08	0	0.001439	0.000857	0.000631	0.000254	0	8.72E-15	0	4.65E-06	0	0	0
F_{69}	8.52E-05	3.86E-07	0.002693	0.001055	0.000708	0.000676	0.000622	3.27E-07	1.60E-17	0.000148	3.85E-08	5.08E-15	0
F_{70}	0	0	0.000931	0.000993	0.000407	0.000507	0	0	0	5.02E-05	0	0	0
F_{71}	0.000364	0	0.16953	0.139554	0.871257	0.631964	0	0.000496	0	0.041995	0.004805	2.12E-07	0
F_{72}	0	0	0	1348.218	0.000497	0	345.6	1362.622	1735.315	0.000762	1727.987	3.66E-11	0
F_{73}	3.57E-15	3.57E-15	3.57E-15	7.37E-05	7.51E-09	4.28E-14	4.26E-14	6.06E-08	4.00E-14	3.54E-07	0.003735	3.97E-14	4.04E-14
F_{74}	2.86E-14	2.86E-14	2.86E-14	7.28E-05	2.86E-14	2.86E-14	2.86E-14	2.86E-14	2.86E-14	2.86E-14	0.002397	2.86E-14	2.86E-14
F_{75}	3.57E-15	3.57E-15	3.57E-15	0.000132	5.50E-09	3.57E-15	3.57E-15	1.06E-07	3.57E-15	6.31E-07	0.006701	3.57E-15	3.57E-15
F_{76}	0.00144	0.007344	0.007187	2.62E-05	0.009936	0.00954	0.040251	0.008108	0.027941	0.009063	0.000919	0.003901	0.009791
F_{77}	2.89E-11	0	0.000148	6.05E-07	4.84E-09	0	0	2.57E-09	0	1.00E-08	0	0	0
F_{78}	0.064298	0.010008	0.196888	0.027406	0.155113	0.150292	0.245944	0.016926	0.061487	0.061321	0.02401	0.066914	0.009873
F_{79}	0.007106	0.603358	0.216588	0.142678	0.747996	0.441418	0.582228	0.782323	0.805035	0.171437	0.125502	0.402015	0.550849
F_{80}	1.16E-14	1.16E-14	1.16E-14	3.45E-08	3.56E-10	1.16E-14	1.16E-14	3.64E-10	1.16E-14	3.76E-09	2.13E-08	1.16E-14	1.16E-14
F_{81}	0	0	0.000987	4.58E-05	5.58E-07	0	0	8.00E-06	0	2.03E-05	0	3.37E-09	0
F_{82}	0	0	0	3.36E-05	5.68E-07	0	0	5.47E-06	0	1.11E-05	0	1.90E-09	0
F_{83}	1.07E-14	1.07E-14	1.07E-14	4.38E-08	0.37188	0.000703	0.000748	0.001345	0.74634	0.001292	0.000898	0.001378	0.000274
F_{84}	2.25E-11	0	0.093105	0.003509	3.20E-08	0	0	1.45E-07	0	7.68E-07	0	0	0
F_{85}	0.000153	0	39.62369	1.96E+09	0.070197	0.07464	0	0.001811	0	0.115012	0.000867	8.70E-11	0
F_{86}	3.46E-07	0	153.6117	40729.86	0.013368	1.66E-05	0	2.09E-05	0	0.018478	2.52E-06	1.13E-12	0
F_{87}	4.28E-14	4.28E-14	3.136498	2530.442	4.28E-14	4.28E-14	4.28E-14	4.28E-14	4.28E-14	4.28E-14	4.28E-14	4.28E-14	4.28E-14
F_{88}	6.97E-18	6.97E-18	0.000126	1.07E-06	6.97E-18	6.97E-18	6.97E-18	6.97E-18	6.97E-18	1.43E-11	6.97E-18	6.97E-18	6.97E-18
F_{89}	3.35E-16	3.35E-16	0.000285	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16	3.35E-16
F_{90}	2.67E-09	0	0.784605	19.68525	3.65E-06	0	0	1.23E-05	0	3.22E-05	0	4.12E-12	0
F_{91}	1.25E-06	0	0.004129	1.22E-05	1.18E-05	7.21E-05	0	5.80E-08	0	1.83E-06	0.10221	0	0
F_{92}	0	9.19E-05	0.001694	0.000209	0.005643	0.001137	0.000875	1.83E-06	0	0.000972	0.000746	3.21E-05	0
F_{93}	5.60E-14	5.69E-15	0.000803	1.07E-08	1.49E-08	5.79E-15	5.67E-15	2.01E-08	5.73E-15	1.79E-06	0.002517	6.43E-15	5.69E-15
F_{94}	2.35E-05	1.02E-06	0.193789	8.44E-05	0.054952	0.332837	6.92E-08	2.62E-06	0	6.02E-05	3.76E-07	0	0
F_{95}	1.69E-07	4.99E-08	0.001608	6.08E-15	2.04E-13	2.10E-09	1.05E-17	5.14E-13	1.05E-17	1.83E-11	5.00E-09	4.40E-18	1.05E-17
F_{96}	2.76E-07	4.05E-09	0.158962	6.29E-07	3.30E-05	0	0	0.007586	0	8.93E-07	0.032566	1.10E-11	0
F_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{98}	0.048168	0.231158	0.019022	9.59E-05	0.02873	0.014006	0.019873	0.011322	0.014511	0.000473	0.014401	0.013574	0

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F_{99}	0.027984	0.517235	4.203665	0.037731	3.506177	2.512569	1.333925	0.000158	0.805062	0.0954	0.63961	0.882507	0
F_{100}	0	0	25079.94	711166.5	0.040196	0	0	0.377509	0	18.2525	0	2.236163	0
F_{101}	0	150.7788	8275.253	811.3904	7820.57	337.6875	13165.09	1335.226	10411.04	122.5878	503.0337	921.329	0
F_{102}	6.29E-11	2.73E-11	1.27E-09	8.81E-11	1.85E-08	4.30E-09	1.13E-08	4.51E-10	1.91E-09	3.19E-10	4.91E-10	4.86E-10	1.00E-11
F_{103}	9.34E-05	0.000195	0.000564	0.000122	0.005305	0.003685	0.00213	0.008411	0.002825	0.000213	0.000443	0.000315	0.000182
F_{104}	6.57E-07	4.14E-09	3.58E-06	0	4.03E-10	8.74E-09	0	0	0	0	0	0	0
F_{105}	0.170137	3.581706	2.155337	1.02275	3.57341	3.519469	3.481803	2.58932	3.394838	1.786631	2.533978	2.100292	3.475888
F_{106}	1.79E-14	1.774716	1.861631	0.00491	3.453562	3.613047	2.988019	2.505742	3.446014	0.033256	2.212135	1.277374	1.95991
F_{107}	1.61E-14	1.864413	2.055325	0.004752	3.646925	3.453311	3.242926	2.308642	3.638866	0.933252	1.813489	1.717767	1.110213
F_{108}	0.001395	0.003173	0.007936	0.006858	0.006828	0.00461	0.012695	0.005901	0.009338	0.006221	0.276786	0.009928	0.005371
F_{109}	0	0	1211.323	81.17334	921.1006	15.79048	3411.496	14.26696	1768.682	16.84632	56.96372	209.4644	2333.621
F_{110}	1.37E-12	1.37E-12	1.37E-12	2.980047	0.326181	0.014505	2.020508	57.12894	65.05756	0.08756	2.622658	2.53787	0.628276
F_{111}	1.00E-15	0.00802	0.077528	7.06E-06	0.14884	0.082118	0.1334	0.020838	0.053064	0.021794	0.189331	0.031534	0.011881
F_{112}	0.025945	0.003159	0.052591	0.001729	0.083282	0.053283	0.033227	0.002073	0.012487	0.000588	0.009912	0.030352	0.002814
F_{113}	2.68E-15	0.058149	0.039691	0.051999	0.058432	0.041569	0.058879	0.058447	0.05714	0.051417	0.247421	0.046025	0.055883
F_{114}	4.44E-06	0	9.769266	0.527168	0.001209	0.004061	0	4.17E-06	0	0.183078	1.237407	3.74E-12	5.63E-10
F_{115}	8.93E-16	8.93E-16	8.93E-16	0.002829	0.005053	0.002051	0.003017	0.004309	0.00543	0.003362	0.002529	0.002385	0.002738
F_{116}	6.43E-14	0.019277	3.905028	0.001423	6.43E-14	6.43E-14	6.43E-14	6.43E-14	6.43E-14	0.075135	2.66828	6.43E-14	6.43E-14
F_{117}	0.069911	7.20E-13	376.9157	176.0505	0.684119	2.841448	459.7645	0.001289	1.04E-11	7.930871	48.91673	3.25E-10	1.62E-05

TABLE S4. The mean values of the function evaluations for the 2D mathematical functions in different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_1	150000	4044	150000	150000	54390.17	95762	7038.5	150000	6665.5	150000	12621.5	150000	5136.06
F_2	3364.372	1865	147278.4	150000	2579.225	6659.4	2076.5	143979.5	2885	41625	108364	81057	2575.4
F_3	113.3603	51.5	916.1677	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_4	150000	3367.5	150000	150000	43596.91	97721	5763.5	150000	6307	150000	10038.5	150000	4105.24
F_5	150000	4193.5	150000	150000	132208.1	52355.6	4616	150000	16169.5	150000	150000	125284	2625.2
F_6	150000	5270	150000	150000	28015.98	30837.6	3715	150000	4296.5	150000	150000	128170.5	8644.66
F_7	145111.3	1977.5	150000	150000	42997.11	29086	3167	150000	4073	150000	150000	127278	2367.26
F_8	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_9	150000	2454.5	150000	150000	27558.99	46455	4208	150000	4885	150000	6284.5	147366	2825.72
F_{10}	150000	2486.5	150000	150000	33243.79	54346.2	4233	150000	4873.5	150000	6488.5	147515	2770.02
F_{11}	150000	3643.5	150000	150000	124867.2	57561.4	5143.5	150000	5166	150000	9009.5	145017	2751.88
F_{12}	150000	2618.5	150000	150000	104009.5	39142.4	3906.5	150000	4412	150000	150000	131742.5	2465.44
F_{13}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{14}	149365.4	150000	111440.1	102682.9	95108.47	63497.4	65518	103609	114504	56550	144212	64963	106676.3
F_{15}	136479.8	1947	150000	1256.098	237.6743	539.6	202.5	152	170	3750	52.5	50.5	50.36
F_{16}	150000	2574.5	150000	150000	32309.42	63591	4379.5	150000	5369	150000	150000	149977.5	3562.36
F_{17}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{18}	94067.81	1932	150000	150000	39979.26	28985.4	3294	150000	4137.5	150000	4461.5	124630.5	2227.42
F_{19}	2134.615	1584	103140.7	64646.34	640.0884	2011.6	1636.5	26300	2085	11887.5	54182.5	58975	2267.18
F_{20}	101.2146	50	898.2036	150000	6602.346	15255.2	2613.5	149945.5	3846.5	144600	150000	102964	9852.9
F_{21}	150000	150000	150000	150000	150000	150000	150000	150000	148981.5	150000	150000	150000	16303.06
F_{22}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	8518.4
F_{23}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{24}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000

F_{25}	150000	146287.5	150000	150000	150000	150000	147585.5	150000	12407.5	150000	150000	134010	4386.8
F_{26}	150000	150000	150000	150000	150000	147861	150000	150000	148553	150000	150000	147449.5	148686.6
F_{27}	1762.146	1355.5	87413.17	90658.54	606.9364	1503.6	1311	15255.5	1854.5	7425	5328.5	42858.5	1119.98
F_{28}	150000	2964	150000	150000	29506.8	40331.4	4888.5	150000	4687.5	150000	150000	140923	3086.8
F_{29}	84055.67	2097	150000	150000	98050.83	26200	3176	149998	3908.5	149325	150000	115277.5	4828.54
F_{30}	149152.8	2156	150000	150000	14270.15	34206.8	3674.5	150000	4371.5	150000	4857	133385.5	2547.42
F_{31}	140484.8	1979.5	150000	150000	41502.21	28493.2	3209.5	150000	4073	150000	150000	126632	2361.46
F_{32}	150000	43468	150000	150000	74480.79	42781.2	8691	150000	13449.5	150000	150000	138547.5	4807.94
F_{33}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{34}	150000	2275.5	150000	150000	50590.45	36316.8	3662.5	150000	4459.5	150000	150000	134625	2909.12
F_{35}	102936.2	106344	75718.56	81073.17	23422.14	27081.6	13786	28070	1969.5	9187.5	103820	48832.5	3304.66
F_{36}	150000	3611	150000	150000	49061.88	35072.8	3969.5	150000	4440	150000	150000	133606.5	3687.18
F_{37}	106.2753	52	1230.539	11975.61	473.8184	1205	1124.5	81128	1538.5	6825	138462.5	52380.5	672.08
F_{38}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{39}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{40}	150000	148647	150000	150000	150000	150000	144115	150000	7872	150000	150000	121558.5	3199.24
F_{41}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{42}	17423.08	2169	148994	150000	16872.83	44566.4	9827	144346.5	27986.5	85425	150000	91173	1939.2
F_{43}	150000	3208.5	150000	150000	118858	39056.2	4139	150000	4232	150000	5895	120258	2123.1
F_{44}	274.2915	21086	1293.413	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{45}	1012.146	464.5	2281.437	2048.78	1717.273	422.4	535	1270	192	3750	2079.5	2244.5	322.72
F_{46}	1728.745	1089.5	119371.3	35865.85	728.3237	3118.8	1548	135812.5	2317.5	18825	149989	76331.5	1161.64
F_{47}	437.247	199.5	3961.078	150000	144587.6	148903.8	82695	150000	150000	150000	150000	149898	11276.72
F_{48}	453.4413	196	4562.874	150000	126383	150000	22242	150000	146340	150000	150000	150000	5581.44
F_{49}	152.834	88	1257.485	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{50}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{51}	535.4251	309.5	3359.281	2317.073	200.442	413	550	2780.5	351	3750	2002.5	1877	213.88
F_{52}	141998	142526.5	146685.6	144646.3	18227.98	23327	3174.5	147878.5	4635.5	145350	129661.5	99442	28418.8
F_{53}	150000	150000	150000	150000	22003.74	27638.8	3663	150000	4137.5	150000	149551.5	123278.5	19440.4
F_{54}	101.2146	50	898.2036	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{55}	150000	149192.5	150000	150000	73383.71	50984.4	68363	150000	27710	150000	6096.5	128308.5	3907.78
F_{56}	150000	6230.5	150000	150000	36478.41	45932.2	4608.5	150000	5006.5	150000	150000	147024	7113.5
F_{57}	150000	149192.5	150000	150000	73383.71	50984.4	68363	150000	27710	150000	6096.5	128308.5	3907.78
F_{58}	150000	60294.5	150000	150000	136680.6	96081	73572	150000	78854	150000	150000	149966.5	38874.98
F_{59}	150000	148911.5	150000	150000	135162.7	143573.8	129587	150000	125698	150000	94268.5	146964	81334.4
F_{60}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{61}	150000	8078.5	150000	150000	52100.14	43506.6	48778.5	150000	86106.5	150000	150000	147632.5	14087.48
F_{62}	150000	3895	150000	150000	27510.03	31498.4	5248	150000	85825.5	150000	150000	134221.5	6883.58
F_{63}	107909.9	147490.5	43976.05	12560.98	112672.9	33018.6	80150.5	114799	99323.5	72750	33818	14311	46787.9
F_{64}	150000	2780.5	150000	150000	87794.63	52087.8	4593.5	150000	5102	150000	7525	149431	2837.5
F_{65}	150000	2588	150000	150000	69958.86	48494	4324.5	150000	4921.5	150000	6676	147270	2589.96
F_{66}	150000	108924	150000	150000	39395.27	94282.6	6737.5	150000	7381	150000	28803	150000	29188.98
F_{67}	129472.7	1725	148365.3	150000	1746.855	9919.2	2253	61763	2682	35100	2085.5	81912	1350.66
F_{68}	149716.6	2393	149928.1	150000	72679.87	97767.2	3029	146059.5	3315.5	147787.5	4032.5	93738.5	1679.9
F_{69}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{70}	17560.73	2141	112401.2	144707.3	38634.82	58793.2	2256.5	39721.5	2301.5	62775	5562	47400	1657.48
F_{71}	150000	5369	150000	150000	139746.3	118906.4	8124	150000	7031.5	150000	150000	150000	5433.82
F_{72}	101.2146	50	898.2036	150000	87855.83	52577.8	6892.5	150000	75004	150000	150000	149877.5	3231.8
F_{73}	101.2146	50	898.2036	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{74}	101.2146	50	898.2036	148073.2	1212.343	6679	2214.5	145454.5	3054	50437.5	150000	83503	7204.52
F_{75}	101.2146	50	898.2036	150000	6602.346	15255.2	2613.5	149945.5	3846.5	144600	150000	102964	9852.9
F_{76}	150000	103540	147889.2	150000	86177.32	105796.4	98419.5	149463.5	88000	129487.5	87175	97031	92254.56

F_{77}	142687.2	3518.5	150000	150000	28356.17	29269	3488	150000	4154	150000	6358.5	125840	4842.94
F_{78}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{79}	150000	112766	150000	150000	89481.81	138716.8	54441.5	150000	78336.5	150000	150000	150000	116903.8
F_{80}	130.5668	62	1248.503	150000	10004.25	26978.4	3044	150000	3975.5	150000	150000	120948.5	2257.62
F_{81}	466.5992	2556	32146.71	150000	23272.19	80512.8	6146	150000	6129.5	150000	9655	150000	4902.5
F_{82}	138.664	252	2892.216	150000	38261.99	78104.4	6097.5	150000	6077	150000	10527	150000	3869.1
F_{83}	110.3239	51	916.1677	150000	65275.25	43138	138298	150000	58296.5	150000	150000	138245	148793
F_{84}	144299.6	2302.5	150000	150000	23118.16	37203.2	3778	150000	4509.5	150000	5580.5	136836	2496.82
F_{85}	150000	7843.5	150000	150000	141154.5	133565	6635	150000	5906.5	150000	150000	149920	5672.68
F_{86}	150000	10751	150000	150000	138187.2	147501.2	7152	150000	5742	150000	150000	147498	6489.88
F_{87}	1373.482	649.5	79535.93	150000	642.1285	1601.6	1216.5	13246.5	1677	7312.5	4819	44847	464.54
F_{88}	4630.567	1188.5	146371.3	144878	2143.149	7090.4	1900.5	94854	2716.5	41362.5	9592.5	82808.5	1361.82
F_{89}	1547.571	706.5	85149.7	48792.68	515.6409	1322.6	1224	14171	1662	7875	5736.5	43082.5	639.96
F_{90}	150000	2422.5	150000	150000	35530.77	47028.2	4188	150000	4884.5	150000	6327	147667	2597.02
F_{91}	150000	20351.5	150000	150000	150000	122201.6	8012	150000	5013.5	150000	150000	135988	4336.4
F_{92}	190.2834	150000	147035.9	150000	150000	150000	148930	150000	34754	150000	150000	149782.5	4562.26
F_{93}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{94}	150000	81467.5	150000	150000	142631.1	140557	24408	150000	7089	150000	150000	135350	20006.32
F_{95}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{96}	150000	26618	150000	150000	44758.76	38701	4833.5	149797.5	5760	148950	150000	121512.5	10002.42
F_{97}	135.6275	80.5	1562.874	2829.268	589.0853	1129.2	326	224	308	3750	75.5	57.5	100.72
F_{98}	150000	148936	150000	150000	150000	150000	146717	150000	54951.5	150000	150000	150000	9041.48
F_{99}	150000	149883.5	150000	150000	150000	150000	150000	150000	66604	150000	150000	149943	9752.86
F_{100}	10434.21	5356.5	150000	150000	112379.6	37472.4	9200	149705.5	4980.5	149812.5	21212.5	102803.5	1565.92
F_{101}	122.4696	8011	44488.02	150000	150000	150000	107308.5	150000	9865.5	150000	150000	150000	36351.78
F_{102}	116185.2	79264	132790.4	119292.7	145509.7	138161.4	148521	134304.5	125360.5	135637.5	146166	139448	64679.4
F_{103}	150000	150000	150000	150000	150000	150000	149111.5	149999.5	44043.5	150000	150000	147879.5	14664.96
F_{104}	150000	54946.5	150000	7146.341	147012.8	150000	2289	6329	2596	5512.5	1262.5	22654.5	1306.16
F_{105}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{106}	13065.79	14658.5	149515	150000	62509.35	81469.8	38093.5	149939	64977	150000	150000	107490.5	13609.9
F_{107}	13936.23	13319.5	150000	150000	75833.39	65989.2	48601	149994	65232	150000	150000	114216.5	6179.36
F_{108}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	133854.5
F_{109}	101.2146	4013	91814.37	150000	150000	150000	150000	150000	150000	150000	150000	150000	140441.3
F_{110}	178.1377	109	4311.377	150000	150000	138283.4	125883.5	149991.5	98759	150000	150000	142904	77145.02
F_{111}	7390.688	6694.5	127095.8	7158.537	100735.6	69678.2	118979	100231	79856	40350	149178.5	101872	11664.44
F_{112}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	144702.2
F_{113}	18591.09	59103.5	150000	109146.3	60730.87	24615.4	106736.5	149581	54645	150000	150000	140184	54768.28
F_{114}	150000	24131	150000	150000	150000	115199.2	32750.5	150000	6831	150000	150000	149972	60605.84
F_{115}	375.5061	279	9655.689	117402.4	129518.2	141786	108587.5	129599.5	129535.5	121612.5	134010.5	134474	119267.6
F_{116}	173.0769	150000	150000	150000	6276.437	6298.6	10063	149269	3808.5	150000	150000	100920.5	13170.66
F_{117}	150000	149939	150000	150000	150000	150000	148889.5	150000	150000	150000	150000	150000	150000

TABLE S5. **The minimum values of the 50D mathematical functions for different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{118}	17.115407	20.704612	19.884261	20.38123	0.226361	3.655346	16.13841	0.069087	1.63E-08	10.71924	5.93E-05	2.41E-05	0
F_{119}	61.275397	75.360332	52.521659	5.423342	0.007075	0.408871	5.18E-09	1.955532	0.002168	22.26846	2.43E-10	0.240687	1.52E-05
F_{120}	290.43381	784.1567	215.34007	0.036081	8.75E-06	0.248708	16	0.000225	0	2.888847	0	3.03E-11	0
F_{121}	385848.34	19933773	1.767E+09	1.09E+09	1.76E-05	31448.5	0	0.006724	0	3202826	0	0	0
F_{122}	0.498926	2.5392723	0.1787038	3.27E-12	0	5.68E-07	4.88E-10	0	5.68E-09	0.000197	0	0	0
F_{123}	-0.594137	-0.580815	-0.73783	-0.993424	-0.999982	-0.998795	-1	-0.999869	-1	-0.627428	-0.594867	-1	-1
F_{124}	-1	-0.206207	-0.560843	-0.999628	-0.999995	-0.998702	-1	-0.999843	-1	-0.680081	-0.560407	-1	-0.957564
F_{125}	543899.84	604972.77	322396.76	264.902	1.096248	80.40436	0.010636	0.720064	0.666667	2973.612	0.6672628	0.666667	0.205001
F_{126}	-3.31E-159	-6.72E-262	-4.42E-182	-0.208962	-0.999905	-0.254841	-0.997351	-0.999261	-1	-3.34E-21	-5.10E-106	-1	-1
F_{127}	-0.980174	-0.799926	-0.122274	-0.999744	-1	-0.991172	-1	-0.999996	-1	-0.792158	-1	-1	-1
F_{128}	1.2334737	2.1161814	11.507466	9.24373	0.000476	1.037503	3.13E-12	0.007917	0	1.680525	4.63E-10	7.38E-10	3.23E-07
F_{129}	148942.95	201831.67	96270.898	25.45138	4.22E-09	10.45924	0	1.39E-06	0	454.1945	1.33E-11	0	0
F_{130}	3.481E+14	7.56E+15	2.24E+17	2.29E+18	3.45E+08	1.32E+11	4.36E+14	3.43E+15	0	8.28E+15	6.80E-05	1.63E+14	1.43E+10
F_{131}	-6.28891	-4.852631	-7.397765	-13.54793	-33.0243	-39.3724	-24.9662	-13.6182	-25.50935	-9.544449	-49	-28.45886	-49
F_{132}	115.61132	133.81139	125.94328	1.836757	1.356691	0.62392	28.36551	6.54965	0.543852	9.776273	3.6138616	2.898318	0.000364
F_{133}	2	2	6.249E+14	2.01937	2	3.053328	2	2.024689	2	2.868284	2.44E+15	2.000011	2
F_{134}	2	2	3.214E+14	2.01904	2	3.088493	2	2.032522	2	2.97392	1.14E+16	2.000019	2
F_{135}	5.12E+118	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128
F_{136}	0.2612805	0.2889089	0.0152831	2.69E-07	0	5.14E-07	0	0	0	1.58E-05	0.0147237	0	0
F_{137}	21.491004	22.234748	20.157378	11.70646	8.819908	5.588792	6.308752	16.86281	12.04677	17.03131	15.433238	13.9567	0
F_{138}	12877.797	15547.54	26678.318	4403.94	574.5956	651.3584	3127.734	2841.421	1524.471	10927.93	2.29E-11	2877.064	3.50E-05
F_{139}	3507.5261	9348.2396	1825.5535	4.253944	0.191927	3.694348	96.20493	0.657633	0.000794	114.1071	3.01E-05	0.251311	0.103734
F_{140}	10667.579	23834.663	23658.272	19.10382	0.018174	38.84903	7.31E-09	0.021402	0	541.1549	1.74E-10	0.003808	2.07E-06
F_{141}	0.074729	0.2656087	0.0027088	1.39E-09	0	0	0	3.78E-09	0	4.54E-08	0	2.31E-09	0
F_{142}	459.75461	511.80674	391.61351	112.6812	4.173633	15.96008	159.193	110.5245	49.74793	299.3909	8.70E-12	33.8286	8.06E-06
F_{143}	6.117E+10	1.57E+11	4.721E+10	2.67E+11	19.77534	3451810	3.42E-07	253.6222	0	2.45E+08	23266	0.00022	0.864618
F_{144}	54852.879	270205.84	41251.061	116.1182	0.876909	32.10266	3.46E-07	10.32173	0.067372	203.3366	124.16198	22.5722	0.019237
F_{145}	68907412	197406188	52198484	968483.3	80.26309	5691.214	1.11468	42.77962	0.340719	280074.5	47.742164	38.51206	0.000158
F_{146}	7.3082695	10.054568	25.60001	18.11648	0.999873	2.599891	2.899873	0.599873	0.799873	6.504566	0.0998794	0.699873	1.02E-05
F_{147}	97974196	251380738	75622891	53770073	5.43E-06	6414.884	0	0.000696	0	365026.2	7.34E-06	0	0
F_{148}	8.58E+15	7.92E+21	5.51E+27	1.29E+27	0	3.11E+13	0	2.51E-08	0	1.07E+20	0	0	0
F_{149}	3.28E-09	8.49E-07	0	5.35E-11	0	1.08E-09	0	0	0	2.55E-11	1.94E-10	0	0
F_{150}	2068.8357	2372.4738	14294.51	1.791322	0.001909	5.029641	21.90563	0.006009	0	18.45791	45.035841	8.76E-10	0.119431
F_{151}	15.407046	89.728956	1103.922	979.1411	2.25824	34.19162	2.97E-07	2.079096	0.069851	302.9992	0	0.05688	0.001342
F_{152}	68.601683	77.503655	74.805573	71.56383	1.990625	14.6766	56.08004	0.460693	0.259886	18.31413	10.226183	4.109379	0
F_{153}	1.27E+47	7.77E+59	3.41E+50	1.58E+51	4.262342	14.12769	200	7.65E+16	0.430453	1.15E+28	0	416.8082	0.211321
F_{154}	2.002E+09	5.573E+09	248352224	0.359828	0	0.33866	0	0	0	540.4825	8.21E-09	0	0
F_{155}	1529.3666	2420.4741	14991.492	1.833791	0.004448	5.05769	20.47313	0.006096	1.12E-10	18.47382	44.035841	8.64E-10	0.198812
F_{156}	-418.983	-418.983	-241.8645	-156.5671	-365.2239	-416.731	-333.1301	-303.8824	-268.1324	-232.8495	-128.6022	-297.3528	-418.9829
F_{157}	7.5999957	22.37744	647.31859	0.05404	0	1.402402	0	0.000292	0	1.182145	0	6.57E-11	0.000278
F_{158}	4	69	1216	981	0	16	0	0	0	237	0	3	0
F_{159}	872	4501	39325	30456	0	184	0	5	2	2291	0	7	0
F_{160}	902	5183	42519	31922	0	162	0	3	0	2172	4	4	0
F_{161}	-275	-275	-113	-275	-275	-274	-275	-258	-261	-256	-183	-254	-275
F_{162}	44.544981	52.129616	30.597138	20.07656	10.14591	5.748371	12.53484	11.45235	6.502144	23.68987	8.6790533	7.368844	0
F_{163}	202.77157	837.04297	7842.5641	114.4709	0.056831	26.18298	4.33E-11	0.044349	0	580.7981	6.38E-11	9.68E-06	3.99E-06

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

F													

F_{164}	-1059.204	-913.375	-1384.204	-1867.043	-1831.077	-1954.614	-1816.941	-1816.934	-1788.668	-1691.114	-996.7384	-1816.941	-1958.29
F_{165}	9303665.9	25869441	14817537	43784778	-5554.275	169987.3	-1984.163	-16727.02	-19315.56	1618360	2.5391805	-18020.96	-20819.03
F_{166}	16497.626	32466.607	28874.986	0.370112	4.380985	44.14838	0	0.982075	14.72043	61.02442	0	8.31289	49.20797
F_{167}	20949.939	120274.41	1042713.5	2482351	29.24161	4168.16	66.50678	247.2805	101.0643	59870.29	146.44846	105.489	1.000126
F_{168}	0.7363825	0.7797681	0.5982099	0.318961	0.0224	0.01117	0.183502	0.420824	0.221659	0.606428	2.03E-12	0.121701	4.03E-09
F_{169}	0	0	0	0	0	0	0	0	0	0	0	0	-1
F_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{171}	1.68E-10	6.56E-09	0	0	0	0	0	0	0	0	0	0	0
F_{172}	1.132E+14	9.22E+15	5.618E+12	0.000521	0.003599	0.030753	132261.3	0.01998	3.91E-07	1.955703	7.29E-07	1.97E-07	3.18E-10
F_{173}	4.00E-17	5.84E-16	5.62E-18	4.08E-21	1.54E-25	1.09E-21	1.38E-32	1.03E-24	9.17E-41	3.89E-20	1.04E-17	1.65E-31	-1
F_{174}	1.53E-05	1.22E-05	1.72E-05	9.05E-07	1.87E-06	7.02E-06	3.56E-06	7.52E-06	5.16E-06	1.83E-06	3.15E-06	3.96E-06	5.15E-06
F_{175}	1127.5713	1657.1601	499.08069	43.05743	331.0613	94.16721	101.936	0.033303	0.001044	145.7924	1.5919177	5.14E-08	5.12E-10

TABLE S6. **The mean values of the 50D mathematical functions for different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{118}	19.40182	20.97241	20.303699	20.50565	1.111686	4.372899	19.93105	3.203908	2.104718	11.92346	19.972762	2.283903	1.745419
F_{119}	79.416307	95.462007	73.884755	8.546599	0.050886	0.767617	9.911618	7.655639	0.059464	29.36743	0.2700079	4.277821	0.21562
F_{120}	1878.6079	45013.733	2808.0318	0.050099	4.82E-05	0.384547	98.87	0.000402	7.85E-07	4.123768	8.70E-07	4.72E-11	0.002213
F_{121}	1888432.5	75539630	4.118E+09	1.72E+09	0.001718	67983.71	1.01E+08	0.024618	2.51E-10	12912652	9.44E-05	0	1.15E-06
F_{122}	2.4789431	6.4194451	1.3944393	1.11E-11	0	2.24E-06	0.069512	0	2.71E-07	0.000566	7.17E-05	0	5.06E-07
F_{123}	-0.551589	-0.547911	-0.601127	-0.990587	-0.990548	-0.997855	-0.954	-0.989784	-0.9996	-0.581653	-0.510564	-0.960738	-0.990535
F_{124}	-1	-0.176442	-0.402818	-0.997335	-0.991149	-0.997994	-0.92685	-0.991312	-1	-0.634675	-0.481712	-0.954035	-0.939278
F_{125}	1086943.8	2542554.6	1668727.3	490.3996	12.07584	204.4145	280651.2	3.041391	0.715495	7830.366	4.3463603	0.914635	2.966192
F_{126}	-3.31E-161	-1.21E-263	-4.42E-184	-0.01718	-0.181686	-0.128316	-0.026879	-0.249753	-0.490798	-8.00E-23	-9.21E-108	-0.050544	-0.959867
F_{127}	-0.966256	-0.657912	-0.045621	-0.999668	-0.999998	-0.987242	-0.79695	-0.999992	-1	-0.722972	-1	-1	-0.999679
F_{128}	1.5254576	3.1171153	16.902049	11.31371	0.013305	1.064283	1.704358	0.021806	0.009114	1.98247	0.066516	0.006942	0.002814
F_{129}	274990.84	673935.96	420887.48	82.16855	6.14E-07	26.97812	51900	1.15E-05	0	1808.744	6.0970297	0	1.92E-05
F_{130}	8.135E+14	1.44E+16	1.18E+18	8.73E+18	3.7E+11	6.99E+11	9.06E+16	1.26E+16	72907.3	2.53E+16	31.152038	7.45E+15	1.08E+13
F_{131}	-4.536875	-2.999024	-5.41331	-10.03256	-27.43647	-36.60633	-17.93042	-8.734473	-19.56688	-8.064489	-46.52209	-17.69624	-45.77882
F_{132}	175.26204	210.61038	191.53819	2.660342	8.472031	0.939074	58.61352	33.86282	5.457659	15.52962	4.0953339	10.34334	1.442194
F_{133}	2	2	1.23E+20	2.024891	2	3.732827	2	2.232081	2	3.850493	5.63E+26	2.029913	2
F_{134}	2	2	1.66E+20	2.02534	2	3.909418	2	2.358089	2	4.070052	7.26E+26	2.141498	2
F_{135}	2.58E+123	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128
F_{136}	0.4201111	0.5901758	0.0949211	8.95E-07	4.66E-13	1.28E-06	4.46E-07	1.98E-12	0	6.99E-05	0.0602881	0	0
F_{137}	22.436507	22.823921	21.391758	13.12133	11.58711	7.065165	8.126004	19.01225	15.21277	18.30883	16.904158	15.94676	0.055795
F_{138}	15413.681	18151.535	34878.315	5395.454	5016.777	1500.041	10218.51	6914.513	4069.187	12725.53	0.0001707	6981.349	1439.717
F_{139}	6499.7135	15330.958	5060.1547	9.935475	0.565916	11.72493	1550.835	1.440063	0.013619	198.7328	2.2452353	0.661296	2.049922
F_{140}	24822.105	64750.889	56270.943	27.68775	0.432378	64.71576	11306.81	0.149561	6.18E-10	974.7189	0.0415169	0.047538	1.113144
F_{141}	0.3769163	0.9448624	0.0560315	1.28E-08	6.11E-10	3.37E-10	0	2.71E-08	0	2.28E-06	6.10E-07	4.30E-08	2.36E-08
F_{142}	528.89822	587.60854	557.65635	155.6537	8.174678	21.72945	277.3696	211.2711	86.54141	360.6257	18.873578	84.86987	11.6083
F_{143}	1.17E+11	3.157E+11	1.208E+11	3.74E+11	162.5558	8472930	3.75E+09	669.7863	0.025505	5.12E+08	732316.84	2.094801	5.698438
F_{144}	168529.62	457171.72	119514.83	150.026	5.355107	42.6829	9487.6	24.27905	4.103448	295.2548	172.0276	59.58372	42.39186
F_{145}	135215998	371554430	146185085	1864485	412.9542	14710.07	5615204	459.8805	70.01599	688477.2	1697.4303	92.05365	68.20301
F_{146}	9.4520066	12.345691	29.683898	21.19853	1.208077	3.134814	10.09587	0.830873	1.391873	7.541142	0.4101407	1.011873	0.32129
F_{147}	186455269	510874877	195870147	1.11E+08	0.00014	14392.12	6000000	0.003137	0	796691	735.17913	0	1.06E-06
F_{148}	2.61E+18	7.31E+23	8.39E+28	5.52E+27	2.08E-07	4.46E+14	6.16E+25	3.18E-05	0	2.45E+21	1.97E-09	0	0

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

F_{149}	0.0006232	0.0029835	8.77E-07	2.54E-05	6.77E-07	1.72E-05	0	9.25E-09	3.96E-10	4.37E-07	2.04E-05	0	0
F_{150}	4200.8833	5339.4413	21794.144	2.475727	0.0091	13.29967	29.51354	0.011879	0.000245	22.76345	46.851723	1.72E-09	0.29535
F_{151}	26.19331	155.93624	1436.282	1096.032	22.46229	49.95441	233	4.122351	3.07426	374.1819	1.53E-08	14.26094	18.87974
F_{152}	84.609868	91.587687	81.946879	83.56584	3.152774	17.55798	77.49465	1.319453	0.607331	24.42314	37.101274	9.317174	5.9141
F_{153}	3.87E+57	7.86E+66	2.44E+60	3.62E+56	60.84896	20.65112	838	2.65E+44	312.796	2.19E+42	2.67E-08	5.84E+28	31.80445
F_{154}	6.229E+09	1.785E+10	1.406E+09	7.798554	0	2.122973	9.92E-12	0	0	4983.347	594401.65	0	7.27E-11
F_{155}	4156.2622	5407.2687	21677.444	2.573681	0.011775	13.3641	29.04415	0.01336	0.000288	22.47355	45.851989	1.91E-09	0.382021
F_{156}	-418.983	-418.983	-164.2208	-133.7004	-322.9354	-415.1678	-279.2633	-261.5968	-213.362	-203.6391	-109.0789	-248.7864	-338.6572
F_{157}	14.862472	41.828595	810.99161	0.078156	0	2.563221	0	0.000656	0	2.076542	0	1.13E-10	0.000896
F_{158}	11.83	130.78	1449.62	1076.81	14.43	26.48	221.97	6.91	4.64	340.71	0.01	31.98	0.25
F_{159}	1957.65	8582.01	64688.57	41443.82	4.19	265.63	5200.55	14.96	31.64	3606.93	0.15	23.18	6.26
F_{160}	2095.99	8877.77	68110.18	41086.85	4.11	252.39	6394.23	16.12	33.68	3659	8.04	24.51	4.03
F_{161}	-275	-275	-82.36	-273.9	-275	-268.86	-274.89	-250.01	-228.99	-253.39	-165.05	-234.15	-275
F_{162}	52.615087	58.530487	41.695128	27.23315	13.87136	7.44419	17.48889	17.18191	8.432815	29.9674	40.769231	9.690253	2.701206
F_{163}	345.81203	1596.0068	13868.016	153.5335	1.202342	47.88789	2834	0.721726	8.64E-07	898.8876	0.0001918	0.173989	0.751827
F_{164}	-881.4242	-674.2096	-1186.014	-1770.141	-1707.663	-1951.366	-1708.056	-1685.605	-1673.595	-1619.258	-899.7664	-1662.851	-1911.064
F_{165}	15427864	37675572	29756392	63850282	44429.67	417435.9	4076123	48435.16	-2222.151	2471195	2210.9317	167.1344	-9329.129
F_{166}	32663.974	60487.765	47867.064	3.836854	52.17777	71.36089	1.172542	14.60744	68.77643	108.2809	0	73.37087	91.02908
F_{167}	52032.439	210033.38	1588279.6	3002533	91.84935	6961.246	150465.1	338.8702	597.5366	91384.31	201.34811	230.8234	59.52747
F_{168}	0.7905762	0.8436391	0.7198913	0.39861	0.057987	0.02897	0.383731	0.533291	0.352506	0.664914	0.0180959	0.349684	0.02306
F_{169}	0	0	0	0	0	0	0	5.28E-280	4.6e-322	2.37E-302	0	0	-1
F_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{171}	2.24E-08	2.07E-07	1.57E-10	8.89E-14	0	0	0	0	0	0	0	0	0
F_{172}	1.72E+17	1.03E+20	8.14E+16	2.869414	7.64E+10	263.7135	1.60E+17	1.96E+10	6.262739	583.2442	1981.0337	30447.08	0.000331
F_{173}	9.84E-16	5.68E-15	1.62E-15	5.62E-21	8.46E-25	1.61E-21	9.69E-20	2.31E-24	1.21E-26	1.24E-19	2.26E-16	3.40E-31	-0.257182
F_{174}	0.0028791	0.0088382	0.0009519	0.000714	0.000698	0.000669	0.001016	0.000856	0.000551	0.000811	0.0009545	0.000782	0.000942
F_{175}	5650.4878	11223.99	791.85059	99.40362	536.6168	143.9092	666.5562	0.065079	0.005908	280.9615	19.084993	2.78E-06	0.279898

TABLE S7. The standard deviation values of the 50D mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{118}	0.684914	0.086512	0.121584	0.050224	0.2602	0.313316	0.383269	6.133982	0.637222	0.440251	2.868035	0.684552	1.087813
F_{119}	5.840935	6.486329	7.305176	1.509117	0.04356	0.150857	8.147383	3.045156	0.167142	2.75843	1.454399	1.891412	0.277777
F_{120}	1096.979	144505.3	4804.165	0.005506	3.43E-05	0.067126	135.6789	8.81E-05	7.34E-06	0.550266	3.00E-06	7.58E-12	0.001757
F_{121}	1076548	34603873	1.07E+09	3.04E+08	0.003008	24225.86	2.23E+08	0.010664	2.46E-09	3324095	0.00062	0	1.30E-06
F_{122}	0.705621	1.318535	0.555437	5.55E-12	0	1.01E-06	0.321837	0	3.07E-07	0.000232	0.000247	0	6.08E-07
F_{123}	0.013663	0.013119	0.036168	0.001114	0.011849	0.000464	0.034757	0.015327	0.002814	0.014446	0.022749	0.024882	0.01956
F_{124}	0	0.016064	0.15719	0.005749	0.012433	0.000373	0.041575	0.01308	3.53E-08	0.015853	0.02756	0.027507	0.007901
F_{125}	254437.9	769758	449395.8	102.6909	7.189871	59.70666	424664.4	3.598259	0.341348	2121.373	9.475153	0.510259	3.627702
F_{126}	3.31E-160	0	0	0.050291	0.385149	0.050782	0.154177	0.43469	0.501633	4.19E-22	6.51E-107	0.218921	0.196432
F_{127}	0.007413	0.063929	0.021164	3.13E-05	1.14E-06	0.002225	0.250803	1.65E-06	1.68E-06	0.018615	4.76E-07	5.37E-13	0.000257
F_{128}	0.146192	0.49139	2.131935	0.932221	0.016208	0.011433	2.280675	0.008409	0.014467	0.144451	0.155274	0.009319	0.006353
F_{129}	64185.06	242811.5	115981.8	21.96334	8.74E-07	8.007719	71388.75	6.27E-06	0	499.907	27.66151	0	2.11E-05
F_{130}	2.69E+14	5.20E+15	6.00E+17	2.24E+18	1.25E+12	5.92E+11	1.42E+17	5.56E+15	583509.4	9.83E+15	108.6685	5.08E+15	1.47E+13
F_{131}	0.504711	0.411566	0.664565	1.273413	2.286376	1.04247	3.404953	1.508705	2.969567	0.564951	6.55176	3.902412	6.52497
F_{132}	25.36563	37.98454	27.93861	0.358945	4.115593	0.173089	21.01025	13.73918	3.174216	2.932419	0.528696	4.10063	1.002663
F_{133}	0	0	4.26E+20	0.002282	0	0.326037	0	1.435351	1.49E-12	0.463529	2.93E+27	0.098329	0

F_{134}	0	0	6.19E+20	0.002279	0	0.452594	0	2.223839	3.98E-13	0.476669	2.38E+27	0.247446	0
F_{135}	4.86E+123	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113	7.73E+113
F_{136}	0.070596	0.137709	0.037879	2.37E-07	1.30E-12	5.67E-07	3.30E-06	1.09E-12	0	3.69E-05	0.024249	0	0
F_{137}	0.198331	0.190065	0.346477	0.458267	0.906758	0.69533	1.017495	0.821637	1.32467	0.428453	0.513856	0.758812	0.158698
F_{138}	979.2929	1093.599	1940.738	411.362	2038.056	584.8083	3558.774	2225.258	1470.221	737.1597	0.000717	1544.12	1290.669
F_{139}	1177.997	2786.31	1388.378	2.823786	0.148072	4.047435	1178.385	0.374095	0.037978	43.08427	6.767522	0.227453	1.01834
F_{140}	5737.557	22043.17	14540.36	3.270047	0.230638	13.77671	10915.66	0.101167	5.75E-09	157.1419	0.315344	0.021289	0.834847
F_{141}	0.167066	0.294469	0.03812	9.39E-09	1.84E-09	1.23E-09	0	1.56E-08	0	2.17E-06	4.02E-06	2.55E-08	3.15E-08
F_{142}	24.85595	25.18078	44.54785	15.83632	1.562745	2.781115	52.70647	42.39096	18.93823	18.04087	28.8979	22.93592	11.35756
F_{143}	2.35E+10	5.66E+10	3.2E+10	3.78E+10	142.7532	3008800	1.49E+10	212.1246	0.187439	1.12E+08	2860548	6.874423	11.85088
F_{144}	43491.27	64925.09	38242.74	13.01407	2.908578	4.740992	26451.35	9.412322	4.007501	37.30043	46.90935	18.88619	25.08388
F_{145}	27629079	81251368	41827183	435984.7	462.8349	4509.79	20514405	715.8034	39.13399	193062.8	7931.85	87.42163	83.69783
F_{146}	0.859361	0.966472	1.14066	0.822021	0.131797	0.27839	4.919019	0.099184	0.307706	0.445421	0.259905	0.150608	0.117855
F_{147}	41802814	91440695	51230506	18359676	0.000138	4316.001	23868326	0.001918	0	207745.9	2484.966	0	1.04E-06
F_{148}	3.74E+18	1.00E+24	6.13E+28	2.72E+27	1.84E-06	5.54E+14	4.20E+26	4.30E-05	0	1.72E+21	1.84E-08	0	0
F_{149}	0.00145	0.007704	3.06E-06	7.88E-05	3.86E-06	3.90E-05	0	1.75E-08	1.92E-09	7.75E-07	4.12E-05	0	0
F_{150}	806.9536	1442.516	3635.905	0.266992	0.003123	4.831311	2.595315	0.003942	0.001224	1.640912	0.788832	4.56E-10	0.071412
F_{151}	5.346871	20.65522	104.1589	46.30883	14.8746	6.808391	183.1569	1.39594	3.042545	25.89586	6.60E-08	13.3024	19.58601
F_{152}	3.947156	2.434241	2.271104	4.747136	0.565357	1.236382	6.673991	0.505237	0.194445	1.61258	11.26537	2.328283	3.218648
F_{153}	1.41E+58	3.36E+67	1.00E+61	9.27E+56	36.76119	3.383883	319.0199	2.29E+45	388.9973	1.73E+43	2.41E-07	4.12E+29	31.95237
F_{154}	2.35E+09	4.16E+09	9.66E+08	5.409731	0	1.492805	9.88E-11	0	0	3221.808	3316395	0	2.15E-10
F_{155}	991.9518	1574.487	3594.309	0.308288	0.003287	4.352477	3.0159	0.005294	0.00104	1.774559	0.7946	5.31E-10	0.100812
F_{156}	1.14E-13	1.14E-13	20.44992	6.369048	13.4271	0.851663	29.2177	17.46831	26.47692	9.500071	6.660084	20.73726	37.17983
F_{157}	4.480967	10.75861	60.47796	0.010985	0	0.498588	0	0.000179	0	0.465367	0	2.03E-11	0.000306
F_{158}	4.942201	20.73234	94.31473	39.58974	14.23274	5.573168	150.1089	5.335407	4.69799	29.23029	0.1	17.91787	1.233988
F_{159}	607.3874	2145.538	8698.064	3587.548	1.801767	51.69777	7313.631	6.941196	27.7436	435.9462	0.435194	10.58432	4.576223
F_{160}	511.0036	2290.618	9566.849	3547.009	1.979159	52.28455	7719.555	7.935701	24.1815	509.7214	3.113736	10.96781	4.154722
F_{161}	0	0	6.873342	0.745356	0	2.251913	1.1	4.939012	11.01743	1.118215	6.918538	10.81	0
F_{162}	2.203079	2.625224	3.779583	2.763368	1.72733	0.784343	2.428613	2.254187	1.093047	1.906266	8.336864	1.470039	5.731932
F_{163}	88.2401	402.9592	2352.88	15.23495	1.183623	10.10451	2277.213	0.612532	5.74E-06	120.1772	0.001278	0.269881	1.084589
F_{164}	69.93247	81.17004	82.35055	48.11296	57.35702	1.35495	53.20484	52.98244	43.7466	29.85022	53.95108	57.09753	37.5487
F_{165}	2621352	6375348	4720233	6532203	31694.02	133640.9	3405838	51242.44	9393.244	350078.9	8596.776	9202.788	14090.57
F_{166}	6034.147	8983.535	7741.284	3.202337	35.00407	14.24055	11.72542	14.07071	51.42458	24.57744	0	54.05888	28.55147
F_{167}	16301.5	49148.29	246741.5	203135.9	25.88208	1367.591	204025.3	56.11409	352.6286	11451.32	28.66076	53.40162	41.12593
F_{168}	0.016184	0.013709	0.030334	0.031347	0.018745	0.006854	0.088188	0.045449	0.074429	0.02077	0.053637	0.098651	0.044122
F_{169}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{171}	2.55E-08	2.14E-07	3.11E-10	3.68E-13	0	0	0	0	0	0	0	0	0
F_{172}	2.44E+17	2.57E+20	1.36E+17	10.93318	7.64E+11	1157.909	1.33E+18	1.96E+11	32.5155	2082.234	16391.19	244294	0.002068
F_{173}	6.73E-16	2.90E-15	2.01E-15	6.27E-22	5.36E-25	2.05E-22	2.36E-19	8.02E-25	1.02E-25	5.24E-20	2.00E-16	1.64E-31	0.419362
F_{174}	0.002688	0.007845	0.000851	0.000679	0.00066	0.00077	0.001166	0.00088	0.000451	0.000807	0.001007	0.000905	0.000685
F_{175}	2381.55	10096.52	103.2517	21.9485	103.9273	18.46808	198.9078	0.018169	0.004718	61.72362	14.54401	5.51E-06	0.263768

TABLE S8. **The mean values of the function evaluations for the 50D mathematical functions in different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{118}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	132091.6
F_{119}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{120}	150000	150000	150000	150000	150000	150000	150000	150000	52100	150000	149215.5	150000	149106
F_{121}	150000	150000	150000	150000	150000	150000	138907.5	150000	30805	150000	139349.5	140276.5	145274.6
F_{122}	150000	150000	150000	150000	41766.41	150000	150000	147521.5	150000	150000	148562	97067.5	131443.3
F_{123}	150000	150000	150000	150000	150000	150000	148774.5	150000	125714.5	150000	150000	150000	142901.7
F_{124}	101.2146	150000	150000	150000	150000	150000	149734.5	150000	120993	150000	150000	150000	150000
F_{125}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{126}	150000	150000	150000	150000	150000	150000	150000	150000	142451.5	150000	150000	150000	149979.2
F_{127}	150000	150000	150000	150000	150000	150000	145852.5	150000	114524.5	150000	147229	149747.5	148714.3
F_{128}	150000	150000	150000	150000	150000	150000	150000	150000	142576	150000	150000	150000	150000
F_{129}	150000	150000	150000	150000	150000	150000	148417	150000	23152	150000	150000	129305	148894.2
F_{130}	150000	150000	150000	150000	150000	150000	150000	150000	130025	150000	150000	150000	150000
F_{131}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{132}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{133}	243.9271	201	150000	150000	15703.33	150000	10895	150000	23197.5	150000	150000	150000	85.72
F_{134}	238.8664	199	150000	150000	15933.87	150000	11195.5	150000	19961	150000	150000	150000	85.72
F_{135}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{136}	150000	150000	150000	150000	120558.5	150000	17476	149991	15850	150000	150000	111302	55.9
F_{137}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	143858.4
F_{138}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{139}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{140}	150000	150000	150000	150000	150000	150000	150000	150000	148582.5	150000	150000	150000	150000
F_{141}	150000	150000	150000	150000	94860.59	143420.4	65846.5	150000	5710.5	150000	141844	150000	146670.6
F_{142}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{143}	150000	150000	150000	150000	150000	150000	150000	150000	122143.5	150000	150000	150000	150000
F_{144}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{145}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{146}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{147}	150000	150000	150000	150000	150000	150000	149952	150000	26785.5	150000	150000	137140.5	147463.4
F_{148}	150000	150000	150000	150000	149266.6	150000	117567.5	150000	16181	150000	94791.5	114572.5	50010.92
F_{149}	150000	150000	149748.5	150000	90855.83	150000	5358.5	149997.5	69382	150000	150000	123065.5	1010.56
F_{150}	150000	150000	150000	150000	150000	150000	150000	150000	149760.5	150000	150000	150000	150000
F_{151}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	147668	150000	150000
F_{152}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	149391.8
F_{153}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	147196.5	150000	150000
F_{154}	150000	150000	150000	150000	61049.13	150000	136833	148466	18211.5	150000	150000	99465.5	134719
F_{155}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{156}	696.3563	341	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{157}	150000	150000	150000	150000	14458.86	150000	13165.5	150000	13175.5	150000	866	150000	150000
F_{158}	150000	150000	150000	150000	141662.5	150000	140680.5	149910	136689.5	150000	71646	150000	41521.72
F_{159}	150000	150000	150000	150000	149246.2	150000	120430	150000	150000	150000	97432.5	150000	126524.5
F_{160}	150000	150000	150000	150000	148822.3	150000	125623.5	150000	149860.5	150000	150000	150000	99777.3
F_{161}	246.9636	209	150000	149195.1	4429.106	150000	6501	150000	150000	150000	150000	150000	95.4
F_{162}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	117788.1
F_{163}	150000	150000	150000	150000	150000	150000	150000	150000	67443.5	150000	150000	150000	150000

***F*₁₆₄**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₆₅**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₆₆**	150000	150000	150000	150000	150000	150000	15908	150000	150000	150000	868	150000	150000
***F*₁₆₇**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₆₈**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₆₉**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	9.38
***F*₁₇₀**	101.2146	50	898.2036	1219.512	51.00306	20	50	50	50	3750	50	50	2
***F*₁₇₁**	150000	150000	141071.9	96743.9	1023.121	565.8	438	2852.5	2274	12037.5	80	58	9.32
***F*₁₇₂**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₇₃**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₇₄**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₁₇₅**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000

TABLE S9. **The minimum values of the 100D mathematical functions for different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
***F*₁₇₆**	20.89596	20.93462	20.45163	20.5866	2.17242	10.92787	19.90531	1.471331	2.348442	14.93472	0.134755	2.344727	0
***F*₁₇₇**	210.434	221.4995	163.2754	50.40556	0.967872	11.64745	0.038548	16.73482	0.335449	87.59161	0.026219	5.514185	1.05E-05
***F*₁₇₈**	13026596	2.48E+08	74153.12	0.477271	0.000174	17.98591	232	0.004405	2.86E-07	17.52525	0.000579	2.11E-10	1.27E-07
***F*₁₇₉**	3.77E+10	4.99E+10	2.65E+10	1.78E+10	0.017347	68257512	0.640726	2.56421	0	2.47E+08	0.152026	0	0
***F*₁₈₀**	10.25021	10.25021	4.162479	6.10E-06	0	0.016441	3.27E-07	1.12E-11	6.36E-08	0.007358	0.003902	3.35E-11	0
***F*₁₈₁**	-0.52832	-0.525356	-0.58909	-0.96588	-0.99967	-0.97865	-0.99999	-0.99839	-1	-0.52263	-0.48132	-0.98008	-1
***F*₁₈₂**	-1	-0.283461	-0.464488	-0.997313	-0.99004	-0.97887	-0.97375	-0.99776	-1	-0.56098	-0.48896	-0.99	-0.89219
***F*₁₈₃**	16308362	18181374	9432252	29900.16	29.50961	104826.3	196.0569	2.608706	0.6667	160556.5	1463.005	0.682417	0.259289
***F*₁₈₄**	0	0	0	0	0	0	0	0	0	0	0	0	-1
***F*₁₈₅**	-7.82E-05	-1.41E-05	-0.000292	-0.996165	-0.99999	-0.6616	-0.99996	-0.99991	-1	-0.24912	-0.99998	-1	-1
***F*₁₈₆**	46.07033	56.83658	41.69738	34.3526	0.010456	3.065453	0.015818	0.088745	3.02E-08	5.604099	0.164562	7.34E-09	3.44E-10
***F*₁₈₇**	4505297	4505297	2314961	6058.917	1.77E-05	27231.63	0.505467	0.001183	0	34676.12	174.7608	0	4.31E-10
***F*₁₈₈**	1.18E+30	4.34E+31	5.28E+32	2.58E+33	4.63E+23	1.40E+26	4.45E+29	2.07E+30	2.08E+08	1.16E+31	1.91E+13	1.34E+30	1.08E+24
***F*₁₈₉**	-6.976021	-5.39471	-9.766152	-15.43269	-41.3594	-69.6636	-50.2169	-16.5915	-49.6918	-11.0615	-98.6048	-60.0013	-99
***F*₁₉₀**	679.0422	792.8587	384.1515	35.3898	19.75085	21.35196	83.7425	56.67836	4.357945	79.31911	10.69817	7.954225	0.013189
***F*₁₉₁**	2	2	3.39E+54	2.09823	2	678.7932	2	2.31004	2	116.6447	2.13E+61	2.001678	2
***F*₁₉₂**	2	2	4.40E+54	2.105227	2	2521.474	2	2.28386	2	295.6178	5.76E+61	2.001319	2
***F*₁₉₃**	0	0	0	0	0	0	0	0	0	0	0	0	0
***F*₁₉₄**	0.41282	0.55269	0.10512	1.11E-05	3.05E-12	0.000294	0	1.08E-10	0	0.001693	0.219069	0	0
***F*₁₉₅**	46.02936	46.40994	43.50457	27.90756	26.95662	17.19637	14.53504	40.07528	28.86502	40.6067	35.73577	34.06033	0
***F*₁₉₆**	141540	173548.8	150077.8	42154.22	23498.87	19258.21	31366.09	28784.24	11528.62	69182.4	0.012522	20128.29	7.702008
***F*₁₉₇**	27014.92	36967.89	15765.12	117.6082	3.196483	580.3716	813.7357	6.945084	0.018717	1053.796	5.344801	1.212761	3.900297
***F*₁₉₈**	333012.9	333012.9	154909.2	284.4142	0.154683	3805.585	535.5444	0.424549	1.61E-05	6025.846	0.152571	0.002657	0.007504
***F*₁₉₉**	0.451022	0.451022	0.028822	3.73E-08	0	1.32E-10	0	1.44E-08	0	1.90E-06	4.89E-07	5.43E-09	0
***F*₂₀₀**	1440.284	1491.802	1145.241	522.0891	38.18557	132.9979	427.105	388.408	110.1731	858.6743	1.512274	62.6824	0.000349
***F*₂₀₁**	6.57E+11	6.57E+11	3.72E+11	6.57E+11	1452.665	4.35E+09	30431.45	8884.005	0.020276	5.94E+09	85748565	0.521437	67.04614
***F*₂₀₂**	884535.9	912746.7	368864.8	613.3301	43.70947	2719.022	5.043051	64.55151	28.20927	2647.178	461.503	93.24978	0.69697
***F*₂₀₃**	8.47E+08	8.47E+08	4.79E+08	36057720	487.3531	7265701	982.2989	119.8934	85.68383	6628239	611203.5	92.29443	0.000333
***F*₂₀₄**	45.38333	49.08555	44.10004	35.72866	2.339662	9.919722	13.89987	1.499873	1.699873	15.11964	1.500128	2.199873	0.081718
***F*₂₀₅**	1.05E+09	1.05E+09	5.96E+08	5.38E+08	0.001903	8042041	91.6712	0.188584	0	8120718	142223.8	0	0
***F*₂₀₆**	5.52E+31	1.24E+32	1.86E+31	5.64E+30	5.22E-06	3.18E+23	0.263037	15.41746	0	7.99E+24	0.003514	0	0

F_{207}	9.86E-08	6.03E-08	4.56E-12	2.90E-09	0	7.61E-10	0	3.53E-12	0	9.80E-10	8.98E-11	0	0
F_{208}	45469.84	71819.64	46017.91	15.43768	0.005149	954.7067	70.54275	0.132864	0.062016	67.63235	80.73163	1.40E-08	11.15298
F_{209}	2078.136	3971.691	3171.686	2779.674	83.29787	338.9585	101.1943	17.85311	20.85703	1140.816	1.57E-07	11.50819	0.141442
F_{210}	92.48447	92.82625	85.47049	92.82625	17.78914	45.71347	85.39617	8.948306	6.371002	46.03091	66.48675	14.20515	0.147862
F_{211}	6.80E+119	1.86E+134	2.24E+120	5.34E+118	143.0136	129.6503	802.3115	7.00E+68	137.215	2.94E+90	5.87E-08	1.69E+41	0.079772
F_{212}	2.79E+10	3.23E+10	4.89E+09	113932.5	2.70E-12	1819339	1.746243	6.70E-11	0	1503891	34620928	0	0
F_{213}	43396.05	70339.89	45995.68	16.00331	0.003656	888.4887	68.16797	0.130584	0.01827	70.49078	79.33921	1.32E-08	11.53787
F_{214}	-418.983	-418.983	-164.2404	-108.9406	-294.364	-398.82	-320.041	-276.173	-258.916	-165.068	-92.7076	-305.726	-418.983
F_{215}	1098.147	1872.414	1672.404	0.527636	0	85.29447	0	0.008924	0	15.82846	0	5.14E-10	0.013082
F_{216}	2252	3922	3287	2681	69	297	18	15	33	1177	0	71	0
F_{217}	185674	223510	158619	129887	16	8154	31	43	206	18706	2	46	0
F_{218}	187473	223877	167415	130660	14	8707	11	36	124	16897	28	59	0
F_{219}	-575	-575	-185	-563	-575	-523	-575	-512	-489	-506	-294	-470	-575
F_{220}	105.5261	109.3918	88.90376	70.47091	26.40239	26.94958	29.10177	32.71195	13.60358	71.45726	36.965	17.03246	0
F_{221}	66849.26	102537.3	73822.58	2382.539	7.34899	3020.655	1204.926	3.272758	4.97E-06	8121.605	0.408471	0.119647	1.61E-05
F_{222}	-1543.396	-1455.445	-2168.354	-3505.919	-3534.89	-3823.02	-3563.01	-3506.54	-3549.05	-2765.25	-1774.87	-3492.51	-3916.58
F_{223}	1.62E+09	2E+09	1.16E+09	2E+09	855798.8	81397791	19857691	1079657	74084.7	1.5E+08	260684.3	21391.8	-88024.4
F_{224}	333640	710308.5	405801.9	23.70293	82.01857	2968.322	0	216.4425	80.59963	2471.825	0	226.5979	1023.476
F_{225}	5086435	5588993	4100536	5588993	269.502	195795.3	668.7044	867.3468	2445.109	430581.5	506.4798	554.1788	1.003556
F_{226}	0.817745	0.857868	0.703502	0.47082	0.119539	0.083849	0.277957	0.504413	0.239103	0.747206	9.74E-05	0.096257	0
F_{227}	0	0	0	0	0	0	0	0	0	0	0	0	-1
F_{228}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{229}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{230}	4.05E+45	5.12E+48	1.55E+39	58691612	1.9E+12	3.73E+11	1.42E+35	71555904	0.016525	2.45E+14	1345.72	0.05095	1.40E-08
F_{231}	2.84E-32	9.55E-29	2.66E-30	9.53E-41	1.14E-42	5.05E-42	3.14E-42	1.15E-42	1.14E-42	5.49E-37	2.47E-32	1.14E-42	-0.92995
F_{232}	3.23E-05	0.000205	1.11E-05	2.35E-05	3.89E-06	2.79E-06	2.64E-05	1.99E-05	6.98E-07	1.54E-07	2.67E-05	1.01E-05	1.05E-05
F_{233}	8196.895	5688.103	1380.746	603.0868	900.9533	615.6945	1307.55	3.645146	27.57664	825.8996	103.0208	96.94373	0.001714

TABLE S10. The mean values of the 100D mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{176}	21.07813	21.11641	20.69164	20.71885	2.617836	11.84391	19.96214	12.12367	3.864352	15.84191	20.37482	3.675916	3.76174
F_{177}	233.3898	249.1393	186.6833	64.10933	5.398022	15.25338	25.10505	29.48524	2.256215	103.5661	7.459412	11.77882	3.754582
F_{178}	3.7E+10	5.79E+12	1.14E+08	0.680487	0.000811	22.38114	1702.96	0.00665	0.00576	21.9173	0.346841	2.91E-10	0.069832
F_{179}	5.73E+10	7.2E+10	3.94E+10	2.29E+10	0.352729	1.17E+08	6.26E+08	6.625545	717.7019	4.08E+08	1593967	0	0.000655
F_{180}	16.7472	17.45345	7.795704	3.20E-05	3.90E-14	0.029701	0.488869	8.11E-10	1.13E-06	0.032897	0.196039	2.02E-07	9.06E-06
F_{181}	-0.479691	-0.479445	-0.529808	-0.950316	-0.94449	-0.9714	-0.94118	-0.974328	-0.99719	-0.493804	-0.44267	-0.94436	-0.97979
F_{182}	-1	-0.161596	-0.276881	-0.990482	-0.95263	-0.97205	-0.9066	-0.96529	-0.99697	-0.538184	-0.43503	-0.93447	-0.85757
F_{183}	26020161	27210531	15187106	55885.03	76.01499	178043.1	1775663	27.72207	20.61683	266535.8	141629.8	5.625352	28.82514
F_{184}	0	0	0	0	0	0	0	0	0	0	0	0	-1
F_{185}	-6.23E-06	-1.96E-06	-6.30E-05	-0.993911	-0.99997	-0.58497	-0.43328	-0.999874	-0.99965	-0.197847	-0.96413	-1	-0.99414
F_{186}	60.76361	68.01628	50.49601	38.76511	0.033491	3.687095	5.930224	0.148389	0.081905	6.971821	1.112249	0.003276	0.006986
F_{187}	6516133	6781921	3647003	13693.94	0.000181	42560.13	446925.7	0.003632	0	63592.87	29333.13	0	0.018678
F_{188}	2.01E+30	7.09E+31	2.28E+33	1.03E+34	5.27E+26	8.08E+26	1.31E+32	1.44E+31	1.52E+20	2.85E+31	4.34E+17	7.69E+30	1.42E+28
F_{189}	-5.397696	-3.677028	-7.288278	-10.68408	-35.2754	-65.4355	-31.4205	-11.84984	-34.2504	-9.378408	-65.9831	-32.9195	-89.2355
F_{190}	844.3482	951.0533	567.75	57.92845	41.59467	28.48154	154.1385	105.8916	15.00436	127.4225	60.56308	18.55855	6.919981
F_{191}	2	2	3.07E+63	2.116226	2	16189.81	80.23	2.613721	2	1387.417	6.10E+73	2.39771	2

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

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F_{192}	2	2	6.63E+63	2.117519	2	15650.38	3.319177	2.726324	2	3131.457	3.28E+73	2.804208	2
F_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{194}	0.803752	0.866963	0.270761	3.20E-05	4.03E-11	0.000635	3.53E-05	2.09E-10	3.73E-08	0.004525	0.387955	0	0
F_{195}	46.80828	47.34441	45.32491	30.32896	31.64453	20.35608	18.23	42.39087	33.30477	42.49258	37.91084	36.94009	0.254986
F_{196}	161217.2	200555.7	166562	50103.32	38728.99	25298.55	60808.54	45524.26	23902.14	73673.03	65.44364	31453.53	12544.74
F_{197}	49674.05	61283.55	26670.17	206.0606	9.235068	873.7601	6918.246	14.14625	0.479662	1373.943	422.3927	4.154698	11.92993
F_{198}	445463.4	474307.6	260286.6	471.6474	1.581901	5370.455	29381.25	0.904045	1.365555	8660.795	1444.405	0.165422	5.860861
F_{199}	1.183457	1.302846	0.214572	1.50E-07	1.11E-09	3.66E-08	8.15E-10	6.45E-08	0	5.50E-05	0.005428	4.98E-08	3.04E-08
F_{200}	1567.92	1617.462	1318.956	610.8014	54.91293	155.0488	624.9662	549.6488	178.1936	971.6875	153.3871	147.9701	53.62747
F_{201}	8.6E+11	8.87E+11	5.29E+11	8.87E+11	4170.474	7.44E+09	3.14E+10	15181.68	25456.53	8.81E+09	1.12E+10	79.66955	186.464
F_{202}	1137684	1164996	619887.8	1013.829	88.06371	4279.963	47036.43	114.0298	67.41478	4951.072	16545.57	167.8945	210.1058
F_{203}	1.12E+09	1.14E+09	6.73E+08	61493505	882.6253	9399397	32786787	544.1943	358.3396	11722287	14635218	196.2993	277.3173
F_{204}	49.81411	52.51466	47.19555	39.57762	2.988897	11.48018	25.65387	1.916873	3.328873	17.87065	4.981423	2.750873	0.772582
F_{205}	1.39E+09	1.42E+09	8.52E+08	6.99E+08	0.018083	12104315	59362775	0.501756	0	14237903	17516361	0	0.000682
F_{206}	2.20E+32	3.85E+32	6.40E+31	1.22E+31	0.224299	1.74E+24	2.11E+27	328.502	3.89E+11	6.44E+25	4.90E+20	0	1.05E-09
F_{207}	0.001391	0.010235	1.76E-06	4.35E-05	3.82E-07	3.66E-05	0	1.82E-08	2.00E-09	1.60E-06	3.79E-05	0	0
F_{208}	66123.5	97864.79	74966.87	19.82098	0.01863	1262.973	9340.091	0.269682	1.107763	97.57622	95.78998	6.08E-08	15.84393
F_{209}	2497.106	4299.386	3632.104	2976.799	180.7334	414.5626	535.3266	27.63941	59.61958	1344.116	0.026099	61.28983	138.746
F_{210}	95.53149	95.75496	90.91855	95.75496	31.27479	48.71824	91.73194	22.17755	7.947813	52.34997	76.82625	18.59507	14.20714
F_{211}	5.65E+136	1.81E+147	2.74E+133	8.37E+126	341.2303	211.8277	1755.139	1.82E+121	1305.099	9.04E+108	0.021136	1.03E+71	309.8501
F_{212}	4.72E+10	5.02E+10	1.5E+10	777593.8	2.42E-09	4042059	1E+08	1.30E-09	0	5554166	7E+08	0	0.000125
F_{213}	65599.24	97836.47	74761.17	19.52131	0.024318	1240.962	9737.528	0.279098	1.043199	98.12526	94.7721	1.18E-07	16.62155
F_{214}	-418.983	-418.983	-122.8638	-96.67194	-270.594	-392.253	-251.211	-248.6631	-206.95	-140.2465	-78.0743	-246.564	-291.165
F_{215}	1327.413	2199.699	2163.381	0.668883	0	108.5625	31	0.013944	0	25.2061	0	7.93E-10	0.023988
F_{216}	2475.86	4265.16	3602.84	2936.95	163.81	368.29	532.99	54.7	90.87	1293.46	1.89	164.41	108.52
F_{217}	236466.5	268025.5	201972.8	149515.4	30.14	10530.27	22302.34	84.28	553.16	22806.39	779.92	130.66	65.47
F_{218}	235689.7	268117.9	199881.2	148673.9	31.16	10812.77	21157.96	84.51	502.55	22512.84	908.44	116.37	57.22
F_{219}	-575	-575	-140.07	-557.62	-575	-509.7	-573.13	-477.28	-430.55	-497.23	-264.51	-420.58	-575
F_{220}	127.2437	134.2523	103.3707	82.89332	35.60419	30.48904	37.11001	39.50084	18.00161	85.82308	98.67219	21.21227	18.66609
F_{221}	93836.9	129953.5	92239	3193.669	31.58082	4045.381	14895.24	21.1228	2.708942	10756.37	229.7063	3.969445	12.16475
F_{222}	-1140.626	-1101.693	-1864.449	-3347.853	-3360.44	-3776.79	-3349.75	-3346.24	-3313.71	-2603.598	-1479.74	-3314.47	-3719.93
F_{223}	2.04E+09	2.39E+09	1.62E+09	2.39E+09	2900466	1.16E+08	1.9E+08	3770209	461261.5	2.02E+08	20176597	362938.9	77988.78
F_{224}	540900.8	911738	684690.8	103.7407	461.8877	3892.528	3836.232	421.8379	567.2002	3423.645	0	580.439	1565.34
F_{225}	5931384	6702584	4982508	6702584	511.6174	267328.7	422560.1	1195.725	7443.692	569554.3	20733.39	886.0713	311.0989
F_{226}	0.855487	0.892505	0.79068	0.541511	0.168648	0.106005	0.427615	0.580524	0.362691	0.781505	0.115582	0.375136	0.087171
F_{227}	0	0	0	0	0	0	0	0	1.1525e-319	0	0	0	-1
F_{228}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{229}	3.94E-14	1.39E-11	0	0	0	0	0	0	0	0	0	0	0
F_{230}	1.97E+50	2.12E+55	6.24E+45	6.9E+14	1.25E+32	2.27E+16	4.34E+49	9.28E+21	1.55E+12	3.32E+20	3.72E+24	3.74E+14	0.265472
F_{231}	3.57E-29	1.66E-27	4.07E-28	5.52E-40	1.30E-42	9.17E-42	7.58E-37	1.19E-42	1.15E-42	1.19E-35	5.84E-29	1.49E-42	-0.05574
F_{232}	0.00631	0.018252	0.001643	0.001444	0.001632	0.001851	0.002323	0.002034	0.001256	0.001624	0.001815	0.001859	0.001555
F_{233}	31064.42	105208.5	1.66E+12	831.8967	1249.729	912.565	1973.833	6.612307	56.61749	1229.378	222.8324	205.4873	36.67934

TABLE S11. **The standard deviation values of the 100D mathematical functions for different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	***ACO***	***BA***	***FA***	***GA***	***HS***	***MFO***	***MVO***	***PSO***	***SA***	***SCA***	***SSA***	***CryStAl***
F_{176}	0.062751	0.058223	0.081825	0.033957	0.21153	0.356841	0.014509	8.644434	0.587858	0.242654	2.044851	0.693456	1.129202
F_{177}	8.762598	10.07569	12.08928	5.932246	2.895168	1.455119	14.11509	5.772204	1.362994	6.68551	8.125233	3.324052	3.621405
F_{178}	7.69E+10	1.46E+13	2.13E+08	0.101094	0.000448	1.854499	851.0099	0.001011	0.013496	1.799916	0.446559	3.70E-11	0.03753
F_{179}	7.96E+09	7.13E+09	5.06E+09	2.03E+09	0.321671	21230732	7.53E+08	2.286176	3840.843	79267128	4415621	0	0.000572
F_{180}	1.988681	2.021415	1.532549	2.69E-05	3.90E-13	0.005171	0.88146	1.70E-09	7.12E-07	0.010623	0.157115	2.33E-07	9.35E-06
F_{181}	0.010411	0.010372	0.020631	0.006897	0.021685	0.003037	0.028015	0.015657	0.004607	0.009585	0.014864	0.022144	0.039052
F_{182}	0	0.14496	0.107737	0.007068	0.018802	0.003335	0.037396	0.01801	0.005107	0.009086	0.02006	0.026782	0.014405
F_{183}	3063898	3046914	2167646	13236.32	36.29405	26829.05	1455315	19.90129	32.57516	47378.9	171135.6	5.774793	27.26641
F_{184}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{185}	8.73E-06	1.94E-06	5.13E-05	0.001107	1.18E-05	0.02909	0.251631	1.89E-05	0.001026	0.015911	0.046157	6.68E-13	0.003759
F_{186}	4.376614	3.374358	3.235832	1.686266	0.014	0.247853	3.653589	0.023546	0.143694	0.514413	0.339717	0.005568	0.005315
F_{187}	697989.2	753942.8	553504.3	3880.846	0.000146	5659.864	389920.1	0.001483	0	11506.09	28970.75	0	0.014031
F_{188}	4.45E+29	1.64E+31	9.95E+32	2.80E+33	1.21E+27	5.83E+26	1.81E+32	6.41E+30	1.46E+21	9.98E+30	1.31E+18	5.61E+30	1.45E+28
F_{189}	0.504552	0.55056	0.898547	1.50344	2.495694	2.003742	7.217722	2.056875	5.381337	0.59697	17.9783	8.387703	14.36188
F_{190}	59.02967	61.91329	61.5002	11.45553	13.15083	2.389959	35.48267	21.29752	4.878555	23.76722	34.84678	5.60252	3.790308
F_{191}	0	0	2.83E+64	0.00666	0	14135.16	777.3526	0.357002	3.79E-12	1188.34	4.84E+74	0.527528	0
F_{192}	0	0	6.44E+64	0.005133	0	11312.63	6.704834	0.401119	2.66E-12	6674.581	2.67E+74	1.011699	0
F_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{194}	0.111168	0.121682	0.061624	1.33E-05	3.68E-11	0.000111	0.000158	6.10E-11	2.27E-07	0.002378	0.10007	0	0
F_{195}	0.251374	0.251631	0.482346	0.715345	1.278566	0.985996	1.508346	0.853183	1.607096	0.401927	0.632297	0.786486	0.43151
F_{196}	7991.553	8997.94	5615.485	2908.107	6209.906	2887.383	13109.29	7500.139	4998.456	1956.577	145.2643	4348.979	8741.962
F_{197}	6981.437	8931.01	3933.083	43.23439	2.496527	127.5914	3523.079	2.998599	0.921573	159.1504	369.5591	1.380072	3.797194
F_{198}	41321.24	44714.95	36047.05	90.6102	0.944567	635.0582	16613.95	0.30938	3.194661	1156.337	1774.172	0.084636	2.726384
F_{199}	0.310921	0.337126	0.084549	8.03E-08	3.94E-09	7.47E-08	5.06E-09	3.09E-08	0	5.41E-05	0.010425	3.00E-08	3.74E-08
F_{200}	45.83161	47.84714	62.20738	35.61609	5.482409	12.39901	74.614	68.51942	36.57467	31.90534	85.75176	38.89353	35.99495
F_{201}	6.21E+10	6.97E+10	6.88E+10	6.97E+10	2182.164	1.08E+09	4.01E+10	2938.742	146940.7	1.58E+09	1.28E+10	117.6768	168.9942
F_{202}	98462.42	103565.4	95182.17	298.4557	18.55348	602.5861	59780.26	23.37994	20.37641	1167.358	15488.84	37.76543	78.40406
F_{203}	83793599	86908569	78874377	11188416	502.0054	1155442	48202899	645.3308	615.8465	2067571	14044492	154.8443	236.4193
F_{204}	1.673518	1.295179	1.046033	0.987371	0.258805	0.555469	4.48374	0.156383	0.763206	0.770496	2.120013	0.244741	0.152075
F_{205}	97908074	1.12E+08	1.07E+08	58432546	0.016882	1604566	73249509	0.160563	0	2772700	20590262	0	0.000439
F_{206}	7.90E+31	1.08E+32	2.37E+31	3.24E+30	0.791137	9.21E+23	7.16E+27	315.3449	2.79E+12	3.72E+25	3.09E+21	0	2.06E-09
F_{207}	0.004119	0.018842	6.36E-06	8.61E-05	1.42E-06	6.52E-05	0	3.49E-08	9.27E-09	4.59E-06	0.000102	0	0
F_{208}	6582.758	11197	7587.073	1.451369	0.012055	142.6338	8867.558	0.075034	1.310517	14.12109	2.246457	3.68E-08	2.229428
F_{209}	133.3707	128.4396	140.3959	68.73471	49.93512	27.91526	232.6409	6.270386	21.38355	62.46868	0.067804	26.05626	85.32797
F_{210}	1.048294	1.031102	1.348714	1.031102	4.611436	1.244179	2.267845	5.233596	0.79585	4.633564	3.784938	2.3574	5.330837
F_{211}	4.72E+137	8.43E+147	1.75E+134	2.42E+127	80.38215	24.83825	594.9243	1.82E+122	681.449	8.87E+109	0.06986	9.53E+71	230.8464
F_{212}	7.24E+09	7.1E+09	4.86E+09	497836.4	7.12E-09	973997.9	1E+09	1.88E-09	0	2954970	7.5E+08	0	0.000183
F_{213}	7193.398	10159.17	7564.908	1.460158	0.01416	139.9793	8952.955	0.072831	0.997731	12.91699	2.268297	1.26E-07	2.248722
F_{214}	1.14E-13	1.14E-13	14.20604	3.839308	10.77834	2.938355	29.35251	12.15664	20.09688	6.51318	5.231894	17.6439	55.74372
F_{215}	104.6847	134.2236	105.852	0.058032	0	10.19812	72.04796	0.002377	0	3.335505	0	1.13E-10	0.005692
F_{216}	110.7477	137.3616	125.2157	76.98529	47.72897	29.91986	276.613	20.62092	27.9941	56.92906	3.749465	42.22573	68.56059
F_{217}	15396.01	13507.84	13062.25	7204.313	7.526411	1001.738	18590.02	27.53623	236.8583	2032.407	990.0557	45.56097	30.07447
F_{218}	18407.81	13451.02	12065.86	7049.15	7.831199	969.7187	14057.89	31.88759	214.5193	2483.774	1259.24	41.89637	36.85642
F_{219}	0	0	11.13667	1.963197	0	4.975699	4.95771	15.04544	26.78661	2.884494	11.40485	30.79492	0
F_{220}	4.870605	5.299779	4.730912	4.836989	4.05723	1.789147	4.656532	4.046768	1.946236	3.228695	21.43131	2.744525	16.99084
F_{221}	7038.284	8662.723	7019.302	380.1375	18.44878	363.1347	8419.106	14.30112	12.4544	1061.03	308.2272	3.302856	12.23333

F_{222}	127.4814	124.8166	145.6073	62.42103	71.28491	16.28049	88.98996	67.48794	73.95122	56.98365	85.89584	76.6777	114.0596
F_{223}	1.79E+08	1.58E+08	1.39E+08	1.58E+08	956416.8	11140313	1.07E+08	1902743	281757.6	22832491	24381912	190860.9	827574.2
F_{224}	72219.16	67576.09	67963.17	55.16836	216.5779	380.6381	6394.996	135.7919	349.5291	407.9491	0	254.7199	247.6562
F_{225}	392620.2	337357.7	324093.4	337357.7	129.2365	25817.66	346860.9	140.2667	3746.549	61030.95	31350.34	159.5107	150.7318
F_{226}	0.012162	0.009596	0.026523	0.024226	0.021315	0.008691	0.078456	0.031767	0.059766	0.011813	0.073433	0.087819	0.079953
F_{227}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{228}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{229}	2.82E-13	1.95E-11	0	0	0	0	0	0	0	0	0	0	0
F_{230}	7.01E+50	8.66E+55	1.98E+46	4.86E+15	1.25E+33	1.39E+17	3.89E+50	5.76E+22	1.44E+13	1.33E+21	3.05E+25	3.74E+15	1.580565
F_{231}	3.92E-29	1.28E-27	8.63E-28	4.11E-40	5.14E-43	2.00E-42	7.47E-36	2.61E-43	1.47E-44	1.58E-35	1.50E-28	8.98E-43	0.208101
F_{232}	0.006255	0.021241	0.001495	0.00114	0.001619	0.002028	0.0022	0.001801	0.001231	0.001591	0.001652	0.001941	0.001432
F_{233}	14028.11	80412.21	2.33E+12	74.33198	176.1808	77.32751	203.8172	1.431509	18.98281	151.2516	58.81132	48.21157	9.052479

TABLE S12. **The mean values of the function evaluations for the 100D mathematical functions in different metaheuristics.**

No.	*Alternative Metaheuristic Algorithms*												
	ABC	*ACO*	*BA*	*FA*	*GA*	*HS*	*MFO*	*MVO*	*PSO*	*SA*	*SCA*	*SSA*	*CryStAl*
F_{176}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	147026.1
F_{177}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{178}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{179}	150000	150000	150000	150000	150000	150000	150000	150000	149300	150000	150000	144749	149135.1
F_{180}	150000	150000	150000	150000	121089.9	150000	150000	150000	150000	150000	150000	150000	145577.2
F_{181}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	144915
F_{182}	101.2146	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{183}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{184}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	2
F_{185}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{186}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{187}	150000	150000	150000	150000	150000	150000	150000	150000	76572	150000	150000	134839.5	150000
F_{188}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{189}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{190}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{191}	297.5709	241	150000	150000	20468.04	150000	28450.5	150000	72671	150000	150000	150000	101.08
F_{192}	301.6194	249.5	150000	150000	20570.55	150000	29741.5	150000	63776.5	150000	150000	150000	101.08
F_{193}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{194}	150000	150000	150000	150000	150000	150000	39694	150000	124170	150000	150000	115740	81.62
F_{195}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	146200.7
F_{196}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{197}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{198}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{199}	150000	150000	150000	150000	96550.83	150000	141150.5	150000	6115.5	150000	150000	150000	147419.1
F_{200}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{201}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{202}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{203}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{204}	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
F_{205}	150000	150000	150000	150000	150000	150000	150000	150000	87799	150000	150000	141277.5	149624.1
F_{206}	150000	150000	150000	150000	150000	150000	150000	150000	131524	150000	150000	119979	137397.4

***F*₂₀₇**	150000	150000	150000	150000	130420.4	150000	5978.5	150000	86098.5	150000	150000	124636	1465.36
***F*₂₀₈**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₀₉**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₁₀**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₁₁**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₁₂**	150000	150000	150000	150000	150000	150000	150000	150000	63846	150000	150000	107322.5	141172.3
***F*₂₁₃**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₁₄**	951.417	410.5	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₁₅**	150000	150000	150000	150000	18392.21	150000	49759	150000	18547.5	150000	1151.5	150000	150000
***F*₂₁₆**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	122122	150000	141272.9
***F*₂₁₇**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	141339.6
***F*₂₁₈**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	129970.4
***F*₂₁₉**	355.2632	252	150000	150000	6276.947	150000	28836.5	150000	150000	150000	150000	150000	117.08
***F*₂₂₀**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	144139.9
***F*₂₂₁**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₂₂**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₂₃**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₂₄**	150000	150000	150000	150000	150000	150000	110874	150000	150000	150000	1163	150000	150000
***F*₂₂₅**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₂₆**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	149342
***F*₂₂₇**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	998.92
***F*₂₂₈**	101.2146	50	898.2036	1219.512	51.00306	20	50	50	50	3750	50	50	2
***F*₂₂₉**	24982.79	130588.5	3179.641	1219.512	98.43591	139	148	182.5	162.5	3750	50	50	3.96
***F*₂₃₀**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₃₁**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₃₂**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000
***F*₂₃₃**	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000	150000

TABLE S13. The mean values of the composite mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*							
	ABC	*BA [25]*	*FA [25]*	*GA [25]*	*MFO [25]*	*MVO [21]*	*PSO [25]*	*CryStAl*
***F*₂₃₄**	8.813585	130.3125	175.9715	92.13909	8.25E-31	10.00017	137.7789	38.00006
***F*₂₃₅**	71.4911	544.1045	353.6269	96.70927	66.73272	30.00705	166.6643	58.77178
***F*₂₃₆**	280.0985	696.9752	308.0516	369.1036	119.0146	50.00061	394.507	118.6454
***F*₂₃₇**	347.8401	745.1403	548.5276	450.829	345.4688	190.3	468.3534	286.0303
***F*₂₃₈**	41.11309	543.8894	175.1975	95.92017	10.4086	160.5312	256.5258	47.0754
***F*₂₃₉**	548.5086	896.355	829.5929	523.7037	706.9953	440.005	790.1284	722.4695

TABLE S14. The standard deviation values of the composite mathematical functions for different metaheuristics.

No.	*Alternative Metaheuristic Algorithms*							
	ABC	*BA [25]*	*FA [25]*	*GA [25]*	*MFO [25]*	*MVO [21]*	*PSO [25]*	*CryStAl*
***F*₂₃₄**	16.62675	118.8206	86.928	27.90131	1.08E-30	31.62288	116.3128	56.46129
***F*₂₃₅**	16.32421	149.381	103.423	9.703147	53.22555	48.30615	164.3894	80.75401
***F*₂₃₆**	29.50073	190.5441	37.435	42.84275	28.3318	52.70461	121.949	40.92701
***F*₂₃₇**	11.24064	143.1577	162.8993	31.54446	43.11578	128.6659	67.31685	75.70852
***F*₂₃₈**	15.01647	198.8883	83.15078	53.79146	3.747669	158.2887	200.3816	48.95327
***F*₂₃₉**	23.92378	86.29955	157.2787	22.92001	194.9068	51.64	189.4915	198.2014

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3079161, IEEE Access

TABLE S15. Comparative results of the CEC 2017 test functions.

Fun.	Metaheuristics																			
	EBO with CMAR [87]					***LSHADE-cnEpSin*** [88]					***MM_OED*** [89]					***CryStAl*** (the present study)				
	Best	***Worst***	***Median***	***Mean***	***Std***	***Best***	***Worst***	***Median***	***Mean***	***Std***	***Best***	***Worst***	***Median***	***Mean***	***Std***	***Best***	***Worst***	***Median***	***Mean***	***Std***
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_5	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.99E+00	1.99E+00	1.69E+00	7.53E-01	0.00E+00	2.99E+00	9.95E-01	1.11E+00	7.35E-01	1.99E+00	1.19E+01	5.97E+00	6.38E+00	2.27E+00
G_6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.84E-04	2.72E-01	6.03E-03	2.02E-02	5.04E-02
G_7	1.04E+01	1.10E+01	1.05E+01	1.06E+01	1.75E-01	1.06E+01	1.29E+01	1.20E+01	1.20E+01	4.80E-01	1.04E+01	1.31E+01	1.15E+01	1.15E+01	6.71E-01	1.08E+01	1.92E+01	1.38E+01	1.40E+01	1.83E+00
G_8	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.70E-03	2.99E+00	1.99E+00	1.80E+00	7.71E-01	0.00E+00	2.99E+00	9.95E-01	1.11E+00	9.68E-01	1.99E+00	8.95E+00	5.97E+00	5.66E+00	2.02E+00
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.48E-08	2.25E+01	1.81E-06	5.84E-01	3.17E+00
G_{10}	1.25E-01	2.17E+02	1.36E+01	3.72E+01	5.39E+01	3.71E-01	1.55E+02	1.51E+01	4.30E+01	5.57E+01	2.50E-01	1.42E+02	6.83E+00	1.79E+01	3.64E+01	1.44E+02	1.27E+03	6.60E+02	6.79E+02	2.78E+02
G_{11}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.96E-01	4.43E+01	6.69E+00	9.53E+00	8.40E+00
G_{12}	0.00E+00	2.37E+02	1.18E+02	9.02E+01	7.44E+01	2.08E-01	2.38E+02	1.19E+02	1.01E+02	7.30E+01	0.00E+00	2.64E+02	1.19E+02	1.02E+02	5.96E+01	4.28E+02	1.10E+04	2.91E+03	3.55E+03	2.97E+03
G_{13}	0.00E+00	7.95E+00	3.13E-02	2.17E+00	2.53E+00	0.00E+00	8.32E+00	4.84E+00	3.66E+00	2.66E+00	6.82E-05	9.63E+00	5.05E+00	4.19E+00	2.66E+00	7.63E+00	4.08E+02	6.42E+01	9.65E+01	9.78E+01
G_{14}	0.00E+00	9.95E-01	0.00E+00	6.05E-02	2.36E-01	0.00E+00	9.95E-01	0.00E+00	7.80E-02	2.70E-01	0.00E+00	1.03E+00	0.00E+00	8.80E-02	2.74E-01	2.21E+00	3.39E+01	2.20E+01	1.83E+01	9.23E+00
G_{15}	3.81E-05	5.00E-01	3.07E-02	1.09E-01	1.74E-01	7.03E-06	5.00E-01	4.91E-01	3.24E-01	2.16E-01	2.00E-05	5.00E-01	2.13E-02	6.71E-02	1.19E-01	1.76E+00	8.46E+01	1.46E+01	2.28E+01	2.06E+01
G_{16}	2.62E-02	9.35E-01	4.43E-01	4.17E-01	1.98E-01	3.92E-03	1.08E+00	4.98E-01	5.37E-01	2.93E-01	2.11E-02	8.80E-01	2.16E-01	2.53E-01	2.01E-01	5.03E-01	2.18E+02	3.10E+00	3.06E+01	5.00E+01
G_{17}	1.00E-02	1.01E+00	4.94E-02	1.47E-01	2.03E-01	2.66E-03	2.63E+00	3.23E-01	3.07E-01	3.81E-01	0.00E+00	3.76E-01	1.97E-02	5.64E-02	1.13E-01	3.24E-01	2.52E+02	2.18E+01	3.40E+01	4.18E+01
G_{18}	3.92E-04	2.00E+01	4.09E-01	7.00E-01	2.77E+00	2.22E-04	2.05E+01	4.62E-01	3.86E+00	7.63E+00	1.11E-04	2.00E+01	9.00E-02	9.69E-01	3.89E+00	9.76E+00	2.77E+02	3.78E+01	5.18E+01	4.61E+01
G_{19}	0.00E+00	1.22E-01	1.79E-02	1.50E-02	1.88E-02	0.00E+00	1.50E+00	1.97E-02	4.47E-02	2.09E-01	0.00E+00	1.94E-02	0.00E+00	3.80E-03	7.49E-03	1.20E+00	2.12E+01	4.66E+00	6.04E+00	4.67E+00
G_{20}	0.00E+00	3.12E-01	0.00E+00	1.47E-01	1.57E-01	0.00E+00	6.24E-01	3.12E-01	2.57E-01	2.31E-01	0.00E+00	6.24E-01	0.00E+00	6.73E-02	1.57E-01	3.15E-01	4.67E+01	1.03E+01	1.22E+01	1.15E+01
G_{21}	1.00E+02	2.02E+02	1.00E+02	1.14E+02	3.52E+01	1.00E+02	2.04E+02	1.00E+02	1.46E+02	5.17E+01	1.00E+02	2.03E+02	1.00E+02	1.04E+02	1.99E+01	1.00E+02	2.10E+02	2.05E+02	2.00E+02	2.50E+01
G_{22}	2.17E+01	1.00E+02	1.00E+02	9.85E+01	1.10E+01	1.00E+02	1.00E+02	1.00E+02	1.00E+02	6.80E-02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	6.88E-02	1.00E+02	1.01E+02	1.00E+02	1.00E+02	2.49E-01
G_{23}	3.00E+02	3.03E+02	3.00E+02	3.00E+02	7.07E-01	3.00E+02	3.05E+02	3.03E+02	3.02E+02	1.64E+00	1.00E+02	3.08E+02	3.03E+02	2.98E+02	2.84E+01	3.03E+02	3.17E+02	3.06E+02	3.07E+02	3.51E+00
G_{24}	1.00E+02	3.30E+02	1.00E+02	1.66E+02	9.97E+01	1.00E+02	3.32E+02	3.30E+02	3.16E+02	5.45E+01	1.00E+02	2.01E+02	1.00E+02	1.04E+02	1.97E+01	3.30E+02	3.40E+02	3.34E+02	3.35E+02	2.65E+00
G_{25}	3.98E+02	4.43E+02	3.98E+02	4.12E+02	2.12E+01	3.98E+02	4.43E+02	4.43E+02	4.26E+02	2.24E+01	3.98E+02	4.43E+02	3.98E+02	4.14E+02	2.19E+01	3.98E+02	4.47E+02	4.44E+02	4.28E+02	2.19E+01
G_{26}	2.00E+02	3.00E+02	3.00E+02	2.65E+02	4.74E+01	3.00E+02	3.00E+02	3.00E+02	3.00E+02	0.00E+00	2.00E+02	3.00E+02	3.00E+02	2.94E+02	2.38E+01	2.00E+02	1.24E+03	4.44E+02	4.53E+02	2.16E+02
G_{27}	3.90E+02	3.95E+02	3.90E+02	3.92E+02	2.40E+00	3.84E+02	3.95E+02	3.89E+02	3.89E+02	1.96E+00	3.89E+02	3.90E+02	3.90E+02	3.90E+02	1.22E-01	3.90E+02	4.03E+02	3.97E+02	3.97E+02	2.42E+00
G_{28}	0.00E+00	5.84E+02	3.00E+02	3.07E+02	7.18E+01	3.00E+02	6.11E+02	3.00E+02	3.85E+02	1.19E+02	3.00E+02	6.47E+02	3.00E+02	3.37E+02	1.02E+02	3.00E+02	6.12E+02	6.12E+02	5.69E+02	1.03E+02
G_{29}	2.27E+02	2.45E+02	2.30E+02	2.31E+02	3.77E+00	2.26E+02	2.33E+02	2.28E+02	2.28E+02	1.72E+00	2.30E+02	2.48E+02	2.34E+02	2.36E+02	4.19E+00	2.32E+02	2.71E+02	2.43E+02	2.46E+02	9.65E+00
G_{30}	3.95E+02	4.43E+02	3.95E+02	4.07E+02	1.78E+01	3.39E+02	4.65E+05	4.07E+02	1.76E+04	8.61E+04	3.95E+02	1.25E+06	3.95E+02	5.69E+04	2.34E+05	4.14E+02	8.18E+05	5.96E+02	9.69E+04	2.66E+05

EBO with CMAR: *Effective Butterfly Optimizer with Covariance Matrix Adapted Retreat*
LSHADE-cnEpSin: *Ensemble Sinusoidal Differential Covariance Matrix Adaptation with Euclidean Neighborhood*
MM-OED: *Multi-Method Based Orthogonal Experimental Design*

TABLE S16. Statistical results for the M_1 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	5.52 E-02	7.66 E-02	1.17 E-01	2.41 E-02
(MAES) Kumar et al. [90]	3.80 E-02	5.01 E-02	6.42 E-02	9.33 E-03
(LSHADE) Kumar et al. [90]	7.71 E-02	1.08 E-01	1.64 E-01	2.97 E-02
Present study (CryStAl)	2.76E-02	4.78E-02	1.34E-01	2.98E-02

IUDE: *Improved Unified Differential Evolution Algorithm*
MAES: *Matrix Adaptation Evolution Strategy*
LSHADE: *Linear Success-History based Adaptive Differential Evolution*

TABLE S17. Statistical results for the M_2 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	4.31 E-02	5.48 E-02	6.71 E-02	9.07 E-03
(MAES) Kumar et al. [90]	2.36 E-02	2.91 E-02	3.37 E-02	4.13 E-03
(LSHADE) Kumar et al. [90]	6.67 E-02	9.20 E-02	2.04 E-01	4.45 E-02
Present study (CryStAl)	2.02E-02	3.31E-02	8.20E-02	1.86E-02

TABLE S18. Statistical results for the M_3 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	3.44 E-02	6.44 E-02	2.05 E-01	5.64 E-02
(MAES) Kumar et al. [90]	1.51 E-02	1.98 E-02	2.46 E-02	2.99 E-03
(LSHADE) Kumar et al. [90]	2.71 E-02	4.57 E-02	6.98 E-02	1.12 E-02
Present study (CryStAl)	1.28E-02	1.87E-02	2.73E-02	4.88E-03

TABLE S19. Statistical results for the M_4 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	4.50 E-02	6.53 E-02	2.09 E-01	5.38 E-02
(MAES) Kumar et al. [90]	1.68 E-02	1.74 E-02	2.24 E-02	1.87 E-03
(LSHADE) Kumar et al. [90]	4.74 E-02	2.23 E-01	4.94 E-01	1.62 E-01
Present study (CryStAl)	1.68E-02	2.29E-02	4.20E-02	7.14E-03

TABLE S20. Statistical results for the M_5 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	2.85 E-02	5.50 E-02	9.58 E-02	2.33 E-02
(MAES) Kumar et al. [90]	9.83 E-03	3.06 E-02	5.96 E-02	1.69 E-02
(LSHADE) Kumar et al. [90]	6.78 E-02	1.61 E-01	3.20 E-01	8.89 E-02
Present study (CryStAl)	1.93E-02	3.21E-02	4.84E-02	9.13E-03

TABLE S21. Statistical results for the M_6 real-world optimization problems considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [90]	6.23 E-02	2.53 E-01	3.78 E-01	1.13 E-01
(MAES) Kumar et al. [90]	1.56 E-02	2.94 E-02	7.77 E-02	2.34 E-02
(LSHADE) Kumar et al. [90]	2.62 E-01	3.07 E-01	3.53 E-01	3.17 E-02
Present study (CryStAl)	3.50E-02	1.81E-01	3.33E-01	1.24E-01

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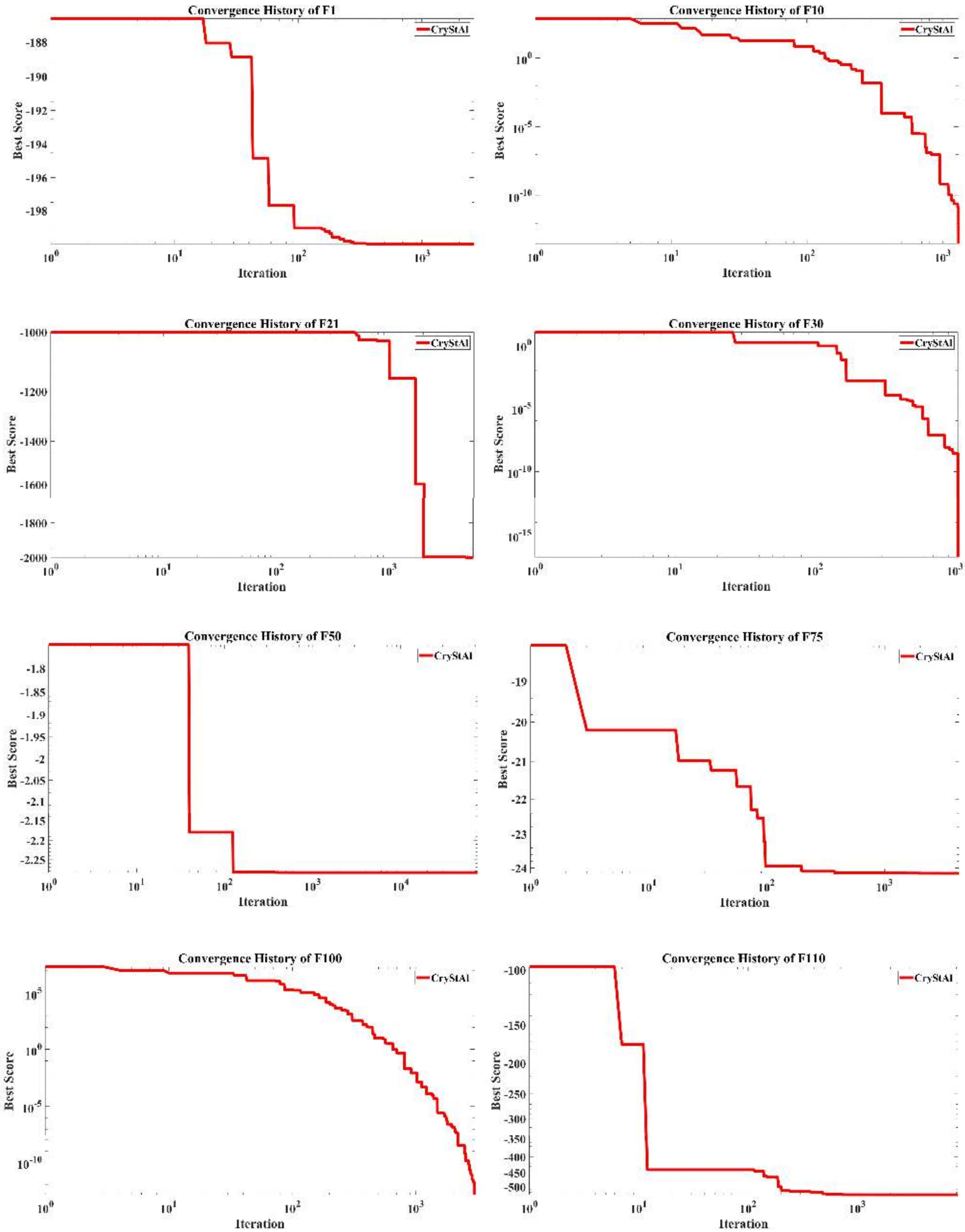

FIGURE S1. Convergence history of the proposed algorithm for the mathematical test functions **(Continues on the next page →)**

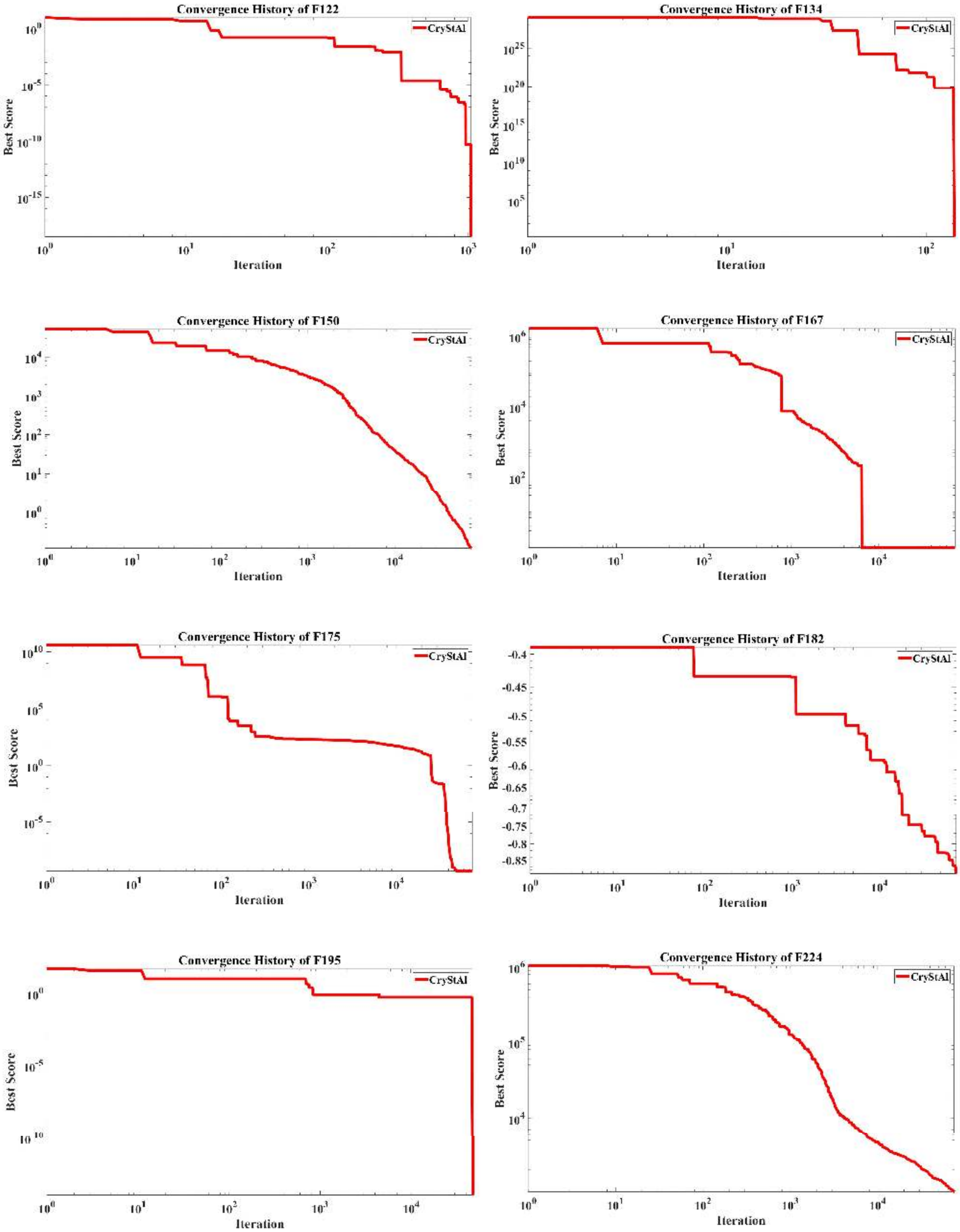

FIGURE S1. (Continued). Convergence history of the proposed algorithm for the mathematical test functions.

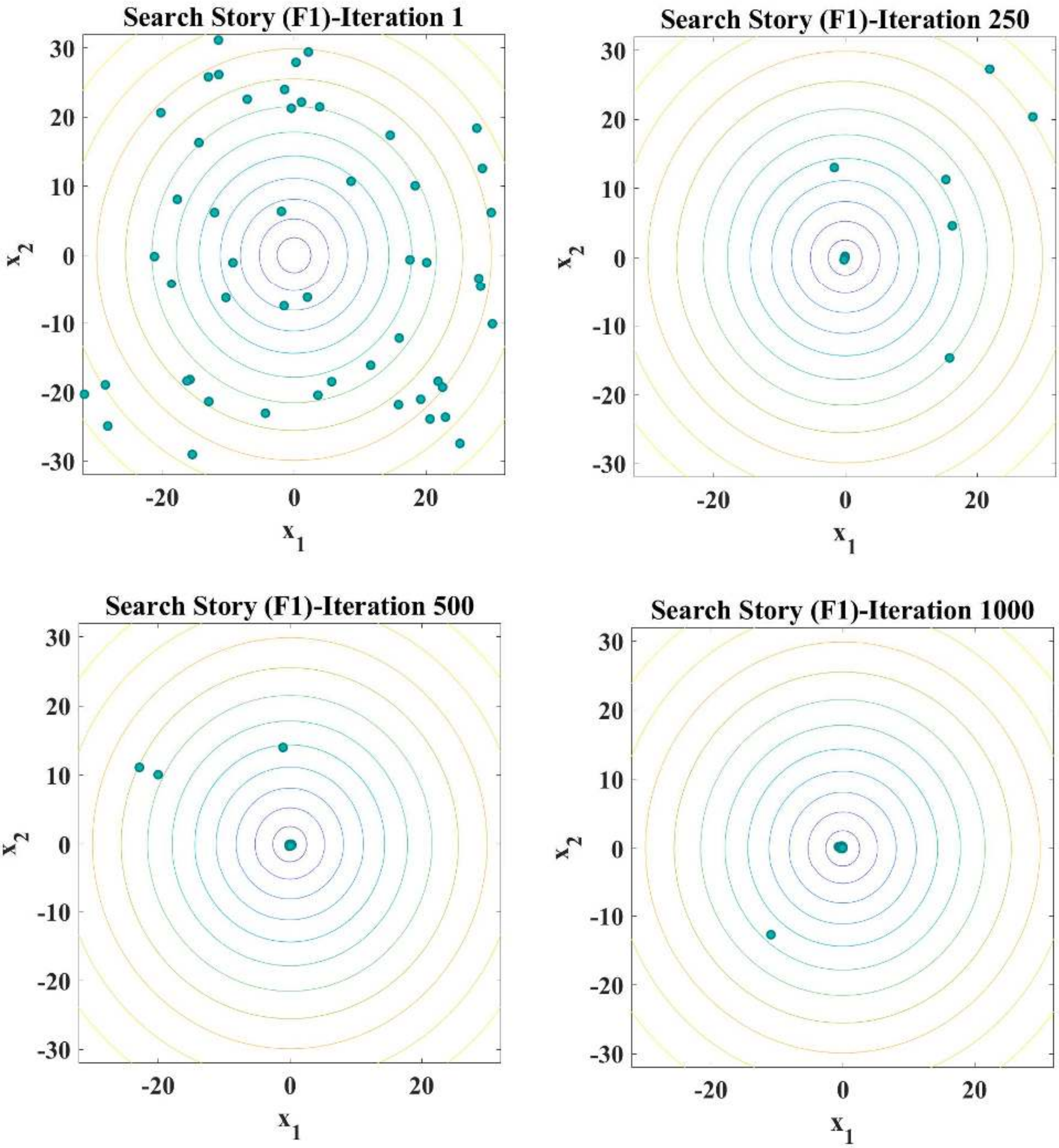

FIGURE S2. Diversity graphs of the proposed algorithm for the mathematical test functions. (Continues on the next page →)

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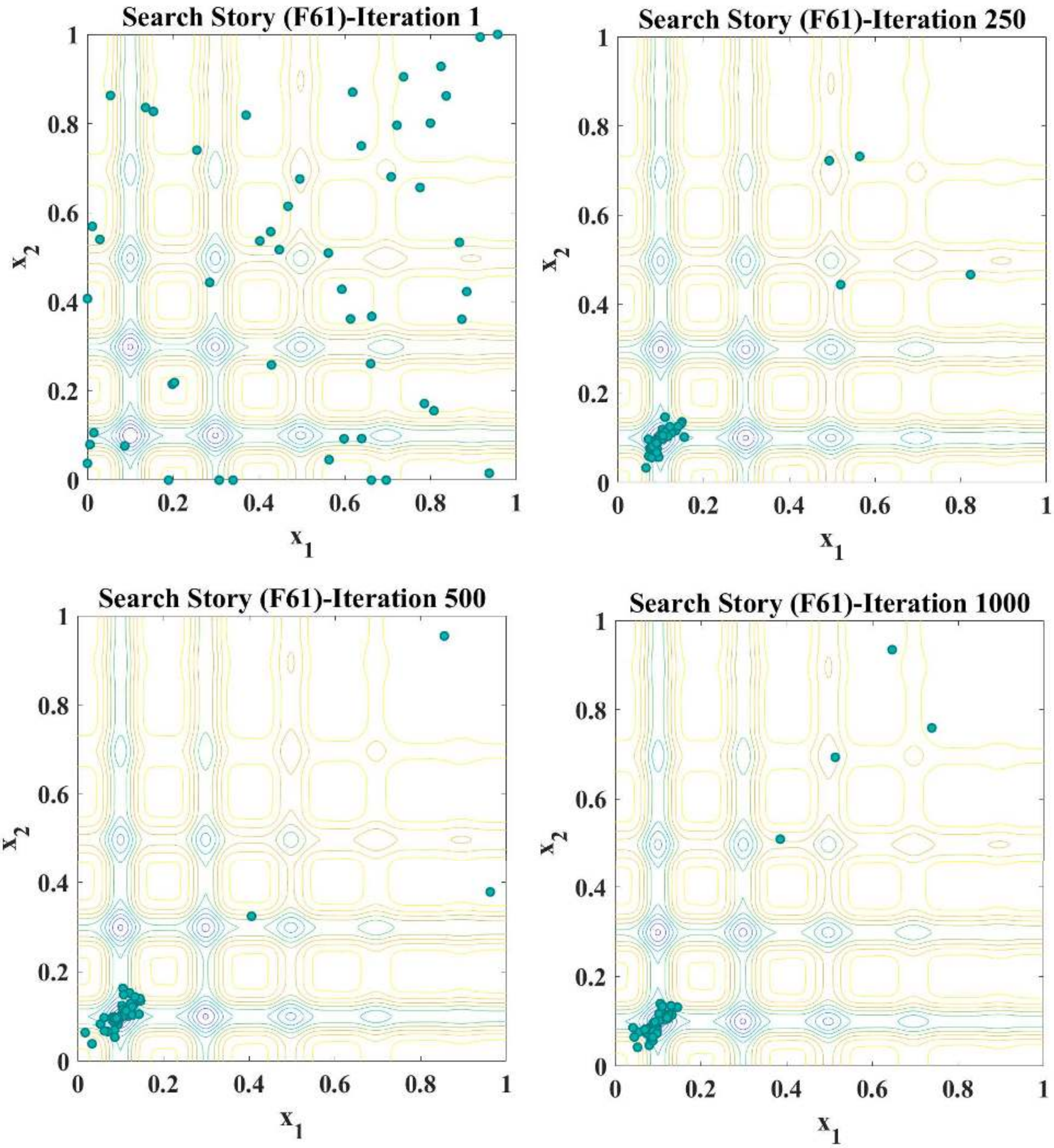

FIGURE S2. (Continued). Diversity graphs of the proposed algorithm for the mathematical test functions. **(Continues on the next page →)**

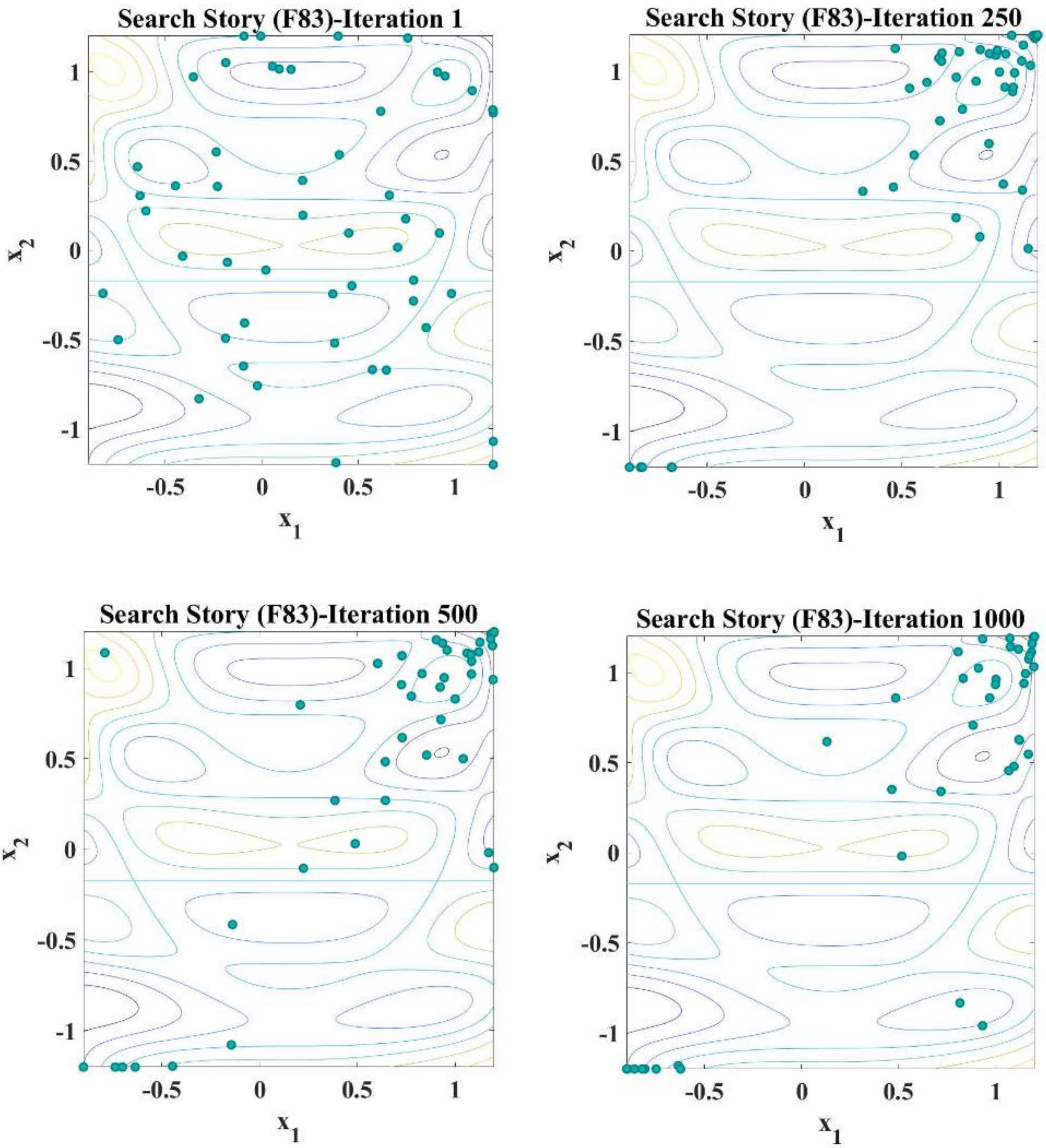

FIGURE S2. (Continued). Diversity graphs of the proposed algorithm for the mathematical test functions.