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CVaR models with selective hedging for international asset allocation ☆

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Abstract

We develop an integrated simulation and optimization framework for multicurrency asset allocation problems. The simulation applies principal component analysis to generate scenarios depicting the discrete joint distributions of uncertain asset returns and exchange rates. We then develop and implement models that optimize the conditional-value-at-risk (CVaR) metric. The scenario-based optimization models encompass alternative hedging strategies, including selective hedging that incorporates currency hedging decisions within the portfolio selection problem. Thus, the selective hedging model determines jointly the portfolio composition and the level of currency hedging for each market via forward exchanges. We examine empirically the benefits of international diversification and the impact of hedging policies on risk–return profiles of portfolios. We assess the effectiveness of the scenario generation procedure and the stability of the model's results by means of out-of-sample simulations. We also compare the performance of the CVaR model against that of a model that employs the mean absolute deviation (MAD) risk measure. We investigate empirically the ex post performance of the models on international portfolios of stock and bond indices using historical market data. Selective hedging proves to be the superior hedging strategy that improves the risk–return profile of portfolios regardless of the risk measurement metric. Although in static tests the MAD and CVaR models often select portfolios that trace practically indistinguishable ex ante risk–return efficient frontiers, in successive applications over several consecutive time periods the CVaR model attains superior ex post results in terms of both higher returns and lower volatility.

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1. Introduction

Asset managers aim to select investment portfolios that yield the maximum possible return, while at the same time ensuring an acceptable level of risk exposure. Risk derives from potential losses in portfolio value due to possible reductions in the market value of financial assets resulting from changes in equity prices, interest rates, foreign exchange rates, credit ratings of security issuers, etc. Diversification into multiple securities can practically eliminate idiosyncratic risk, that is, potential severe losses from any individual security. However, domestic diversification cannot mitigate systematic market risk. This is the risk associated with concurrent losses in most domestic securities due to high correlations between their returns. Since market risk differs from country to country, international diversification can reduce the overall risk exposure of investment portfolios.

International diversification is practiced by institutional investors to improve the risk–return profiles of their portfolios. The inclusion of securities denominated in foreign currencies in the asset holdings can provide dual benefits: (1) The prospect for higher profit in the event of favorable performance of foreign markets and (2) the potential reduction in the portfolio's exposure to market risk. However, international investments introduce a new element of risk (currency risk). The volatility of return from a foreign asset depends not only on the differential change of its price within any given period (domestic return) but also on the variation of the foreign exchange rate to the reference currency, as well as on the correlation between the two. Exchange rates between currencies are correlated with domestic returns of assets in the respective countries – in particular with the returns of interest-sensitive securities (e.g. bonds). The effects of exchange rate changes on the overall risk profile of international portfolios are discussed in Eun and Resnick (1988). As fluctuating exchange rates can mitigate the potential gains from international diversification, holistic risk management approaches are needed that account for all risk factors affecting the performance of international portfolios.

Currency risk is typically hedged via forward contracts. However, the portfolio selection and hedging decisions are often considered separately. An extensive body of literature has studied the merits of hedging exchange rate risk. (See, for example, Perold and Schulman, 1988; Eun and Resnick, 1988; Jorion, 1989; Black, 1990; Filatov and Rappoport, 1992; Glen and Jorion, 1993; Abken and Shrikhande, 1997; Solnik, 1998; Beltratti et al., in press.) This literature presents somewhat different views as to the optimal course of action for international portfolio management depending on the focus of each study with regard to factors such as the investment opportunity set, the risk aversion preference and time horizon of the decision maker, the reference currency of the investor, the investment strategy (passive vs active), the distribution of asset returns and exchange rates in the time frame of the study (i.e., the historical data used in calibrating the distributions), and the hedging strategies that were compared. The overall conclusions from these studies are: (a) the relative merits of hedging strategies remain mostly an empirical issue and depend on the factors mentioned above, (b) currency hedging becomes more important for foreign investments whose domestic returns exhibit considerable correlation with the exchange rate to the reference currency. These observations point to the need for integrated simulation and optimization approaches that determine jointly portfolio structures and flexible hedging policies. Our models move exactly in this direction.

With the exception of Beltratti et al. (in press) who internalized selective hedging decisions within a portfolio optimization model, in all the other studies cited above the hedging policy was prespecified at the portfolio selection stage. Hedging policies were then contrasted by comparing ex post the performance of portfolio selections made with alternative policies. Three major hedging policies are discussed in the literature: unitary, partial, and selective hedging. Unitary hedging concerns the selection of either no hedging or complete hedging of the currency risk associated with all foreign asset holdings. In partial hedging, the hedge ratio can be different from zero or one, but it is common across all foreign markets. This approach follows from the theory proposed by Black (1990) that suggests the existence of a universal hedge ratio that is optimal for all investors. Selective hedging is the more general approach as it permits the hedge ratio to be different across markets and to take any value between zero and one.

In this study we adopt the approach of Beltratti et al. (in press) as we apply scenario-based optimization models that simultaneously determine the portfolio composition and the appropriate hedging level for each position in a foreign asset. The models prescribe optimal selective hedging policies by means of forward currency exchanges. Our models encompass all three hedging strategies mentioned above. They can yield as a special case solutions that imply a uniform hedge ratio across markets. Any value of the hedge ratio between zero and one is allowable, including the two extreme values that correspond to no hedging and complete hedging, respectively. Our aim is to assess the performance of the optimization models as risk management tools in selecting internationally diversified portfolios.

Risk management entails the exercise of control over some statistical characteristic(s) of the uncertain portfolio return. The aim is to avoid portfolios that may likely be susceptible to severe losses. We focus on the development, implementation, and testing of a model that employs the conditional value-at-risk (CVaR) metric (Rockafellar and Uryasev, 2000, 2002). By optimizing CVaR we maximize the conditional expectation of portfolio returns below a prespecified low percentile of the distribution, thus we minimize the expected losses in severe circumstances. Our motivation for applying a CVaR model stems from the observation that returns of international assets and proportional changes of exchange rates exhibit asymmetric distributions; empirical evidence supporting this assertion is given in Section 4.2.

This study extends the work of Beltratti et al. (in press) in several directions. We employ the CVaR risk metric that accounts for asymmetric return distributions. Due to the observed asymmetry of asset returns in the international asset allocation problem, CVaR should be a more appropriate metric than alternative risk measures that are geared towards symmetric distributions (see, for example, Jobst and Zenios, 2001). We also apply a more rigorous scenario generation method than Beltratti et al. (in press) who relied on bootstrapping of historical data. We test the effectiveness of the scenario generation procedure and the stability of the model's results in out-of-sample simulations. Moreover, we contrast the performance of the CVaR model against that of a mean absolute deviation (MAD) model. We conduct backtests using historical market data to investigate empirically the ex post performance of the models in selecting international portfolios of stock and bond indices.

Essential to the application of the optimization models is an effective representation of the random returns of the assets. We devise a sampling procedure based on principal component analysis (PCA) to jointly generate scenarios of domestic holding period returns for international assets, as well as spot exchange rates at the end of the holding period. On the basis of these – and the currently quoted spot and forward exchange rates – we compute corresponding scenarios of returns, in terms of a reference currency, for hedged and unhedged positions in each asset. Our sampling procedure is superior to random sampling in terms of approximating the statistical properties of historical data sets. It also leads to more stable risk–return efficient frontiers as we demonstrate with out-of-sample simulations.

The scenarios of asset returns and their associated probabilities constitute the necessary inputs to the optimization models that determine portfolio compositions. The parametric optimization models trade off expected portfolio return against the relevant risk measure. We thus trace the efficient risk–return frontiers for the respective risk measures. Comparisons of these frontiers enable a relative assessment of alternative models. These static evaluations compare potential performance profiles at a single point in time.

We also carried out backtesting experiments, whereby the models were repeatedly applied in several successive time periods and the ex post returns of their selected portfolios were determined on the basis of observed market data. The results of backtests provide a more reliable basis for comparative assessment of the models as they reflect realized performance over longer time periods. Although in static tests the MAD and CVaR models often select portfolios that trace almost indistinguishable ex ante risk–return frontiers, in backtests the CVaR model attains superior performance, and the CVaR-optimized portfolios yield higher growth rates and lower volatility than the MAD-optimized portfolios.

The rest of the paper is organized as follows. In Section 2 we discuss the formulation of the optimization models with selective hedging for international portfolio selection. In Section 3 we present our scenario generation method based on PCA. In Section 4 we examine the statistical characteristics of the historical data set, we describe our computational tests, and we discuss the empirical results. Finally, Section 5 concludes the paper.

The contributions of our study are: the development of a CVaR model for optimal selection of international portfolios incorporating currency hedging decisions within the portfolio selection model; the empirical comparison, using historical market data, of the CVaR model with a MAD model; the development of a scenario generation method based on PCA for depicting hedged and unhedged returns for international securities. The empirical results indicate that this integrated simulation and optimization framework can provide an effective decision support tool in international investment management.

2. Risk management models

Consider a set of investment opportunities indexed by $i = 1, 2, \dots, n$. At the end of a certain holding period the assets generate returns $\tilde{\mathbf{r}} = (\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n)^{\mathrm{T}}$. The returns are unknown at the beginning of the holding period – i.e., at the time of the portfolio selection – and are treated as random variables. Denote their mean value by $\bar{\mathbf{r}} = \mathscr{E}(\tilde{\mathbf{r}}) = (\bar{r}_1, \bar{r}_2, \dots, \bar{r}_n)^{\mathrm{T}}$. At the beginning of the holding period the investor wishes to apportion his budget to these assets by deciding on a specific allocation $\mathbf{x} = (x_1, x_2, \dots, x_n)^{\mathrm{T}}$, such that $x_i \geqslant 0$ (i.e., short sales are disallowed) and $\sum_{i=1}^{n} x_i = 1$ (budget constraint). Using the vector $\mathbf{1} = (1, 1, \dots, 1)^{\mathrm{T}}$ of ones, we express the basic portfolio constraints in vector notation as

$$X = \{\mathbf{x} : \mathbf{x}^{\mathrm{T}}\mathbf{1} = 1, \mathbf{x} \geqslant 0\}.$$

Throughout the paper we use boldface characters to denote vectors.

For portfolios that involve both hedged and unhedged international positions the investment opportunity set is partitioned into these two categories. This implies a corresponding partitioning of the vectors $\tilde{\mathbf{r}}$, $\bar{\mathbf{r}}$, $\mathbf{x}$. Thus, these vectors are composed of a concatenation of two subvectors corresponding to the hedged and unhedged asset positions, respectively. Assets denominated in the investor's base currency can be assigned to either of the two subvectors. Total asset returns are expressed in terms of the base currency.

The uncertain return of the portfolio at the end of the holding period is $R(\mathbf{x}, \tilde{\mathbf{r}}) = \mathbf{x}^{\mathrm{T}}\tilde{\mathbf{r}} = \sum_{i=1}^{n} x_i \tilde{r}_i$. This is a random variable with a distribution function, say F, i.e., $F(\mathbf{x}, u) = P\{R(\mathbf{x}, \tilde{\mathbf{r}}) \leqslant u\}$. Of course the distribution function F depends on the portfolio composition $\mathbf{x}$. The expected return of the portfolio is $\mathscr{E} \times (R(\mathbf{x}, \tilde{\mathbf{r}})) = R(\mathbf{x}, \bar{\mathbf{r}}) = \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}}$. Suppose the uncertain returns of the assets, $\tilde{\mathbf{r}}$, are represented by a finite set of discrete scenarios $\Omega = \{s : s = 1, 2, \dots, S\}$, whereby the returns under a particular scenario $s \in \Omega$ take the values $\mathbf{r}_s = (r_{1s}, r_{2s}, \dots, r_{ns})^{\mathrm{T}}$ with associated probability $p_s > 0$, $\sum_{s=1}^{S} p_s = 1$. The mean return of the assets is $\bar{\mathbf{r}} = \sum_{s=1}^{S} p_s \mathbf{r}_s$. The portfolio return under a particular realization of asset returns $\mathbf{r}_s$ (i.e., scenarios $s \in \Omega$) is denoted $R(\mathbf{x}, \mathbf{r}_s) = \mathbf{x}^{\mathrm{T}}\mathbf{r}_s = \sum_{i=1}^{n} x_i r_{is}$. The expected portfolio return is expressed as $R(\mathbf{x}, \bar{\mathbf{r}}) = \sum_{s=1}^{S} p_s R(\mathbf{x}, \mathbf{r}_s) = \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}} = \sum_{i=1}^{n} x_i \bar{r}_i$.

Suppose φ is some risk measure. Then for a certain minimal expected portfolio return μ, the φ-efficient portfolio is obtained from the solution of the following problem:

$$\begin{array}{ll} \min_{\mathbf{x}\in X} & \varphi(\mathbf{x}^{\mathrm{T}}\tilde{\mathbf{r}}) \\ \text{s.t.} & \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}} \geqslant \mu. \end{array} \tag{1}$$

The curve that depicts the dependence of the optimal value of this parametric program on the required minimal expected portfolio return μ is the φ-efficient frontier.

This is a generalization of the classical concept of the mean–variance efficient frontier to an arbitrary risk measure φ. The choice of the risk measure generally depends on the preferences of the decision maker or, in some cases, on regulatory specifications. Matters of computational tractability also affect this choice.

Value-at-risk (VaR) is a percentile based metric that has become an industry standard for risk measurement purposes (Riskmetrics, 1996). It is usually defined as the maximal allowable loss with a certain confidence level $\alpha \times 100\%$. Here we define VaR equivalently, in terms of returns, as the minimal portfolio return for a prespecified confidence level $\alpha \times 100\%$. Thus,

$$\mathrm{VaR}(\mathbf{x},\alpha) = \min\{u : F(\mathbf{x},u) \geqslant 1-\alpha\} = \min\{u : P\{R(\mathbf{x},\tilde{\mathbf{r}}) \leqslant u\} \geqslant 1-\alpha\}. \tag{2}$$

$\mathrm{VaR}(\mathbf{x},\alpha)$ is the $(1-\alpha)\times 100\%$ percentile of the distribution of portfolio return.

Despite its popular use in risk measurement, VaR is not typically used in mathematical models for optimal portfolio selection. While its calculation for a certain portfolio $\mathbf{x}$ reveals that the portfolio return will be below $\mathrm{VaR}(\mathbf{x},\alpha)$ with likelihood $(1-\alpha)\times 100\%$, it provides no information on the extent of the distribution's tail which may be quite long; in such cases, the portfolio return may take substantially lower values than VaR and result in severe losses. VaR lacks a theoretical property for coherent risk measures (Artzner et al., 1999), namely, subadditivity. Moreover, VaR is difficult to optimize. When the asset returns are specified in terms of scenarios the VaR function is non-smooth and non-convex with respect to the portfolio positions $\mathbf{x}$ and exhibits multiple local extrema. Efficient algorithms for solving problems with such objective functions are lacking.

CVaR is a related risk measure. It is usually defined as the conditional expectation of losses exceeding VaR at a given confidence level (VaR is also defined as a percentile of a loss function in this case). Here, we define CVaR equivalently as the conditional expectation of portfolio returns below the VaR return. As introduced by Rockafellar and Uryasev (2000), for continuous distributions, CVaR is defined as

$$\mathrm{CVaR}(\mathbf{x},\alpha) = \mathscr{E}[R(\mathbf{x},\tilde{\mathbf{r}})\,|\,R(\mathbf{x},\tilde{\mathbf{r}}) \leqslant \mathrm{VaR}(\mathbf{x},\alpha)]. \tag{3}$$

Hence, this definition of CVaR that is applicable to continuous distributions measures the expected value of the $(1-\alpha)\times 100\%$ lowest returns for portfolio $\mathbf{x}$ (i.e., the conditional expectation of portfolio returns below $\mathrm{VaR}(\mathbf{x},\alpha)$).

For discrete distributions, the formula in (3) gives a non-convex function in portfolio positions $\mathbf{x}$, and is not a subadditive risk measure. A definition of CVaR for general distributions (including discrete distributions) has been introduced by Rockafellar and Uryasev (2002):

$$\mathrm{CVaR}(\mathbf{x},\alpha) = \left(1 - \frac{\sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s}{1-\alpha}\right) z + \frac{1}{1-\alpha} \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s R(\mathbf{x},\mathbf{r}_s), \tag{4}$$

where $z = \mathrm{VaR}(\mathbf{x},\alpha)$. As we consider discrete distributions (i.e., scenarios) in this paper, we will utilize this alternative definition of CVaR. Note that CVaR as defined for discrete distributions in (4) may not be equal to the conditional expectation of portfolio returns below $\mathrm{VaR}(\mathbf{x},\alpha)$. This definition of CVaR for discrete distributions measures only *approximately* the conditional portfolio returns below the respective $\mathrm{VaR}(\mathbf{x},\alpha)$ value.

As indicated by Pflug (2001) and Rockafellar and Uryasev (2002), unlike VaR, CVaR is a coherent risk measure in the sense of Artzner et al. (1999). CVaR quantifies the expected portfolio return in a low percentile of the distribution. Hence, it can be used to exercise some control on the lower tail of the return distribution and thus, it is a suitable risk measure for skewed distributions. As it was shown by Rockafellar and Uryasev (2002), when the uncertain asset returns are represented by a discrete distribution CVaR can be optimized by linear programming (LP). We follow their approach in the derivation below.

Let us define for every scenario $s \in \Omega$ an auxiliary variable

$$y_s^+ = \max[0, z - R(\mathbf{x},\mathbf{r}_s)],$$

which is equal to zero when the portfolio return for the particular scenario exceeds $\mathrm{VaR}(\mathbf{x},\alpha)$, and is equal to the return shortfall in relation to VaR when the portfolio return is below $\mathrm{VaR}(\mathbf{x},\alpha)$. Using these auxiliary variables we have

$$\begin{aligned}
\sum_{s\in\Omega} p_s y_s^+ &= \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s y_s^+ + \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)> z\}} p_s y_s^+ \\
&= \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s (z - R(\mathbf{x},\mathbf{r}_s)) \\
&= z \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s - \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s R(\mathbf{x},\mathbf{r}_s) \\
&= z(1-\alpha) - \left(\left(1 - \alpha - \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s\right) z + \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s R(\mathbf{x},\mathbf{r}_s)\right).
\end{aligned}$$

Dividing both sides of the equation by $(1-\alpha)$ and rearranging terms we get

$$z - \frac{\sum_{s\in\Omega} p_s y_s^+}{1-\alpha} = \left(1 - \frac{\sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s}{1-\alpha}\right) z + \frac{1}{1-\alpha} \sum_{\{s\in\Omega\,|\,R(\mathbf{x},\mathbf{r}_s)\leqslant z\}} p_s R(\mathbf{x},\mathbf{r}_s). \tag{5}$$

From Eqs. (4) and (5) we observe that the right hand side term of (5) is $\mathrm{CVaR}(\mathbf{x},\alpha)$. Therefore, the CVaR of portfolio return can be optimized using a linear program with the left hand side expression of (5) as the objective function. The resulting LP that trades off the optimal CVaR-measure of portfolio return at a prespecified confidence level $\alpha \times 100\%$ against the expected portfolio return μ is written as

$$\begin{array}{lll}
\max & z - \frac{1}{1-\alpha}\sum_{s=1}^{S} p_s y_s^+, & \\
\text{s.t.} & \mathbf{x} \in X, z \in \mathscr{R}, & \\
 & \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}} \geqslant \mu, & \\
 & y_s^+ \geqslant z - \mathbf{x}^{\mathrm{T}}\mathbf{r}_s, & s = 1,2,\ldots,S, \\
 & y_s^+ \geqslant 0, & s = 1,2,\ldots,S.
\end{array} \tag{6}$$

Solving the parametric program (6) for different values of the expected portfolio return μ yields the CVAR-efficient frontier. For each expected return target μ, the optimal value of program (6) is the corresponding $\mathrm{CVaR}(\mathbf{x}, \alpha)$. The value of the free variable z at the optimal solution of (6) is the corresponding $\mathrm{VaR}(\mathbf{x}, \alpha)$ value. A formal proof is provided in Rockafellar and Uryasev (2002, Theorem 14 and Corollary 15).

Efficient LP solvers for large-scale programs make possible the optimization of CVaR in a variety of portfolio management problems. Program (6) optimizes the CVaR risk measure for portfolio return and simultaneously determines the corresponding VaR value (z). As defined in (4), in terms of portfolio return, CVaR is a lower bound for VaR (i.e., $\mathrm{CVaR}(\mathbf{x}, \alpha) \leqslant \mathrm{VaR}(\mathbf{x}, \alpha)$). Hence, by maximizing CVaR program (6) should be expected to yield larger values for VaR as well. As we illustrate in the empirical results of Section 4 the bound of CVaR on VaR is not necessarily tight. Still, the extent to which optimizing CVaR also yields near-optimal results from the VaR perspective in practical settings remains an unresolved empirical question due to the computational complexity of optimizing VaR in such cases.

Computational issues aside, there is an ongoing debate among academics and practitioners whether VaR or CVaR is the most appropriate metric for risk management. VaR is the industry standard for *risk measurement*. On the other hand, CVaR has achieved popularity as a suitable risk measure in the insurance industry and is gradually gaining acceptance in the financial community. Its appeal lies not only in its theoretical properties of coherence, but also in its ease of implementation in portfolio optimization models and its ability to reduce the tail of the distribution, thus exercising risk management control (Jobst and Zenios, 2001). Several recent studies have applied CVaR for portfolio selection in various applications (for example, Anderson et al., 2001; Rockafellar and Uryasev, 2000, 2002).

In the MAD framework of Konno and Yamazaki (1991) risk is defined as the mean absolute deviation of portfolio return from its expected value:

$$\mathrm{MAD}(\mathbf{x}) = \mathscr{E}[|R(\mathbf{x}, \tilde{\mathbf{r}}) - R(\mathbf{x}, \bar{\mathbf{r}})|].$$

When the uncertain asset returns are represented in terms of a discrete scenario set the MAD metric becomes:

$$\mathrm{MAD}(\mathbf{x}) = \sum_{s=1}^{S} p_s |\mathbf{x}^{\mathrm{T}}\mathbf{r}_s - \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}}|.$$

In this case MAD can be optimized by the following linear program:

$$\begin{array}{lll}
\min & \sum_{s=1}^{S} p_s y_s, & \\
\text{s.t.} & \mathbf{x} \in X, & \\
& \mathbf{x}^{\mathrm{T}}\bar{\mathbf{r}} \geqslant \mu, & \\
& y_s \geqslant \mathbf{x}^{\mathrm{T}}(\mathbf{r}_s - \bar{\mathbf{r}}), & s = 1, 2, \ldots, S, \\
& y_s \geqslant \mathbf{x}^{\mathrm{T}}(\bar{\mathbf{r}} - \mathbf{r}_s), & s = 1, 2, \ldots, S, \\
& y_s \geqslant 0, & s = 1, 2, \ldots, S.
\end{array} \tag{7}$$

The auxiliary variables y_s are introduced to linearize the absolute value expression, akin to the approach followed earlier to linearize the non-smooth function $\max[0, z - R(\mathbf{x}, \mathbf{r}_s)]$ in the CVaR case. Again, by solving the parametric program (7) for various values of expected portfolio return μ we can construct the MAD-efficient frontier. MAD models have been applied to various portfolio optimization problems. (See, for example, Beltratti et al., in press; Kang and Zenios, 1993; Konno and Yamazaki, 1991; Kouwenberg and Zenios, in press.)

3. Scenario generation

A central issue in any portfolio selection model is a depiction of the uncertain returns of the investment alternatives. Usually this issue is addressed by defining either the expected returns and the covariance matrix of the assets, or a set of possible realizations (scenarios) of the random returns. In the latter case, the scenarios of plausible asset returns can be generated by a model, can be obtained from experts' opinions, or by bootstrapping observed past returns (see the review by Kouwenberg and Zenios, in press).

We use past observations of asset returns and exchange rates in our scenario generation. However, as we demonstrate in Section 4, empirical evidence indicates that these random variables do not follow a multinormal, or log-normal, distribution; they, in fact, exhibit asymmetries. As the distribution of asset returns is unknown, we make no assumption regarding either their joint or their marginal distributions. We resort to a sampling procedure based on PCA. Thus we derive a relatively small number of uncorrelated factors that capture to a great degree the overall variability exhibited in past observations of the random variables. By combining samples from empirical distributions of these uncorrelated random factors we can obtain scenarios of asset returns and exchange rates through a simple transformation. In this manner, we generate the required scenarios of exchange rates and domestic returns for the assets with statistical properties similar to those of the historical observations. Our scenario generation procedure is described next.

3.1. Principal component analysis

PCA is geared to reduce the dimensionality of a multivariate forecasting problem and to overcome the difficulty posed by the correlation of the random variables, while it preserves the covariation structure in the derived samples. This is achieved with a linear transformation to a new set of variables (principal components, PCs) which are uncorrelated and ordered so that a reduced set of them captures most of the variability exhibited by all the original variables. The theory of PCA is covered in Jollife (1986).

Let the relevant random variables be $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_m)^{\mathrm{T}}$. Their mean values are $\bar{\mathbf{u}} = (\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m)^{\mathrm{T}}$ and their covariance matrix is $\mathbf{Q}$. In our case, the random variables are the domestic returns of the assets and the proportional changes of spot exchange rates to the base currency over a certain time horizon. Consider a linear transformation

$$\tilde{v}_j = \mathbf{c}_j^{\mathrm{T}} \tilde{\mathbf{u}}, \quad j = 1, 2, \ldots, m; \qquad \tilde{\mathbf{v}} = \mathbf{C}\tilde{\mathbf{u}}, \tag{8}$$

where $\mathbf{C}$ is an $(m \times m)$ matrix with rows $\mathbf{c}_j^{\mathrm{T}}$. The mean of random variable $\tilde{v}_j$ is $\bar{v}_j = \mathbf{c}_j^{\mathrm{T}}\bar{\mathbf{u}}$, while its variance is given by

$$\sigma_j^2 = \mathrm{var}(\mathbf{c}_j^{\mathrm{T}}\tilde{\mathbf{u}}) = \mathbf{c}_j^{\mathrm{T}}\mathbf{Q}\mathbf{c}_j. \tag{9}$$

Similarly, the covariance σ_{ij} of the random variables $\tilde{v}_i$, $\tilde{v}_j$ is given by

$$\sigma_{ij} = \mathrm{covar}(\mathbf{c}_i^{\mathrm{T}}\tilde{\mathbf{u}}, \mathbf{c}_j^{\mathrm{T}}\tilde{\mathbf{u}}) = \mathbf{c}_i^{\mathrm{T}}\mathbf{Q}\mathbf{c}_j.$$

The aim is to derive random variables $\tilde{\mathbf{v}}$ which are uncorrelated. We want $\sigma_{ij} = 0$, for $i \neq j$; thus, $\mathbf{c}_i$, $\mathbf{c}_j$ must be $\mathbf{Q}$-orthogonal. The eigenvectors of the covariance matrix $\mathbf{Q}$ satisfy the orthogonality requirement. Therefore, the coefficient vectors $\mathbf{c}_j^{\mathrm{T}}$ in (8) are the eigenvectors from the normalized solutions of the eigenproblems

$$\mathbf{Q}\mathbf{c}_j = \lambda_j\mathbf{c}_j, \quad j = 1, 2, \ldots, m, \tag{10}$$

where λ_j is the jth eigenvalue of $\mathbf{Q}$. The resulting PCs $\tilde{\mathbf{v}}$ obtained by (8) are uncorrelated. In view of (9) and (10) the variance of $\tilde{v}_j$ is $\sigma_j^2 = \mathbf{c}_j^{\mathrm{T}}\mathbf{Q}\mathbf{c}_j = \lambda_j$; the last equation follows from the normalization of the eigenvectors (i.e., $\mathbf{c}_j^{\mathrm{T}}\mathbf{c}_j = 1$, $j = 1, 2, \ldots, m$).

Additionally, we want to capture as much of the variability in the original random variables $\tilde{\mathbf{u}}$ as possible with few principal components. The explanatory power of each PC is associated with its variance. Consequently, the PCs $\tilde{v}_j$ are sorted in descending order of variance (eigenvalue λ_j). A measure of the original variables' variability explained by the set of first k PCs ($k < m$) is given by the ratio $\sum_{j=1}^{k}\lambda_j / \sum_{j=1}^{m}\lambda_j$. Hence, the degree of variability in the original variables captured by a retained set of the first k PCs is controlled by selecting $k = \min\{\kappa\colon \sum_{j=1}^{\kappa}\lambda_j / \sum_{j=1}^{m}\lambda_j \geqslant \delta\}$; $\delta \approx 1$ is a desired accuracy level.

Once we determine the set of PCs to retain alternative sampling techniques can be applied. If the initial random variables $\tilde{\mathbf{u}}$ follow a multinormal distribution then the PCs, $\tilde{\mathbf{v}}$, will be univariate normal with the parameters indicated above. If one assumes that the PCs have approximately normal distributions then random samples can be drawn independently from the assumed distribution of each PC and can be combined to construct a set of scenarios for the PCs. Applying the inverse transformation of (8) on each PC scenario yields a corresponding scenario for the initial random variables. A scenario $\mathbf{u}_s$ for the initial random variables $\tilde{\mathbf{u}}$ is typically computed by

$$\mathbf{u}_s = \mathbf{C}_k^{-1}\mathbf{v}_s^k, \tag{11}$$

where $\mathbf{C}_k^{-1}$ is the matrix of the first k columns of $\mathbf{C}^{-1}$, and $\mathbf{v}_s^k$ denotes the vector of values for the k retained PCs under scenario s. So, in (11) only the subset of retained PCs is used, while the remaining PCs are ignored.

3.2. Sampling procedure

We make no distributional assumptions, as empirical evidence indicates that the random variables we consider in this study cannot be well approximated by normal distributions. Instead, we devise an alternative empirical sampling procedure. We substitute the values of past observations of the random variables $\tilde{\mathbf{u}}$ in the definition of PCs (Eq. (8)) and obtain corresponding values for the PCs; these are termed *principal component scores* and reflect the historical implied values of the PCs. We assume that the distribution of each PC is completely described by its historical scores; hence, an empirical distribution is constructed for each PC. We sample from the empirical distribution of each retained PC and combine these samples to generate representative joint scenarios $\mathbf{v}_s^k$ of the PCs, which can then be applied in (11) to obtain corresponding scenarios $\mathbf{u}_s$ for the random variables $\tilde{\mathbf{u}}$. When a sufficient number of PCs is retained and representative samples per PC are taken, the resulting scenario set for $\tilde{\mathbf{u}}$ has very similar statistical characteristics with those of the historical observations of these random variables.

Because only a small number of samples per PC can be afforded – so as to limit the size of the joint scenario set for computational tractability – we do not sample randomly the PC values. Instead, we obtain representative sample sets of small size. The procedure works as follows. First, we determine for each PC the minimum, the maximum, and the mean value from its principal component scores. For the jth PC let us denote these values as $v_j^{\min}$, $v_j^{\max}$ and $\bar{v}_j$, respectively. We specify for each of the retained PCs a required number of samples; say that for the jth PC this number is n_j. Next, for each of the retained PCs we divide the range $[v_j^{\min}, \bar{v}_j]$ into $n_j/2$ intervals of equal width; similarly, we partition the range $[\bar{v}_j, v_j^{\max}]$ into $n_j/2$ equal segments. In both cases the positioning of the segments starts from the extreme points $v_j^{\min}$ and $v_j^{\max}$, respectively, and proceeds towards the mean. We assign to the midpoint of each of the n_j segments the entire probability mass associated with its respective interval, as determined from the empirical distribution of the principal component scores. Note that when n_j is even, the mean value $\bar{v}_j$ of the PC is at the boundary of the two adjacent intervals in the middle of the distribution. When n_j is odd, $\bar{v}_j$ lies in the middle interval which, in this case, is composed of two subintervals, of potentially unequal width.

Each scenario of PC values, $\mathbf{v}_s^k$, is constructed from a specific combination of samples from the k retained PCs. As the PCs are independent, the probability of a scenario is simply the product of the marginal probabilities of its constituent PC samples. The total number of scenarios is $S = \prod_{j=1}^{k} n_j$, arising from all possible combinations of PC samples.

This sampling procedure has some clear advantages. The number of samples per PC can be directly controlled. We take more samples for the first PCs that have higher explanatory power and reduce the number of samples for subsequent PCs. The differential partitioning scheme of the PC's empirical distributions aims at a more effective approximation of potentially skewed distributions; note also that the samples for each PC are not equiprobable. With a small number of samples we are able to obtain a representative approximation of each PC's distribution.

Another novelty in our scenario generation procedure is that we do not completely ignore the PCs with low variance from which we do not sample. Instead of (11), we compute the corresponding scenario $\mathbf{u}_s$ for the initial random variables by

$$\mathbf{u}_s = \mathbf{C}_k^{-1}\mathbf{v}_s^k + \mathbf{C}_{m-k}^{-1}\bar{\mathbf{v}}^{m-k}. \tag{12}$$

$\mathbf{C}_k^{-1}$ is again the matrix of the first k columns of $\mathbf{C}^{-1}$ that correspond to the retained PCs, while $\mathbf{C}_{m-k}^{-1}$ is the matrix of the remaining $(m-k)$ columns of $\mathbf{C}^{-1}$. Similarly, $\mathbf{v}_s^k$ denotes the values of the k retained PCs in scenario s, while $\bar{\mathbf{v}}^{m-k}$ denotes the mean values of the remaining $(m-k)$ PCs, as computed from their historical scores. The second term in the right hand side of (12) is a constant. However, we have found that its inclusion in the generation of the scenarios $\mathbf{u}_s$ yields an estimate of the mean in the resulting scenario set (i.e., $\sum_{s=1}^{S} p_s \mathbf{u}_s$) that approximates more closely the mean obtained directly from historical observations.

Through empirical tests we validated that our selective sampling procedure leads to superior approximations of the statistical properties of historical data sets for variables $\tilde{\mathbf{u}}$ in comparison to random sampling. In turn, as we illustrate in Section 4, scenario sets generated with this sampling procedure lead to more stable and reliable identification of risk–return efficient frontiers in contrast to random samples.

Ideally one wishes to associate some economic interpretation with the PCs, such as the aggregate effect of identifiable economic factors on individual market variables. However, identifying an economic interpretation for the PCs is not always possible. Here we employ PCs only to reduce the dimensionality of the random variables so as to facilitate the scenario generation procedure, without attempting to deduct economic interpretations.

3.3. Hedged and unhedged asset returns

This study considers an asset allocation problem concerned with hedged and unhedged investments in stock and bond indices denominated in multiple currencies. The observable market data are the asset values (index levels) and the currency exchange rates, both spot and forward. From past observations of the index levels we compute domestic returns for the indices over specific time intervals; in this study we use monthly time periods. So, the monthly returns of the indices, expressed in their respective domestic terms, and the monthly proportional changes in the spot exchange rates of the foreign currencies to a base currency, are the relevant random variables. These constitute the random vector $\tilde{\mathbf{u}}$ in the earlier discussion of the PCA approach. Applying PCA on historical values of these variables we obtain scenarios for the following quantities:

r_{id}^s : estimated monthly return for asset i, in domestic terms, under scenario s,
e_i^s : estimated spot exchange rate of the denomination currency of asset i to the base currency at the end of the monthly holding period under scenario s.

At any point in time we also know:

e_i : the currently quoted spot exchange rate of the denomination currency of asset i to the base currency,
f_i : the currently quoted one-month forward exchange rate of the denomination currency of asset i to the base currency.

Using these data we compute the monthly return of an *unhedged* position in asset i, in terms of the base currency, for each scenario $s \in \Omega$ as follows:

$$r^s_{i\mathrm{u}} = \frac{e_i}{e^s_i}(1 + r^s_{i\mathrm{d}}) - 1. \tag{13}$$

This computation takes into account the equivalent foreign value of a base currency transfer at the current spot exchange rate e_i, the growth factor $(1 + r^s_{i\mathrm{d}})$ of the investment in the foreign asset, and the conversion of the final proceeds back to the base currency at the end of the holding period using the spot exchange rate e^s_i applicable at that time. Similarly, the monthly return of a *hedged* position in asset i, in base-currency terms, for each scenario $s \in \Omega$ is computed by

$$r^s_{i\mathrm{h}} = \frac{e_i}{f_i}(1 + r^s_{i\mathrm{d}}) - 1. \tag{14}$$

The difference in (14) is that the final proceeds from a foreign investment i are converted back to the base currency using the known forward exchange rate f_i. The scenarios of hedged and unhedged returns are fed as inputs to the optimization models discussed in Section 2.

Here we assume that the total proceeds at the end of the holding period from an allocation in a "completely hedged" foreign investment, which are uncertain (scenario dependent), can be converted back to the base currency with the known forward exchange rate f_i. In fact, a scenario-invariant amount (say the expected value of such proceeds) should be specified in a forward exchange contract, while the residual amounts from the scenario dependent proceeds are converted back with the spot exchange rates e^s_i prevailing at the end of the period. To accurately capture the value of variable forward transfers of foreign currency to the base currency we could resort to the use of quantos, but that would complicate the model. The simplifying approximation we use here is rather commonplace in the literature. As discussed in Eun and Resnick (1988) the error from this approximation should be very small, especially if we assume that forward exchange rates are a fair estimate of the future spot exchange rates.

Modeling extensions to handle accurately the forward exchange contracts are possible with the use of more advanced optimization models. In a companion paper (Topaloglou et al., 2002) we develop stochastic programs to capture decision dynamics and we generalize the models so as to account for transaction costs in a multi-period portfolio management setting.

3.4. Bayes–Stein estimation corrections

As the PCA procedure is calibrated based on a limited number of recent market observations, the statistical characteristics of a scenario set carry a residual estimation risk. Particularly, as indicated by Jorion (1985) and Eun and Resnick (1988), the mean-return vector of international assets exhibits intertemporal instability, while the variance–covariance matrix of international asset returns demonstrates greater stability through time. Thus, the expected-return vector is more prone to estimation error. The mean-return vector has a major influence on the results of portfolio optimization models and, consequently, its accurate estimation is of primary importance. The Bayes–Stein approach for determining the expected asset returns aims to mitigate the effects of estimation risk. It yields a uniform improvement on the classical sample mean as it relies on a more general estimation model. The approach was formalized by Jorion (1985, 1986) (see also Eun and Resnick, 1988).

A revised estimate $\hat{\mathbf{r}}$ for the mean-return vector is computed from

$$\hat{\mathbf{r}} = (1 - \vartheta)\bar{\boldsymbol{\rho}} + \vartheta \mathbf{1} \rho_0, \tag{15}$$

where $\bar{\boldsymbol{\rho}}$ is the sample mean-return vector, determined from historical observations of asset returns, ρ_0 denotes the mean return of the minimum-variance portfolio based on the same historical observations, $\mathbf{1}$ is the vector of ones, and ϑ represents the estimated shrinkage factor for shrinking the elements of $\bar{\boldsymbol{\rho}}$ toward ρ_0. The shrinkage parameter is estimated by

$$\vartheta = \frac{(N+2)(T-1)}{(N+2)(T-1) + (\bar{\boldsymbol{\rho}} - \rho_0 \mathbf{1})^{\mathrm{T}} T \mathbf{V}^{-1} (T-N-2)(\bar{\boldsymbol{\rho}} - \rho_0 \mathbf{1})}, \tag{16}$$

where T is the length of the time series of sample observations, N is the number of random variables (i.e., hedged and unhedged asset returns), and $\mathbf{V}$ is their sample variance–covariance matrix computed on the basis of the historical observations.

In order to attain the revised mean-return estimate $\hat{\mathbf{r}}$ of Eq. (15) the asset returns under each scenario $s \in \Omega$ must be modified by adding to their initial values $\mathbf{r}^s$ the correction term $(\hat{\mathbf{r}} - \bar{\mathbf{r}})$. This results in an update of the scenarios of returns for the unhedged as well as the hedged asset positions.

We apply the Bayes–Stein procedure in all subsequent experiments to revise the values of asset returns in the postulated scenarios before we solve the portfolio optimization models. We first solve the minimum-variance problem – calibrated on the basis of market observations over a prespecified historical period – to determine the shrinkage target return ρ_0. We then apply Eqs. (15) and (16) to determine the revised estimate of the mean-return vector on the basis of which we update the values of asset returns under all scenarios. We then proceed to solve the portfolio optimization model using the revised scenarios.

4. Empirical analysis

4.1. Data sources

We consider portfolios composed of positions in stock indices and bond indices of short-term (1–3 years) and long-term (7–10 years) maturity ranges in the United States (US), United Kingdom (UK), Germany (GR) and Japan (JPN). The following investment instruments are considered:

USS: US stock index,
UKS: UK stock index,

GRS: German stock index,
JPS: Japanese stock index,
US1: US government bond index (1–3 years maturity),
US7: US government bond index (7–10 years maturity),
UK1: UK government bond index (1–3 years maturity),
UK7: UK government bond index (7–10 years maturity),
GR1: German government bond index (1–3 years maturity),
GR7: German government bond index (7–10 years maturity),
JP1: Japanese government bond index (1–3 years maturity),
JP7: Japanese government bond index (7–10 years maturity).

The values of the stock indices were obtained from the Morgan Stanley Capital International database. The values of the bond indices and the exchange rates were obtained from Datastream. The collected time series involve monthly data for the period from April 1988 through May 2001.

4.2. Statistical characteristics of historical data

First we analyze the statistical characteristics of the data covering the period 01/1990–08/2000 that were used in static tests for the determination of risk–return efficient frontiers on September 2000. As we can see from Table 1, both the domestic returns of the indices and the proportional changes of exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean.

Table 1
Statistical characteristics of historical monthly data for domestic returns of assets and proportional changes of spot exchange rates over the period 01/1990–08/2000

Asset class	Mean (%)	Std. dev. (%)	Skewness	Kurtosis	Jarque–Bera statistic
Statistical characteristics of monthly domestic returns of assets					
USS	1.519	3.900	−0.465	4.271	11.769
UKS	1.164	4.166	−0.233	3.285	1.391
GRS	1.213	5.773	−0.511	4.503	15.738
JPS	−0.133	6.336	0.022	3.609	1.546
US1	0.537	0.473	−0.144	2.801	0.727
US7	0.688	1.646	−0.047	3.276	0.299
UK1	0.723	0.710	1.330	7.209	121.156
UK7	0.913	1.932	0.108	3.482	1.157
GR1	0.537	0.458	0.655	5.319	34.052
GR7	0.670	1.390	−0.863	4.482	25.421
JP1	0.327	0.522	0.492	4.147	10.891
JP7	0.608	1.731	−0.514	5.149	27.039
Statistical characteristics of monthly proportional spot exchange rate changes					
Exchange rate	Mean	Std. dev.	Skewness	Kurtosis	Jarque–Bera statistic
US to UK	−0.074%	0.081%	−1.084	6.790	189.330
US to GR	−0.167%	0.088%	−0.398	3.908	18.842
US to JP	0.303%	0.133%	1.123	6.904	35.966

Jarque–Bera tests on these data indicated that normality and log-normality hypotheses cannot be accepted for the majority of them. [1] This clearly influenced our choice of the scenario generation procedure; it was also a primary motivation for our decision to consider the CVaR risk metric that is suitable for skewed distributions. Even though the skewness and kurtosis values approach somewhat those of the normal distribution when the asset returns are expressed in terms of the base currency, still the normality hypothesis does not hold.

We applied PCA using the 128 monthly observations over the period 01/1990–08/2000. We retained seven PCs that explain about 97% of the total variability. We took 4, 4, 4, 3, 3, 3, 3 samples, respectively, for the retained PCs for a total of 5184 scenarios. Using this scenario set we considered the asset allocation problem in the beginning of September 2000. We generated the efficient risk–return frontiers with the CVaR and the MAD models for different combinations of the investment opportunity set: US assets only, all assets without hedging, all assets with complete hedging, and all assets with selective hedging. Based on the results of these tests we are able to address several questions relating to the management of international portfolios.

4.3. Investigation issues

4.3.1. International diversification benefits

We examine the effects of international diversification from the perspective of a US investor. Fig. 1 contrasts the efficient frontiers of portfolios composed of US assets only against those of internationally diversified portfolios. We observe that international diversification improves the risk–return profiles of the portfolios regardless of the risk metric and regardless of whether hedging is employed or not, although the selective hedging strategy clearly exhibits the best performance. The risk–return efficient frontiers of international portfolios clearly dominate the efficient frontiers of portfolios composed solely of US assets. The same behavior is observed whether we use the CVaR or the MAD risk metric. The benefits of international diversification are verified through backtests discussed later in this section.

4.3.2. Appropriateness of hedging strategies

Fig. 1 depicts efficient frontiers of international stock and bond portfolios on September 2000 constructed with the CVAR and MAD models, respectively, using three alternative hedging policies (no-, complete- and selective-hedging). Both models indicate that at the low-risk end of the spectrum completely hedged portfolios are preferable to unhedged portfolios as they yield dominant risk–return profiles over the overlapping range of their efficient frontiers. However, completely hedged portfolios can reach only a limited range of expected return. The efficient frontiers of the no-hedging strategy extend into a range of higher expected returns (and risk) than are

[1] The Jarque–Bera statistic has a χ^2 distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jarque–Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

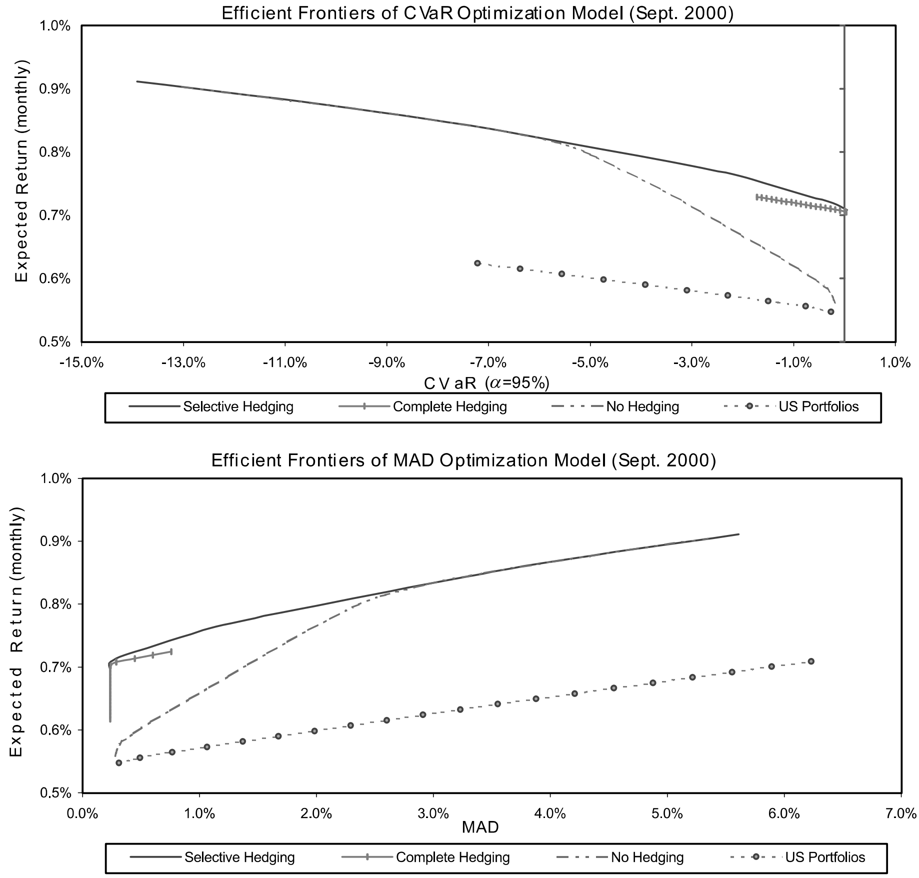

Fig. 1. Efficient frontiers of CVaR and MAD optimization models for US and internationally diversified portfolios of stocks and bonds (with alternative hedging strategies).

attainable by complete hedging. Hence, more aggressive return targets are reached only with riskier unhedged investments; increasing target returns necessitate increasing exposure to currency risk.

In these static tests, selective hedging is the superior strategy as it leads to efficient frontiers that envelope those of the other two hedging strategies. This observation obviously holds for both optimization models as the selective hedging strategy encompasses both of the other two hedging alternatives. We note that in these tests both the CVaR and the MAD models lead to consistent assessments regarding the order of preference of the alternative hedging strategies at any level of target return.

Fig. 2 illustrates the compositions of several selectively hedged international portfolios corresponding to different points of the CVaR-efficient frontier (for confidence level $\alpha = 95\%$) on September 2000. The postfix _u or _h is used on the asset symbols listed in the legend of the graph to denote unhedged, respectively hedged, positions in the corresponding assets. In the risk neutral case the expected return of the

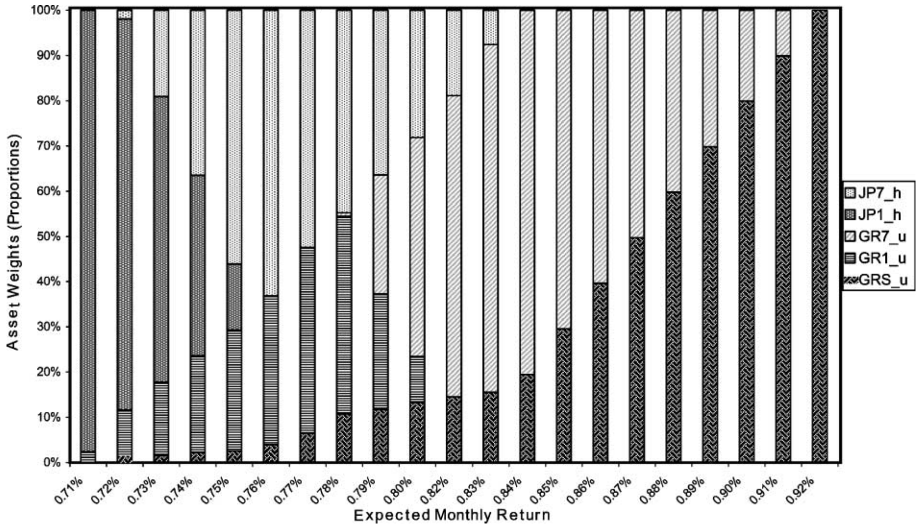

Fig. 2. Compositions of selectively hedged international portfolios from the CVaR-efficient frontier of September 2000 (at confidence level $\alpha = 95\%$).

portfolio is maximized without any consideration on risk. Hence, the entire budget is allocated to the asset with the highest expected return (in this case the unhedged position in the German stock index). In the minimum risk case, the risk measure is optimized without any constraint on the target expected return. The minimum risk portfolio involves almost exclusively a hedged allocation in the short-term Japanese bond index (JP1_h). Greater levels of diversification are exhibited in efficient portfolios with intermediate levels of expected return (and risk) which include both hedged and unhedged positions in multiple international assets. Note that the hedge ratio varies across countries (e.g., Japanese assets are hedged while German assets are unhedged). The hedged proportions also vary between points on the efficient frontier that correspond to different levels of expected return and risk. This points to the advantageous flexibility of the selective hedging approach in comparison to the more restrictive unitary and partial hedging policies.

4.3.3. Effectiveness of scenario generation and model stability

We carried out several tests to validate the effectiveness of our scenario generation procedure and the stability of the models' results. Using the PCA results from the data of the period 01/1990–08/2000, and following our selective sampling approach, we generated a larger set of 33,075 scenarios by taking 7, 7, 5, 5, 3, 3, 3 samples from the seven retained PCs, respectively. We also generated sets of 5184 scenarios by taking random samples from the PCs. In each of these scenario sets we took 4, 4, 4, 3, 3, 3, 3 samples from the corresponding PCs, similar to the scenario set that we had constructed initially. However, in these sets we took random samples rather than employing our selective sampling procedure. We produced the efficient frontiers by employing each of these scenario sets in the two optimization models.

The efficient frontiers for the corresponding scenario sets are plotted in the upper graphs of Figs. 3 and 4 for the CVaR and MAD models, respectively. We note that the efficient frontiers for the 5184-scenario set and the 33,075-scenario set that were generated with the selective sampling procedure are very close for both risk metrics. This illustrates the consistency of the selective sampling approach. We also observe that the random scenario sets imply dominant efficient frontiers in both models. This, however, is misleading as we demonstrate in the middle graphs of Figs. 3 and 4. We took the "efficient" portfolios obtained with the various scenario sets and simulated their performance over the large set of 33,075 scenarios which played the role of an out-of-sample scenario set. For each portfolio we determined the expected return and risk measure (95%-CVaR or MAD) over the out-of-sample simulation. The results of these simulations are plotted in the middle graphs of Figs. 3 and 4 for the CVaR and the MAD metric, respectively. Again, we observe that the simulated frontiers of the initial 5184-scenario set practically retrace the efficient (optimal) frontiers of the 33,075-scenario set in both the CVaR and the MAD models. This is a strong indication of the effectiveness and consistency of our selective sampling procedure, as the resulting frontiers remain quite stable. On the contrary, the simulated frontiers for the random scenario sets are far from efficient with respect to the out-of-sample scenarios.

Finally, we generated 18 different scenario sets ranging in size from 25,000 to 60,000 scenarios. These scenario sets were produced on the basis of the selective sampling procedure by varying the number of samples for each PC. The optimal portfolios of the 33,075-scenario models were then simulated on each of these out-of-sample scenario sets and their corresponding risk–return characteristics were recorded. The results of these simulations are depicted in the bottom graphs of Figs. 3 and 4. Again we observe that the resulting risk–return curves of the simulations remain rather stable; they remain within a narrow band around the efficient frontiers optimized on the 33,075-scenario set, both in the CVaR and in the MAD model. The width of this band collapses at the minimum risk end of the frontiers, indicating that the minimum risk portfolio is practically invariant with respect to sample. All out-of-sample tests indicate that the selective sampling procedure is effective and the results of the models are stable.

4.3.4. Tightness of CVaR bound on VaR

We now examine empirically the relation between CVaR and VaR. In Fig. 5 we plot the efficient frontiers for the CVaR-optimal solutions at two different confidence levels, $\alpha = 95\%$ and 99%. In the same figure we also plot the corresponding VaR estimates for these solutions, denoted as VaR(CVaR*); these curves depict the VaR values of the CVaR-optimized portfolios at the respective confidence levels. As expected, CVaR provides a lower bound for VaR (recall that they are both expressed in terms of portfolio return) at the corresponding confidence level α. However, note that this bound is not tight, although the difference between CVaR and VaR is reduced at increasing confidence levels. It should be kept in mind that the frontiers of VaR against expected return for the CVaR-optimal portfolios need not be efficient from the VaR perspective; the exact VaR-efficient frontiers are not available.

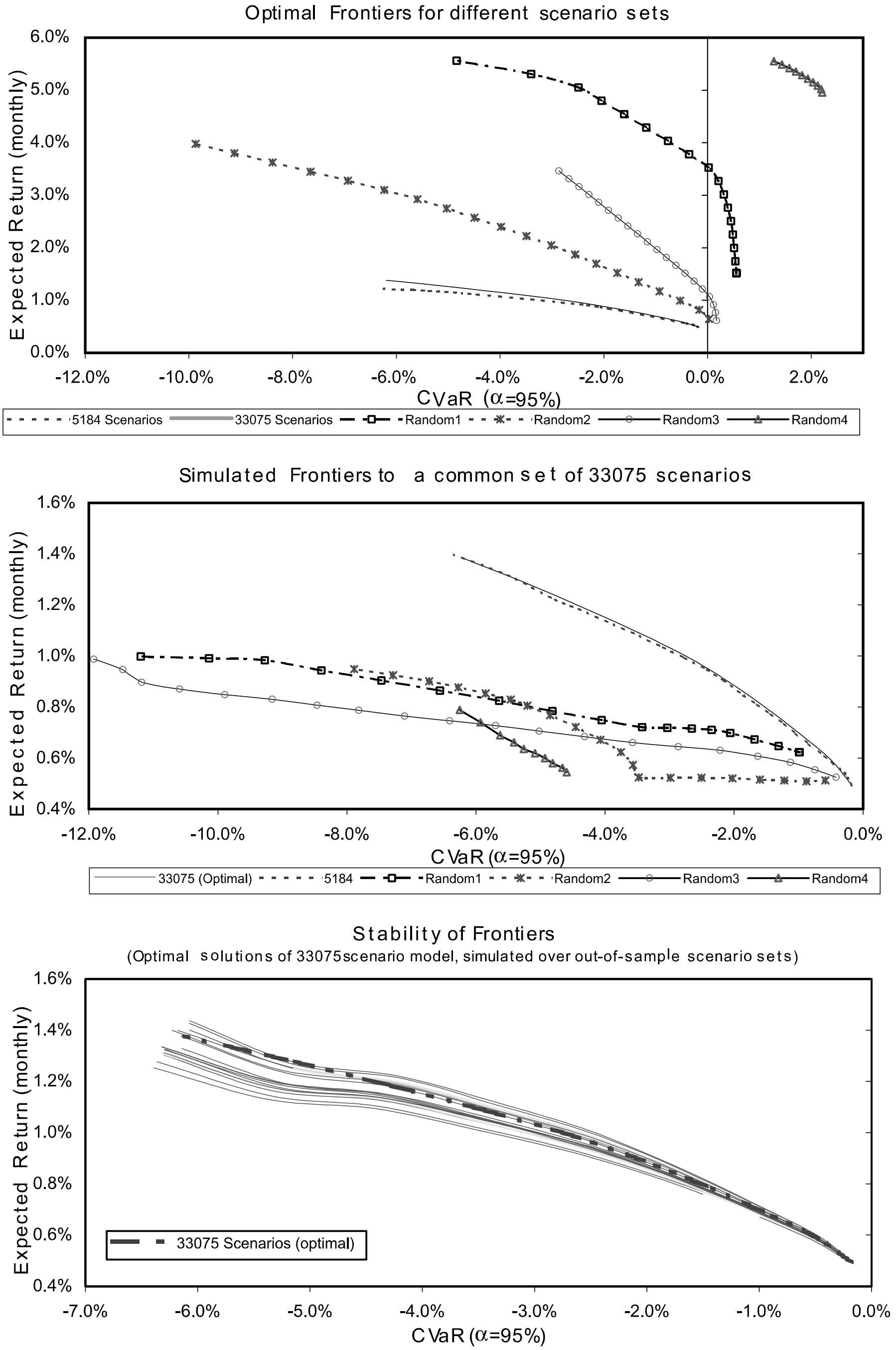

Fig. 3. Risk–return frontiers of portfolios generated with the CVaR model for several in-sample and out-of-sample test cases.

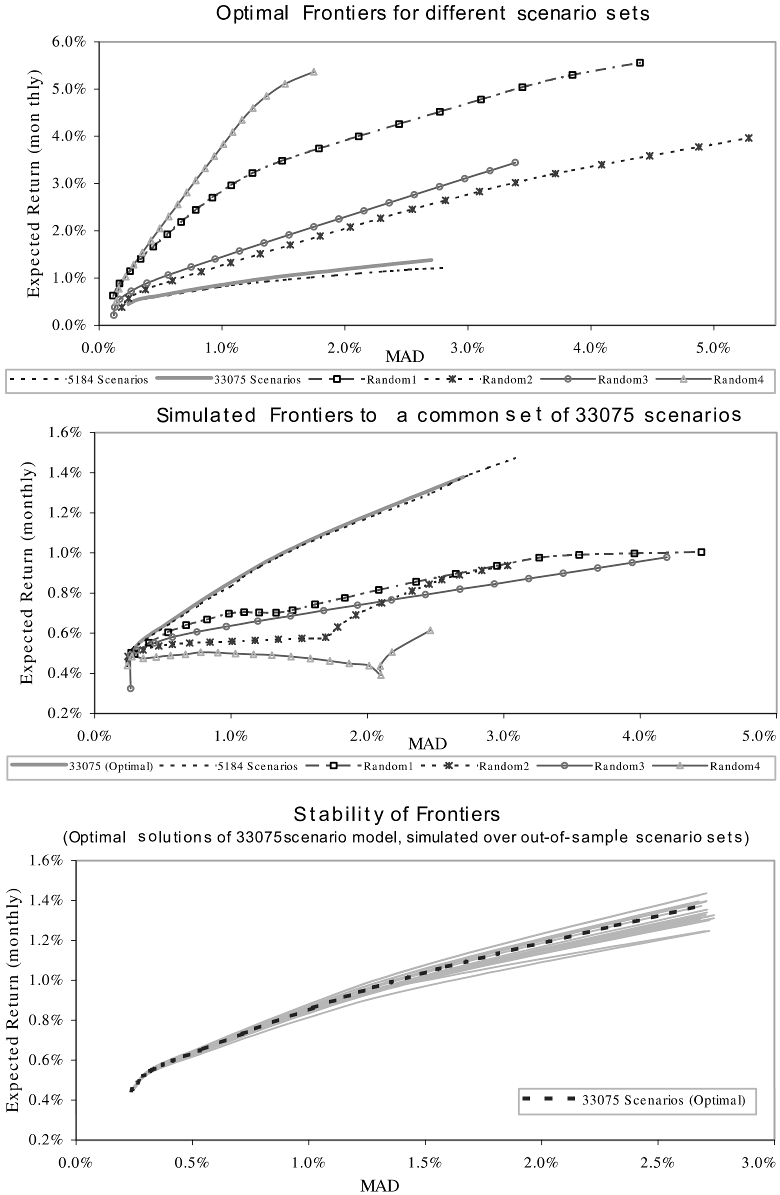

Fig. 4. Risk–return frontiers of portfolios generated with the MAD model for several in-sample and out-of-sample test cases.

Similar results are reported by Jobst and Zenios (2001) for portfolios of corporate bonds. In fact, the differences of their CVaR-efficient frontiers from the VaR-(CVaR*) curves are more pronounced because the returns of corporate-bond

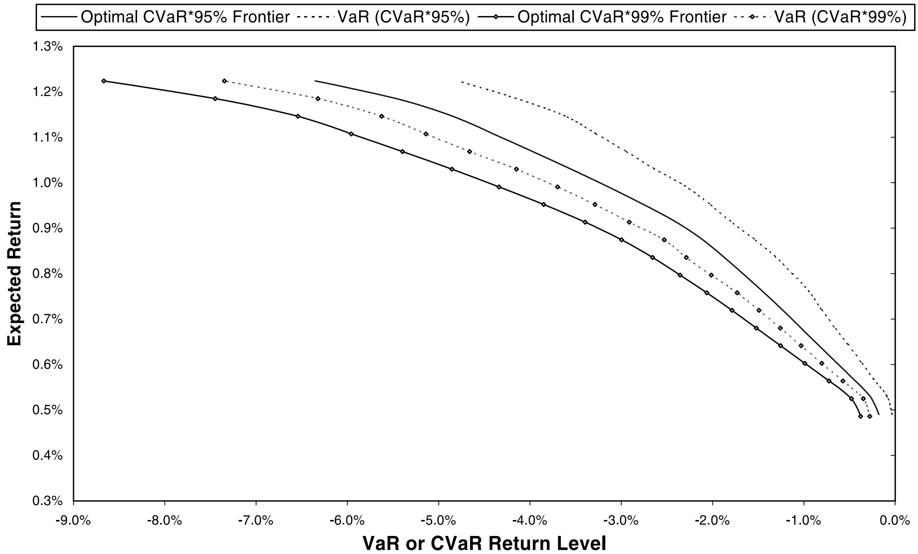

Fig. 5. Comparison of CVaR*-optimal and VaR frontiers at confidence levels $\alpha = 95\%$ and 99%.

portfolios are more skewed (due to credit and default risk) than those of international indices and foreign exchange rates.

4.3.5. Dynamic tests: Comparative performance of risk management metrics

So far we have observed that in the static tests the CVaR and the MAD models exhibit practically the same behavior. For example, in the asset allocation problem of September 2000 they rank the same way, in order of relative preference, the alternative hedging strategies at all levels of target return. Moreover, the two models yield almost indistinguishable frontiers for the asset selection problem of September 2000. That is, the frontier of CVaR for the MAD-optimized portfolios is almost indistinguishable from the CVaR-efficient frontier for the same scenario set. Conversely, the frontier of MAD for the CVaR-optimized portfolios is almost identical to the MAD-efficient frontier. The results of the two models also exhibit quite similar stability in out-of-sample simulations. However, a definitive comparison between the CVaR and the MAD models cannot be reliably made based on static tests alone.

Thus, we resort to backtesting experiments on a rolling horizon basis for a more substantive comparison between the two models. The rolling horizon simulations cover the 37-month period from 04/1998 to 04/2001. At each month, we use the historical data from the previous 10 years (120 monthly observations) to calibrate the PCA procedure and to generate 5184 scenarios by the selective sampling procedure described in Sections 3.1–3.3. The asset returns of these scenarios are updated according to the Bayes–Stein approach of Section 3.4. We then solve the resulting optimization model and record the optimal portfolio. The clock is advanced and the realized return of the portfolio is determined from the actual market values of the assets and the observed exchange rates. The same procedure is then repeated for the next time period and the ex post realized returns are compounded. We ran such backtesting experiments for both the CVaR and the MAD models using various values of target monthly return μ.

The results for the CVaR model are depicted in Fig. 6. The minimum risk portfolio ($\mu = 0.0\%$) attains a stable growth path representing a 0.6% geometric mean of monthly returns (7.2% annual) over the test period. Increasing ex post returns are achieved with increasing levels of target return μ, obviously at the expense of increasing volatility. The MAD model generates growth paths with similar patterns to those of the CVaR model. However, as illustrated in Fig. 7, its ex post performance is somewhat less successful in comparison to that of the CVaR model. The CVaR model consistently outperforms the MAD model, especially for the low risk strategies. As the value of the target return parameter μ is increased the two models behave more similarly. This is due to the fact that as μ increases, meeting the requested target return level becomes the governing factor over the minimization of the respective risk metric.

The superiority of the CVaR model over the MAD model is evident in Fig. 8. This figure depicts the geometric mean against the standard deviation of ex post realized monthly returns over the 37-month test period for all simulation experiments. Clearly, the CVaR model outperforms the MAD model at all levels of target return μ by generating steeper and more stable growth paths (i.e., higher returns with lower volatility). The difference in the performance of the two models is gradually reduced with increasing values of the parameter μ. On the same graph we also plot the ex post performance of individual assets over the same test period. Both optimization

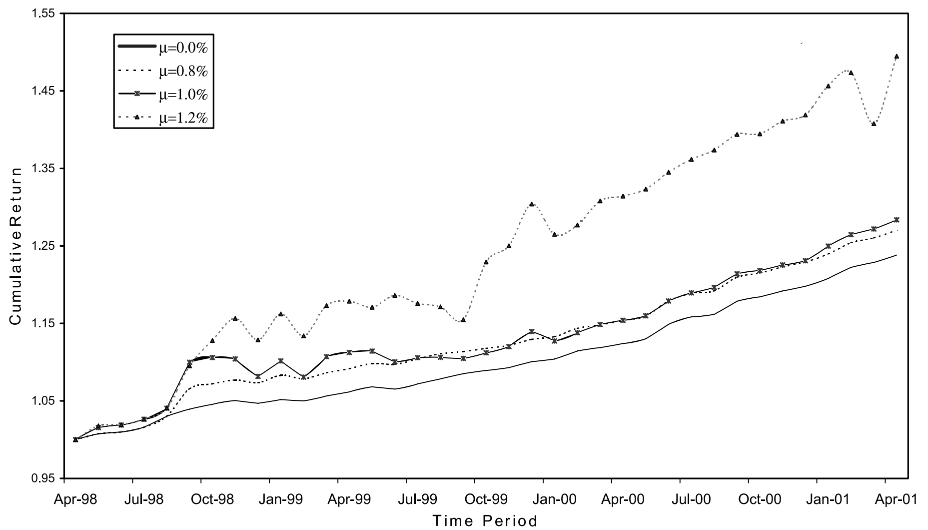

Fig. 6. Ex post realized returns with the selective hedging CVaR model (at confidence level $\alpha = 95\%$) over the period 04/1998–04/2001 for different values of target monthly return (μ).

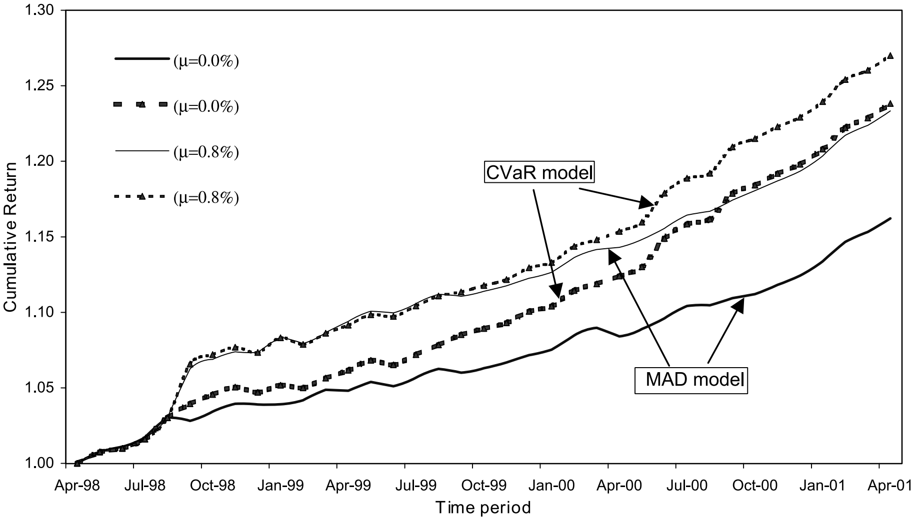

Fig. 7. Ex post realized return paths with the selective hedging CVaR and MAD models over the period 04/1998–04/2001 (for target monthly return values $\mu = 0.0\%$ and 0.8%).

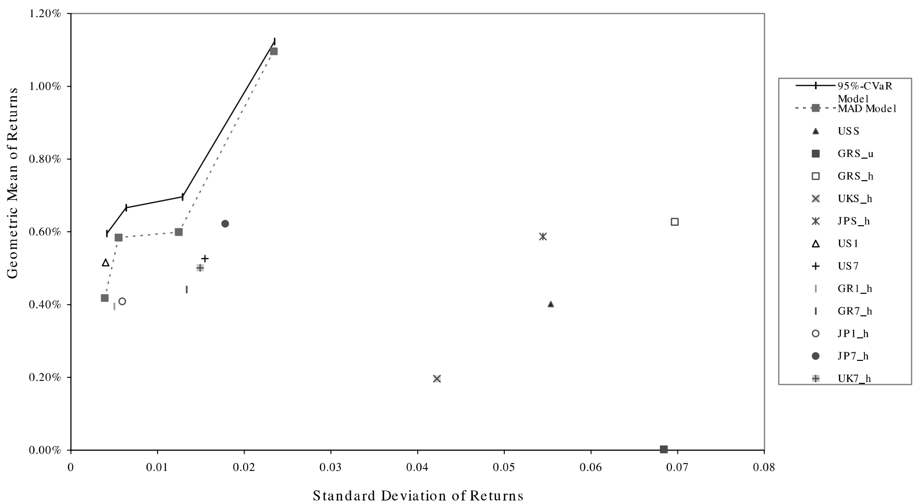

Fig. 8. Comparative ex post performance – in terms of the realized geometric mean and standard deviation of monthly returns – of individual assets, and international portfolios selected by the CVaR and MAD models over the period 04/1998–04/2001.

models outperform all individual assets, with the CVaR model being the most successful in realizing effective growth performance while limiting risk (as evidenced by the volatility levels).

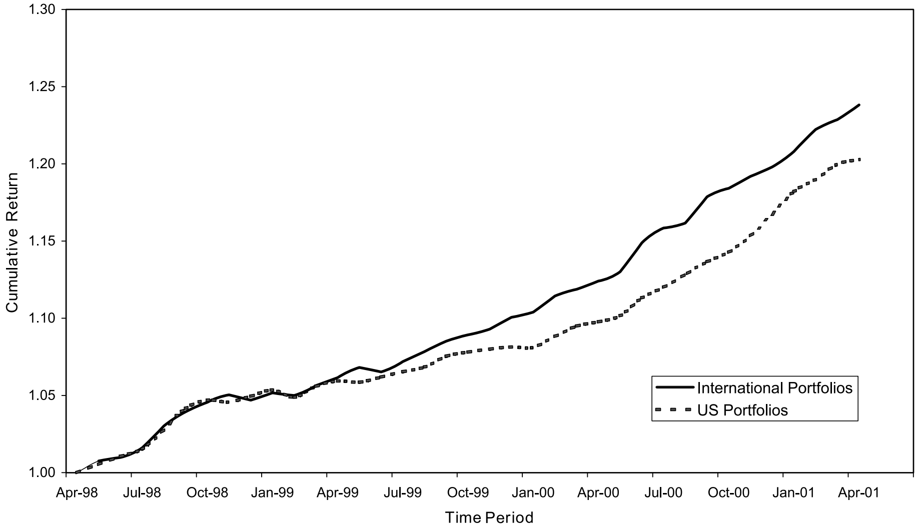

Fig. 9. Ex post realized return paths for US and selectively hedged international portfolios of stocks and bonds obtained with the CVaR model (with parameters $\alpha = 95\%$ and $\mu = 0.0\%$) over the backtest period 04/1998–04/2001.

Finally, we revisit the question regarding the potential of international diversification. We conducted backtesting experiments with the CVaR model (at confidence level $\alpha = 95\%$) allowing portfolios of US assets only and contrasted the results with corresponding experiments that allow selectively hedged international portfolios. The results are summarized in Fig. 9 for the minimum risk case (i.e., $\mu = 0.0\%$). While the selected US and international portfolios perform similarly for the first year of the simulation, after that the international portfolios clearly outperform the US portfolios.

5. Conclusions

We developed an integrated simulation and optimization framework for international asset allocation and we demonstrated its effectiveness as a decision support tool through a series of empirical tests. This framework involves both a scenario generation approach for depicting the uncertainty in asset returns and exchange rates, and suitable portfolio optimization models for risk management.

The scenario generation procedure employs a selective sampling approach with PCA to capture the joint variation of domestic asset returns and exchange rates (i.e., market and currency risk). It is combined with the Bayes–Stein estimation correction to counter estimation risk for the mean returns. The consistency and effectiveness of the scenario generation method, and its superiority over random sampling, were established by means of out-of-sample simulations. For sufficiently large scenario sets the efficient portfolios at any level of target return and, consequently, the risk–return efficient frontiers, remain quite stable with respect to sample.

The risk management optimization model uses the CVaR metric that is suitable for asymmetric return distributions. This choice is consistent with the asymmetric distributions exhibited in the data of multicurrency stock and bond indices. Our model internalizes selective hedging decisions within the portfolio selection context to yield flexible investment recommendations. We verified empirically earlier findings regarding the value of the selective hedging strategy and we demonstrated the benefits of international diversification in improving the risk–return performance of investment portfolios.

We compared the CVaR model with the MAD model both in static tests as well as in dynamic backtesting experiments. We observe that in static tests both models behave quite similarly; they exhibit similar stability with respect to scenario samples and they trace almost indistinguishable risk–return profiles. However, when the models were repeatedly applied over successive time periods in the context of backtesting experiments with real market data, the CVaR model clearly outperformed the MAD model especially for low-risk portfolios. In the backtesting simulations the CVaR model produced more effective ex post realized return paths, both in terms of higher growth rates and lower volatility.

The next step in our research is the extension of the optimization models in a multiperiod decision framework. This involves the development and implementation of stochastic programming models that allow portfolio rebalancing decisions within a longer time horizon. The stochastic programming models capture decision dynamics, include an operational treatment of hedging decisions by means of implementable forward exhange contracts, and they account for the effect of transaction costs. The development of these models and empirical findings are reported in Topaloglou et al. (2002).

References

Abken, P.A., Shrikhande, M.M., 1997. The role of currency derivatives in internationally diversified portfolios. Federal Reserve Bank of Atlanta Economic Review (Third Quarter), 34–59.

Anderson, F., Mausser, H., Rosen, D., Uryasev, S., 2001. Credit risk optimization with conditional value-at-risk criterion. Mathematical Programming Series B 89 (2), 273–291.

Artzner, P., Delbaen, F., Eber, J.M., Heath, D., 1999. Coherent measures of risk. Mathematical Finance 9 (3), 203–228.

Beltratti, A., Laurent, A., Zenios, S.A., (in press). Scenario modeling of selective hedging strategies. Journal of Economic Dynamics and Control.

Black, F., 1990. Equilibrium exchange rate hedging. Journal of Finance 45 (3), 899–907.

Eun, C.S., Resnick, B.G., 1988. Exchange rate uncertainty, forward contracts and international portfolio selection. Journal of Finance 43 (1), 197–215.

Filatov, V.S., Rappoport, P., 1992. Is complete hedging optimal for international bond portfolios? Financial Analysts Journal (July–August), 37–47.

Glen, J., Jorion, P., 1993. Currency hedging for international portfolios. Journal of Finance 48 (5), 1865–1886.

Jobst, N., Zenios, S.A., 2001. The tail that wags the dog: Integrating credit risk in asset portfolios. The Journal of Risk Finance 3 (1), 31–43.

Jollife, I.T., 1986. Principal Component Analysis. Springer, New York.

Jorion, P., 1985. International portfolio diversification with estimation risk. Journal of Business 58 (3), 259–278.

Jorion, P., 1986. Bayes–Stein estimation for portfolio analysis. Journal of Financial and Quantitative Analysis 21 (3), 279–292.

Jorion, P., 1989. Asset allocation with hedged and unhedged foreign stocks and bonds. Journal of Portfolio Management (Summer), 49–54.

Kang, P., Zenios, S.A., 1993. Mean-absolute deviation portfolio optimization for mortgage-backed securities. Annals of Operations Research 45, 443–450.

Konno, H., Yamazaki, H., 1991. Mean absolute deviation portfolio optimization model and its application to Tokyo stock market. Management Science 37 (5), 519–531.

Kouwenberg, R., Zenios, S.A., (in press). Stochastic programming models for asset–liability management. In: Zenios, S.A., Ziemba, W.T. (Eds.), Handbook of Asset and Liability Management. North-Holland, Amsterdam.

Perold, A., Schulman, E., 1988. The free lunch in currency hedging: Implications for investment policy and performance standards. Financial Analysts Journal 44, 45–50.

Pflug, G., 2001. Some remarks on the value-at-risk and the conditional value-at-risk. In: Uryasev, S. (Ed.), Probabilistic Constrained Optimization: Methodology and Applications. Kluwer Academic Publishers, Dordrecht.

Riskmetrics. Technical document, 4th edition. J.P. Morgan Inc., December 1996.

Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. The Journal of Risk 2 (3), 21–41.

Rockafellar, R.T., Uryasev, S., 2002. Conditional value-at-risk for general distributions. Journal of Banking and Finance 26 (7), this issue.

Solnik, B., 1998. Global asset management. Journal of Portfolio Management (Summer), 43–51.

Topaloglou, N., Vladimirou, H., Zenios, S.A., 2002. Stochastic programming models for managing international investment portfolios. Working Paper 02-02, "Hermes" European Center of Excellence on Computational Finance and Economics, School of Economics and Management, University of Cyprus, Nicosia, Cyprus, 2002.