Current-induced magnetization dynamics in disordered itinerant ferromagnets

Yaroslav Tserkovnyak
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA and Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

Hans Joakim Skadsem
Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

Arne Brataas
Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway and Centre for Advanced Study at the Norwegian Academy of Science and Letters, Drammensveien 78, NO-0271 Oslo, Norway

Gerrit E. W. Bauer
Kavli Institute of NanoScience, Delft University of Technology, 2628 CJ Delft, The Netherlands
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Current-driven magnetization dynamics in ferromagnetic metals is studied in a self-consistent adiabatic local-density approximation in the presence of spin-conserving and spin-dephasing impurity scattering. Based on a quantum kinetic equation, we derive Gilbert damping and spin-transfer torques entering the Landau-Lifshitz equation to linear order in frequency and wave vector. Gilbert damping and a current-driven dissipative torque scale identically and compete, with the result that a steady current-driven domain-wall motion is insensitive to spin dephasing in the limit of weak ferromagnetism. A uniform magnetization is found to be much more stable against spin torques in the itinerant than in the *s*-*d* model for ferromagnetism. A dynamic spin-transfer torque reminiscent of the spin pumping in multilayers is identified and shown to govern the current-induced domain-wall distortion.

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I. INTRODUCTION

Metallic ferromagnets, notably the transition metals Fe, Co, and Ni, seem to be well understood, at least at temperatures sufficiently below criticality. Ground state properties such as cohesive energies, elastic constants,[1] magnetic anisotropies in multilayers,[2] but also low-energy excitations that define Fermi surfaces,[3] spin-wave dispersions, and Curie temperatures[4] are computed accurately and without adjustable parameters in the framework of local spin-density-functional theory (SDFT).[5] Transport properties such as electric resistances due to random impurities are accessible to *ab initio* band-structure calculations as well.[6] However, important issues are still under discussion. Consensus has not been reached, e.g., on the nature and modeling of the Gilbert damping of the magnetization dynamics,[7,8] the anomalous Hall effect,[9] and the current-induced magnetization dynamics.[10–18] The fundamental nature and technological importance of these effects make them attractive research topics.

In this paper, we hope to contribute to a better understanding of the interaction of an electric current with a magnetization order parameter in dirty ferromagnets, motivated in part by the sophistication with which the analogous systems of dirty superconductors have been mastered.[19] To this end, we proceed from time-dependent SDFT in an adiabatic local-density approximation (ALDA) and the Keldysh Green's function method in a quasiparticle approximation. We restrict ourselves to weak, diffusive ferromagnets where spin dynamics take place near the Fermi surface, an approximation that enables us to microscopically derive a simple quantum kinetic equation for the electronic spin distribution. The kinetic equation is used to derive a Landau-Lifshitz-Gilbert equation for the spatiotemporal magnetization that significantly differs from earlier phenomenological approaches based on the *s*-*d* model. We apply the general theory to the current-driven spin-wave excitation and domain-wall motion. Recently, Kohno *et al.*[20] treated the same problem by diagrammatic perturbation theory. For weak ferromagnets, i.e., when the exchange potential is small compared to the Fermi energy, their results agree with ours. For strong ferromagnets, they report small corrections.

The convincing evidence that transition-metal ground and weakly excited states are well described by the mean-field Stoner model provided by local-SDFT can be rationalized by the strong hybridization between the nearly free *s*-*p* bands and the localized *d* electrons.[5] It implies that the orbital angular momentum is completely quenched on time scales typical for the transport and magnetization dynamics. Both electric current and magnetization are therefore carried by the same itinerant Bloch states. The alternative *s*-*d* model, in which only the localized *d* electrons are intrinsically magnetic and affect the delocalized *s* electrons via a local spin-dependent exchange potential, is often used because it is amenable to sophisticated many-body treatments. On a mean-field level and with adjustable parameters, both models are completely equivalent for static properties. We find that the magnetization dynamics shows drastic and experimentally testable differences that derive from the necessity of a self-consistent treatment of the exchange potential in itinerant ferromagnets that is not required in the *s*-*d* model.

The paper is organized as follows: In Sec. II, we discuss the model and the basic assumptions of the theory. In Sec. III, the quantum kinetic equation is derived in the real-time Green's function formalism, which is then used to obtain the magnetic equation of motion in Sec. IV. The implications for the macroscopic dynamics are discussed in Sec. V, before the paper is briefly summarized in Sec. VI.

II. MODEL

In time-dependent SDFT (Refs. 21–23), the magnetic response is formally reduced to a one-body Hamiltonian in 2×2 Pauli spin space spanned by the unit matrix $\hat{1}$ and $\hat{\boldsymbol{\sigma}}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$, the vector of *one-half* of the Pauli matrices:

$$\hat{\mathcal{H}}=[\mathcal{H}_0+U(\mathbf{r},t)+V[\hat{\rho}](\mathbf{r},t)]\hat{1}+\gamma\hbar\hat{\boldsymbol{\sigma}}\cdot(\mathbf{H}+\mathbf{H}_{\mathrm{xc}}[\hat{\rho}])(\mathbf{r},t)+\hat{\mathcal{H}}_\sigma, \tag{1}$$

where $\mathcal{H}_0$ is the crystal Hamiltonian, U is the scalar disorder potential including an external electric field, and V the spin-independent part of the exchange-correlation potential. We recognize on the right-hand side the Zeeman energy due to the sum of externally applied and anisotropy magnetic fields $\mathbf{H}$ as well as an exchange-correlation contribution $\mathbf{H}_{\mathrm{xc}}$, disregarding an exchange-correlation magnetic field coupled to the orbital motion. Here, $\gamma>0$ is (minus) the gyromagnetic ratio and $\mathbf{H}_{\mathrm{xc}}$ and V are functionals of the time-dependent spin-density matrix

$$\rho_{\alpha\beta}(\mathbf{r},t)=\langle\Psi^\dagger_\beta(\mathbf{r})\Psi_\alpha(\mathbf{r})\rangle_t \tag{2}$$

that should be computed self-consistently from the Schrödinger equation corresponding to $\hat{\mathcal{H}}$. $\hat{\mathcal{H}}_\sigma$ is the spin-nondiagonal Hamiltonian accompanying magnetic and spin-orbit interaction potential disorder, thereby disregarding the "intrinsic" spin-orbit interaction in the bulk band structure, apart from the crystal anisotropy contribution to $\mathbf{H}$. Since we focus on low-energy magnetic fluctuations that are long range and transverse, we may restrict our attention to a single band with effective mass m_e. Systematic improvements for realistic band structures can be made from this starting point. We furthermore adapt the ALDA form for the exchange-correlation field:

$$\gamma\hbar\mathbf{H}_{\mathrm{xc}}[\hat{\rho}](\mathbf{r},t)\approx\Delta_{\mathrm{xc}}\mathbf{m}(\mathbf{r},t), \tag{3}$$

where $\mathbf{m}$ is the local magnetization direction with $|\mathbf{m}|=1$ and Δ_{xc} is the exchange splitting averaged over the unit cell. In terms of the spin density

$$\mathbf{s}(\mathbf{r})=\hbar\mathrm{Tr}[\hat{\boldsymbol{\sigma}}\hat{\rho}(\mathbf{r})], \tag{4}$$

$\mathbf{m}=-\mathbf{s}/s_0$, where s_0 is the equilibrium value of $|\mathbf{s}|$. For simplicity, the spin-independent random component of the potential $U(\mathbf{r})$ is described as a zero-average, Gaussian white noise correlator:

$$\langle U(\mathbf{r})U(\mathbf{r}')\rangle=\xi\delta(\mathbf{r}-\mathbf{r}'). \tag{5}$$

A characteristic scattering time τ is defined by

$$\xi=\frac{\hbar}{\pi(\nu_\uparrow+\nu_\downarrow)\tau}, \tag{6}$$

where ν_s is the spin-s density of states at the Fermi level. We consider two contributions to the spin-dephasing Hamiltonian $\hat{\mathcal{H}}_\sigma$: spin-orbit scattering associated with the impurities and scattering at magnetic disorder that is modeled as a static random exchange field $\mathbf{h}(\mathbf{r})$ with white-noise correlator

$$\langle h_\alpha(\mathbf{r})h_\beta(\mathbf{r}')\rangle=\xi_\alpha\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}'). \tag{7}$$

It turns out both can be captured in terms of a properly averaged, single parameter τ_σ for the characteristic transverse spin-dephasing time in the equation of motion for the magnetization. Derivation of the phenomenological τ_σ for concrete microscopic models and dephasing mechanisms will be the topic of future correspondence.

The ALDA is appropriate to describe corrections to the magnetization dynamics linear in $\partial_{\mathbf{r}}$ that, although vanishing for homogeneous systems,[22] are important in the presence of a current bias. The second-order correction (in homogeneous isotropic systems) is $H'_{\mathrm{ex}}\propto\hat{\boldsymbol{\sigma}}\cdot\partial^2_{\mathbf{r}}\mathbf{m}$, which contributes to the spin-wave stiffness [and can be taken into account via the effective field, see Eq. (29) below]. Not much is known about the importance of nonadiabatic many-body corrections that in principle contribute to the magnetization damping. However, for slowly varying perturbations of a homogeneous ferromagnet in time and space, the corrections to the ALDA are usually small.[23] Here we concentrate on dirty ferromagnets in which the impurity (or phonon) scattering dominates quasiparticle scattering due to electron-electron interactions.

In the next section, we derive the quantum kinetic equation for ferromagnetic dynamics by adiabatically turning on a uniform electric field until a steady state is established for a given current bias. The magnetization $\mathbf{m}$ is then perturbed with respect to a uniform ground state configuration $\mathbf{m}_0=\mathbf{z}$. We then compute small deviations of the spin density $\delta\mathbf{s}=\mathbf{s}+s_0\mathbf{z}$, and replace $\mathbf{s}$ by $-s_0\mathbf{m}$ in the resulting equations of motion, completing the self-consistency loop. A natural approach to carry out these steps is the Keldysh Green's function formalism, which we briefly outline in the following. If the reader is not interested in the technical details, we recommend jumping to Sec. IV for the discussion of the resulting equation of motion for the magnetization dynamics and Sec. V for the physical consequences for macroscopic dynamics.

III. QUANTUM KINETIC EQUATION

The Keldysh matrix Green's function can be represented by the retarded $\hat{G}^R(x,x')$, advanced $\hat{G}^A(x,x')$, and Keldysh $\hat{G}^K(x,x')$ components,[24] where x denotes position and time arguments. In the mixed (Wigner) representation $(\mathbf{r},t;\mathbf{k},\varepsilon)$, in which $(\mathbf{r},t)$ are the center of mass coordinates, and using the gradient approximation (valid when $\hbar\partial_t\ll\Delta_{\mathrm{xc}}$ and $\partial_r\ll k_F$, a characteristic Fermi wave number), the Keldysh component of the Dyson equation reads in what is called the semiclassical approximation

$$[\hat{G}_0^{-1},\hat{G}^K]_p-[\hat{G}^K,\hat{G}_0^{-1}]_p-2i[\hat{G}_0^{-1},\hat{G}^K]=\{\hat{\Sigma}^K,\hat{A}\}-\{\hat{\Gamma},\hat{G}^K\}. \tag{8}$$

The left-hand side (l.h.s.) is the kinetic equation in the clean limit and the right-hand side (r.h.s.) is the collision integral. In the derivation of this equation, self-energy renormalization effects on the l.h.s and gradient corrections to the collision integral have been disregarded. This requires that $\Delta_{xc}/\mu \ll 1$, where μ is the Fermi energy, although the corrections for large Δ_{xc} appear to be very small, see below. $\hat{\Sigma}$ is the self-energy due to disorder, which has three nontrivial components (R, A, and K) along the Keldysh contour. Here,

$$[\hat{B},\hat{C}]_p=\partial_x\hat{B}\cdot\partial_p\hat{C}-\partial_p\hat{B}\cdot\partial_x\hat{C} \tag{9}$$

is the generalized Poisson bracket (where $\partial_x\cdot\partial_p=\partial_\mathbf{r}\cdot\partial_\mathbf{k}-\hbar\partial_t\partial_\varepsilon$), [,] and {,} are matrix commutators and anticommutators,

$$\hat{A}=i(\hat{G}^R-\hat{G}^A) \tag{10}$$

and

$$\hat{\Gamma}=i(\hat{\Sigma}^R-\hat{\Sigma}^A). \tag{11}$$

$\hat{G}_0^{-1}$ is the inverse of the (retarded or advanced) Green's function in the absence of disorder:

$$\hat{G}_0^{-1}(\mathbf{r},t;\mathbf{k},\varepsilon)=[\varepsilon-\varepsilon_k+e\varphi(\mathbf{r},t)]\hat{1}-\Delta_{xc}\hat{\boldsymbol{\sigma}}\cdot\mathbf{m}(\mathbf{r},t), \tag{12}$$

where φ is the potential due to an applied electric field, and

$$\varepsilon_k=\frac{(\hbar k)^2}{2m_e}-\mu \tag{13}$$

are the eigenvalues of $\mathcal{H}_0$. We have disregarded the magnetic field for the moment. In the self-consistent Born approximation for scalar disorder scattering, the self-energy becomes

$$\hat{\Sigma}(\mathbf{r},t;\mathbf{k},\varepsilon)=\xi\int dk'\,\hat{G}(\mathbf{r},t;\mathbf{k}',\varepsilon) \tag{14}$$

for each of the three components, where $dk'=d^3\mathbf{k}'/(2\pi)^3$. Self-energies for spin-dependent scattering channels can be calculated analogously. For $\Delta_{xc}/\mu\ll 1$, we approximate the spectral function by Dirac delta functions at the two spin bands. Note that even though we are considering weak ferromagnets, the impurity concentration is still considered dilute, so that $\hbar/\tau,\hbar/\tau_\sigma\ll\Delta_{xc},\mu$. By disregarding gradient terms of self-energies and the spectral function in the derivation of Eq. (8), the Wigner representation transformed the collision integral into a local form. Gradient corrections disappear when the system is spatiotemporally homogeneous and/or we restrict our attention to weak ferromagnets, thereby discarding corrections of order $\mathcal{O}((\hbar/\tau,\Delta_{xc})/\mu)$. In spite of this restriction, we believe that our formalism still captures the essential physics of the model (and therefore transition-metal ferromagnets) in a clear and coherent fashion. Assessing the leading corrections to our treatment would require one to reconsider as well the simple ALDA mean-field treatment we are relying on.

We concentrate now on the spin dynamics for small deviations of the magnetization direction $\mathbf{m}=\mathbf{z}+\mathbf{u}$ from the z axis ($\mathbf{u}\perp\mathbf{z}$) in the presence of a weak uniform electric field $\mathbf{E}=-\partial_\mathbf{r}\varphi$ in the quasiparticle approximation for the Keldysh Green's function,

$$\hat{G}^K(\mathbf{r},t;\mathbf{k},\varepsilon)=-2\pi i\sum_s\delta(\varepsilon-\varepsilon_{ks})\hat{g}_{\mathbf{k}s}(\mathbf{r},t), \tag{15}$$

where

$$\varepsilon_{ks}=\varepsilon_k+\frac{s}{2}\Delta_{xc}. \tag{16}$$

Two spin bands labeled by $s=\uparrow,\downarrow=\pm$ become separated when the disorder is weak. Note that in equilibrium,

$$\hat{g}_{\mathbf{k}s}=\left(\frac{1}{2}+s\hat{\sigma}_z\right)\tanh\left(\frac{\varepsilon_{ks}}{2k_BT}\right), \tag{17}$$

where T is the temperature and k_B the Boltzmann constant. The electric field applied to a rigidly uniform ferromagnet, $\mathbf{u}=0$, excites a nonequilibrium distribution $\hat{g}_{\mathbf{k}s}$ that is also diagonal in the spin indices. Interband spin-flip scattering vanishes upon momentum integration, since a weak uniform electric field induces only a p-wave distribution. The transport in each spin band [obtained by integrating Eq. (8) over energy ε at fixed $\mathbf{k}$ near ε_{ks}] is thus described by the conventional Boltzmann equation,[25] at $T\to 0$ solved by the "drift" distribution

$$\delta\hat{g}_{\mathbf{k}s}=\frac{\hbar e}{\pi\xi\nu_s}\left(\frac{1}{2}+s\hat{\sigma}_z\right)\mathbf{E}\cdot\mathbf{v}_\mathbf{k}\delta(\varepsilon_{ks}). \tag{18}$$

The distribution functions $\hat{g}_{\mathbf{k}s}$ acquire off-diagonal components (describing transverse spins) in the presence of a finite $\mathbf{u}$ (so that out of equilibrium the spin subscript should not be taken literally). Equation (8) leads to the linearized kinetic equation for the transverse component $\hat{g}^T_{\mathbf{k}s}=\mathbf{g}_{\mathbf{k}s}\cdot\hat{\boldsymbol{\sigma}}$ ($\mathbf{g}_{\mathbf{k}s}\perp\mathbf{z}$):

$$\begin{aligned}&\hbar\partial_t\mathbf{g}_{\mathbf{k}s}+\hbar(\mathbf{v}_\mathbf{k}\cdot\partial_\mathbf{r})[\mathbf{g}_{\mathbf{k}s}-\Delta_{xc}\mathbf{u}\delta(\varepsilon_{ks})]-\Delta_{xc}\mathbf{z}\times\mathbf{g}_{\mathbf{k}s}+s\Delta_{xc}\mathbf{z}\\&\quad\times\mathbf{u}\,\mathrm{sign}(\varepsilon_{ks})+\frac{s\hbar e}{\pi\xi\nu_s}(\mathbf{E}\cdot\mathbf{v}_\mathbf{k})\Delta_{xc}\mathbf{z}\times\mathbf{u}\delta(\varepsilon_{ks})\\&\quad-e(\mathbf{E}\cdot\partial_\mathbf{k})\mathbf{g}_{\mathbf{k}s}=\pi\xi\sum_{s'}\int dk'\,\delta(\varepsilon_{k's'}-\varepsilon_{ks})[\mathbf{g}_{\mathbf{k}'s'}-\mathbf{g}_{\mathbf{k}s}\\&\quad+(s-s')\mathbf{u}\,\mathrm{sign}(\varepsilon_{ks})]+(\nu_{-s}/\nu_s-1)\hbar e(\mathbf{E}\cdot\mathbf{v}_\mathbf{k})\mathbf{u}\delta(\varepsilon_{ks})\\&\quad-\frac{\hbar}{\tau_\sigma}\left(\mathbf{g}_{\mathbf{k}s}-s\mathbf{u}\left[\mathrm{sign}(\varepsilon_{ks})+\frac{\hbar e}{\pi\xi\nu_s}\mathbf{E}\cdot\mathbf{v}_\mathbf{k}\delta(\varepsilon_{ks})\right]\right).\end{aligned} \tag{19}$$

Quasiparticles propagate with group velocity $\mathbf{v}_\mathbf{k}=\partial_\mathbf{k}\varepsilon_k/\hbar$. On the l.h.s., an inhomogeneous exchange field is seen to cause electron acceleration and spin precession. The second term on the second line describes spin precession of electrons accelerated by the electric field and the following term acceleration of the precessed electrons. On the r.h.s. we recognize elastic disorder scattering and transverse spin relaxation, the latter in terms of the spin-dephasing time τ_σ. Energy-conserving mixing between the spin bands is allowed by disorder (in the presence of transverse fields), as reflected in the $s'=-s$ part of the collision integral. We also took into account the contribution to the r.h.s. of Eq. (8) from anticommuting the current-induced drift Keldysh component with the spectral-function correction due to the magnetization deviation $\mathbf{u}$:

$$\delta\hat{A}=2\pi\hat{\boldsymbol{\sigma}}\cdot\mathbf{u}\sum_s s\,\delta(\varepsilon-\varepsilon_{ks}). \tag{20}$$

IV. MAGNETIC EQUATION OF MOTION

Integrating the kinetic equation (19) over momentum yields the equation of motion for the nonequilibrium spin density $\delta\mathbf{s}=-(\hbar/4)\Sigma_s\int dk\mathbf{g}_{\mathbf{k}s}$:

$$\partial_t\delta\mathbf{s}-\frac{\Delta_{\rm xc}}{\hbar}\mathbf{z}\times\delta\mathbf{s}-\frac{\Delta_{\rm xc}}{\hbar}\mathbf{z}\times\mathbf{u}s_0 =\frac{\hbar}{4}\sum_s\int dk(\mathbf{v}_{\mathbf{k}}\cdot\partial_{\mathbf{r}})\mathbf{g}_{\mathbf{k}s}-\frac{\delta\mathbf{s}+\mathbf{u}s_0}{\tau_\sigma}. \tag{21}$$

The integral on the r.h.s. is the divergence of the spin-current density, determined by the p-wave component of $\mathbf{g}_{\mathbf{k}s}$, which can be found by a tedious (but straightforward) manipulation of the kinetic equation. Confining our interest to spatially slowly varying phenomena results in a major simplification: since $\partial_{\mathbf{r}}$ already appears in Eq. (21), we can disregard spatial derivatives in the p-wave component of $\mathbf{g}_{\mathbf{k}s}$. We can now also include a static field $H\ll\Delta_{\rm xc}$ along the z axis by substituting primed quantities $\Delta'_{\rm xc}=\Delta_{\rm xc}+\gamma H$ and $\mathbf{u}'=-(1-\gamma H/\Delta'_{\rm xc})\delta\mathbf{s}/s_0$ for the corresponding unprimed ones in the above expressions. The final result for the small-angle transverse spin dynamics is

$$\partial_t\mathbf{u}=\omega_0\mathbf{z}\times\mathbf{u}-\beta\omega_0\mathbf{u}+\mathcal{P}\left[1-\mathbf{z}\times\frac{\hbar\partial_t}{\Delta_{\rm xc}}\right](\mathbf{j}\cdot\partial_{\mathbf{r}})\mathbf{u}, \tag{22}$$

disregarding the $\mathcal{O}(1/\Delta_{\rm xc}^2)$ terms inside the square brackets. Here $\mathbf{j}$ is the applied current density bias, $\gamma\mathbf{H}=\omega_0\mathbf{z}$,

$$\beta=\frac{\hbar}{\tau_\sigma\Delta_{\rm xc}}, \tag{23}$$

and $\mathcal{P}=(\hbar/2e)P/s_0$, where $P=(\sigma_\uparrow-\sigma_\downarrow)/(\sigma_\uparrow+\sigma_\downarrow)$ is the conductivity spin polarization, σ_s being the conductivity for spin s along $-\mathbf{m}$. For a Drude conductivity of parabolic bands, $P=\Delta_{\rm xc}/(\varepsilon_{F\uparrow}+\varepsilon_{F\downarrow})$. We can transform the Bloch-like damping term in Eq. (22) to the Gilbert form by multiplying the equation by $1-\beta\mathbf{z}\times$ from the left, which brings us to our central result:

$$\partial_t\mathbf{m}=\partial_t\mathbf{m}|_{\rm LLG}+\partial_t\mathbf{m}|_j, \tag{24}$$

where

$$\partial_t\mathbf{m}|_{\rm LLG}=-\gamma\mathbf{m}\times\mathbf{H}+\beta\mathbf{m}\times\partial_t\mathbf{m} \tag{25}$$

is the usual Landau-Lifshitz-Gilbert (LLG) with Gilbert damping

$$\alpha_{\rm LDA}=\beta, \tag{26}$$

$$\partial_t\mathbf{m}|_j=\mathcal{P}\left[1-\mathbf{m}\times\left(\beta+\frac{\hbar\partial_t}{\Delta_{\rm xc}}\right)\right](\mathbf{j}\cdot\partial_{\mathbf{r}})\mathbf{m}, \tag{27}$$

where, as before, we neglect the $\mathcal{O}(1/\Delta_{\rm xc}^2)$ terms. $\alpha_{\rm LDA}=\hbar/\tau_\sigma\Delta_{\rm xc}$ relates the collective magnetization damping to the single-electron spin relaxation that can be measured independently[8] and is consistent with experiments in permalloy films.[26] Equations (25) and (27) hold for small deviations from a homogeneous equilibrium state, but have the correct spin-rotationally invariant form valid also for long-wavelength large-angle dynamics when the magnetic state is locally close to the equilibrium configuration (which requires a large exchange splitting in comparison with other relevant energy scales). In particular, Eq. (27) should correctly describe domain walls wider and spin-wave lengths longer than the magnetic coherence length $\hbar v_F/\Delta_{\rm xc}$. For the same reason, the field $\mathbf{H}$ does not have to be nearly collinear with $\mathbf{m}$.

We can apply our method also to the mean-field s-d model[27] which leads to interesting differences. We reproduced the phenomenologically derived Eq. (11) of Ref. 17 (plus the dynamic term linear in ∂_t). The Gilbert damping becomes reduced by the fraction η of the total spin angular momentum carried by the s electrons, while β is unmodified:

$$\alpha_{s-d}=\eta\beta, \tag{28}$$

assuming $\eta\ll1$ (Ref. 27). We will see in the following that the ratio β/α determines several interesting physical quantities with $\beta/\alpha_{\rm LDA}=1$ being a very special point. A sizable s-d character of the ferromagnetism alters this ratio, which could also be affected by a possible d-magnetization damping in addition to the s-electron dephasing treated here.

Recently, a diagrammatic treatment of spin torques in static weakly disordered localized and itinerant ferromagnets has been reported by Kohno *et al.*[20] Their calculation is not restricted to weak ferromagnets [although it misses dynamic current-driven torques such as the last term in Eq. (27)], and they find that in contrast to our result β is not universally identical to α in the LDA approximation. However, the ratio $\beta/\alpha_{\rm LDA}$, in Ref. 20 expressed by the ratio between the density of states averaged over the two Fermi surfaces and the energy range spanned by $\Delta_{\rm xc}$, is close to unity for almost all systems of interest. In particular, at low temperatures and in three dimensions, Kohno *et al.*'s expressions can be evaluated to be $\beta/\alpha_{\rm LDA}\approx1+(1/48)(\Delta_{\rm xc}/\mu)^2$ (with the same correction for the β/α_{s-d} ratio). This quadratic deviation from unity is very small; even for $\Delta_{\rm xc}/\mu\sim1/2$ it only amounts to about half a percent. The present quasiparticle treatment is not well suited to study ferromagnets exhibiting an arbitrarily strong exchange splitting (due to the increasing importance of gradient corrections to the semiclassical approximation with stronger exchange splittings). We are therefore hesitant to make predictions for half-metallic ferromagnets. We are, however, confident that we capture the important physics of most experimental systems to date. For this reason, the present framework can also be used in studies of, e.g., relevant spin-dephasing mechanisms and microscopically derived scattering rates. The influence of realistic band structure effects, intrinsic spin-orbit, and Coulomb interaction, as well as corrections beyond the mean-field description might be more important than the gradient corrections to the ratio β/α. Furthermore, it is in general possible that other than impurity-related dephasing processes may contribute differently to α and β, especially in the presence of strong anisotropies.

V. CURRENT-DRIVEN DOMAIN-WALL MOTION AND BULK INSTABILITIES

Let us proceed by discussing the influence of β/α on the magnetization dynamics, and in particular the limiting case in which this ratio is unity. The dominant term $\boldsymbol{\tau}=\mathcal{P}(\mathbf{j}\cdot\partial_{\mathbf{r}})\mathbf{m}$ in Eq. (27) is the conventional spin-transfer torque that, as far as the equation of motion is concerned, can be absorbed into the magnetic free energy.[11–13] The (dissipative) term proportional to β acts like a magnetic field parallel to the direction of the magnetization gradient in the current direction. This term appears in our treatment by transforming Eq. (22) into the LLG form (25). Zhang and Li[17] noted that although this "effective field" is much smaller than $\boldsymbol{\tau}$ when $\beta\ll 1$, it has a qualitative effect on the domain-wall motion. For example, in the absence of an external magnetic field, a finite terminal velocity of a current-driven Néel wall is found for all currents only when the effective field does not vanish. Judging from the importance of dynamic corrections to the spin torques in multilayer structures,[28] the dynamic contribution in Eq. (27) could be as significant since the typical frequencies of ferromagnetic dynamics are $\omega\sim\tau_\sigma^{-1}$.

In this section, we discuss several experimental consequences for $\mathbf{j}=j\mathbf{z}$, and a net effective field

$$\mathbf{H}=(Km_z+H)\mathbf{z}-K_\perp m_x\mathbf{x}+A\nabla^2\mathbf{m}. \tag{29}$$

Here, K is an easy axis and $K_\perp$ an easy-plane anisotropy constant, A is the exchange-stiffness, and H is the applied magnetic field. $K,K_\perp,A,H\geq 0$.

Let us first consider current-driven domain-wall motion in the absence of applied field, $H=0$. At the onset of the applied current density, a Néel wall along the z direction of width $W=\sqrt{A/K}$ with magnetization in the yz plane (pointing along z at $z\to-\infty$ and in the opposite direction at $z\to\infty$) starts to move[17] with velocity (for not too large currents)

$$v_i=-\mathcal{P}j, \tag{30}$$

acquiring a terminal steady velocity for a constant current density given by

$$v_f=-\frac{\beta}{\alpha}\mathcal{P}j. \tag{31}$$

We find that the terminal velocity (31) is not influenced by the dynamic term on the r.h.s. of Eq. (27), and we get $v_f/v_i=\beta/\alpha_{\rm LDA}=1$ for the self-consistent LDA model of itinerant ferromagnetism. The initial velocity (30) agrees with expectations based on angular-momentum conservation, and, curiously, for our model, the terminal velocity is the same. According to Ref. 20, in three dimensions, the correction to $\beta/\alpha_{\rm LDA}$ of order $(\Delta_{\rm xc}/\mu)^2$ is positive, which means that $v_f\gtrsim v_i$. Yamaguchi *et al.*[29] expressed the current-induced domain-wall velocity

$$v_f=-\zeta\mathcal{P}j \tag{32}$$

in terms of an "efficiency" ζ of spin-current conversion into magnetization dynamics. Their experimental value $\zeta\sim 0.1$ is much smaller than our result of $\zeta=1$ in the absence of bulk or interface pinning (which, if smooth enough, could in principle be added to the effective field $\mathbf{H}$). For currents in excess of a threshold imposed by extrinsic pinning defects, Barnes and Maekawa[18] predicted $\zeta=1$ for an s-d model, in contrast to a nonuniversal mean-field result $\zeta=\beta/\alpha_{s-d}=1/\eta$ of Ref. 17 which we confirm here.

Under the action of the current-induced spin torque, the shape of the moving domain wall distorts somewhat with respect to the equilibrium configuration. The corresponding domain-wall change from the equilibrium value W to the steady-state value W_f was calculated in Ref. 15 using the Walker's ansatz. After generalizing their method to include the effects of β as well as the dynamic term in the magnetic equation of motion (27), we find

$$1-\frac{W_f}{W}\approx\frac{(\mathcal{P}j)^2}{2\gamma A}\left[\frac{1}{\gamma K_\perp}\left(1-\frac{\beta}{\alpha}\right)^2-\frac{\hbar}{\Delta_{\rm xc}}\frac{\beta}{\alpha}\right], \tag{33}$$

where the first (second) term on the r.h.s. describes the wall deformation due to the static (dynamic) part of Eq. (27). Now, considering $\alpha_{\rm LDA}=\beta$, the first term vanishes and the wall slightly broadens, unlike the wall compression predicted for the s-d model with a finite damping α but setting $\beta=0$ (Ref. 15).

Finally, we discuss small-amplitude spin-wave solutions of Eqs. (24), (25), and (27), of the form

$$\mathbf{m}(\mathbf{r},t)=\mathbf{z}+\mathbf{u}_0\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)]. \tag{34}$$

We are especially interested in solutions with ${\rm Im}\,\omega>0$, which describe exponentially growing spin-wave amplitude, signaling the onset of current-driven instabilities. We find that the critical current corresponding to ${\rm Im}\,\omega=0$ is determined from

$$b'^2\left(1-\frac{\beta}{\alpha}\right)^2=\left(H'+\frac{\beta}{\alpha}\frac{b'^2\hbar}{\Delta_{\rm xc}}\right)\left(K'+H'+\frac{\beta}{\alpha}\frac{b'^2\hbar}{\Delta_{\rm xc}}\right), \tag{35}$$

where $b'=\mathcal{P}(\mathbf{q}\cdot\mathbf{j})$, $H'=\gamma(H+K+Aq^2)$, and $K'=\gamma K_\perp$. For $\beta\to 0$, this reduces to

$$|b'|\to\sqrt{H'(K'+H')} \tag{36}$$

which can be thought of as the Doppler shift due to drifting spins necessary to overcome the natural spin-wave frequency.[11,12,14,15] Our result that $\alpha_{\rm LDA}=\beta$ for weak ferromagnets, however, implies that a uniform magnetic state is stable against current-driven torques. In general, the critical current density j_c determined from Eq. (35) can be significantly enhanced (depending on how close α and β are) with respect to the "Doppler-shift value" j_{c0} calculated from Eq. (36):

$$j_c = \frac{j_{c0}}{|1 - \beta/\alpha|}, \tag{37}$$

where small corrections proportional to β on the r.h.s. of Eq. (35) have been disregarded.

VI. SUMMARY

In conclusion, we have used a quasiparticle approximation, valid for weak ferromagnets, to derive an equation of motion for the magnetization dynamics of disordered ferromagnets similar to the conventional LLG equation (25) with Gilbert damping α and a current-induced contribution (27) that is parametrized by a normalized single-electron spin-dephasing rate $\beta=\hbar/\tau_\sigma\Delta_{xc}$. By virtue of the quasiparticle approximation, we obtain intuitively appealing kinetic equations that clearly reflect the physical processes involved.

Within a self-consistent picture based on the local density approximation, we related the macroscopic damping in weak itinerant ferromagnets to the microscopic spin dephasing: $\alpha_{LDA}=\beta$, and pointed out striking implications for current-driven macroscopic dynamics when the ratio β/α is close to unity (which can also be expected for strong ferromagnets in the ALDA approximation). We furthermore noted remarkable differences in the dynamics of itinerant ferromagnets, supposedly well-described by the local-density approximation, and those with localized d or f electron magnetic moments.

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