CYCLIC VARIATION OF THE GLOBAL MAGNETIC FIELD INDICES

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Abstract. The energetical aspect of solar phenomena of different spatial and time scales has been studied with special attention to global magnetic fields. Cyclic regularities in the heliosphere are determined by energetics of global magnetic fields. The energy variation of global fields consists of a number of maxima and minima coinciding with reference points of the sunspot cycle. The correlations of a number of well-known indices in the heliosphere with Wolf numbers and with indices of energetics of the global magnetic field have been investigated. The results can be used to identify more exactly the reference points of the cycle.

1. Introduction

The cyclic variation of solar activity is usually characterized by special indices associated with local magnetic fields. The primary index that allowed the phenomenon of solar activity to be discovered was that of the Wolf number. Other widely used indices are the sunspot area, the numbers of flares and radio bursts, the characteristics of active regions, faculae and flocculi, as well as the integral radio-, X-ray, and UV-fluxes. All indices and their various combinations are determined by local magnetic fields. At the same time, there is an obvious deficit of indices to characterize cyclic variations of the structure and energetics of global magnetic fields. In some recently performed studies (e.g., see Makarov and Sivaraman, 1989a, b; Hoeksema and Scherrer, 1986) the cyclic variations of global and background fields have been investigated. However, neither in the articles mentioned above, nor in other publications has any attempt been made to create a generally usable index to characterize the global magnetic field. Note that even the terminology is not quite definite. By 'local fields' we shall, henceforth, mean the fields with characteristic spatial scale comparable with a developed active region, and by 'global fields' – the fields with characteristic spatial scale comparable with the solar radius. In this paper, we briefly touch upon the intermediate-scale fields called 'background fields', and we will not mention at all the fields of smaller scale than local.

Why do we especially emphasize the shortage of data on global fields? The fact is that many authors (Bumba, 1976, 1986; Bumba and Gesztelyi, 1988; Ivanov, 1986, 1987; Kuklin, 1971, 1986; Kuklin and Obridko, 1982, 1988; Obridko, 1984) have shown in their work that:

(1) There are two systems of magnetic fields existing in the Sun that differ in their evolution, rotation and energy characteristics (see Bumba, 1986; Ivanov, 1986). One

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system rotates as a solid body with a period of 27.3 days and consists of very large cells (> 90°). Another is characterized by differential rotation with a period changing from 29 to 27.8 days during the solar cycle and consists of smaller cells (about 20°). There is a lot of evidence that these systems are associated with global and local fields whose interplay determines the main variety of solar active phenomena.

(2) The long-term characteristics of solar activity are determined by global fields, both directly and through their effect on local fields (active longitudes, complexes of activity, impulses of activity, etc.). Of course, there is also a reverse effect of local fields on the global ones.

(3) It is the structure of global fields that determines many geophysical and interplanetary events.

What then are the indicators of the global field structure? First of all, these are coronal holes. Coronal holes are determined by the entire pattern of large-scale magnetic field distribution in the Sun which manifests itself in the corona and in interplanetary space, and they are closely associated with high-velocity streams and the IMF sector structure (Obridko and Shelting, 1987; Shelting and Obridko, 1988). As shown in Obridko and Shelting (1989a, b, 1990), coronal holes can be used as indicators to trace the location of the neutral line on the source surface in the corona. At the same time, coronal holes are shown to concentrate in regions of enhanced magnetic field at the source surface. This provides a simple method to predict the interplanetary current sheet and sector structure, which, in turn, determine the location of the proton complexes and the outflow regions of high-velocity streams.

Coronal holes are, however, inappropriate to characterize quantitatively the general energetics of the global magnetic field.

2. Energy Indices of the Global Magnetic Field

A quantitative index that can be introduced for global fields is the energy index. For analyzing cyclic variations of the global solar magnetic field, it is convenient to introduce magnetic indices which are quantities defined by the global distribution.

Let $i(B_r)$ denote the square of the radial component of the magnetic field averaged over the solar surface $\langle B_r^2 \rangle$:

$$i(B_r)|_R = \langle B_r^2 \rangle , \tag{1}$$

where R is the radius of the sphere over which the averaging is performed.

In order to calculate $i(B_r)$, one should know the magnetic field B_r. We have used magnetic field data obtained at Stanford with 3 arc min resolution for the period of June 1976 to September 1985, i.e., for Carrington rotations Nos. 1641–1766 (Hoeksema and Scherrer, 1986). Instead of using directly measured data, their representation in the form of Legendre polynomials has been used for the convenience of calculations:

$$B_r = \sum_{lm} P_l^m(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi) \times$$
$$\times \left\{(l+1)\left[\frac{R_\odot}{R}\right]^{l+2} + l\left[\frac{R}{R_s}\right]^{l-1}\zeta^{l+2}\right\}, \tag{2}$$

where l and m are the indices of spherical harmonics, $\zeta = R_\odot/R_s$ and $R_\odot$ is the radius of the Sun, $R_s = 2.5\,R_\odot$ is the radius of the source surface, P_l^m is the Legendre polynomial; g_{lm} and h_{lm} are tabulated in Hoeksema and Scherrer (1986).

For the surface of the photosphere ($R = R_\odot$) and the source surface ($R = R_s$) we have, respectively,

$$B_r|_{R_\odot} = \sum_{lm} P_l^m(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi)(l + 1 + l\zeta^{2l+1}) \tag{3}$$

$$B_r|_{R_s} = \sum_{lm} P_l^m(\cos\theta)(g_{lm}\cos m\phi + h_{lm}\sin m\phi)(2l+1)\zeta^{l+2}\,. \tag{4}$$

If we apply the averaging procedure (1) to formulae (3) and (4) we obtain the following expressions for the indices $i(B_r)|_{R_\odot}$ and $i(B_r)|_{R_s}$ (Obridko and Yermakov, 1989)

$$i(B_r)|_{R_\odot} = \sum_{lm} \frac{(l + 1 + l\zeta^{2l+1})^2}{2l+1}(g_{lm}^2 + h_{lm}^2), \tag{5}$$

$$i(B_r)|_{R_s} = \sum_{lm} (2l+1)\zeta^{2l+4}(g_{lm}^2 + h_{lm}^2)\,. \tag{6}$$

There are several points to be taken into account:

(1) Coefficients g_{lm} and h_{lm} have been calculated under the assumption that the field above the photosphere is potential and that there exists a source surface at $R = 2.5\,R_\odot$, where the field lines are radial and the potential goes to zero. However, in calculating both the field and its indices at the surface of the photosphere these coefficients may be regarded as mere approximation parameters of the original field and, thus, calculation at the $R_\odot$ level is independent of any assumptions.

(2) Index $i(B_r)$ is the square of the field intensity averaged over a spherical surface of radius R. One might introduce as an index the mean energy in the layer between the photosphere and the source surface, $W(B_r)$. However, since the field intensity drops rapidly with height, the cyclic curve of index $W(B_r)$ closely resembles that of index $i(B_r)$.

(3) The calculation of indices above the photosphere depends in principle on the assumptions of a potential field and source surface. However, the potential approximation for large-scale fields above the photosphere seems quite reasonable and the assumption of the existence of a source surface does not appreciably change the cyclic curve.

Let us also introduce partial field indices, namely, zonal-even, ZE ($m = 0$, $1 = 2k$); zonal-odd, ZO ($m = 0, l = 2k + 1$); sectorial-even, SE ($m = l = 2k$); and sectorial-odd, SO ($m = l = 2k + l$). These indices are determined from Equations (5) or (6) taking into account the above-mentioned limitations on m and l.

The physical meaning of the partial field indices is as follows: index ZO denotes the part of the field with a zonal-odd type of symmetry, i.e., the same as in a dipole aligned with the rotation axis of the Sun. It should be noted that the cyclic behaviour of the ZO field is to a large extent determined by the correction for the polar field used by Hoeksema and Scherrer (1986). The zonal-even index, ZE, is always small as a result of Hale's law and, therefore, will not be considered below. The sectorial-odd index, SO, characterizes, for example, a tilted dipole and manifests itself in the occurrence of 2- and 6-sector structure. The sectorial-even index, SE, is usually associated with the 4-sector structure. The qualitative behaviour of these indices is illustrated in Figures 1 and 2 (see also Kuklin *et al.*, 1990) and in Table I. All curves in Figures 1 and 2 are smoothed over an interval of 6 solar rotations.

TABLE I

The qualitative behaviour of the indices

Ref. point	$i(B_r)$	ZO	SO	SE	Dates
Min		const., maximal	const., very weak	const., weak	
t_{mA}	sharp increase	beginning of sharp decrease	const., weak	sharp jump	Rot 1661–1665 (1977.9–78.3)
t_{AM}	max	min	local max	narrow max	Rot 1678–1681 (1979.2–79.4)
Max	local min		local min	local min	Rot 1691 (1979.9)
t_{MD}	max		some narrow max	wide max	Rot 1712 (1981.8)
t_{Pm}	secondary max		wide max		Rot 1722 (1982.5)
t_{Dm}	beginning of rapid decrease	beginning of sharp growth of the $ZO/i(B_r)$ ratio	local max		Rot 1748–1757 (1984.4–84.7)

3. Reference Points of the Solar Cycle and Global Magnetic Field Indices

The usefulness of the indices considered above is proved best of all by the fact that they are good at characterizing the solar cycle reference points. In Vitinsky, Kuklin, and Obridko (1986), definitions of the basic phases and reference points are given mostly in terms of the local magnetic field. Now we can describe these basic phases and reference points using not only the indices of the local field, but also the global field indices.

The reference point t_{mA} appears 1 or 2 years after the calendar date of the cycle minimum according to the Wolf numbers and is characterized by disappearance of sunspot groups belonging to the preceding cycle, rapidly increased sunspot activity and

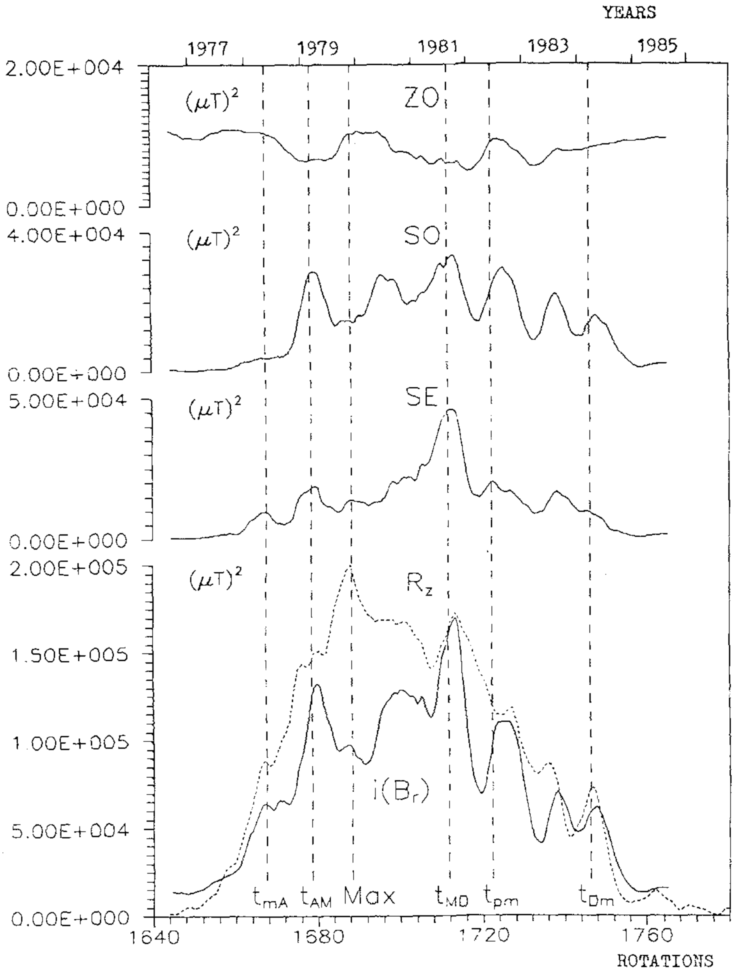

Fig. 1. Cyclic variation of the $i(B_r)|_{R_\odot}$ and the various partial field indices (see text) on the photosphere surface in units of $(\mu T)^2$. The dashed curve in the lower panel shows the variation of the Wolf numbers. The solar cycle reference points have been indicated by vertical dashed lines. The Carrington rotation numbers and the relevant years are indicated along the abscissae.

a sharply decelerated drift of the sunspot formation zone towards the equator. It is also characterized by a sharp increase of the global magnetic field energy and by the decrease of the *ZO* symmetry field which was dominant before.

The reference point t_{AM} occurs 1 year before the cycle maximum according to the Wolf numbers and coincides with the start of the process of polar magnetic field reversal. At this time, a local maximum occurs in the number of active regions and the total number of flares, and we see strong fluctuations of indices. This point was called the pre-maximum, although in some cycles a separate maximum of some indices may be revealed at this time.

At the same time, the local maximum of total energy of the global field is reached. The *ZO* field which was dominant at the beginning of the cycle declines to a minimum.

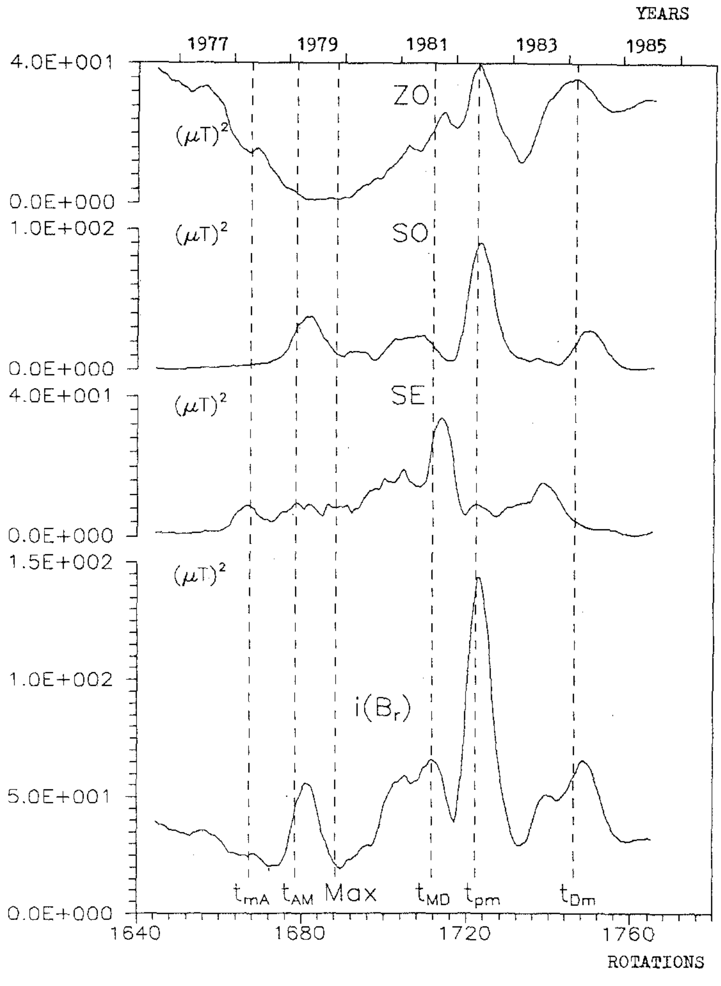

Fig. 2. Cyclic variation of the $i(B_r)|_{R_s}$ partial field indices on the source surface in units of $(\mu T)^2$. The solar cycle reference points have been indicated by vertical dashed lines. The Carrington rotation numbers and the relevant years are indicated along the abscissae.

Simultaneously, a sharp increase of the *SO* field begins, leading to a pronounced maximum of the *SO* field and to a less pronounced maximum of the *SE* field after several solar rotations. In cycle 21, this situation was reached in solar rotation 1678 as a transition from a 4-sector structure of the weak interplanetary magnetic field to a 2-sector structure of the strong one. We should note that the *SE* field increases before t_{AM}, but that the *SO*-type field increased abruptly at t_{AM}. This probably provides a formal mathematical definition of the start of the sign reversal.

The maximum phase (*Max*) is situated between the reference points t_{AM} and t_{MD}, i.e., it covers the whole interval of the sign reversal. This includes the maximum of the sunspot indices, the maximum width (in latitude) of the sunspot formation zone and the symmetric shape of the solar corona according to eclipse observations.

The global magnetic field index decreases and the field minimum closely coincides with the date of maximum of the Wolf numbers. Around this time, fields of all types of symmetry have decreased. However, the maximum of the Wolf numbers is determined basically by the behaviour of local magnetic fields and thus the intensity drop at the source surface is most noticeable. This field shows that it is after t_{AM} that a gradual revival of the *ZO* field occurs at the source surface, appearing to be the generation of a new global magnetic field. In this sense, the new global field originates not at the moment t_{MD} as is often accepted, but at the moment t_{AM}. However, it should be noted that at the photospheric surface, this revival is not noticeable and the ratio of the *ZO* symmetry field index to the total index does not exceed 5–10% for a long time, which lends support to the deep localization of the new global magnetic field.

From the viewpoint of the global magnetic field, an epoch t_{MD} is most probably a time interval of 10 rotations with some fine structure inside. The photospheric magnetic field reached its maximum in cycle 21 during rotation 1712, about 1.5 years after the maximum of the Wolf numbers. It should be noted that around t_{MD} the index of the *SE*-type field is larger that that of the *SO*-type field, while around t_{AM} the latter dominates (see Table I).

After a short but comparatively deep decrease (rotation 1718), the magnetic field energy increases again, reaching a new maximum in rotation 1722 (t_{pm} – 'pre-minimum'). Here the magnetic field of type *SO* exceeds the *SE*-type component to an obvious extent, especially at the source surface where the energy of the *SO* field exceeds all other types of field energy. Moreover, in rotation 1722 an absolute maximum of the *SO* field on the source surface occurs. From rotation 1712 to rotation 1722, we observe a transition in the interplanetary magnetic field from a four-sector structure to a sharply pronounced two-sector structure. After rotation 1722 the total energy decreases gradually, passing through local maxima and minima. In rotations 1748–1751, the energetics of the solar magnetic fields jumps into a state typical of the cycle minimum. This interval corresponds to the reference point t_{Dm} – the beginning of the minimum phase.

The reference point t_{Dm} lies 1 or 2 years before the date of the cycle minimum according to the Wolf sunspot numbers and it is characterized by the appearance of the first high-latitude sunspot groups belonging to a new cycle and of ephemeral active regions of the new cycle, accompanied by a sharp decrease in the number of the old cycle active regions.

After this time, the total index decreases and the ratio of the *ZO* field index to the total index at the photospheric level increases abruptly. During most of the cycle after rotation 1670, this ratio was systematically less than or of the order of 10%. After rotation 1746, the sharp increase of the *ZO* field ratio begins at the source surface and after 5–6 rotations the field of this type becomes dominant at all levels, as it was at the minimum phase. Simultaneously, the last burst of the *SO* field takes place on both boundary surfaces. It is at this moment that a very complicated active region arose which gave a strong X13/3B flare on 24–25 April, 1984. Bumba and Gesztelyi (1988) have shown that the origin of this active region must be regarded as a part of a global process of large-scale magnetic field reorganization.

Thus, these global field indices allow us to ascertain the location of the reference points of the cycle. These points were introduced earlier by Vitinsky, Kuklin, and Obridko (1986) based on general characteristics of the solar cycle but they were believed difficult to see on the curve of the Wolf numbers. Moreover, after the points are identified with the help of global magnetic field indices, one can note that the curve of the Wolf numbers smoothed over 6 solar rotations displays some features that coincide with the reference points of the cycle.

4. Index of the Mean Magnetic Field

In the preceding section we computed the indices by averaging the squared field over a sphere of radius R. However, an alternative procedure might be to average the field over the solar hemisphere on a given day and then to find the mean of the squared average field for one rotation. In fact, this means that we use as directly observed value the magnetic field of the Sun as a star measured at Stanford, which is regularly published in *Solar Geophysical Data*. This new index will be henceforth called the index of the mean magnetic field (Shelting, Obridko, and Yermakov, 1989). It is defined as the square magnetic field of the Sun as a star averaged over one rotation:

$$i(\overline{B}_{\|}) = \langle \overline{B}_{\|}^2 \rangle = \frac{\sum_{k=1}^{27} (\overline{B}_{\|})_k^2}{27} . \tag{7}$$

Figure 3 illustrates the cyclic curve of this index together with the index of the source surface field $i(B_r)|_{R_s}$. Both curves are seen to be very much alike.

It should be once more emphasized that $i(B_r)|_{R_s}$ is derived from the radial component

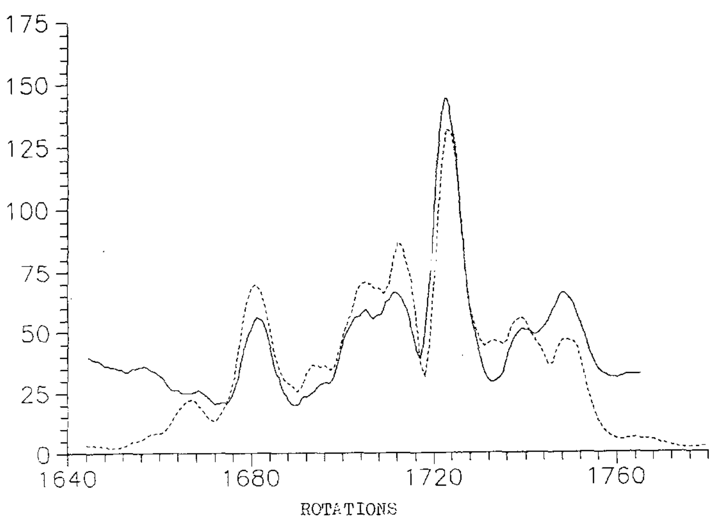

Fig. 3. Cyclic variation of the $i(\overline{B}_{\|})$ index plotted from the Stanford routine data on the mean longitudinal magnetic field of the Sun as a star $i(\overline{B}_{\|})$ (dashed) and $i(B_r)|_{R_s}$ (solid line).

of the source surface field ($R = 2.5\,R_\odot$) under the assumption of a potential field, whereas to calculate $i(\overline{B}_\parallel)$ the observed longitudinal component of the Sun as a star at $R = R_\odot$ is used. Mathematical averaging in the former case is not equivalent to the instrumental averaging during the observation of the Sun as a star in the latter, and, therefore, the averaging regions are different. So it would be no wonder if the curves had nothing in common, which makes their high degree of coincidence the more interesting. Note that in Figure 3 the values of $i(\overline{B}_\parallel)$ are reduced by factor of 50.

5. Comparison with Other Heliogeophysical Indices

As we have shown above, the heliophysical indices are determined both by global and by local fields in the Sun, the contribution of which is different. In each particular case, the contribution of the two types of the field can be estimated by analyzing the cyclic

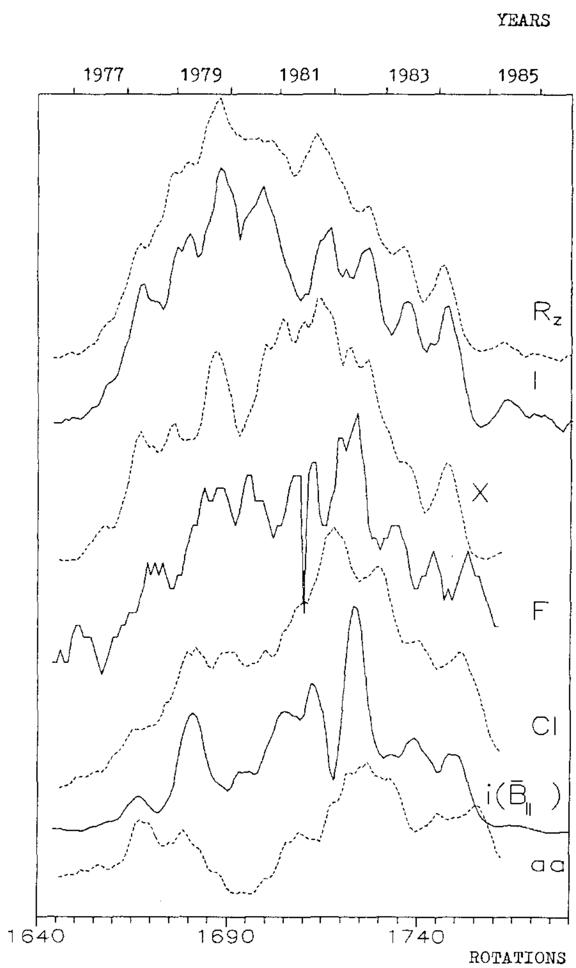

Fig. 4. Cyclic variation of some heliogeophysical indices: the Wolf number R_z, the Ca II K plage index I, the 1–8 Å solar X-ray flux X, the Ca II K full disk flux F, the coronal index CI, the mean solar magnetic field index $i(B_\parallel)$, and the geomagnetic *aa* index.

variation. Figure 4 illustrates the time variation of some helio- and geophysical indices for solar cycle 21. Plotted from top to bottom are: the Wolf number R_z, the Ca II K plage index I, the 1–8 Å solar X-ray flux X (Wagner, 1988), the Ca II K full disk flux F (Foukal and Lean, 1988), the index of green coronal line intensity CI (Rybansky, Kušin, and Dzifčakova, 1988), the mean solar magnetic field index $i(\overline{B}_{\parallel})$, and the geomagnetic *aa* index. All indices, except for the full disk index F, have been smoothed by using the running average over 6 Carrington rotations. The flux F has been smoothed over 3 rotations only.

We have arranged the indices from the top to the bottom according to the growing contribution of global fields as we understand it. The Wolf number R_z is conventionally identified with the local magnetic fields, though the global fields also make some contribution to the cyclic variation (see the secondary R_z maximum in 1982). The I and X indices are also directly associated with the active regions, i.e., with the local fields. The other indices are obtained from the full disk emission and magnetic field observations. The global events in the solar atmosphere that are characterized by low intensity, but cover a very large area compared to local active regions must contribute significantly to the F, CI, $i(\overline{B}_{\parallel})$, and *aa*-indices. This gradual transition from 'local' to 'global' is readily seen in Figure 4. The largest maximum in the cyclic curves moves from the position characteristic of the maximum of the local fields (end of 1979) to that characteristic of the maximum of the global fields (1982). The most global index is naturally the geomagnetic *aa*-index.

Our conception is corroborated sufficiently well by the cross correlation between different indices (see Table II).

TABLE II

Correlation coefficients

Indices	R_z	Ca II plage I	Ca II flux F	X-ray	*aa*-ind.	Corona CI	$i(B_r)\vert_{R_\odot}$	$i(\overline{B}_{\parallel})$	$i(B_r)\vert_{R_s}$
Sunspot number, R_z	1.00	0.98	0.83	0.94	0.15	0.74	0.95	0.74	0.15
Photospheric magnetic index, $i(B_r)\vert_{R_\odot}$	0.95	0.90	0.84	0.96	0.01	0.81	1.00	0.77	0.33
Mean solar magnetic index, $i(\overline{B}_{\parallel})$	0.74	0.68	0.60	0.88	0.65	0.97	0.77	1.00	0.81
Source surface magnetic index, $i(B_r)\vert_{R_s}$	0.15	0.15	0.43	0.42	0.73	0.7	0.33	0.81	1.00

6. Conclusion

(1) A detailed study of the behaviour of global fields in the solar atmosphere made it possible to define the phases of the solar cycle more exactly. The reference points that were formerly determined from the behaviour of the local features prove to depend on the global phenomena as well. Therefore, the global field data can be used to ascertain the location of the reference points.

(2) The mean field index $i(\overline{B}_{\parallel})$ is closely associated with global fields. It is easy to calculate and can be used to ascertain the location of the reference points.

(3) By the type of their cyclic variation, all helio- and geophysical indices fall into one of the two groups – indices similar to R_z and those similar to $i(\overline{B}_{\parallel})$. This helps us to understand the contribution made to them by local and global fields.

(4) A large cross-correlation coefficient between the $i(\overline{B}_{\parallel})$ and the *aa* indices may be helpful in studying solar-geophysical coupling.

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