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Customized Bundle Pricing for Information Goods: A Nonlinear Mixed-Integer Programming Approach

Abstract

This paper proposes using nonlinear mixed-integer programming to solve the customized bundle-pricing problem in which consumers are allowed to choose up to N goods out of a larger pool of J goods. Prior work has suggested that this mechanism has attractive features for the pricing of information and other low-marginal cost goods. Although closed-form solutions exist for this problem for certain cases of consumer preferences, many interesting scenarios cannot be easily handled without a numerical solution procedure. In this paper, we investigate the efficiency gains created by customized bundling over the alternatives of pure bundling or individual sale under different assumptions about customer preferences and firm cost structure, as well as the potential loss of efficiency caused by pricing with incomplete information about consumer reservation values. Our analysis suggests that customized bundling enhances sellers' profits and enhances welfare when consumers do not place positive values on all goods, and that this consumer characteristic is much more important than the shape of the valuation distribution in determining the optimal pricing scheme. We also find that customized bundling outperforms both pure bundling and individual sale in the presence of incomplete information, and that customized bundling still outperforms other simpler pricing schemes even when exact consumer valuations are not known ex ante.

Keywords

nformation goods, electronic commerce, customized bundle, pricing, nonlinear programming, integer programming

Disciplines

Non-linear Dynamics | Other Economics | Other Mathematics | Other Social and Behavioral Sciences

Comments

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Optimal Customized Bundle Pricing for Information Goods

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Optimal Customized Bundle Pricing for Information Goods

Abstract

In this paper, we provide a model for choosing the optimal number of bundles and their prices in the context of designing markets for information goods. Selling bundled goods is a widespread phenomenon, and a recent paper by Bakos and Brynjolfsson (1999) showed that under conditions of zero marginal cost, and independent and identically distributed customer valuations, pure bundling is optimal for a multi-product monopolist. In this paper, we show, using a nonlinear programming framework and numerical analyses, that in many cases, even with iid customers and zero marginal cost, offering multiple customized bundles may be better. In cases with non-iid customers, offering different customized bundles is clearly optimal; a pure bundling approach may lead to significant loss of profit.

1. Introduction

In this paper, we provide a model for choosing the optimal number of bundles and their prices in the context of designing markets for information goods. The emergence and rapid growth of the Internet has led researchers and information goods providers to rethink the economics of selling information goods. Many researchers have argued that such electronic distribution of information goods enables information goods providers to engage in more efficient and profitable pricing schemes. For example, Bakos and Brynjolfsson (1999) showed that under certain conditions: zero marginal costs, and independent and identically distributed customer valuations for all goods, pure bundling (offering all goods for a fixed price) is optimal for a multiproduct monopolist.

However, there are some practical difficulties associated with a pure bundling strategy. First, a large bundle of information goods may price out consumers with budget constraints. For example, when the information goods provider has 300 different kinds of goods and all customers have i.i.d. U(0,1) valuation on each good, by the law of large number, it is suggested that the provider should sell the pure bundle at a price of $150 and that almost all customers will be willing to spend this much! But in reality, we would hardly expect that all customers will be willing to spend nearly $150 on a bundle of 300 different kinds of goods, many of which would not add any value at all. Second, valuable demand information may be obscured when all goods are sold in a whole bundle. Third, in reality, we may seldom incur situations where customer valuations for all goods are i.i.d. Customers usually have high variation in valuations and tastes.

The i.i.d. assumption itself can be violated in many cases. For example, all people may not positively value all information goods (Chuang and Sirbu, 1999). Different people usually have very different tastes in consuming information goods; they only positively value some of the information goods that interest them. In addition to this reason of heterogeneous personal tastes, due to budget (time, attention) constraints, and bounded rationality, customers can only consume a finite number of information goods, usually small. For example, we seldom read every article or column in the newspaper; the empirical study performed by King and Griffiths (1995) show that out of the 80 to 100 articles in an average journal, over 40% of those surveyed read no more than five articles. This, again, indicates that consumers only put positive values on some goods among all the goods available.

Further, Hitt and Chen (2001) show that offering one or more customized bundles (and letting customers self-select goods up to a certain number) can further increase a firm's profit while reducing inefficiency when sellers face heterogeneous demand, e.g. when customers do not positively value all goods or face budget constraints. However, they could only find optimal bundle sizes and prices under certain customer valuation functions (e.g. for valuations with the single-crossing property). Unfortunately, this property seldom holds in reality, and so it is still unclear to many information goods providers how to sell and price their goods in practice. In particular, it is important to decide how many bundles to offer, and at what price(s) to charge given various kinds of valuation functions.

A previous paper (Hanson and Martin, 1990) tried to solve the bundle pricing problem for a product line as a mixed integer linear program. A significant difference between our framework and theirs is that they assume that supplying customers with bundles cost money; i.e. there is a non-zero marginal cost for producing items. This assumption does not hold true for most information goods. Further, their assumptions lead to a formulation where there is an exponential growth of variables with the number of goods. Thus, the best they can do is to solve examples with no more than 21 goods. Given the fact that almost all, if not all, information goods providers offer much more than 21 goods, this technique may be inappropriate for information goods providers. In addition, they do not consider menu costs for offering multiple bundles.

This paper aims at providing guidance to a monopolist selling a large number of information goods on how to optimally sell and price their goods by formulating the problem as a nonlinear mixed integer program based on the concept developed by Hitt and Chen's (2001) customized bundling strategy. We note that pure bundling is an extreme example of customized bundling, and thus is treated as a special case in our model. Our model can accommodate not only any kind of demand distribution or consumer valuation functions, but also the marginal bundle cost of a bundle and the menu cost which may occur in offering multiple bundles. Moreover, since the variables in our model grow only linearly with the number of goods, our model can be solved for any feasible number of information goods. Given any demand structure and cost structure, our model can determine how many bundles an information goods provider should offer, and at what price it should charge for each bundle.

We use pure bundling (selling all goods at a fixed price) and individual sale (selling each good at its profit maximization price) as benchmark to demonstrate the performance of the solution from our model. Overall, our numerical analyses suggest that, under independently and identically distributed (i.i.d.) customer valuations and zero marginal cost, pure bundling is optimal, as shown in a recent paper by Bako and Brynjolfsson (1999), however, pure bundling is only one of the optimal solutions. In cases with non i.i.d. customers, the solution given by our model (i.e., offering different customized bundles) outperforms pure bundling and selling individual goods. In addition, number of bundles to be offered is a decreasing function of both bundle costs and menu costs.

We introduce our model formulation in section 2, and discuss the solution approach in section 3. Numerical results and case analysis are presented in section 4, followed by concluding remarks in section 5.

2. Problem Formulation

In this section, we formulate the optimal bundling and pricing problem for an information goods provider that distributes J goods to I consumers. The model is developed from the seller's perspective. The problem for the seller is to decide how many goods to include in each bundle, and how to price these bundles in order to maximize its profit subject to a set of consumer participation and incentive constraints. The problem for the buyer (i.e. customer) is to choose exactly what items go into each bundle. Following Stigler (1963), we assume that customer demand information is captured by a vector of reservation prices of the items that go into a bundle. Customers will want to maximize consumer surplus based on the difference between the total reservation price for items in a bundle, and the price they pay. The seller has take into account dimensions of the customer's decision problem which appear as constraints in the seller's problem.

Note that the price is only determined by the size of the bundle and not its contents. For example, the seller might decide to bundle 3 CDs for $20.00, and the buyer will choose which 3 CDs to buy. Different buyers could choose different CDs to buy. This is similar to what many CD and DVD clubs like Columbia House offer. There is an overhead cost for having bundles for sales. This overhead cost is due to the need by the provider to search and package different sizes of goods, and to keep track of a heterogeneous demand set of the customers. The more bundles offered in a sales menu, the larger the overhead cost. In addition, the cognitive costs consumers face when evaluating large sets of offers (Shugan, 1980) are also one motivation for menu costs. Thus, the number of bundles for sale will be limited. Therefore the provider/seller of bundles will try to maximize revenue net of the cost of providing a menu of bundles.

We will model the problem of optimizing the customized bundles for information goods and pricing them as a nonlinear integer mathematical programming problem. Table 1 provides the definitions of all the parameters and variables used in this model. The primal problem is given by:

Table 1 Definitions of the parameters and variables used in the model

Given Parameters
B_j: Bundle cost of creating a bundle of j goods. This would include the sum of marginal production cost, distribution cost, transaction cost, any binding cost, etc. We assume this is the same for any kind of bundles of j goods.
I: There are total I potential customers in our target market.
J: The vendor has total J different kinds of information goods in hand.
M: Marginal menu cost if we add one more bundle choice on the menu.
R_{ij}: Total reservation price of customer i's top j favorite goods.
V_{ik}: Customer i's reservation price for his kth favorite information goods.
Decision Variables
P_j: The price assigned to the bundle of j goods.
S_i: Consumer surplus of customer i.
X_{ij}: The decision variable which is 1 if consumer i chooses to buy the bundle of j goods and 0 otherwise.
Y_j: The decision variable which is 1 if the vendor chooses to offer the bundle of j goods on the menu and 0 otherwise. $\left(\sum_{j=1,\dots,J} Y_j = \text{\# of customized bundles offered}\right)$

Primal Problem IP:

$$\text{Max}\sum_{i=1,\dots I}\sum_{j=1,\dots,J}(P_j - B_j)X_{ij} - \sum_{j=1,\dots,J} MY_j \qquad (1)$$

s.t.

$$R_{ij} = \sum_{k=1,\dots,j} V_{ik}, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (2)$$

$$S_i \geq \left(R_{ij} - P_j\right)Y_j, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (3)$$

$$S_i = \sum_{j=1,\dots,J}(R_{ij} - P_j)X_{ij}, \qquad i=1,\dots,I \qquad (4)$$

$$(R_{ij} - P_j)X_{ij} \geq 0, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (5)$$

$$\sum_{j=1,\dots,J} X_{ij} \leq 1, \qquad i=1,\dots,I \qquad (6)$$

$$X_{ij} \leq Y_j, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (7)$$

$$S_i \geq 0, \qquad i=1,\dots,I \qquad (8)$$

$$P_j \geq 0, \qquad j=1,\dots,J \qquad (9)$$

$$X_{ij} = 0 \text{ or } 1, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (10)$$

$$Y_j = 0 \text{ or } 1, \qquad j=1,\dots,J \qquad (11)$$

The objective function (1) is to maximize the total profits of the vendor. This is calculated by the summation of profit obtained from each customer minus the menu cost of the vendor. Each constraint is explained in the following.

Constraint (2) defines R_{ij} as the total reservation price of customer *i*'s top *j* favorite goods. In regards to the assumption on complete information of consumer reservation price vector, such truth-telling could be implemented through mechanisms like the Groves-Clarks mechanism. Constraint (3) ensures that each customer maximizes her surplus S_i. This is achieved by requiring the final consumer surplus is no less than the consumer surplus from any other bundle offered by the seller (incentive compatible constraints). Constraint (4) defines consumer surplus as the difference between customer *i*'s reservation price and the market price of the bundle she chooses. Constraint (5) ensures that consumer will choose a bundle only when her surplus on this bundle is nonnegative (individual rationality constraints); otherwise, she won't choose this bundle. Constraint (6) ensures the assumption that each customer will purchase exactly one bundle, or won't make a purchase at all. This constraint could be relaxed and this relaxation would further favor our customized bundle setting. Constraint (7) ensures that only when the vendor offers the bundle of j goods, customers can choose this kind of bundle; otherwise, no such choice is available. Constraint (8) and (9) are nonnegative constraints for consumer surplus and bundle price. Constraint (10) enforces the integer property of the decision variables with respect to consumer purchasing, and constraint (11) enforces the integer property of the decision variables with respect to bundle offering.

We will now provide a method to solve this problem for cases where the number of potential items that can be bundled is great.

3. Solution Approach

3.1. Lagrangean Relaxation

By using the Lagrangean relaxation method, we can transform the primal problem (IP) mentioned above into the following Lagrangean relaxation problem (LR) where constraint (6) is relaxed:

Problem LR

$$\phi(a)=\text{Max}\sum_{i=1,\dots,I}\sum_{j=1,\dots,J}(P_j-B_j)X_{ij}-\sum_{j=1,\dots,J}MY_j+\sum_{i=1,\dots,I}a_i(1-\sum_{j=1,\dots,J}X_{ij}) \quad (12)$$

s.t.

$$R_{ij}=\sum_{k=1,\dots,j}V_{ik}, \quad i=1,\dots,I;\ j=1,\dots,J \quad (13)$$

$$S_i \geq \left(R_{ij}-P_j\right)Y_j, \quad i=1,\dots,I;\ j=1,\dots,J \quad (14)$$

$$S_i=\sum_{j=1,\dots,J}(R_{ij}-P_j)X_{ij}, \quad i=1,\dots,I \quad (15)$$

$$(R_{ij}-P_j)X_{ij}\geq 0, \quad i=1,\dots,I;\ j=1,\dots,J \quad (16)$$

$$X_{ij}\leq Y_j, \quad i=1,\dots,I;\ j=1,\dots,J \quad (17)$$

$$S_i\geq 0, \quad i=1,\dots,I \quad (18)$$

$$P_j\geq 0, \quad j=1,\dots,J \quad (19)$$

$$X_{ij}=0 \text{ or } 1, \quad i=1,\dots,I;\ j=1,\dots,J \quad (20)$$

$$Y_j=0 \text{ or } 1, \quad j=1,\dots,J \quad (21)$$

Since we relax only one constraint here, we have one corresponding Lagrangean Multiplier a_i which is nonnegative because the constraint we relaxed is an inequality constraint.

Note that we can reorder the terms in objective function of *Problem LR* and obtain the following objective function:

$$\phi(a)=\text{Max}\sum_{j=1,\ldots,J}[\sum_{i=1,\ldots,I}(P_j - B_j - a_i)X_{ij} - MY_j]+\sum_{i=1,\ldots,I} a_i \qquad (22)$$

This nonlinear integer programming problem *LR* is complicated and hard to solve. But we have the following observations. First, given constraint (15) and (16), constraint (18) is redundant, so we can remove it. Second, constraint (15) simply defines the consumer surplus. If we replace S_i in constraint (14) with $\sum_{k=1,\ldots,J}(R_{ik}-P_k)X_{ik}$, we can remove constraint (15). After doing these, we will get an equivalent problem as following:

Problem LR

$$\phi(a)=\text{Max}\sum_{j=1,\dots,J}[\sum_{i=1,\dots,I}(P_j - B_j - a_i)X_{ij} - MY_j]+\sum_{i=1,\dots,I}a_i \quad (23)$$

s.t.

$$R_{ij} = \sum_{k=1,\dots,j}V_{ik}, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (24)$$

$$\sum_{k=1,\dots,J}(R_{ik} - P_k)X_{ik} \ge \left(R_{ij} - P_j\right)Y_j, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (25)$$

$$(R_{ij} - P_j)X_{ij} \ge 0, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (26)$$

$$X_{ij} \le Y_j, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (27)$$

$$P_j \ge 0, \qquad j=1,\dots,J \qquad (28)$$

$$X_{ij} = 0 \text{ or } 1, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (29)$$

$$Y_j = 0 \text{ or } 1, \qquad j=1,\dots,J \qquad (30)$$

Note *LR* can be decomposed into *J* different problems except for the complicated constraint (25). Because the left-hand side is summed over *J*, one cannot decompose this particular constraint. We will drop constraint (25) and solve a relaxed version of *LR*, which we call *Problem R*.

Problem R

$$V(a)=\text{Max}\sum_{j=1,\dots,J}[\sum_{i=1,\dots,I}(P_j - B_j - a_i)X_{ij} - MY_j]+\sum_{i=1,\dots,I}a_i \quad (31)$$

s.t.

$$R_{ij} = \sum_{k=1,\dots,j}V_{ik}, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (32)$$

$$(R_{ij} - P_j)X_{ij} \ge 0, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (33)$$

$$X_{ij} \le Y_j, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (34)$$

$$P_j \ge 0, \qquad j=1,\dots,J \qquad (35)$$

$$X_{ij} = 0 \text{ or } 1, \qquad i=1,\dots,I;\ j=1,\dots,J \qquad (36)$$

$$Y_j = 0 \text{ or } 1, \qquad j=1,\dots,J \qquad (37)$$

It is not hard to see that *Problem R* is actually the summation of *J* independent subproblems (one for each *j*, as shown in (38)-(44)) plus a constant term $\sum_{i=1,\dots,I}a_i$.

Subproblem j

$$\text{Max} \sum_{i=1,\ldots,I} (P_j - B_j - a_i) X_{ij} - MY_j \quad (38)$$

s.t.

$$R_{ij} = \sum_{k=1,\ldots,j} V_{ik}, \quad i = 1,\ldots,I \quad (39)$$

$$(R_{ij} - P_j) X_{ij} \geq 0, \quad i = 1,\ldots,I \quad (40)$$

$$X_{ij} \leq Y_j, \quad i = 1,\ldots,I \quad (41)$$

$$P_j \geq 0, \quad (42)$$

$$X_{ij} = 0 \text{ or } 1, \quad i = 1,\ldots,I \quad (43)$$

$$Y_j = 0 \text{ or } 1, \quad (44)$$

Each of such subproblem j is actually equivalent to the following setting. Given each customer's reservation price for the bundle of j goods (constraint (39)), we are trying to maximize vendor's total profit by deciding whether to offer the bundle of j goods (i.e. determine Y_j) and the price of the bundle P_j. The fixed cost of offering the bundle j is M while the marginal bundle cost to provide this bundle to customer i is $B_j + a_i$ (a_i is Lagrangean Multiplier). Again, each customer is assumed to maximize her surplus and she is willing to purchase the bundle (set X_{ij} to 1) when the surplus is nonnegative (constraint (40)).

3.2. Algorithmic Intuition

Under this *Subproblem j*, the vendor should offer a bundle of j goods if the maximum revenue $\sum_{i=1,\ldots,I} (P_j - B_j - a_i)X_{ij}$ achievable by providing this bundle is larger than the fixed cost M; in addition, the vendor is willing to sell the bundle with j goods only when it is profitable (i.e. $P_j - B_j - a_i \geq 0$).

Keeping these in mind, we first try to find the most profitable price P_j and use it to calculate the maximum revenue $\sum_{i=1,\ldots,I} (P_j - B_j - a_i)X_{ij}$ for each subproblem j. If the resulting revenue is larger than M, the vendor should offer the bundle by setting $Y_j = 1$. We then set X_{ij} of those profitable and willing-to-buy customers to 1, others to 0, and the optimal objective function value of this subproblem is equal to (maximum revenue - *M*). On the other hand, if the maximum revenue achievable is smaller than *M*, the vendor should not offer the bundle. We then simply set all decision variables P_j and Y_j to 0, which in turn set all X_{ij} to zero, and the optimal objective function value of this subproblem is equal to 0.

To derive the most profitable price P_j, let's first sort the reservation price R_{ij} of all potential customers in descending order and call them $R_1, R_2, \ldots, R_I$. Obviously, if we set the price above R_1, no customer will be willing to buy and the maximum revenue achievable is 0. So setting the price above R_1 is not profitable. Now assume we set the price between R_k and R_{k+1} ($R_k > R_{k+1}$), then at most *k* customers will be willing to buy. We soon discover that if we raise the price a bit to R_k, the total number of willing-to-buy customers won't change while the maximum profit achievable will rise for sure. In other words, any price setting between R_k and R_{k+1} is no better than R_k, and when considering the best price, we only have to take $R_1, R_2, \ldots, R_I$ as possible candidates. To this point, we successfully decrease the set of possible optimal pricing P_j from a continuous set to a discrete and finite one. Since the set is finite (with size *I*), we can simply compare the maximum revenue achievable by each possible price and pick up the best one as our final choice.

After the above analysis, we can design the following algorithm to solve *Problem R*, the further relaxed problem of *Problem LR* where constraint (25) is removed.

Step 1. $\forall\, j=1,\ldots,J$

{ choose the best price P_j among $R_{1j}, R_{2j}, \ldots, R_{Ij}$ by comparing the maximum revenue achievable;

if maximum revenue $\geq M$, set $Y_j = 1$ and which in turn determines X_{ij} for each *i*;

else set P_j, X_{ij}, and Y_j to 0 }

Step 2. Add the objective function value of *J* subproblems together plus a constant term $\sum_{i=1,\dots,I} a_i$. This is the objective function value of *Problem R*.

3.3. The Dual Problem and the Subgradient Method

According to the algorithm proposed above, although we could not directly solve the Lagrangean relaxation problem *Problem LR*, we can successfully solve *Problem R*. According to the weak Lagrangean duality theorem, the optimal objective value of *Problem LR* is an upper bound of the optimal objective value of *Primal Problem IP*. And since *Problem R* is a relaxation of *Problem LR*, the optimal objective value of *Problem R* is also an upper bound of the optimal objective value of *IP*. We then construct the following dual problem to calculate the tightest upper bound and solve the dual problem by using the subgradient method. More details about the subgradient method can be found in (Bertsekas, 1999).

min $V(a)$ (D)

s.t. $a \geq 0$.

where $V(a)$ is *Problem R* from Page 10.

Let the vector S be a subgradient of $V(a)$ at $(a_1, a_2,\dots, a_I)$. In iteration k of the subgradient optimization procedure, the multiplier a is updated by $a^{k+1} = a^k - \alpha^k S^k$ where $S^k = 1 - \sum_{j=1,\dots,J} X_{ij}$.

The step size α^k is determined by

$$\alpha^k = \beta \frac{V(a^k) - \phi^h}{\left\| S^k \right\|^2}$$

where ϕ^h is the primal objective value, which we get from a heuristic solution (a lower bound of the optimal primal objective value), and β is a constant, $0 \leq \beta \leq 2$. The initial value of the multiplier vector a is chosen to be the **0** vector and ϕ^h is always updated to the best lower bound we can get so far.

After the implementation of the subgradient optimization procedure, we got an upper bound on the optimal objective value of the primal problem. But as expected, no primal feasible solution was found in the process due to the structure and complexity of this bundle pricing problem. In the following section, we try to utilize the dual solutions to develop some heuristic algorithm to get the primal feasible solutions.

3.4. Getting Primal Feasible Solutions

For each iteration of the subgradient method, we get a set of dual feasible solutions which is affected by the updating Lagrangean Multiplier a. But because we relax constraints (6) and (25) above, the dual feasible solutions we get for *Problem R* may not be feasible for the original primal problem. However, it is possible to obtain a good primal solution using heuristics which start from a fairly suboptimal dual solution. We will now describe this heuristic, which we can use to generate a sequence of primal feasible solutions by adjusting the sequence of dual feasible solutions we get from the subgradient method.

Algorithm "Bundling":

Step 1. For each potential customer i, choose the bundle k (only those with $Y_j = 1$) with the largest positive surplus. Set X_{ik} to 1, and other X_{ij} to 0. (This enforces constraint (25) and constraint (6)).

Step 2. For each offered bundle (those with $Y_j = 1$), calculate its revenue achieved $\sum_{i=1,\ldots,I} (P_j - B_j)X_{ij}$. Choose the bundle k with the smallest revenue. If the revenue is smaller than M, we don't offer this bundle by setting the corresponding P_j, X_{ij}, and Y_j to 0 and go to step 1.

Step 3. Calculate consumer surplus S_i according to constraint (4).

4. Numerical Results and Case Analysis

For our case analyses, we examined the cases when customer's reservation price for the goods is i.i.d. as well as when it is non-i.i.d. We compare the optimal customized bundle case (our model) to both the pure bundling strategy, and the case where the items are sold individually and the customers choose to do the bundling themselves based on their reservation prices. We also examined the sensitivity of the number of optimal bundles to the menu cost and marginal bundle cost.

4.1. When Customer's Reservation Price for the Goods is i.i.d.

In this section, we examined the cases when customer valuation for each good is i.i.d. (Table 2). For cases 1 to 4, we assume the customer valuation for each good is i.i.d. uniform distribution between 0 and 1, and for cases 5 to 8, we assume the customer valuation for each goods is i.i.d. exponential distribution with parameter 1. For each of these test sets, we change the number of potential customers I and the number of goods J. Without loss of generality, we set number of goods J as 30% of the number of potential customers I; in these cases, we also set marginal menu cost M and marginal bundle cost B_j as 0. The sensitivity of the number of optimal bundles to the menu cost and marginal bundle cost will be discussed in section 4.3 and 4.4.

The row "# of customized bundles offered" in Table 2 shows the number of different bundles to be offered based on our algorithm. "Best customized bundling profit," "Best pure bundling profit" and "Best individual sale profit" list the net profit that could be made with our customized bundling strategy, pure bundling strategy (as shown in Bakos and Brynjolsson, 1999) and individual sale strategy (set a profit maximization price for a single good; customers will buy many goods so long as her individual reservation price is higher than the set price). "Profit improvement from pure bundling to customized bundling" and "Profit improvement from individual sale to customized bundling" compares the customized bundling strategy to pure bundling and individual sale strategies respectively.

From Table 2, it is clear that when customer valuation is i.i.d., the customized bundling strategy will result in a number of different bundles being offered. The profit is no worse than the pure bundling strategy, and both of these strategies strictly dominate the individual sale strategy, suggesting that pure bundling strategy is not the unique optimal solution for the iid cases, the customized bundling strategy is also one of the optimal solutions. Note that in Table 2, although the profits we get from our customized bundling strategy and the pure bundling strategy are not exactly the same, the differences are negligible and are due to randomly generated customer reservation prices and the convergence behavior of our algorithm.[1]

Table 2 Summary --- Total profit of the customized bundling problem (i.i.d. cases)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
Customer's reservation price for the goods V_{ik}	**i.i.d. U(0,1)**	**i.i.d. U(0,1)**	**i.i.d. U(0,1)**	**i.i.d. U(0,1)**	**i.i.d. Exp(1)**	**i.i.d. Exp(1)**	**i.i.d. Exp(1)**	**i.i.d. Exp(1)**
# of potential customers I	**100**	**200**	**500**	**1000**	**100**	**200**	**500**	**1000**
# of goods J	**30**	**60**	**150**	**300**	**30**	**60**	**150**	**300**
# of customized bundles offered	**14**	**6**	**4**	**11**	**2**	**7**	**11**	**4**
Best customized bundling profit	**1208.3**	**5119.3**	**32836.7**	**138533.5**	**2155.2**	**9368.2**	**63333.8**	**263281.6**
Best pure bundling profit	**1197.4**	**5078.3**	**32960.3**	**138930.6**	**2157.2**	**9394.7**	**63510.9**	**263633.9**
Best individual sale profit	**753.9**	**2970.2**	**18626.6**	**75219.8**	**1114.8**	**4439.9**	**18667.9**	**110767.4**
Profit improvement from pure bundling to customized bundling	**0.9 %**	**0.8 %**	**-0.3 %**	**-0.3 %**	**0 %**	**-0.2 %**	**-0.2 %**	**-0.1 %**
Profit improvement from individual sale to customized bundling	**60.3 %**	**72.4 %**	**76.3 %**	**84.2 %**	**93.3 %**	**110.5 %**	**127.1 %**	**137.7 %**

4.2. When Customer's Reservation Price for the Goods is Non-i.i.d.

From cases 1 to 4 in Table 3, we considered non-i.i.d. customer valuations where consumers positively value only a portion of all information goods: for each customer *i*, we randomly pick an integer *k* between 1 and *J* for her. This *k* presents the number of information goods in the pool this customer will positively value. We then generate *k* random number out of U(0,1) to represent these positive values. For the remaining goods, we assume the customer will only put zero value on them.

As shown in Table 3, when customer valuation is non-i.i.d., offering multiple customized bundles is always more profitable than pure bundling and individual sales. And from cases 3 and 4, we know that pure bundling strategy is no longer always better than individual sale strategy. It is also interesting to note that the profit improvement of customized bundling over pure bundling strategy tends to increase as the size of the problem increases (from case 1 to case 3).

Table 3 Summary --- Total profit of the customized bundling problem (non-i.i.d. cases)

	Case 1	Case 2	Case 3	Case 4
Customer's reservation price for the goods V_{ik}	**Non i.i.d.**	**Non i.i.d.**	**Non i.i.d.**	**Non i.i.d.**
# of potential customers *I*	**100**	**200**	**500**	**1000**
# of goods *J*	**30**	**60**	**150**	**300**
# of customized bundles offered	**8**	**13**	**15**	**16**
Best customized bundling profit	**432.4**	**1884.6**	**11932.2**	**43169.6**
Best pure bundling profit	**379.9**	**1594.4**	**9809.4**	**37385.1**
Best individual sale profit	**361.1**	**1522.2**	**9855.0**	**37447.8**
Profit improvement from pure bundling to customized bundling	**13.8 %**	**18.2 %**	**21.6 %**	**15.5 %**
Profit improvement from individual sale to customized bundling	**19.8 %**	**23.8 %**	**21.1 %**	**15.3 %**

4.3. Sensitivity of the Number of Optimal Bundles to the Menu Cost

We also examined the sensitivity of the number of optimal bundles to the menu cost (both i.i.d. and non-i.i.d. cases). For each of these, two test examples are presented here. For the i.i.d. cases, as

[1] Since we only run a finite number of iterations; but as the number of iterations increases, the difference will shrink.

shown in Fig. 1, test example 1 is i.i.d. Exp(1) with $I = 120$, $J = 60$, $B_j = 0.1$ while test example 2 is i.i.d. U(0,1) with $I = 200$, $J = 30$, $B_j = 0$. For the non-i.i.d. cases, as shown in Fig. 2, test example 1 is non-i.i.d. with $I = 1000$, $J = 100$, $B_j = 0.1$ while test example 2 is i.i.d. U(0,1) with $I = 200$, $J = 100$, $B_j = 0.1$.

From Fig. 1 and Fig. 2, we can see that when menu cost increases, the vender should decrease the number of bundles offered, no matter the customer valuation is i.i.d. or not. As also evident from the figure, i.i.d. cases are much more sensitive to the menu cost, in the sense that the number of optimal bundles drops faster with small increase of menu cost, and therefore we use different scales in Fig. 1 and Fig. 2. Interestingly, even when the menu cost is zero, our customized bundling strategy will not include all kinds of possible bundles, but only chooses to offer a reasonably small number of choices. For example, in test example 2 of Fig. 2, our customized bundling strategy chooses to offer the bundles of size 1, 2, 3, 5, 11, 14, 29, 49, and 100 with 13.6 % profit improvement from the pure bundling strategy. Note that this is only 9 % of the 100 possible bundle choices.

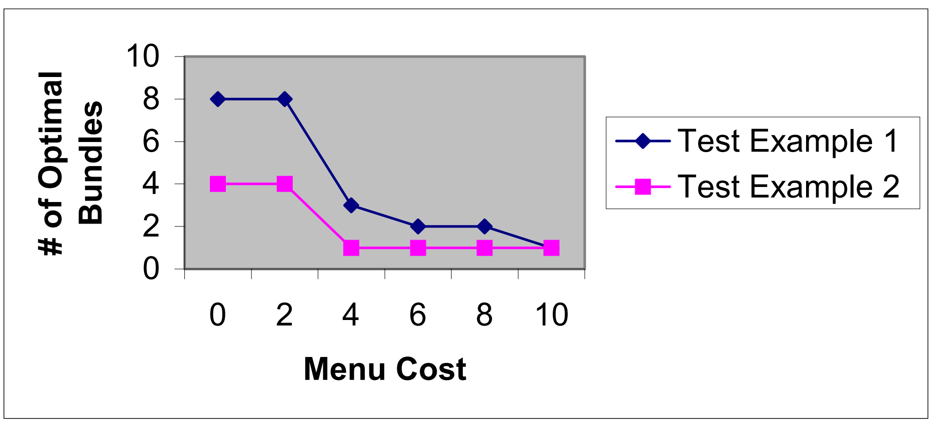

Figure 1. Relationship of Number of Bundles to Menu Cost (i.i.d. customers).

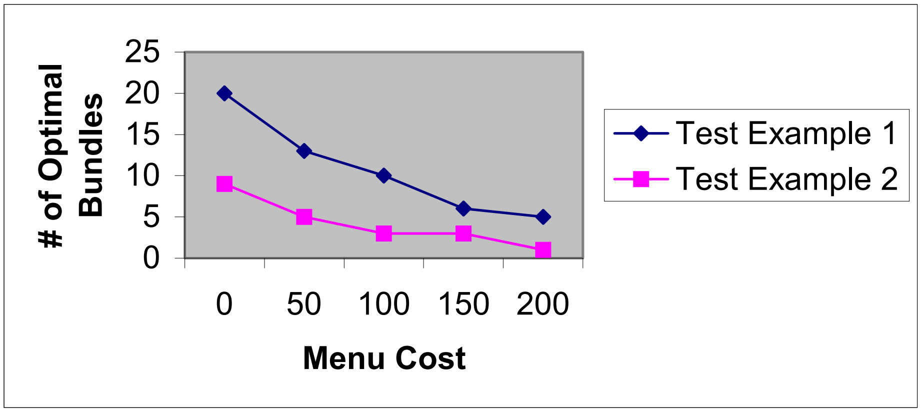

Figure 2. Relationship of Number of Bundles to Menu Cost (non i.i.d. customers).

4.4. Sensitivity of the Number of Optimal Bundles to the Marginal Bundle Cost

In this section, we consider the relationship between the number of optimal bundles and the marginal bundle cost B_j. Since if marginal bundle cost is increasing in number of goods in the bundle (when marginal production cost for each good is non-zero), pure bundling is clearly worse. We consider the case when the marginal production cost for each good is zero and that marginal bundle cost B_j is the same for any kind of bundles for vendors, even individual sales.[2] This is the case when marginal production cost for each good is zero, and marginal bundle cost consists of only transaction cost or distribution cost which is independent of number of goods in the bundle.[3] When the marginal bundle cost is near zero, the vender is willing to provide several different sizes of bundles, as shown in Table 1 and 2. But when the marginal bundle cost is high, the vendor will have less incentive to sell small size bundles. Because only when the size of the bundle is large, the customers will attach large enough valuation for the bundle, which will then in turn gives the vender some room to set a profitable price. As the marginal bundle cost increases to some point,

[2] This can be the case when marginal bundle cost is dominated by distribution cost or transaction cost associated with the bundle rather than marginal production cost for each good in the bundle.
[3] For example, a CD with one song in it and with 10 songs in it may have the same marginal bundle cost, since the marginal cost for copying one more song to the CD is negligible, and most of the bundle cost arises from the CD (the

the vender will be only willing to offer the pure bundle as this is the only profitable bundle.

We demonstrate this idea with a non-i.i.d. case with $I = 100$, $J = 30$, and $M = 0$ in Table 4. We can clearly see that as marginal bundle cost increases, both the number of optimal bundles and the profit decrease, and our customized bundling strategy will get the same results as the pure bundling strategy when the marginal bundle cost is large enough.

Table 4. Relationship of number of optimal bundles to marginal bundle cost

	$B_j = 0$	$B_j = 5$	$B_j = 10$	$B_j = 15$
# of customized bundles offered	**7**	**4**	**3**	**1**
Best customized bundling profit	**432.4**	**161.5**	**37.9**	**3.2**
Best pure bundling profit	**379.9**	**155.3**	**36.0**	**3.2**
Best individual sale profit	**361.1**	**0**	**0**	**0**
Profit improvement from pure bundling to customized bundling	**13.8 %**	**4.0 %**	**5.2 %**	**0 %**
Profit improvement from individual sale to customized bundling	**19.8 %**	**N.A.**	**N.A.**	**N.A.**

5. Concluding Remarks

In this paper we demonstrate how to solve the customized bundling pricing problem by determining what bundle size(s) to offer and at what price(s) for a monopolist selling a large number of information goods. This research not only adds to the information goods pricing literature but also is of high practical use in guiding firms how to bundle and price information goods given their demand and cost structure.

Our model can accommodate any kind of demand distribution or valuation function. Moreover, we consider not only the marginal bundle cost of a bundle but also menu cost for offering one more bundle. Given that the variables in our model grow only linearly with the number of goods, our model can solve any feasible number of information goods and can determine how many bundles an information goods provider should offer and what price it should charge for each bundle. The numerical analyses show that when customer valuation is i.i.d., pure bundling is one of the optimal

distribution media) itself.

solutions. When, however, customer valuation is non-i.i.d., offering multiple customized bundles is a better choice. We also demonstrate the sensitivity of the number of optimal bundles to the menu cost as well as marginal bundle cost and show that number of bundle offered is a decreasing function of menu cost and marginal bundle cost.

Our model also offers a mechanism to decide whether we should add a certain good to the selection pool from which customers could select up to a certain number of goods. Comparing the largest profits we can extract from selling a selection pool of *n* goods plus one more goods sold individually with selling a selection pool of *n*+1 goods directly could give us some insight into deciding which goods are better offered as a pool, and what goods are better sold individually. The research on this part in currently in progress.

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