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Current generation by minority species heating

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ABSTRACT

It is proposed that electric currents be generated from the preferential heating of ions traveling in one direction but with no net momentum injected into the system. This can be accomplished with, for example, traveling waves in a two-ion-species plasma. The current can be generated efficiently enough for the scheme to be of interest in maintaining steady-state toroidal currents in a reactor.

1. INTRODUCTION

The confinement of plasma in tokamak devices relies upon the maintenance of toroidal electric currents which sustain a poloidal magnetic field. There exist several methods for driving these toroidal currents in the plasma. The method most usually employed is the inducing of a toroidal electric field by means of a time-varying magnetic field. This method suffers, however, from the liability that it cannot operate in the steady state. The most promising other methods that have been proposed rely upon pushing electrons with fast waves,[1,2] slow waves,[3] or neutral beams.[4] Interest in these other schemes arises because of their steady-state nature which affords the possibility of continuous operation of tokamak reactors.

This paper describes yet another way to drive toroidal current continuously. *This new scheme differs fundamentally from* the other schemes in that it employs waves that interact with ions rather than electrons, and yet these waves need not have net toroidal momentum. The operation of this scheme may be thought of as comprising of two steps. The first step is the inducement of the minority ion species to drift toroidally relative to the majority ion species. The second step is the reaction of the electrons to the ion distributions. The electrons can drift even when the total ion current vanishes.

To show that electrons can drift when the first step is satisfied, consider the electron response to oppositely drifting ion species with differing ion charge states. The electrons, being so light, do not sense the ion mass, but they do sense the ion charge and velocity. In a neutral plasma the total current is, of course, frame invariant. Consider then the frame of reference in which the total ion current vanishes. There may, however, persist an imbalance in the forces on the electrons since they collide more frequently with the species that has the larger ion charge state. The imbalance in forces gives rise to an electron drift. Since the ion current vanishes, a net current must flow.

The question is how to achieve the first step, or in other words, how to produce a relative drift between the two ion species. The most direct means of producing this relative drift is simply to inject counter-streaming ion beams, one of either species. The beams, which taken together have no net momentum, may enter the plasma as neutrals. This is the neutral beam method of current-drive.[4]

Here, a different method of establishing the relative ion drift is suggested. The basic mechanism is similar to the one used in driving currents with electron cyclotron waves.[2] The idea is to create an asymmetric ion drag so that, for example, minority ions moving to the left collide more frequently with the majority ions than do those minority ions moving to the right. The imbalance in forces resolves itself with minority ions moving, on average, to the right. By momentum conservation, the majority ion species must drift to the left. Thus, the consequence of asymmetric ion drag is the establishment of a relative ion drift.

The means of accomplishing the asymmetric ion drag of the type described above could be the selective heating of those minority species ions that move to the right. To see that this is so, consider a single test minority species ion that moves to the right, which is, say, the direction parallel to a static magnetic field. The collision frequency between the test ion and the background majority ions, which exert a drag force, is sensitive to the test ion energy. By increasing the test ion energy, the drag force can be reduced. The energy increase need not occur as a result of increasing any specific velocity component. Thus, for example, even increasing the perpendicular energy of minority ions that move to the right induces a drift of that species to the right.

The preceding discussion indicates that an electric current can be driven parallel to a static magnetic field simply by increasing the gyration energy of those minority ions that move in one direction parallel to the magnetic field. The increase in gyration energy can be accomplished through the damping of a traveling plasma wave that resonates with the selected ions. An example of such a wave is the ion cyclotron wave at a frequency near the minority ion gyrofrequency. These waves would be launched unidirectionally in the toroidal direction in the tokamak.

The proposed scheme for driving current is especially exciting in view of the confidence in the heating mechanism, established by the recent success of experiments designed to heat minority species ions with ion cyclotron waves.[5] In these experiments, the wave energy was successfully imparted preferentially to the minority species, although with absorbtion in velocity space symmetric with respect to the magnetic field. These results are certainly encouraging even though it remains to be established that asymmetric absorbtion can occur too. The asymmetric absorbtion is necessary, of course, for driving current.

For current-drive schemes to be practical for reactor applications, the current must be sustained at levels substantial enough for confinement while the power cost is kept at a level small compared with the fusion power output. The remainder of this paper is devoted to a calculation of the power requirements for the proposed scheme of current-drive. While some aspects of the problem certainly warrant more consideration than is offered in this analysis, it does appear that the new scheme may be feasible and competitive, in terms of power cost, with other schemes.[1-4]

The analysis is presented as follows: In Sec. 2 we calculate the amount of power that is necessary to establish a given minority species ion drift. In Sec. 3 we calculate the electric current that results from the ion drift. In Sec. 4 we identify the wave regime and plasma parameters for which our scheme operates optimally in reactor applications. In Sec. 5 we conclude with a summary of the salient findings, a reiteration of the assumptions in the analysis and a discussion of additional caveats.

2. ION MOTION

In this section it is shown quantitatively how preferential heating of the minority ion species can lead to a drift of that species relative to the majority species. Let us first assume that some means exist to displace a velocity space element of volume δf from the velocity space location to be associated with subscript one to that to be associated with subscript two (see Fig. 1). Assume that momentum in the interested direction is lost from the minority species ions in the first location at a rate ν_1, while the displaced ions lose the momentum at a rate ν_2. These rates include collisions both with the majority ion species and with electrons. For small minority ion concentrations, which we envision, the self-collisions among them may be neglected.

The amount of energy required to displace the ions from location one to location two is simply the difference in kinetic energy associated with the two locations, i.e.,

$$\Delta E = (E_2 - E_1)\delta f, \tag{1}$$

$$E_j = m_\alpha v_j^2/2 , \tag{2}$$

where v_j is the speed at location j and m_α is the mass of a minority species ion.

The change in parallel momentum in the minority species induced by this displacement is given approximately by

$$\Delta p_z \simeq \delta f\; m_\alpha [v_{z2}\exp(-\nu_2 t) - v_{z1}\exp(-\nu_1 t)], \tag{3}$$

where v_z is the velocity component in the z-direction, which is parallel to an applied magnetic field. The approximation in Eq. (3) arises essentially from the assumption that the collision rates ν_1 and ν_2 are functions of their initial coordinates only. This is tantamount to neglecting energy scattering, which could cause these rates to vary with time. A justification of the simplification is provided in Sec. 4.

Note that at t = 0, the instant in which the displacement occurs, there is no z-directed momentum change unless v_{z2} differs from v_{z1} This is merely a restatement of the obvious argument that unless there is an impulse given to the minority ions in the z-direction, there initially can be no flow in that direction. At $t = \infty$, however, even if $v_{z1} \neq v_{z2}$, all directed momentum has relaxed through collisions with other species, so that there can be no relative drift of the minority ion species with respect to the majority species.

Consider the case that $v_{z1} = v_{z2}$. In view of the above, Δp_z vanishes both at t = 0 and at $t = \infty$. At finite times, however, there will be finite Δp_z if $\nu_1 \neq \nu_2$. Implied here is an asymmetric drag on the minority ion species. By continuously supplying energy, the resulting drift can be continuously maintained.

Consider the directed momentum of the minority ion species time-averaged over a time Δt, i.e.,

$$p_\alpha \equiv \frac{1}{\Delta t} \int_o^{\Delta t} \Delta p_z dt \simeq \frac{m_\alpha \delta f}{\Delta t} \left(\frac{v_{z2}}{\nu_2} - \frac{v_{z1}}{\nu_1}\right), \tag{4}$$

where the second equality is obtained by taking the time-average over an interval Δt that is large compared to the momentum destruction times $1/\nu_1$ and $1/\nu_2$. The rate at which power is dissipated to maintain the directed momentum is simply

$$P_d = \Delta E/\Delta t, \tag{5}$$

which may be viewed as a definition. Combining Eqs. (1), (4) and (5), we find the ratio of minority species momentum to power dissipated

$$\frac{p_\alpha}{P_d} = m_\alpha \left(\frac{v_{z2}/\nu_2 - v_{z1}/\nu_1}{E_2 - E_1}\right). \tag{6}$$

Let us define $\vec{s}$ as the velocity space vector that represents the displacement from location one to location two. Then taking the limit $|\vec{s}| \to 0$ in Eq. (6), we may write

$$\frac{p_\alpha}{P_d} = m_s \frac{\hat{s}\cdot\vec{\nabla}(v_z/\nu)}{\hat{s}\cdot\nabla\vec{E}}, \tag{7}$$

where $\hat{s}$ is the unit vector in the displacement direction and, in view of the limiting procedure, we have dropped the subscript identifying the now-indistinguishable velocity space locations.

3. ELECTRON MOTION

The electric current arises from the motion of electrons reacting to the relative drift between the two ion species. From Eq. (7) it may be seen that there is a drift of the minority species only in the event that $\hat{s}\cdot\vec{\nabla}(v_z/\nu) \neq 0$, which occurs in essentially two ways, i.e., either

$$\hat{s}\cdot\vec{\nabla}v_z \neq 0, \tag{8a}$$

or

$$\hat{s}\cdot\vec{\nabla}\nu \neq 0. \tag{8b}$$

When Eq. (8a) holds, net momentum is imparted from an external source directly to the minority species as the ions are pushed to higher v_z. To keep the plasma from rotating, or simply to define a rest frame, it must be imagined that an equal amount of momentum, but in the opposite direction, is imparted to the majority species, also from an external source. If the minority species is fast enough to collide mainly with the electrons, an electric current can be maintained efficiently. The method described is similar in concept and efficiency to driving currents by injecting neutral beams.[4]

Consider, on the other hand, that Eq. (8b) holds, but Eq. (8a) does not. A minority ion species drift develops even though no net momentum is injected. This is similar to a mechanism previously identified for electrons,[2] except for one complication. Here it is essential that the minority ions collide neither too frequently nor too infrequently with the electrons, where the comparison is with the frequency of collisions with the majority species. They must not collide too infrequently because, after all, the electric current eventually arises from these collisions and all is lost if the minority ions collide first with majority ions. On the other hand, they must not collide too frequently with the electrons because their momentum loss would then be due mainly to the electrons. This momentum loss rate is insensitive to the ion velocity (since the ions are much slower than the electrons) so that $\vec{\nabla}\nu$ vanishes with the result that no minority ion drift develops.

To consider quantitatively the amount of current generated assume that

$$\nu = \nu_e + \nu_i, \tag{9}$$

where ν_e and ν_i are the rates of momentum loss to electrons and to majority ions respectively. The regime of interest here is when

$$v_{Ti} << v << v_{Te}, \tag{10}$$

the first inequality assuring that Eq. (8b) is satisfied and the second inequality arising, in practical situations, simply because the electrons are so light. In this regime, ignoring energy scattering which will be justified later, we have[6]

$$\nu_e = C_e, \tag{11a}$$

$$\nu_i = C_i v_{Te}^3/v^3 = C_i (T_e/E)^{3/2} (m_\alpha/2m_e)^{3/2} , \tag{11b}$$

where C_e and C_i are constants and v_{Te} and v_{Ti} are respectively the electron and majority ion thermal velocities such that, e.g., $v_{Te}^2 \equiv T_e/m_e$. Using Eqs. (9)-(11) in Eq. (7), we may write

$$\begin{aligned}\frac{p_\alpha}{P_d} &= \frac{m_\alpha}{\hat{s}\cdot\vec{\nabla}E} \left(\nu^{-1}\hat{s}\cdot\vec{\nabla}v_z + v_z\hat{s}\cdot\vec{\nabla}\nu^{-1}\right) \\ &= \frac{m_\alpha}{\hat{s}\cdot\vec{\nabla}E} \left(\nu^{-1}\hat{s}\cdot\vec{\nabla}v_z + \frac{3v_z\nu_i}{2\nu^2}\,\frac{\hat{s}\cdot\vec{\nabla}E}{E}\right).\end{aligned} \tag{12}$$

Considering purely perpendicular heating, i.e., such that Eq. (8b) holds but Eq. (8a) does not, we may write

$$\frac{p_\alpha}{P_d} = \frac{3}{2} m_\alpha v_z \left(\frac{\nu_i}{\nu}\right)\left(\frac{1}{\nu E}\right) . \tag{13}$$

It is obvious from Eq. (13) that p_α vanishes for either $\nu_i \to 0$ or $\nu_i \to \infty$. Schemes which generate a drift by injecting momentum directly into the minority ions attain the maximum p_α/P_d when $\nu_i \to 0$. Here, in contrast, the largest and hence most favorable p_α/P_d will be attained when $\nu_i \sim \nu_e \sim \nu$.

It remains to calculate the amount of current generated per power dissipated. Assume that the minority ion species drifts at speed v_α parallel to the magnetic field while the majority ion species drifts at speed v_i. Assume that the minority ion charge state is Z_α while the majority ion charge state is unity. Then, since the electrons collide with the minority ions Z_α^2 times more often than with the majority ions, the electron parallel drift speed, v_e, must obey, in the steady state,

$$n_\alpha Z_\alpha^2 (v_\alpha - v_e) + n_i (v_i - v_e) = 0, \tag{14}$$

where n_α and n_i are respectively the minority and majority ion densities. It is assumed that the plasma has no net parallel momentum so that the drifts must also satisfy

$$n_\alpha m_\alpha v_\alpha + n_i m_i v_i + n_e m_e v_e = 0, \tag{15}$$

which is obeyed automatically in the instance that no net parallel momentum is injected into the minority species. Otherwise Eq. (15) can be considered to apply in the center of mass frame instead of the rest frame. This transformation, does not affect the current so long as charge neutrality is obeyed, i.e.,

$$Z_\alpha n_\alpha + n_i - n_e = 0. \tag{16}$$

The current is given by

$$J = e(-n_e v_e + Z_\alpha n_\alpha v_\alpha + n_i v_i) , \tag{17}$$

so that combining Eqs. (14)-(17), and taking the limit $m_e/m_i \to 0$, we get

$$J = \frac{en_\alpha v_\alpha (1 - Z_\alpha)}{n_i + n_\alpha Z_\alpha^2} \left[Z_\alpha n_i + (n_e - n_i) m_\alpha / m_i \right] . \tag{18}$$

Assuming the practical regime

$$n_\alpha Z_\alpha^2 << n_i \approx n_e, \tag{19}$$

and making use of Eqs. (13) and (18), we finally write

$$\frac{J}{P_d} = \frac{3}{2} e v_z (Z_\alpha - Z_\alpha^2) \frac{\nu_i}{\nu^2 E} . \tag{20}$$

It should be remembered that Eq. (20) applies only in the case that the parallel momentum injected into the minority species from the external source is negligible. In other words, only perpendicular heating is assumed. To generalize this equation to arbitrary diffusion paths is easy, of course, since one need simply to retain both terms in Eq. (12). The simplification is, however, useful in the case of most interest, where ion cyclotron waves induce primarily perpendicular velocity diffusion.

Note that the ratio m_e/m_i does not enter explicitly in Eq. (20). The implication is that Eq. (20) would still hold in the more general case wherein the majority ion charge state is not unity, and where Z_α is then to be interpreted as the ratio of the ion charge states. It would be understood too that ν_i

would be larger for larger majority ion charge states. We forego, however, retaining the more general expression since for a D-T reactor, which is the preeminent application, the majority ion charge state is unity. It may be noticed, however, that the efficiency of the scheme diminishes rapidly with increasing majority ion charge state.

4. APPLICATION TO A REACTOR

In order for the proposed scheme of current generation to be useful in steady-state reactor operation, a number of criteria must be satisfied. Firstly, there are criteria to be satisfied by the charge state of the minority species. From Eq. (20) we note the condition

$$Z_\alpha(Z_\alpha - 1) \neq 0, \qquad (21)$$

which is the basis for any current generation. This condition expresses the fact that while the ion counter-current increases linearly with Z_α, the electron current increases even faster with Z_α, since the collision frequency is quadratic in Z_α. Satisfying this condition implies only that $Z_\alpha \neq 0, 1$, or, in other words, that there simply be present ions of disparate charge states. If ion cyclotron waves are to be employed for current generation via minority heating, a second condition must be imposed on the minority ion charge state, namely,

$$Z_\alpha - m_\alpha/m_i \neq 0, \qquad (22)$$

which expresses the requirement that the ion species gyrofrequencies be disparate. Disparate gyrofrequencies are required for the ion cyclotron wave to distinguish between the two ion species so that energy may be transferred selectively to the minority species.

Let us assume, for the moment, that the restrictions on the minority charge state are met and that the ion cyclotron wave is capable of depositing its energy fully to the minority species. The scheme will still not be practical for reactors unless the power dissipated is minimal. To address this consideration, we look first for the best possible case of J/P_d. In order to facilitate comparison with other schemes for current drive we adopt normalizations characteristic of previous work;[1,2,7,8] specifically, we normalize J to $-env_{Te}$ and P_d to $\nu_o n_e m_e v_{Te}^2$, where $\nu_o = \omega_{pe}^4 \ell n\Lambda/2\pi v_{Te}^3$. Thus, we may rewrite Eq. (20) in a normalized version as

$$\frac{J}{P_d} = 3\left(\frac{m_e}{m_\alpha}\right)\left(\frac{v_{Te}v_z}{v^2}\right)\left(\frac{\nu_o}{\nu_i}\right)\frac{Z_\alpha^2 - Z_\alpha}{(1 + \nu_e/\nu_i)^2}, \tag{23}$$

where J and P_d are now assumed to be normalized. From the discussion following Eq. (13), it is seen that J/P_d attains a maximum with respect to v_z. The task here is to specify where that occurs, and what its value is.

At this point it is necessary to specify ν_e and ν_i. We assume that these rates do not vary as the minority ions slow down. Actually, this is true for ν_e, but not for ν_i, since ν_i is sensitive to the minority ion energy. However, the final answer is fairly insensitive to this approximation as we will later show. A more exact treatment would proceed along the lines outlined in Ref. [illegible] for current-drive with electron cyclotron waves, but here the resulting expressions would be far more complicated. In view of the above, we use the notation given in Eq. (11) and write[6]

$$C_i = \nu_o Z_\alpha^2 \left(\frac{m_e}{m_\alpha}\right)^2 \left(1 + m_\alpha/m_i\right), \tag{24a}$$

$$C_e = \frac{\nu_o Z_\alpha^2}{3} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m_e}{m_\alpha}\right)^2 \left(1 + \frac{m_\alpha}{m_e}\right). \tag{24b}$$

If we make the further normalizations

$$w = v_z/v_{T\alpha}, \tag{25a}$$

$$u = v/v_{T\alpha}, \tag{25b}$$

and define the quantity

$$Y \equiv \nu_e/\nu_i u^3 = \frac{1}{3} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m_e}{m_\alpha}\right)^{1/2} \left(\frac{1}{1 + m_\alpha/m_i}\right), \tag{26}$$

then we may write the normalized J/P_d as

$$\frac{J}{P_d} = 3 \left(\frac{1 - Z_\alpha^{-1}}{1 + m_\alpha/m_i}\right) \frac{wu}{(1 + Yu^3)^2} \; . \tag{27}$$

It will be shown that the maximization of J/P_d occurs in the regime $w \gg 1$. Therefore, for the resonant ions, we have $u \approx w$, and we may replace u with w in Eq. (27). The maximization of J/P_d then depends on the maximization of the quantity

$$G(w) \equiv \frac{w^2}{(1 + Yw^3)^2}, \tag{28}$$

which attains its maximum value,

$$G(w_o) = \frac{4}{9 \cdot 2^{2/3} Y^{2/3}}, \tag{29}$$

when

$$w = w_o \equiv (2Y)^{-(1/3)}. \tag{30}$$

Substituting then Eqs. (26) and (29) into Eq. (27), we may finally write the maximum J/P_d in the particularly simple form,

$$\frac{J}{P_d} = 2\left(\frac{\pi}{3}\right)^{1/3} \left(\frac{M}{m_e}\right)^{1/3} \left(1 - Z_\alpha^{-1}\right), \tag{31}$$

where we have made use of notation for the reduced ion mass, i.e.,

$$M \equiv \frac{m_\alpha m_i}{m_\alpha + m_i}. \tag{32}$$

From Eq. (31), we can see that the most favorable conditions for current generation by minority heating occur when the ions are heavy and the disparity between the two ion charge states is great. For deuterium tokamak reactors, the most favorable minority species admixture is seen to allow a maximum $J/P_d \simeq 30$. This value indicates an efficiency comparable to that obtained when the current-drive is accomplished by means of lower-hybrid waves,[1,7] but somewhat less than the efficiency obtained when Alfvén waves are employed.[8] There is, however, the advantage that the present scheme can be used just as efficiently in high temperature or low beta reactors, where, respectively, lower-hybrid waves and Alfvén waves become less efficient.

Assuming, for the moment, that all the required current can be generated at near the maximum efficiency, we may envision a steady-state reactor with the poloidal magnetic field sustained by a toroidal current generated by means of minority species heating. Assuming that tokamak operation is achievable at $\beta_p = R/a$, where R and a are, respectively, the major and minor radii of the tokamak and

$$\beta_p \equiv \frac{n_e(T_e + T_i)}{B_p^2/2\mu_0} \simeq \frac{4n_eT_e\mu_o}{B_p^2}, \tag{33}$$

then the wave power dissipated over fusion power output may be written as[8]

$$\frac{P_d}{P_f} \simeq \frac{0.95\ \log\Lambda}{(n_{14}T_{10}a_1R_1)^{1/2}}\ \frac{P_d/J}{(3T_{10} - 2)},\ 1 \lesssim T_{10} \lesssim 2, \tag{34}$$

where n_{14} is the density normalized to 10^{14}cm^{-3}, T_{10} is the temperature normalized to 10 KeV and a_1 and R_1 are, respectively, the minor and major radii in meters. If $J/P_d \simeq 30$

is attained in a reactor with parameters $n_{14} \sim 1$, $T_{10} \sim 2$ and $a_1R_1 \sim 25$, then P_d/P_f may be as low as 2%, which implies a level of recirculating power that may indeed be practical.

It is of importance to note that, while J/P_d attains its maximum as a function of w at w_o, J/P_d is not very peaked near its maximum. From Eq. (30) we see that $w_o \simeq 5$; however, were the ions instead pushed at $w \simeq 3$ or $w \simeq 7$, the efficiency (i.e., J/P_d) would be reduced by only 20%. This allows efficient current generation by heating the minority species ions over a very broad range, i.e., $3 \lesssim w \lesssim 7$. This, in turn, implies that large currents, in fact sufficient for steady-state reactor operation, can be generated at near maximum efficiency. To see this quantitatively, consider that the normalized current required to operate a tokamak at $\beta_p = R/a$ may be given by[8]

$$J = 2.1 \times 10^{-4}(n_{14}a_1R_1)^{-(1/2)} \simeq 4 \times 10^{-5}. \tag{35}$$

To obtain this current, we need a minority species drift large enough that

$$\frac{n_\alpha v_\alpha}{n_e v_{Te}} > \frac{4 \times 10^{-5}}{Z_\alpha^2}, \tag{36}$$

where we made use of Eq. (18). We now estimate the magnitude of the minority species drift that is obtainable efficiently. The estimate is admittedly crude, but adequate for the purposes here. We envision that the minority current is of the same order of magnitude as the current of resonant ions, i.e.,

$$n_\alpha v_\alpha \simeq n_r w v_{T\alpha}, \tag{37}$$

where n_r is the density of resonant minority ions. The requirement can then be rewritten as

$$\frac{n_r}{n_e} \gtrsim \frac{4 \times 10^{-5}}{Z_\alpha^2 w} \left(\frac{m_\alpha}{m_e}\right)^{1/2} \simeq 2 \times 10^{-4}, \tag{38}$$

where we assumed $Z_\alpha^2 \sim w \sim 2m_\alpha/m_e \sim 4$. Assuming also that the distribution function flattens in the resonant region of velocity space, we very roughly have

$$\frac{n_r}{n_e} \simeq \frac{n_\alpha}{n_e} f(u_1) \int_{u_1}^{u_2} 4\pi u^2 \, du \tag{39}$$

where $f(u_1) = (2\pi)^{-3/2} \exp(-u_1^2/2)$, and u_1 and u_2 are respectively the minimum and maximum normalized speeds of the resonant ions, which may be taken, respectively, equal to 3 and 7 without jeopardizing the efficiency of current generation. Despite the very approximate nature of Eq. (39), it is clear that even with a minority concentration n_α/n_e of a few per cent, there can be a sufficient number of ions in the range $3 < u < 7$ to satisfy Eq. (38). The dependency of n_r on u_1 is so sensitive that u_1 can always be adjusted slightly to give the required minority current. Similarly, a slight adjustment in u_1 overshadows the effect of any errors made in our estimating n_r. These error effects would have an algebraic rather than exponential dependence on u_1, indicating that our rough estimates are adequate here.

It remains to justify the neglect of energy scattering in finding J/P_d. This approximation is contained in writing Eq. (3) and in the use of Eqs. (11) and (24). The justification is based, in part, on the broadness of J/P_d as a function of w near its maximum value. The inclusion of energy scattering would not affect ν_e, but would introduce an enhancement of ν_i; consider then the implication of $\nu_i \to \Gamma\nu_i$, where Γ is a constant somewhat greater than unity. This means that in Eq. (27) we have $Y \to Y/\Gamma$. Using then Eqs. (23), (29) and (30), we find

$$J/P_d \to (J/P_d)/\Gamma^{1/3}, \tag{40a}$$

$$w_o \to w_o\Gamma^{1/3}, \tag{40b}$$

indicating a dependence on Γ sufficiently weak that no dramatically different results should be expected from a more exact treatment of the energy scattering.

In concluding this section, we consider a practical question for designing experiments, namely, the power (P) in watts necessary to generate a given current (I) in amps in a tokamak through any of the various current generation techniques. A convenient expression answering this question is

$$\frac{I}{P} \simeq 2 \times 10^{-2} \left(\frac{T_{10}}{R_1 n_{14}}\right)\left(\frac{J}{P_d}\right) \ \frac{\text{Amps}}{\text{Watts}} \ , \tag{41}$$

where J/P_d is dimensionless here and depends on the current-drive mechanism. For state-of-the-art tokamaks, say for PLT operating at $T_{10} \simeq 0.2$, $R_1 \simeq 1$, $n_{14} \simeq 0.2$, it is seen that about two watts are necessary to maintain each amp of current, assuming that the current is driven by minority species heating.

5. CONCLUSION

The important finding in this paper is that currents can not only be generated by minority species heating, but that they can be generated efficiently. The criteria for efficiency is that currents sufficient for maintaining the required poloidal magnetic field in a tokamak reactor can be generated at a power cost that is small compared to the fusion power output of the reactor. Efficiencies are found to be comparable to those obtainable through the employment of the other steady-state current generation mechanisms that have previously been suggested.

The calculation of this efficiency assumes , however, that the wave power is absorbed exclusively by the resonant minority species ions. Furthermore, only ions in the most favorable region of velocity space are assumed to be resonant. A justification of these assumptions is beyond the scope of this paper, although the assumptions are certainly plausible. Requirements on the wave propagation are eased somewhat since the most favorable resonant region, as calculated in Sec. 4, is seen to be quite large. There is a condition, however, on the wave parallel phase velocity that arises in order to inhibit power absorbtion by majority species ions. Consider the resonance condition for harmonic heating of the minority species

$$\frac{\omega - \Omega_\alpha}{k_{||}} \sim 3v_{T\alpha}, \quad (42)$$

and the resonance condition to avoid heating a substantial number of majority ions

$$\frac{\omega - \Omega_i}{k_{||}} > 5v_{Ti}, \quad (43)$$

where ω is the wave frequency, $k_{||}$ is the wave parallel wave number, and Ω_i and Ω_α are, respectively, the majority and minority ion gyrofrequencies. The satisfaction of Eqs. (42) and (43) is a necessary condition for minority but not majority ion heating to occur at the first wave harmonic. Equivalently, we may combine Eqs. (42) and (43) to write a condition on the parallel wave phase velocity, i.e.,

$$\frac{\omega}{k_{||}} > \frac{\left[5(m_\alpha/m_i)^{1/2} - 3\right]v_{T\alpha}}{\left[(m_\alpha/Z_\alpha m_i) - 1\right]}, \quad (44)$$

where we limited our consideration here to $\Omega_\alpha > \Omega_i$. A similar condition may be obtained when $\Omega_\alpha < \Omega_i$. Consider the case that $m_i = 2$, $m_\alpha = 3$ and $Z_\alpha = 2$, which corresponds to the use of the minority species ^{3}He for current generation in a D-T reactor. For these parameters, it is seen that $\omega/k_{\parallel} \gtrsim 20\, v_{T\alpha}$.

Note that one implication of the rather large wave parallel phase velocities that must be employed is that there is little momentum content in the wave. Thus, the wave heating results primarily in diffusion of ions in the perpendicular direction, with very little energy going into ion parallel motion. This justifies our concentration on the case that no parallel momentum is injected directly into the ions, i.e., on the case that Eq. (8b) is satisfied but Eq. (8a) is not satified.

Competing with the cyclotron damping of the wave by minority ions is the Landan damping of the wave by electrons. Although the cyclotron damping should dominate near a resonance, the electron damping should dominate off-resonance. The regime off-resonance cannot be avoided since Ω_α is a function of position in an inhomogeneous magnetic field.

In summary, an attractive case has been made for the employment of ion cyclotron waves for generating steady-state current in a tokamak reactor. More precise treatments of the wave propagation and alternate damping mechanisms must be performed, however, to ascertain that the idealized situation envisioned in the present analysis is realizable. It remains too to address practical issues, such as the optimal candidate for the minority species, which conceivably could be a fusion reagent or product, or a deliberately introduced impurity.

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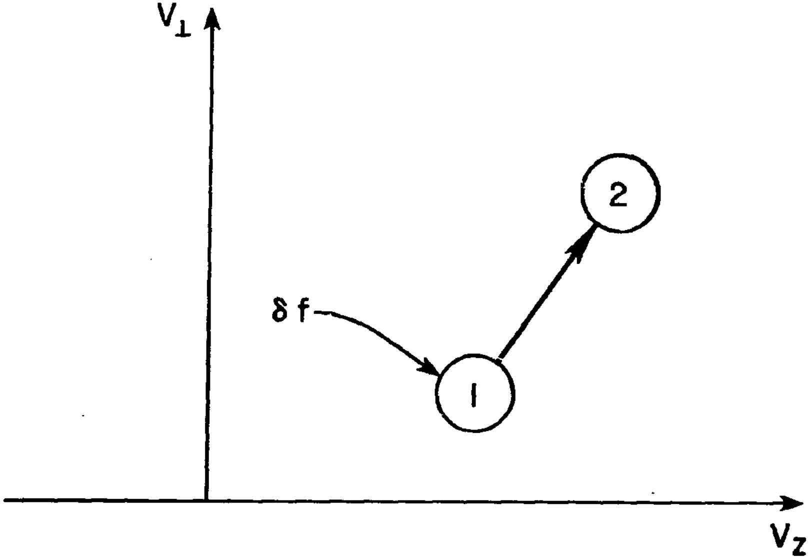

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Fig. 1. A schematic drawing indicating the displacement in velocity space of the minority species element δf from location 1 to location 2. The vector connecting these locations is $\vec{s}$. The axes, $v_\perp$ and v_z are, respectively, the velocities perpendicular and parallel to the magnetic field.