



CVaR sensitivity with respect to tail thickness

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ABSTRACT

The sensitivity of a risk measure with respect to the parameters of the hypothesized distribution is a useful tool in investigating the impact of marginal rebalancing decisions on the portfolio return distribution and also in the analysis of the asymptotic variability of the risk estimator. We calculate the relative importance of the conditional value-at-risk (CVaR) sensitivity with respect to tail thickness and scale of the portfolio return distribution in the case of regularly varying tails and in the case of exponential and faster-than-exponential decay. We discuss the implications for asset return modeling and the asymptotic variability of the risk estimator.

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1. Introduction

There is substantial empirical evidence that financial returns exhibit fat-tails and excess kurtosis after accounting for the clustering of volatility and autocorrelation. Different models have been suggested to explain these empirical facts. Mandelbrot (1963) and Fama (1965) proposed the stable Paretian distribution which was later incorporated as a building block in GARCH-type processes, see for example Mittnik et al. (2002) and Mittnik and Paoletta (2003).

Stable Paretian distributions are fat-tailed; they can explain the observed skewness and excess kurtosis of financial returns and represent a clear improvement over the Gaussian distribution, see Rachev and Mittnik (2000) for empirical studies and further references. Models based on stable Paretian laws, however, do not explain the empirical fact that lower-frequency returns tend to have a higher tail index, see Samorodnitsky and Grabchak (2010). Stable distributions have an infinite variance and this creates technical difficulties in other areas such as option pricing and asset allocation modeling. Recently, tempered stable distributions have been suggested as an alternative model which has some of

the attractive features of the class of stable laws but is more flexible, see Kim et al. (2008, 2010) for additional details. A more recent discussion on risk measurement and Pareto-style tails can be found in Fiori et al. (2011).

Apart from stable and tempered stable laws, other distributional models have been suggested in the literature and are used by practitioners. One alternative is the class of hyperbolic distributions which contains fat-tailed and skewed representatives and can be considered as a building block in financial models both in a discrete and continuous setting, see Bibby and Sorensen (2003). This class contains the Student's *t* distribution which is attractive to practitioners because of its simplicity.

Other examples include distributions used only for tail modeling without providing a basis for more general models. This category of models includes, for example, the approach based on extreme value theory. An application to value-at-risk modeling with a GARCH model for the clustering of volatility effect is available in Kuester et al. (2006). There are also more ad hoc models such as the generalized normal distribution (GND). An application to high-frequency data is available in Chen et al. (2008). For additional examples with the Weibull, the Laplace-Gaussian mixture, and other distributions, see Haas et al. (2006) and Rachev and Mittnik (2000).

An important property distinguishing these classes of models is the rate of tail decay of the distribution. Stable Paretian

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