



# Cubic interactions of $d=4$ irreducible massless higher spin fields within BRST approach

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**Abstract** We develop an approach to constructing the manifestly Lorentz covariant cubic interaction vertices for the four-dimensional massless higher spin bosonic fields with two-component dotted and undotted spinor indices. Such fields automatically satisfy the traceless conditions what simplify form of the equations determining the irreducible massless representation of the Poincaré group with given helicity. The cubic vertex is formulated in the framework of the BRST approach to higher spin field theory. Use of the above spin-tensor fields allows to simplify a form of the BRST-charge and hence to find the cubic vertices just in terms of irreducible higher spin fields. We derive an equation for the cubic vertex and find solutions for arbitrary spins  $s_1, s_2, s_3$  with the number of derivatives  $s_1 + s_2 + s_3$  in the vertex. As an example, we explicitly construct a vertex corresponding to the interaction of a higher spin field with scalars.

## 1 Introduction

Study the various aspects of higher spin field theory still attracts an attention motivated among other things by certain possibilities for the development of new principles for constructing unified models of fundamental interactions, including quantum gravity (see e.g. the reviews [1–4] and the reference therein for current progress).

Cubic interactions are the first approximation in the theory of interacting fields. Their peculiarity is that the cubic interaction for given three fields does not depend on the presence

or absence of any other fields in the full nonlinear theory. Thus, they are model independent and can be classified. The complete classification of consistent cubic interaction vertices of massless and massive fields of arbitrary spin was constructed in the light-cone formalism in the space dimensions  $d \geq 4$  by Metsaev [5–7]. In the simplest case of three massless symmetric fields with spins  $s_1, s_2, s_3$  in Minkowsky space, the cubic vertices are characterized by the number of derivatives  $k$

$$k_{min} = s_1 + s_2 + s_3 - 2s_{min} \leq k \leq s_1 + s_2 + s_3 = k_{max}, \quad d > 4.$$

There are only two vertices in four dimensions  $d = 4$  with  $k = k_{min}, k_{max}$  (see e.g. [4]). Already in this case the Lorentz-covariant realization of cubic interaction vertices for higher spins requires very cumbersome calculations.

As is known the irreducible massless higher spin representation of the Poincaré group with integer spin  $s$  is described by metric-like symmetric tensor fields  $\phi^{\mu_1 \dots \mu_s}$  which satisfy equation  $\partial^2 \phi^{\mu_1 \dots \mu_s} = 0$  and subject transversality  $\partial_\nu \phi^{\mu_1 \dots \mu_{s-1} \nu} = 0$  and tracelessness  $\phi^{\mu_1 \dots \mu_{s-2} \nu \nu} = 0$  constraints. One can expect that in the case of interacting fields these equation and constraints should be modified in some way. One of the generic problems in higher spin field theory is to derive these modified equations and the corresponding constraints within the Lagrangian formulation. This problem has been studied by many authors using different approaches (see e.g. [9–29] and the references therein). Also it is worth noting the papers, where the consistent cubic vertices for massless higher spin fields was constructed in the the frame-like formalism [30–32].<sup>1</sup>

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<sup>1</sup> Recently, there was developed a general approach to gauge invariant deformations of gauge systems [33–36], that opens the new possibilities

Recently the cubic vertices for symmetric massless higher spin fields with integer spins were considered within the BRST approach for  $d \geq 4$  [40] in Minkowski space where the BRST charge takes into account all the constraints. As it turned out, taking into account the of traceless constraints, although it requires overcoming some formal difficulties, can lead to the appearance of new terms in the cubic vertex in comparison with the formulation without using the conditions of tracelessness in the BRST charge. In this paper, we consider the same problem but from a different angle.

We want to pay attention that in  $d4$  all the worries with tracelessness constraints can be avoid if to work within of the two-component spinor formalism. Irreducible massless fields with integer spin  $s$  are described in this case by multispinors  $\phi^{a_1 \dots a_s \dot{a}_1 \dots \dot{a}_s}$  satisfying the equation  $\partial^2 \phi^{a_1 \dots a_s \dot{a}_1 \dots \dot{a}_s} = 0$  and subject to transversality condition  $\partial_{b\dot{b}} \phi^{a_1 \dots a_{s-1} b \dot{a}_1 \dots \dot{a}_{s-1} \dot{b}} = 0$ . There are no need to use the tracelessness constraint. Our aim is to apply the BRST approach for construction of cubic interaction for massless bosonic fields in terms of spin-tensor fields with two component dotted and undotted spinor indices.

The paper is organized as follows. In Sect. 2 we present the basic aspects of the BRST approach for constructing the free Lagrangian formulations of irreducible massless fields with an arbitrary integer spin in two-component formalism. In Sect. 3 we describes the general procedure of the BRST approach for constructing interactions and present the BRST-closed condition for cubic vertices. Particular solution to this condition with the number of derivatives  $k_{max} = s_1 + s_2 + s_3$  in the vertex.is given in Sect. 4. One example of the interaction of a higher spin field with scalars in more detail is discussed in Sect. 5. In Sect. 6 we summarize the results obtained.

## 2 BRST charge and free Lagrangian

In space of  $4d$  multispinor tensors the irreducible massless higher spin fields with integer spin  $s$  can be described by fields  $\phi_{a(s)\dot{a}(s)}$  subjected to constraints

$$\partial^{b\dot{b}} \phi_{a(s-1)b\dot{a}(s-1)\dot{b}} = 0, \quad \partial^2 \phi_{a(s)\dot{a}(s)} = 0, \quad (2.1)$$

where  $\partial^2 = \partial_\mu \partial^\mu = -\frac{1}{2} \partial_{a\dot{a}} \partial^{a\dot{a}}$ .

In the framework of the BRST approach, the higher spin fields appear as the coefficients in the vectors of the Fock

space

$$|\phi\rangle = \sum_{s=0}^{\infty} |\phi^{(s)}\rangle, \quad |\phi^{(s)}\rangle = \frac{1}{s!} \phi_{a(s)\dot{a}(s)} c^{a(s)} c^{\dot{a}(s)} |0\rangle, \quad (2.2)$$

$$c^{a(s)} := c^{a_1} \dots c^{a_s} \quad c^{\dot{a}(s)} := c^{\dot{a}_1} \dots c^{\dot{a}_s} \quad (2.3)$$

generated by creation  $c^a, c^{\dot{a}}$  and annihilation operators  $a^a, a^{\dot{a}}$

$$\langle 0|c^a = \langle 0|c^{\dot{a}} = 0, \quad a^a|0\rangle = a^{\dot{a}}|0\rangle = 0, \quad \langle 0|0\rangle = 1 \quad (2.4)$$

with following nonzero commutation relations

$$[a^a, c^b] = \varepsilon^{ab}, \quad [a^{\dot{a}}, c^{\dot{b}}] = -\varepsilon^{\dot{a}\dot{b}}. \quad (2.5)$$

The Hermitian conjugation in a Fock space is defined as follows

$$(a^a)^+ = c^{\dot{a}}, \quad (a^{\dot{a}})^+ = c^a, \quad (c^a)^+ = a^{\dot{a}}, \quad (c^{\dot{a}})^+ = a^a. \quad (2.6)$$

Let us introduce operators

$$p^2 = \partial^2, \quad l = a^a a^{\dot{a}} p_{a\dot{a}}, \quad l^+ = -c^a c^{\dot{a}} p_{a\dot{a}}, \quad p_{a\dot{a}} = \partial_{a\dot{a}}, \quad (2.7)$$

which act on the state of the Fock space as

$$p^2 |\phi^{(s)}\rangle = \frac{1}{s!} \partial^2 \phi_{a(s)\dot{a}(s)} c^{a(s)} c^{\dot{a}(s)} |0\rangle, \\ l |\phi^{(s)}\rangle = -\frac{s^2}{s!} \partial^a \dot{a} \phi_{a(s)\dot{a}(s)} c^{a(s-1)} c^{\dot{a}(s-1)} |0\rangle, \\ l^+ |\phi^{(s)}\rangle = \frac{1}{s!} \partial_b \dot{b} \phi_{a(s)\dot{a}(s)} c^{a(s)b} c^{\dot{a}(s)\dot{b}} |0\rangle.$$

The constraints (2.1) in terms of operators (2.7) take form

$$p^2 |\phi^{(s)}\rangle = 0, \quad l |\phi^{(s)}\rangle = 0. \quad (2.8)$$

Note the set of operators  $F_A = \{p^2, l, l^+\}$  is invariant under Hermitian conjugation with respect to the scalar product

$$\langle \phi || \phi \rangle \quad (2.9)$$

and form a closed algebra  $[F_A, F_B] = f_{AB}{}^C F_C$  with the only nonzero commutation relation

$$[l^+, l] = (N + \bar{N} + 2)p^2, \quad (2.10)$$

where

$$N = c^a a_a, \quad \bar{N} = c_{\dot{a}} a^{\dot{a}}, \quad (N)^+ = \bar{N}.$$

The Hermitian nilpotent BRST charge is constructed in the form

$$Q = \eta^A F_A - \frac{1}{2} \eta^A \eta^B f_{AB}{}^C \mathcal{P}_C, \quad Q^+ = Q, \quad Q^2 = 0, \quad (2.11)$$

Footnote 1 continued

to construct the higher spin field vertices [37,38]. We also point out a new approach to problem of locality in the higher spin field theory that can be related with a structure of the interaction vertices [39].

where  $\eta_A = \{\theta, c^+, c\}$  and  $\mathcal{P}_A = \{\pi, b, b^+\}$  are the fermionic ghosts and corresponding momenta (antighosts) satisfying the anticommutation relations  $\{\eta_A, \mathcal{P}_B\} = \delta_{AB}$ . The ghost and antighost variables have ghost numbers  $gh(\eta_A) = -gh(\mathcal{P}_A) = 1$  while  $gh(F_A) = 0$ . Applying formula (2.11) we obtain the explicit expression for BRST charge

$$Q = \theta p^2 + c^+ l + c l^+ + c^+ c (N + \bar{N} + 2) \pi. \tag{2.12}$$

The operator  $Q$  (2.12) is nilpotent  $Q^2 = 0$  and acts in the extended Fock space of the vectors  $|\Phi\rangle$  including dependence on the fermionic ghosts defined above.

We note here that there are two operators

$$S = c^a a_a + c^+ b + b^+ c, \quad \bar{S} = c_{\dot{a}} a^{\dot{a}} + c^+ b + b^+ c, \tag{2.13}$$

that commute with the BRST operator (2.12). Thus if we want to get Lagrangian for bosonic spin  $s$  field (2.2) the state vector  $|\Phi\rangle$  in the extended Fock space must satisfy

$$S|\Phi\rangle = s|\Phi\rangle, \quad \bar{S}|\Phi\rangle = s|\Phi\rangle. \tag{2.14}$$

The basic equation of motion in the BRST approach is postulated in the extended Fock space as follows

$$Q|\Phi\rangle = 0. \tag{2.15}$$

To reproduce the conditions of the irreducible representation (2.8) from the equation of motion (2.15) we define the vacuum of extended Fock space as

$$c|0\rangle = b|0\rangle = \pi|0\rangle = 0 \tag{2.16}$$

and require that the ghost number of vector  $|\Phi\rangle$  is equal to the ghost number of initial vector (2.2),  $gh(|\Phi\rangle) = gh(|\phi^{(s)}\rangle) = 0$ . Then the most general vector  $|\Phi\rangle$  of the extended Fock space has the form

$$|\Phi\rangle = |\phi^{(s)}\rangle + \theta b^+ |\phi_1^{(s-1)}\rangle + c^+ b^+ |\phi_2^{(s-2)}\rangle \tag{2.17}$$

where the  $|\phi^{(s)}\rangle$  is the initial vector (2.2) and the  $|\phi_1^{(s-1)}\rangle$ ,  $|\phi_2^{(s-2)}\rangle$  have a similar form. After that, the whole construction is completely defined and closed.

Due to the nilpotency of the BRST charge (2.12), solutions to Eq. (2.15) is defined up to the gauge transformations

$$\delta|\Phi\rangle = Q|\Lambda\rangle. \tag{2.18}$$

Since  $gh(Q) = 1$  and  $gh(|\Phi\rangle) = 0$ , the gauge parameter  $|\Lambda\rangle$  has the ghost number  $-1$  and therefore its most general form looks like

$$|\Lambda\rangle = b^+ |\lambda^{(s-1)}\rangle, \tag{2.19}$$

where the vector  $|\lambda^{(s-1)}\rangle$  has decomposition in creation and annihilation operators similar with vector  $|\phi^{(s)}\rangle$  in (2.2).

The equation of motion  $Q|\Phi\rangle = 0$  in terms of the vectors  $|\phi\rangle, |\phi_1\rangle, |\phi_2\rangle$  can be rewritten as follows<sup>2</sup>

$$\begin{aligned} p^2|\phi\rangle - l^+|\phi_1\rangle &= 0, \\ p^2|\phi_2\rangle - l|\phi_1\rangle &= 0, \\ l|\phi\rangle - l^+|\phi_2\rangle + (N + \bar{N} + 2)|\phi_2\rangle &= 0. \end{aligned}$$

In this case, the gauge transformations  $\delta|\Phi\rangle = Q|\Lambda\rangle$  look like

$$\delta|\phi\rangle = l^+|\lambda\rangle, \quad \delta|\phi_1\rangle = p^2|\lambda\rangle, \quad \delta|\phi_2\rangle = l|\lambda\rangle.$$

The gauge invariant Lagrangian is constructed as follows

$$\mathcal{L} = \frac{1}{2} \int d\theta \langle \Phi | Q | \Phi \rangle. \tag{2.20}$$

It is not difficult to rewrite the Lagrangian (2.20) in explicit component form. Putting the expansion (2.17) into the Lagrangian (2.20) and integrating over ghost  $\theta$  according to the rule

$$\int d\theta \langle 0 | \theta | 0 \rangle = 1, \quad \int d\theta \langle 0 | 0 | 0 \rangle = 0, \tag{2.21}$$

one obtains

$$\begin{aligned} \mathcal{L} = \frac{1}{2} \{ & \langle \phi | (p^2|\phi\rangle - l^+|\phi_1\rangle) \\ & - \langle \phi_1 | (l|\phi\rangle - l^+|\phi_2\rangle + (N + \bar{N} + 2)|\phi_1\rangle) \\ & - \langle \phi_2 | (p^2|\phi_2\rangle - l|\phi_1\rangle) \}. \end{aligned} \tag{2.22}$$

Now, using the relation (2.2) and the analogous relations for  $|\phi_1\rangle$  and  $|\phi_2\rangle$ , we define the new Fock space vectors  $|H\rangle, |C\rangle, |D\rangle$  of the form

$$|\phi\rangle = |H\rangle = \frac{1}{s!} H_{a(s)}^{\dot{a}(s)} c^{a(s)} c_{\dot{a}(s)} |0\rangle, \tag{2.23}$$

$$|\phi_1\rangle = |C\rangle = \frac{1}{(s-1)!} C_{a(s-1)}^{\dot{a}(s-1)} c^{a(s-1)} c_{\dot{a}(s-1)} |0\rangle, \tag{2.24}$$

$$|\phi_2\rangle = |D\rangle = \frac{1}{(s-2)!} D_{a(s-2)}^{\dot{a}(s-2)} c^{a(s-2)} c_{\dot{a}(s-2)} |0\rangle. \tag{2.25}$$

Then for given spin  $s$ , the Lagrangian (2.22) takes the form

$$\begin{aligned} \mathcal{L} = \frac{1}{2} \{ & H_{a(s)}^{\dot{a}(s)} (\partial^2 H^{a(s)}_{\dot{a}(s)} - s \partial^a_{\dot{a}} C^{a(s-1)}_{\dot{a}(s-1)}) \\ & - C_{a(s-1)}^{\dot{a}(s-1)} (-s \partial_b^{\dot{b}} H^{a(s-1)b}_{\dot{a}(s-1)b}) \\ & - (s-1) \partial^a_{\dot{a}} D^{a(s-2)}_{\dot{a}(s-2)} + 2s C^{a(s-1)}_{\dot{a}(s-1)} \\ & - D_{a(s-2)}^{\dot{a}(s-2)} (\partial^2 D^{a(s-2)}_{\dot{a}(s-2)}) \\ & + (s-1) \partial_b^{\dot{b}} C^{a(s-2)b}_{\dot{a}(s-2)b} \}. \end{aligned} \tag{2.26}$$

<sup>2</sup> In what follows, we will often omit superscripts.

The corresponding gauge transformations are written as follows

$$\begin{aligned} \delta H_{a(s)}^{\dot{a}(s)} &= \frac{1}{s} \partial_a^{\dot{a}} \lambda_{a(s-1)}^{\dot{a}(s-1)} \\ \delta C_{a(s-1)}^{\dot{a}(s-1)} &= \partial^2 \lambda_{a(s-1)}^{\dot{a}(s-1)} \\ \delta D_{a(s-2)}^{\dot{a}(s-2)} &= -(s-1) \partial_b^{\dot{b}} \lambda_{a(s-1)b}^{\dot{a}(s-1)\dot{b}}. \end{aligned} \tag{2.27}$$

One can show that Lagrangian (2.20) describes massless spin  $s$  field [41,42]. Indeed, after removing field  $C$  from Lagrangian (2.26) with the help of its equation of motion we are left with two traceless fields  $H$  and  $D$ . These two traceless fields can be combined into one double traceless field and Lagrangian for this double traceless field will coincide with the Fronsdal’s Lagrangian [43].

### 3 Construction of cubic interaction

For deriving the cubic interactions we use three copies of the vectors in extended Fock space  $|\Phi_i\rangle, i = 1, 2, 3$  and three corresponding operators. These operators satisfy the commutation relations

$$[a_i^a, c_j^b] = \delta_{ij} \varepsilon^{ab}, \quad [a_i^{\dot{a}}, c_j^{\dot{b}}] = -\delta_{ij} \varepsilon^{\dot{a}\dot{b}}, \tag{3.1}$$

$$\{\theta_i, \pi_j\} = \{c_i, b_j^+\} = \{c_i^+, b_j\} = \delta_{ij}. \tag{3.2}$$

The full interacting Lagrangian up to cubic level is defined as follows

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \sum_i \int d\theta_i \langle \Phi_i | Q_i | \Phi_i \rangle \\ &+ \frac{1}{2} g \int d\theta_1 d\theta_2 d\theta_3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | | V \rangle + h.c., \end{aligned} \tag{3.3}$$

where  $|V\rangle$  is some cubic vertex, which should be found, and  $g$  is a coupling constant. It is easy to check that Lagrangian (3.3) is invariant under the following gauge transformations up to  $g^2$  terms (in what follows  $i \simeq i + 3$ )

$$\begin{aligned} \delta |\Phi_i\rangle &= Q_i |\Lambda_i\rangle - g \int d\theta_{i+1} d\theta_{i+2} (\langle \Phi_{i+1} | \langle \Lambda_{i+2} | \\ &+ \langle \Phi_{i+2} | \langle \Lambda_{i+1} |) | V \rangle, \end{aligned} \tag{3.4}$$

if the following condition takes place

$$\hat{Q} |V\rangle = 0, \quad \hat{Q} = \sum_{i=1}^3 Q_i. \tag{3.5}$$

This condition is considered as an equation for  $|V\rangle$ . To guarantee zeroth ghost number of Lagrangian (3.3), the vertex  $|V\rangle$  must have ghost number 3. We will looking for vertex in the form

$$|V\rangle = V |\Omega\rangle, \quad |\Omega\rangle = \theta_1 \theta_2 \theta_3 |0_1\rangle \otimes |0_2\rangle \otimes |0_3\rangle, \tag{3.6}$$

where the operator  $V$  has the ghost number 0 and depends on operators  $c_i^a, c_i^{\dot{a}}, c_i^+, b_i^+, \pi_i$  as well as on momenta  $p_i^{a\dot{a}}$

satisfying the momenta conservation condition

$$\sum_i p_i^{a\dot{a}} = 0. \tag{3.7}$$

However, the Eq. (3.5) do not determine the vertex  $|V\rangle$  uniquely. Indeed if vertex  $|V\rangle$  satisfies the Eq. (3.5) then the vertex

$$|V\rangle = |V\rangle + \hat{Q} |W\rangle \tag{3.8}$$

also satisfies this equation. Vertices of the form  $\hat{Q} |W\rangle$  (BRST exact) can be obtained from the free theory via field redefinitions

$$|\Phi_i\rangle \rightarrow |\tilde{\Phi}_i\rangle = |\Phi_i\rangle + \int d\theta_{i+1} d\theta_{i+2} \langle \Phi_{i+1} | \langle \Phi_{i+2} | W \rangle. \tag{3.9}$$

Here  $|W\rangle$  is a vector with ghost number 2. Our aim is to find a operator  $V$  in (3.6) which satisfies the BRST invariance condition (3.5) and determined up to transformation (3.8) and can not be removed by the field redefinition (3.9). We will call such vertices as BRST-closed. One can use ambiguity (3.8) in the solution to Eq. (3.5) to obtain different explicit forms of the same physical vertex.

### 4 Solutions to the cubic vertices V

First of all, we note that there are six operators  $S_i, \bar{S}_i$  (2.13) commuting with the BRST operator (3.5). As a consequence of this we can decompose the vertex  $|V\rangle$  as

$$|V\rangle = \sum_{s_i=0}^{\infty} |V(s_1, s_2, s_3)\rangle \tag{4.1}$$

$$S_i |V(s_1, s_2, s_3)\rangle = \bar{S}_i |V(s_1, s_2, s_3)\rangle = s_i |V(s_1, s_2, s_3)\rangle \tag{4.2}$$

and solve equation on the vertex (3.5) for each values of the spins  $s_i$  separately

$$\hat{Q} |V(s_1, s_2, s_3)\rangle = 0. \tag{4.3}$$

Secondly, we note that if we want to construct an interaction for fields with spins  $s_i$  then the operator  $V(s_1, s_2, s_3)$  (3.6) must obligatory have terms without the ghost fields, i.e. terms constructed only from the operators  $c_i^a, c_i^{\dot{a}}$  and the momenta  $p_i^{a\dot{a}}$ . The terms of the operator  $V(s_1, s_2, s_3)$  with the ghost fields are found from Eq. (4.3).

Thirdly, the terms of the operator  $V(s_1, s_2, s_3)$  without the ghost fields must not depend on the operators  $p_i^2$  and  $l_i^+$  since these operators are contained in the BRST operators  $Q_i$  (2.12) and as a consequence such terms can be removed from the vertex operator with the help of transformation (3.8).

Let us turn to finding explicit solutions to Eq. (4.3).

Taking into account the remarks made above and condition (4.2) one can show that for three scalar fields we get the vertex operator  $V(0, 0, 0) = const$ .

Next let us consider the case  $s_1 = 1, s_2 = s_3 = 0$ . In this case the part of the operator  $V(1, 0, 0)$  without the ghost fields must be linear both in  $c_1^a$  and in  $\dot{c}_1^a$  and due to remarks made above and due to condition (3.7) the only possible operator is  $c_1^a(p_2 - p_3)_{a\dot{a}}\dot{c}_1^{\dot{a}}$ . The rest part of the operator  $V(1, 0, 0)$  which depends on the ghost fields are found from Eq. (4.3). The final result for the operator  $V(1, 0, 0)$  is

$$V(1, 0, 0) \equiv L_1 = c_1^a \dot{c}_1^{\dot{a}}(p_2 - p_3)_{a\dot{a}} - 2c_1^+(\pi_2 - \pi_3). \tag{4.4}$$

Similar expression was found in [19], but in our case  $[\hat{Q}, L_1] \neq 0$ . Vanishing of commutator  $[\hat{Q}, L_1]$  in [19] is a consequence that the tracelessness constraint was not taken into account in the BRST charge in [19]. Nonetheless we try to generalize the result of [19] to our case, namely, we will looking for a solution for the vertex operator  $V(s_1, 0, 0)$  in a similar form

$$V(s_1, 0, 0) \equiv \mathbb{L}_1^{(s_1)} = L_1^{s_1} + \text{terms proportional to the ghost fields.} \tag{4.5}$$

Doing so, we find

$$\mathbb{L}_1^{(s_1)} = L_1^{s_1} + s_1(s_1 - 1)L_1^{s_1-2}c_1^+ \times [l_1^+(2\pi_2 + 2\pi_3 - \pi_1) - 2L_1(\pi_2 - \pi_3)]. \tag{4.6}$$

Vertex operators  $V(0, s_2, 0) \equiv \mathbb{L}_2^{(s_2)}$  and  $V(0, 0, s_3) \equiv \mathbb{L}_3^{(s_3)}$  have analogous form

$$\mathbb{L}_i^{(s_i)} = L_i^{s_i} + s_i(s_i - 1)L_i^{s_i-2}c_i^+ \times [l_i^+(2\pi_{i+1} + 2\pi_{i+2} - \pi_i) - 2L_i(\pi_{i+1} - \pi_{i+2})], \tag{4.7}$$

$$L_i = c_i^a \dot{c}_i^{\dot{a}}(p_{i+1} - p_{i+2})_{a\dot{a}} - 2c_i^+(\pi_{i+1} - \pi_{i+2}). \tag{4.8}$$

Since for  $i \neq j$

$$[[\hat{Q}, \mathbb{L}_i^{(s_i)}], \mathbb{L}_j^{(s_j)}] = 0, \quad [\hat{Q}, \mathbb{L}_i^{(s_i)}]|\Omega\rangle = 0 \tag{4.9}$$

then we can construct a vertex for arbitrary values of spins  $s_i$  with the number of derivatives  $k_{max} = s_1 + s_2 + s_3$

$$V(s_1, s_2, s_3; k_{max})|\Omega\rangle = \mathbb{L}_1^{(s_1)}\mathbb{L}_2^{(s_2)}\mathbb{L}_3^{(s_3)}|\Omega\rangle. \tag{4.10}$$

Thus the vertex operator  $V(s_1, s_2, s_3; k_{max})$  for arbitrary values of spins  $s_i$  with the number of derivatives  $k_{max}$  is found.

### 5 Example of higher spin interaction

Various problems of interaction of higher spin fields with scalar fields were considered in many papers (see e.g. [44–46]).

Let us consider an explicit example of cubic interaction of one real massless field with arbitrary spin  $s$

$$|\Phi_3\rangle = |H\rangle + \theta_3 b_3^+ |C\rangle + c_3^+ b_3^+ |D\rangle \tag{5.1}$$

where

$$|H\rangle = \frac{1}{s!} H_{a(s)} \dot{c}^{a(s)} c_{\dot{a}(s)} |0\rangle, \tag{5.2}$$

$$|C\rangle = \frac{1}{(s-1)!} C_{a(s-1)} \dot{c}^{a(s-1)} c_{\dot{a}(s-1)} |0\rangle, \tag{5.3}$$

$$|D\rangle = \frac{1}{(s-2)!} D_{a(s-2)} \dot{c}^{a(s-2)} c_{\dot{a}(s-2)} |0\rangle. \tag{5.4}$$

and two real massless scalar fields

$$|\Phi_1\rangle = \varphi_1 |0\rangle, \quad |\Phi_2\rangle = \varphi_2 |0\rangle. \tag{5.5}$$

The total Lagrangian has form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}. \tag{5.6}$$

Here  $\mathcal{L}_{free}$  is the free Lagrangian for our system of fields (2.26)

$$\begin{aligned} \mathcal{L}_{free} = & \frac{1}{2} \left\{ \varphi_1 \partial^2 \varphi_1 + \varphi_2 \partial^2 \varphi_2 \right. \\ & + H_{a(s)} \dot{c}^{a(s)} (\partial^2 H^{a(s)}_{\dot{a}(s)} - s \partial^a_{\dot{a}} C^{a(s-1)}_{\dot{a}(s-1)}) \\ & + C_{a(s-1)} \dot{c}^{a(s-1)} (s \partial_b^{\dot{b}} H^{a(s-1)b}_{\dot{a}(s-1)b} \\ & + (s-1) \partial^a_{\dot{a}} D^{a(s-2)}_{\dot{a}(s-2)} - 2s C^{a(s-1)}_{\dot{a}(s-1)}) \\ & \left. - D_{a(s-2)} \dot{c}^{a(s-2)} (\partial^2 D^{a(s-2)}_{\dot{a}(s-2)} \right. \\ & \left. + (s-1) \partial_b^{\dot{b}} C^{a(s-2)b}_{\dot{a}(s-2)b} \right\} \end{aligned}$$

The interacting Lagrangian  $\mathcal{L}_{int}$  corresponds to the vertex  $V(0, 0, s; s) = \mathbb{L}_3^{(s)}|\Omega\rangle$  (3.3)

$$\mathcal{L}_{int} = \frac{1}{2} g \int d\theta_1 d\theta_2 d\theta_3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | \mathbb{L}_3^{(s)} | \Omega \rangle + h.c., \tag{5.7}$$

where

$$\mathbb{L}_z^{(s)} = L_z^s + s(s-1)L_z^{s-2}c_z^+ [l_z^+(2\pi_{i+1} + 2\pi_{i+2} - \pi_i) - 2L_z(\pi_{i+1} - \pi_{i+2})], \tag{5.8}$$

$$L_i = c_i^a \dot{c}_i^{\dot{a}}(p_{i+1} - p_{i+2})_{a\dot{a}} - 2c_i^+(\pi_{i+1} - \pi_{i+2}). \tag{5.9}$$

After rewriting this Lagrangian component form, one gets

$$\mathcal{L}_{int} = (-1)^{s+1} s! \left\{ H^{a(s)}_{\dot{a}(s)} j_{a(s)}^{\dot{a}(s)} - (s-1) \partial_a^{\dot{a}} C^{a(s-1)}_{\dot{a}(s-1)} j_{a(s-2)}^{\dot{a}(s-2)} \right\}, \tag{5.10}$$

where  $j_{a(s)}^{\dot{a}(s)}$  are the higher spin currents constructed from two scalar fields

$$\begin{aligned} j_{a(s)}^{\dot{a}(s)} = & \varphi_1 (\overrightarrow{\partial}_a^{\dot{a}} - \overleftarrow{\partial}_a^{\dot{a}})^s \varphi_2 \\ = & \sum_{k=0}^s C_s^k (-\partial_a^{\dot{a}})^{s-k} \varphi_1 (\partial_a^{\dot{a}})^k \varphi_2, \end{aligned}$$

$$C_s^k = \frac{s!}{k!(s-k)!} \quad (5.11)$$

The relevant gauge transformations for higher spin fields remain as in free theory

$$\begin{aligned} \delta H_{a(s)}^{\dot{a}(s)} &= \frac{1}{s} \partial_a^{\dot{a}} \lambda_{a(s-1)}^{\dot{a}(s-1)}, \\ \delta C_{a(s-1)}^{\dot{a}(s-1)} &= \partial^2 \lambda_{a(s-1)}^{\dot{a}(s-1)}, \\ \delta D_{a(s-2)}^{\dot{a}(s-2)} &= -(s-1) \partial_b^{\dot{a}} \lambda_{a(s-1)b}^{\dot{a}(s-1)\dot{b}}. \end{aligned} \quad (5.12)$$

However, the general approach leads to gauge transformations for scalars

$$\begin{aligned} \delta \varphi_1 &= (-1)^s 2s! g \left[ s \sum_{k=0}^{s-1} C_{s-1}^k (\partial_a^{\dot{a}})^k \lambda^{a(s-1)}_{\dot{a}(s-1)} (2\partial_a^{\dot{a}})^{s-k-1} \varphi_2 \right. \\ &\quad \left. - (s-1) \sum_{k=0}^{s-2} C_{s-2}^k (\partial_a^{\dot{a}})^{k+1} \lambda^{a(s-1)}_{\dot{a}(s-1)} (2\partial_a^{\dot{a}})^{s-k-2} \varphi_2 \right], \end{aligned} \quad (5.13)$$

$$\begin{aligned} \delta \varphi_2 &= 2s! g \left[ s \sum_{k=0}^{s-1} C_{s-1}^k (\partial_a^{\dot{a}})^k \lambda^{a(s-1)}_{\dot{a}(s-1)} (2\partial_a^{\dot{a}})^{s-k-1} \varphi_1 \right. \\ &\quad \left. - (s-1) \sum_{k=0}^{s-2} C_{s-2}^k (\partial_a^{\dot{a}})^{k+1} \lambda^{a(s-1)}_{\dot{a}(s-1)} (2\partial_a^{\dot{a}})^{s-k-2} \varphi_1 \right]. \end{aligned} \quad (5.14)$$

Thus we have explicitly constructed a vertex corresponding to the interaction of a field spin  $s$  with two real scalars and deformation of the gauge transformation.

Let us remind once again that there is arbitrariness (3.8) in the explicit form of the interaction vertex and we can use it to get more convenient expressions for interaction (5.10) and/or gauge transformations (5.13) and (5.14).

## 6 Summary

In this paper we have analyzed and constructed the Lorentz covariant cubic interactions for completely unconstrained massless higher spin fields in  $d = 4$  Minkowski space with the maximum number of derivatives. The construction is given in the framework of the BRST approach to higher spin fields adopted to multispinor formalism. Unlike the previous work [40], in the present formulation there is no need to use the tracelessness constraint for irreducible massless higher spin fields since we use spin-tensors with dotted and undotted indices and this constraint is fulfilled identically what in some sense simplify an analysis. However, the corresponding BRST operator has a different structure than that in [40], and the derivation of a cubic vertex now requires a separate analysis. Such an analysis was given in this paper.

Within of the BRST approach, the problem of constructing cubic interaction vertices is reduced to finding a vector  $|V\rangle$  (3.6) which should be BRST-closed  $\hat{Q}|V\rangle = 0$  (3.5).

We have carried out a general analysis of the equation for the cubic vertex and described a procedure of its finding. For three given massless fields with spins  $s_1, s_2, s_3$  we have constructed a cubic vertex with  $k_{max} = s_1 + s_2 + s_3$  numbers of derivatives. The case of constructing a vertex with  $k_{min}$  number of derivatives will be considered in a future paper.

An explicit example of cubic interaction of a field with spin  $s$  with two real massless scalar fields was constructed in details. The interacting Lagrangian and the gauge transformations in explicit component form are given by (5.10) and (5.12), (5.13), (5.14).

It is evident that the BRST approach to constructing the interacting vertices for  $4d$  completely irreducible higher spin fields in terms of spin-tensor fields with dotted and undotted indices can be used for finding the manifestly Lorentz-covariant cubic and higher vertices for various bosonic and fermionic, massive and massless higher spin fields. We hope to study all these issues in the forthcoming papers.

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