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CYCLOTRON ECHO PHENOMENA

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Introduction

First I'd like to remark that not all the effects which I plan to discuss are really echo phenomena, so a slightly more appropriate title might be "Pulse Stimulated Cyclotron Radiation in Plasmas". However, all of the work was in fact motivated by the very interesting and important discovery of R. M. Hill and D. E. Kaplan at the Lockheed Laboratories earlier this year. Their work was published in the June 28 Physical Review Letters[1], and a number of physicists immediately set out to try to understand the effect; the observation of echo radiation when a plasma is subjected to multiple pulses of radiation at the cyclotron frequency. At the November 1965 meeting of the Plasma Physics Division in San Francisco, this had already resulted in four papers: by Hill and Kaplan[2], by J. Hirschfield[3], by W. W. Kegel[4] and by myself[5]. I'll try to summarize some of the important ideas which have been developed to date.

It was the fact that one could not directly carry over the nice ideas of Hahn[6] and Purcell[7] about spin echoes to cyclotron echoes that attracted people to work on this problem. Basically, although precessing spin system and gyrating charge particle systems have formal similarities, there are some essential differences which, I hope, will become more evident as this talk proceeds.

First, let me summarize in a very gross fasion, the experimental results (Refer to Figure 1). A short (10-20 nanosecond) pulse of radiation at the cyclotron frequency causes the plasma to develop a macroscopic current (or polarization) which decays after the removal of the pulse. A second pulse following the first by an interval τ, causes a similar plasma response. This is followed after still another interval τ by a delayed response or _echo_ and, in many cases, a series of echoes with _decreasing amplitude_. I might remind you that in the spin case normally only _one_ echo is observed. If, after a still longer time T --which is long compared with the decay time of the echo response--a third pulse is applied, one obtains, in addition to the immediate plasma response, again a response delayed by time τ and perhaps also a series of responses or "echoes". As T is increased the three-pulse "echoes" become weaker.

It is important to recognize that a _linear_ system (one whose response is strictly proportional to the driving force) cannot exhibit such echoes. In their Physical Review letter[1] Hill and Kaplan report two-pulse echo decay times of 50-200 nanoseconds and three-pulse decay times of 10-20 microseconds, and at San Francisco[2] it was reported that three-pulse response had been observed with T greater than 1 millisecond. The experiments were performed in _afterglow_ rare-gas plasmas, Ne, Ar and in nitrogen and the magnetic field had about a half percent inhomogeneity over the plasma volume. Electron densities were such as to make the plasma frequency well below the cyclotron frequency and presumably one could, as a first approximation, ignore collective effects.

Figure 2 shows schematically the manner in which the effect can be observed in free space using microwave horns, in a waveguide, or in a cavity resonator. Hill and Kaplan have employed the first two configurations and have established that both the electric field and the propagation vector of the incident wave should be perpendicular to the static magnetic field. With the cavity method one can observe the cross polarized response of the plasma and suppress the observation of the driving field in a manner analogous to that employed by Bloch in nuclear magnetic resonance experiments.

Having reviewed certain aspects of the observations, I would turn now to the attempts which have been made to understand this phenomenon, and to some predictions of related effects which arose from these attempts. The remainder of the talk will be in the form of a short excursion into rotating velocity space.

I shall first discuss the linearized equation of motion of single particle in a static magnetic field and an oscillating electric field. Then I'll discuss the behavior of an ensemble of non-interacting particles, i.e. particles which interact only with the applied fields. Although such a linearized theory does not give rise to echoes, it does give a very clear picture of how they can come about when nonlinearities of various types are included.

Figure 3 shows the linear equation of motion which is to be solved. Effects of the spatial variations of $\underline{E}$ and $\underline{B}$ and of the r.f. magnetic field are ignored. As in the case of magnetic resonance problems it is convenient to transform to a rotating system (velocity space in this case) in which one of the rotating components of $\underline{E}$ is stationary. In this system the effect of the other rotating component may be neglected, and the cyclotron frequency appears reduced by the frequency ω of the r.f. electric field. Cyclotron resonance corresponds to a difference cyclotron frequency ω_c' of zero and near resonance to small ω_c'. Since we shall be concerned only with the components of velocity perpendicular to the static magnetic field (which lies along the axis of rotation) it is also convenient to introduce a complex velocity representation in which the real and imaginary parts of V are V_x and V_y respectively.

The solution of this equation consists of free rotation of the difference cyclotron frequency ω_c' plus the response to the electric field which is constant during the pulse. If the pulse is short $\omega_c' t \ll 1$ the effect of the electric field is simply to translate the velocity "vector" by an amount proportional to intensity and duration of the pulse. Between pulses the velocity vector rotates slowly about the axis with the frequency ω_c'.

We will now use this result to discuss the behavior of electrons which have a distribution of cyclotron frequencies by virtue of their being located in regions of slightly different magnetic field.

Figure 4 shows the behavior in our rotating velocity space. If we ignore the thermal velocities of the electrons, which seems also to be a fair approximation since generally $V_{pulse} \gg V_{th}$ — and we shall return to this later — , the velocity vector of each particle is initially at the origin and is translated by an amount V_1' by the first applied pulse. Following the first pulse each velocity vector rotates about the origin at a rate determined by the difference cyclotron frequency ω_c' and after a time they become distributed around a circle of radius V_1'. Bear in mind that vectors at B, for example, consist of particles which drifted directly to B or encircled the origin once, twice or more, either clockwise or counterclockwise, to reach B. Since the plasma current is essentially the sum of velocity vectors, this dispersal represents the decay of the current induced by the first pulse due to the inhomogeneous magnetic field.

The second applied r.f. (equal in strength for simplicity) translates each of the velocity vectors by the same amount, giving diagram (c). Following the second pulse, each velocity vector rotates with its appropriate rate and the circle in diagram (c) breaks into many "circles", since particles at a given point on the circle have arrived there by rotating at different rates. However, after a time exactly τ each vector will have turned through the same angle as it did during the initial interval τ between the two pulses and all vectors which were together at B in diagram (c) are again together at B in diagram (d). They have, in fact, rotated by 45° + some positive or negative multiple of 360°.

Diagram (e) corresponds to the time of the first echo and diagram (f) corresponds to the time of the second echo, diagram (d) to intermediate times. We see that at special times $n\tau$ (measured from the second pulse) velocity vectors which are otherwise distributed over the entire plane, regroup. This regrouping in phase at special times is not sufficient to produce a macroscopic current in the plasma, since the sums of the velocity vectors in diagrams (e) and (f) are still zero, due to an exact cancellation of positive and negative V_x''s (there are more with negative V_x', but those with positive V_x' have larger V_x'.) However, this exact cancellation at the special times $n\tau$ will be spoiled by any of a number of nonlinearities. The important nonlinearities seem to be

a) Spatial inhomogeneities of the radio frequency driving field (both E and B). When the orbit size is not negligible in comparison with the wavelength of the plane wave driving fields, the effectiveness of the driving field decreases with increasing orbit size. This nonlinear effect causes particles at A in diagram (b) to be translated less than particles at E, for example, the diagram (d) is no longer a circle, but elliptical. The curve in diagram (e) is therefore slightly distorted so that particles near A contribute slightly less current and the exact cancellation at time τ is spoiled and an "echo" produced. The amplitude is roughly v_{1}^{2}/c^{2} times smaller than the instantaneous plasma response. The amplitude of subsequent echoes is proportional to higher powers of v_{1}^{2}/c^{2}, and are very weak.

Another nonlinear mechanism which can spoil the exact cancellation and therefore lead to echoes is an energy-dependent cyclotron frequency, such as is caused by the relativistic mass effect. Particles along HAB have a higher rotational energy than those along DEF and would therefore have a slightly reduced rotation rate. They therefore appear rotated clockwise in (e) and (f) and therefore spoil the symmetry with respect to the V_x' axis. This effect, although it is due to the relativistic shift in cyclotron frequency, is, in general, much more important than the first, since it is actually the relativistic change in phase which matters and this is approximately v^2/c^2 times the rotation angle in the laboratory system (typically 10^4).

Furthermore, Hirschfield pointed out in his San Francisco talk that there are other, possibly more important, reasons for the cyclotron frequency to be energy dependent. For example, in a non-uniform static magnetic field the cyclotron period depends on the orbit size and hence upon the rotational energy. As a very simple example, a rotationally symmetric magnetic field whose strength decreases with radius, leads to a cyclotron frequency which decreases with the particle energy. Spatially non-uniform static electric fields, which may also be present, also can give rise to an energy-dependent cyclotron frequency.

Still another possible mechanism suggested originally by James [illegible] Gordon of the Bell Telephone Laboratory and discussed quantitatively by [illegible] and Kaplan at San [illegible], in the [illegible]-dependent collision process. Although we have [illegible] the effect of an elastic collision is to alter the phase of a particle in a random way and effectively remove it from further consideration—at least in the two-pulse case. If, however, the removal process is energy dependent, more particles will be removed, for example, from HAB than from DEF (when the collision frequency increases with energy). Again the cancellation at times $t = n\tau$ does not occur and echoes result.

Now I would like to digress a moment to discuss the spin echo effect from this point of view. Figure 5 shows the equation obeyed by an individual magnetic moment $\underline{M}$ in the absence of relaxation effects. $\underline{\omega}$ is proportional to the applied field $\underline{H}$, which consists of superimposed static and r.f. fields, ω_o and ω_1, respectively. It follows from the first equation that the magnitude of M is a constant and therefore the tip of M vector lies on a sphere. In nuclear induction experiments one observes only the projection of the macroscopic magnetization, which is the sum of the individual moments, in the x-y plane. The last equation describes this perpendicular part of $\underline{M}$, and it is similar in some respects to the previous equation for electron velocity. Note that free precession frequency is independent of M (i.e., "energy"-independent) but that the driving force is nonlinear, since it is proportional to M as well as to ω_1.

The driving force nonlinearity can perhaps be seen more clearly in the diagram, where the circle centered on the origin represents a distribution of moments with different phases. (The vectors describe a cone [illegible] we see the projection of their tips on the x-y plane.) A r.f. pulse which rotates each of the moments about the y axis through angles about 30° and 60° results in the ellipse-like projection. Obviously in this diagram different moments appear to experience different translations in the xy-plane, depending on the value of M_x which they had to begin with. While spin echoes employing pulses which rotate the moments through exactly 90° and 180° have very simple explanations, given by Hahn and Purcell, any combination of two non-infinitesimal pulses produces an echo.

Furthermore, any two-state quantum-mechanical system, spin or otherwise, can be cast into the same mold[3] as Figure 6 indicates. If the wave function can be described as a linear superposition of ψ_a and ψ_b then the amplitudes a and b, or rather certain quadratic combinations of them, obey a simple three-component vector equation which has the same form as the spin equation. Furthermore in the case of electric or magnetic dipole transitions r_1, r_2 and r_3 are the expectation values of the dipole moment, ω_3 is the energy difference, ω_1 and ω_2 are combinations of the matrix elements of the driving perturbation between states a and b.

We may interpret this to mean that any ensemble of two-state quantum-mechanical systems which has a spread in transition frequencies with sufficient lifetimes could exhibit echoes.

Indeed, this is the proposed explanation of the photon-echo observed in ruby by the Columbia group: Kurnit, Abella and Hartmann[7]. The levels involved are electronic states in chromium which are split by the crystal electric field.

I would now like to outline the mathematical treatment that goes with these diagrams(refer to Fig.7). V' is the complex velocity of a particular electron whose difference cyclotron frequency is ω_c' at a time t measured from the second pulse. The first pulse produces velocity V_1 and between the two pulses the velocity vector is rotated through an angle ϕ_1 --remember the coordinate system is rotating with the rotating electric field and ω_c' is the cyclotron rotation frequency relative in the rotating system. The second pulse provides an additional velocity V_2 and the total velocity vector rotates through an angle ϕ in the next interval.

To obtain the macroscopic plasma current $\vec{J}$ we need to multiply both by the number of particles which have a difference cyclotron frequency ω_c' and by the probability that these particles survive until time t without making a phase-destroying collision, and then integrate over ω_c'.

For the three types of nonlinearities already discussed, either V_2, ϕ or $P(t)$ are energy dependent, according to whether the driving force, the cyclotron frequency, or the collision frequency is energy dependent.

Figure 8 outlines a simplified treatment of what are probably the two most important nonlinearities in plasmas. I have assumed, for simplicity, that both the cyclotron and collision frequencies have a weak dependence on V^2, i.e., upon energy. This is a good approximation for the cyclotron frequency and not so good for the collision frequency, but it serves to illustrate the point. Hill and Kaplan[2] have given results for other more realistic dependences of collision frequency on velocity. Between the two pulses all particles have the same energy, V_1^2, and [illegible] effects are unimportant. After the second pulse, however, V^2 depends on V_1, V_2 and ϕ_1, the phase the particle had at the time of the second pulse. Since both ϕ and ν appear in the exponent, [illegible] ϕ_1 will appear in the exponent and we [illegible] the [illegible] Bessel function identity. Note particularly the [illegible] way in which the two effects appear, so that we can [illegible] their relative importance by comparing $\partial\omega_c/\partial v^2$ directly with $\partial\nu/\partial v^2$. Now ω_c is very large compared with ν, but the relative dependence on v^2 is much weaker. It is therefore not clear, a priori, which effect is more important.

After a few minor steps one obtains an expression for the plasma current which has the form of a series of pulses. Centered on The pulse shape is given by the Fourier transform of the cyclotron frequency distribution function. A very inhomogeneous field leads to narrow echo pulses. The amplitude factors contain the Bessel functions and are slowly varying functions of time.

Figure 9 shows the manner in which the amplitude of the first echo pulse depends upon time in these two theories. Note that they all have the same general appearance, rising at first as either nonlinear effect allows the echo to develop, and finally falling as collisions take their toll. The dashed and solid curves are for energy-dependent collision and cyclotron frequencies, respectively. As the magnitude of energy dependence becomes larger, so does the maximum achievable echo pulse amplitude. A pure exponential decay line is shown for comparison and one sees that decay for large times is only approximately exponential. If the two pulses have unequal strengths, the decay curve for energy dependent cyclotron frequency develops periodic local minima and this may be a way to differentiate between the two effects.

Another important effect may also contribute to decay of the echo pulses: As a result of their relatively small thermal velocities, a [illegible] can move along a magnetic field line and change its cyclotron frequency slightly. The resulting change in phase, since it is different for the particles with different speeds, will lead to a decay

$$\sim \exp\left\{-\left(\frac{1}{4}\,\frac{\partial\omega_c}{\partial z}\,v_{th}\,t^2\right)^2\right\}$$

which becomes a very strong effect after a certain time.

Now I'd like to turn to the three-pulse case to see why it is possible, despite many many collisions between the second and third pulses, for the system to still remember the time interval between the first two pulses τ. The central idea here is that these are essentially elastic collisions, since the electrons collide with heavy particles. Figure 10 shows the effect of the three-pulse sequence on an ensemble of particles. The first pulse imparts the same velocity to all (first diagram) and they disperse because of the different cyclotron frequencies (second diagram). The second pulse translates the circle (third diagram). During the long time interval between the second and third pulses, each particle experiences many collisions, distributing particles with the same speed over a spherical shell. Prior to the third pulse, particles with different energies are distributed over different spherical shells ABC··· according to their energy, i.e., according to the phase they had at the time of the second pulse. The third pulse translates each spherical shell, and they disperse as before. Thus we have particles distributed in shells, onion-like in character, with particles in the same shell having experienced the same phase change, modulo 2π, between the first and second pulse. During the next interval τ, the particles on a given shell turn through the same angle as they did in the first interval τ and we achieve the situation in the [illegible]. [illegible] spheres has its center on the [illegible] and the [illegible] would again exactly cancel were it not for nonlinear effects. Energy-dependent collisions will selectively remove particles from various regions of shell A and thereby change its center of charge. An energy-dependent cyclotron frequency will shift its center clockwise, etc. In either case no cancellation is spoiled and an echo response results. The last figure shows the situation at the time of the second echo, which also arises from nonlinear effects. One as yet very puzzling feature is the long decay time observed in the three-pulse case.

When the electron density is higher, earlier in the decay, the echo effect appears to occur at the upper hybrid frequency[2], $\omega^2 = \omega_p^2 + \omega_c^2$ and collective effects are probably important. A major extension of these ideas will be required.

So much for the phenomena ordinarily regarded as <u>echoes</u>. In trying to understand Hill and Kaplan's experiment, several false starts were made which actually led to new predictions. First there is the single pulse case of J. Hirschfield[3]. He observed, as Figure 11 illustrates, that in a <u>very homogeneous field</u> a single applied pulse might produce a train of responses. Consider plasma electrons which all have the same speed perpendicular to the magnetic field B with random phases, and no velocity along B. A single, very weak pulse translates each velocity vector an amount small compared with V_0. Some particles have their energy very slightly increased, some very slightly decreased, and some not at all. If the gyro frequency is energy dependent there is a differential rotation, shown by the arrows, which causes the particles to bunch at $\theta = -\pi/2$, disperse and bunch at θ [illegible] etc. The expression for the resulting current is given on the slide and J^2 is plotted. Note especially that nearly the full plasma current $Ne v_3$ is eventually achieved, but one must wait longer, the smaller the stimulating pulse is. After integrating over a Maxwell distribution of perpendicular velocities, the effect survives, although reduced in amplitude and with but a single maximum in current.

Probably the mono-energetic initial state first considered by Hirschfield could be produced by the application of a pulse itself. However, the interval between the responses produced by the second applied pulse would depend on the amplitude of the second applied pulse, not the time interval between the two applied pulses!. Wilhelm Kegel, also trying to understand the echo experiment, found yet another effect. Suppose that the magnetic field is very uniform, the cyclotron frequency is energy dependent, and the electrons have a thermal distribution of velocities. After the first pulse (refer to Figure 12), the particles disperse because of the energy-dependent cyclotron frequency, together with the thermal energy spread. The second pulse produces the thick ring shown in (c). The interesting result is that following the decay of the instantaneous response to the second applied pulse a series of responses results. I haven't been able to see how to explain this clearly with diagrams yet.

Figure 13, from Kegel's work[4] shows that these responses are not separated by the interval τ but by $\tau/2$! In this computation the two pulses have equal strength. For two unequal pulses, the responses are separated by an interval

$$\tau/(\frac{V_2}{V_1}+1)$$

and there is a second, weaker train whose interval is

$$\sqrt{\left|\frac{v_2}{v_1} - 1\right|}$$

Kegel has also shown that when the energy dependent cyclotron frequency is involved, a sufficiently strong inhomogeneity in the magnetic field will destroy his effect. Similarly, he has shown that sufficiently high thermal speeds (temperature) will destroy the ordinary echo in a non-uniform magnetic field.

He has also shown that for three pulses, without collisions, a multiplicity of responses occur at times

$$t = n\tau + m\tau' \quad , \quad \text{all integral } m, n \text{ for which } t > 0$$

(t measured from third pulse)

Finally, I would like to discuss echo phenomena in a more general context. We saw that two level quantum mechanical systems obeyed equations similar to "classical" spin equation. so by analogy, echoes could be obtained in ensembles of such systems. It is characteristic of two-level systems that an applied perturbation which resonates with the natural frequency of the system alternately increases and decreases its energy and the nonlinearity required for the echo is immediately evident.

Such is not the case for an ensemble of classical (or quantum) harmonic oscillators such as gyrating charged particles, for these the energy is increased indefinitely (or until more subtle nonlinear effects enter) by a resonant driving force. This is a basically different type of system.

Now our treatment of echo phenomena [illegible] qualita-tively at least and quantitatively perhaps, to oscillator systems [illegible] general. Figure 14 illustrates my point. I employ coordinates p and Ωq, the momentum and frequency times coordinate, respectively. In the p, Ωq phase plane, orbits are circles and by introducing a rotating coordinate system p' $\Omega q'$ which rotates with frequency of the driving force, the rotating component is constant and the other has a negligible effect. Observable quantities might be $\langle p \rangle$ or $\langle q \rangle$ for example. In short, all our previous ideas can be carried over to a general oscillator system.

Basically we require for echo phenomena in a system:

a) An ensemble of oscillators with a narrow distribution of natural frequencies, which interact with external forces

b) Sufficiently long relaxation times to permit observation

c) One of a variety of nonlinear effects to "spoil cancellation"

1. energy-dependent driving force
2. energy-dependent natural frequency--anharmonic oscillator
3. energy-dependent relaxation phenomena
4. others?

It is interesting to note that these conditions are essentially the same as required for use in masers and lasers.

You can see that Hill and Kaplan's discovery has indeed stimulated a considerable interest, some new and potentially useful results, particularly in measuring collision rates at low energies < 1 electron volt. Where it will lead remains to be seen, but it will provide many new problems to consider and, I think, ultimately a variety of new experimental techniques.

In closing I would like to acknowledge very fruitful discussions with W. H. Kegel, R. M. Hill, D. E. Kaplan and J. L. Hirschfield.

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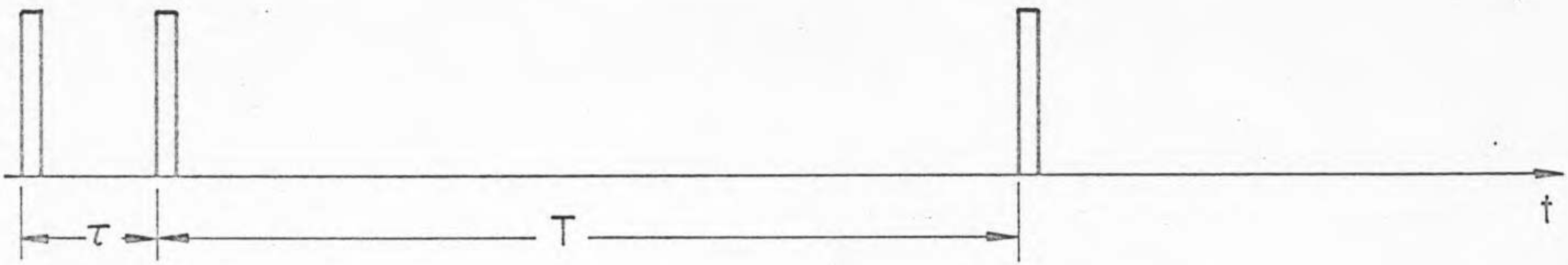
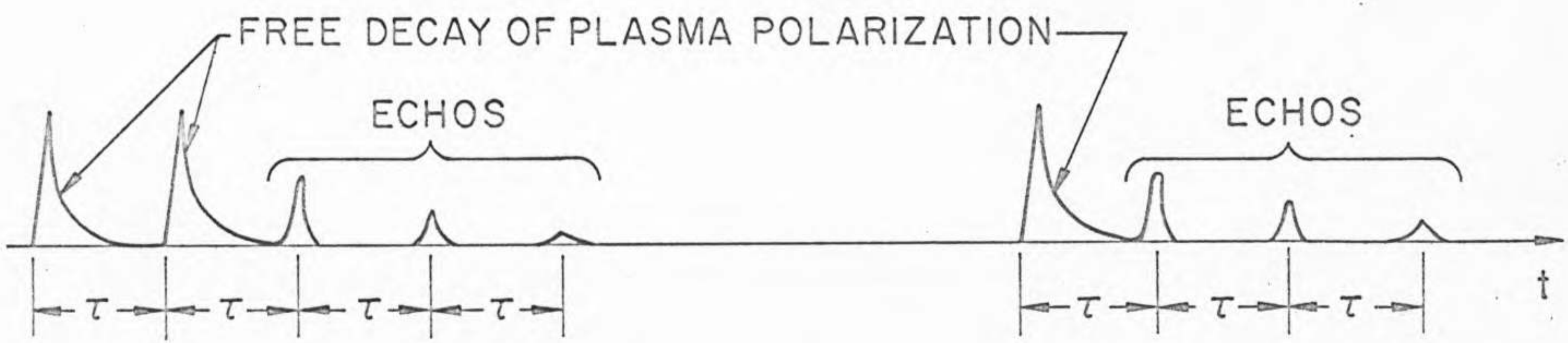
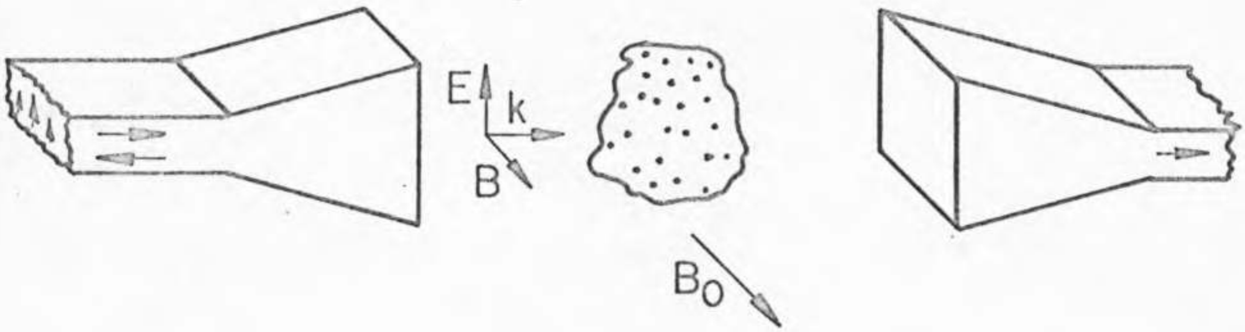


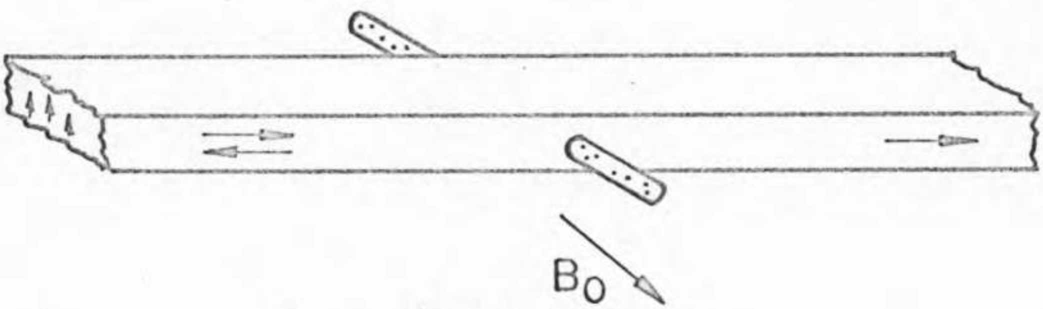
MAGNITUDE OF APPLIED RADIO FREQUENCY $\left(\omega \approx \frac{eB_0}{m}\right)$ FIELD



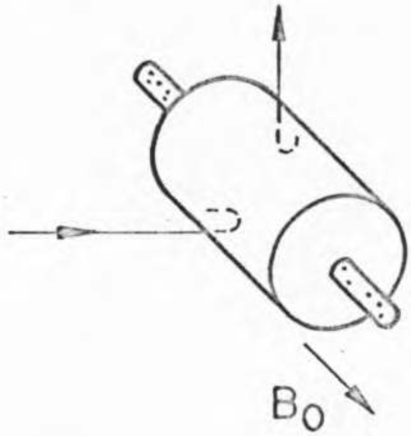
MAGNITUDE OF PLASMA RESPONSE

FIG. 1





TE_{10} WAVEGUIDE



TE_{111} CAVITY RESONATOR

Fig. 2

LINEARIZED EQUATION OF MOTION OF SINGLE ELECTRON:

$$\dot{\underline{V}} - \underline{\omega}_c \times \underline{V} = \frac{-e}{m}\underline{E} \qquad \underline{\omega}_c = \frac{e\underline{B}_0}{m}$$

AFTER TRANSFORMATION TO ROTATING VELOCITY SPACE:

$$\dot{\underline{V}}' - \underline{\omega}_c' \times \underline{V}' = \frac{-e}{m}\underline{E}' \qquad \underline{\omega}_c' = \underline{\omega}_c - \underline{\omega}$$

COMPLEX REPRESENTATION: $\check{V}' = V_x' + i V_y'$ ETC.

$$\dot{\check{V}}' - i\,\omega_c'\,\check{V}' = \frac{-e}{m}\check{E}'$$

SOLUTION:

$$\check{V}'(t) = \check{V}'(0)\,e^{i\omega_c' t} + \int_0^t e^{i\omega_c'(t-s)}\,\frac{-e}{m}\check{E}'(s)\,ds$$

$$\cong \check{V}'(0) + \frac{-e}{m}\check{E}'t \qquad \omega_c' t << 1$$

$\check{E}'$ CONSTANT

FIG. 3

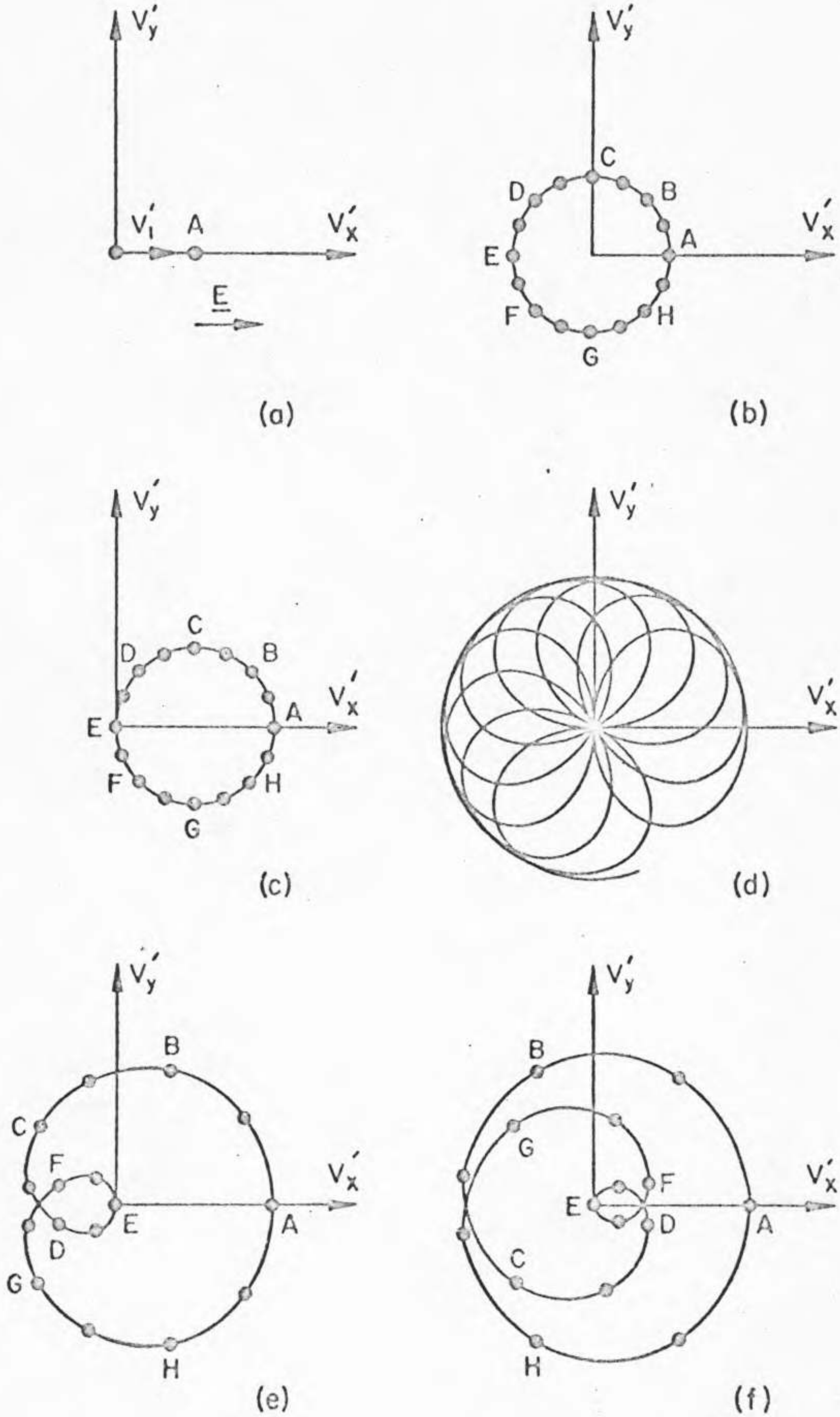


FIG. 4

SPIN ECHO EQUATIONS:

$$\frac{d\underline{M}}{dt} = \underline{M} \times \underline{\omega} \Longrightarrow |\underline{M}| = \text{CONSTANT}$$

$$\underline{\omega} = \gamma\, \underline{H} = \underline{\omega}_0 + \underline{\omega}_1$$

$$\underbrace{\frac{d\underline{M}_\perp}{dt} + \underline{\omega}_0 \times \underline{M}_\perp}_{\text{FREE PRECESSION}} = \underbrace{\underline{M} \times \underline{\omega}_1}_{\text{NON-LINEAR}}$$

FREE PRECESSION NON-LINEAR

ω_0 = FREQUENCY DRIVING FORCE

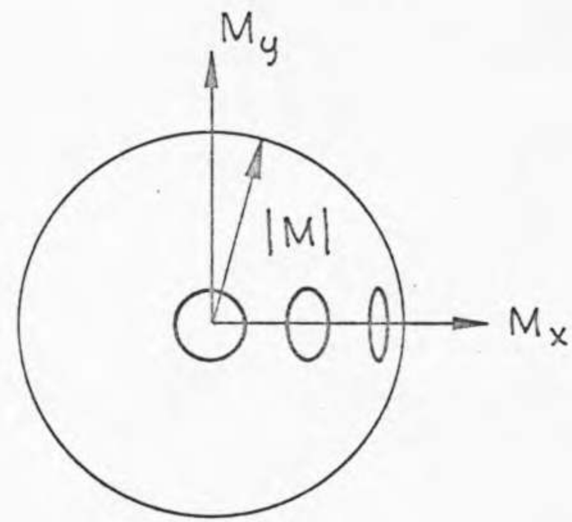


FIG. 5

TWO STATE QUANTUM MECHANICAL SYSTEM

[FEYNMAN, VERNON, HELLWARTH JAP 28 49 (1957)]

$$\psi(t) = a(t)\,\psi_a + b(t)\,\psi_b$$

$$\frac{d\underline{r}}{dt} = \underline{\omega} \times \underline{r}$$

$$r_1 = ab^* + a^*b \qquad \omega_1 = (V_{ab} + V_{ba})/\hbar$$

$$r_2 = i(ab^* - a^*b) \qquad \omega_2 = i(V_{ab} - V_{ba})/\hbar$$

$$r_3 = aa^* - bb^* \qquad \omega_3 = (E_a - E_b)/\hbar$$

Fig. 6

$$\check{V}'(t) = (V_1 e^{i\phi_1} + V_2)\, e^{i\phi}$$

$$\check{J}'(t) = -Ne \int d\omega_c' \, G(\omega_c') \, \check{V}'(t) \, P(t)$$

V_1 = VELOCITY IMPARTED BY 1^{ST} PULSE $\cong \frac{e}{m} E_1 t_1$

V_2 = VELOCITY IMPARTED BY 2^{ND} PULSE $\cong \frac{e}{m} E_2 t_2$

ϕ_1 = ROTATION BETWEEN 1^{ST} AND 2^{ND} PULSES $\cong \omega_c' \tau$

ϕ = ROTATION AFTER 2^{ND} PULSE $\cong \omega_c' t$

$P(t)$ = PROBABILITY OF NO COLLISION BETWEEN 1^{ST} PULSE AND t $\cong e^{-\nu(t+\tau)}$

$G(\omega_c')\, d\omega_c'$ = FRACTION OF ELECTRONS WITH DIFFERENCE GYROFREQUENCIES BETWEEN ω_c' AND $\omega_c' + d\omega_c'$

FIG. 7

ENERGY DEPENDENT CYCLOTRON AND COLLISION FREQUENCIES:

$$J = -Ne \int G(\omega_c') \, d\omega_c' \, [V_1 e^{i\phi_1} \pm V_2] \, e^{i\phi} \, e^{-\nu t}$$

$$\phi = [\omega_c(V^2) - \omega] t \cong [\omega_c' + \frac{\partial \omega_c}{\partial V^2} V^2] \, t$$

$$\nu t \cong [\nu_0 + \frac{\partial \nu}{\partial V^2} V^2] \, t \quad \text{(WEAK DEPENDENCE)}$$

$$V^2 = V_1^2 + V_2^2 + 2 V_1 V_2 \cos \phi_1 \qquad \phi_1 = \omega_c' \tau$$

NOW $e^{-(\alpha + i\beta)\cos\phi_1} = \sum_{-\infty}^{\infty} I_n(\alpha + i\beta) \, e^{-in\phi_1}$

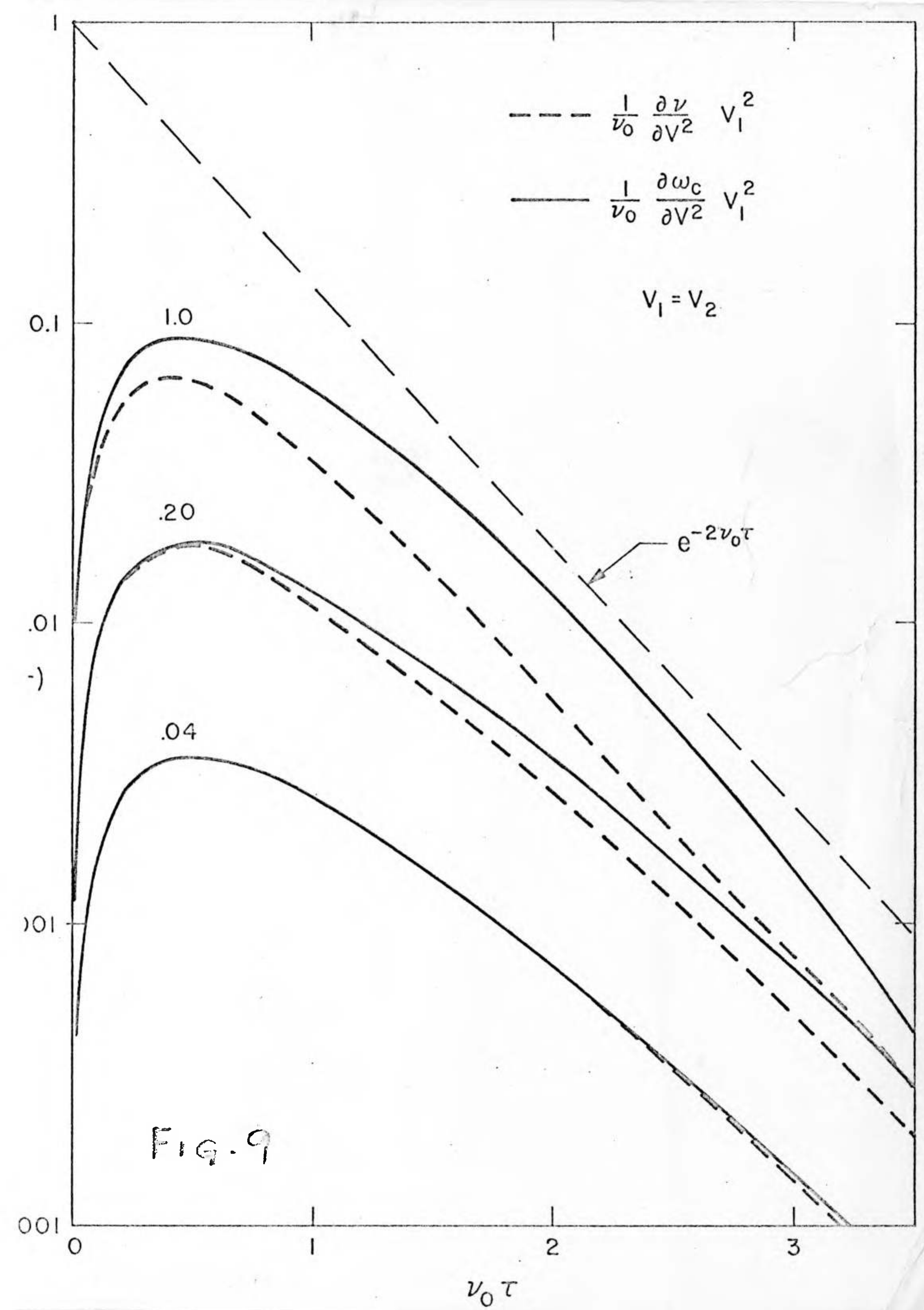
WITH $\alpha = \frac{\partial \nu}{\partial V^2} 2 V_1 V_2 t \qquad \beta = \frac{\partial \omega_c}{\partial V^2} 2 V_1 V_2 t$

$$|J| = Ne \, V_1 \sum_{-\infty}^{\infty} A_n(t) \, g(t - n\tau)$$

$$g(t) = \int_{-\infty}^{\infty} G(\omega_c') \, e^{i\omega_c' t} \, d\omega_c' \quad \text{(PULSE SHAPE)}$$

$A_n(t)$ = PULSE AMPLITUDE FACTOR

FIG 8



1
0.1
.01
-)
)01
001
$\frac{1}{\nu_0}\frac{\partial \nu}{\partial V^2}\ V_1^2$
$\frac{1}{\nu_0}\frac{\partial \omega_c}{\partial V^2}\ V_1^2$
$V_1 = V_2$
1.0
.20
.04
$e^{-2\nu_0\tau}$
0
1
2
3
$\nu_0\tau$

FIG. 9

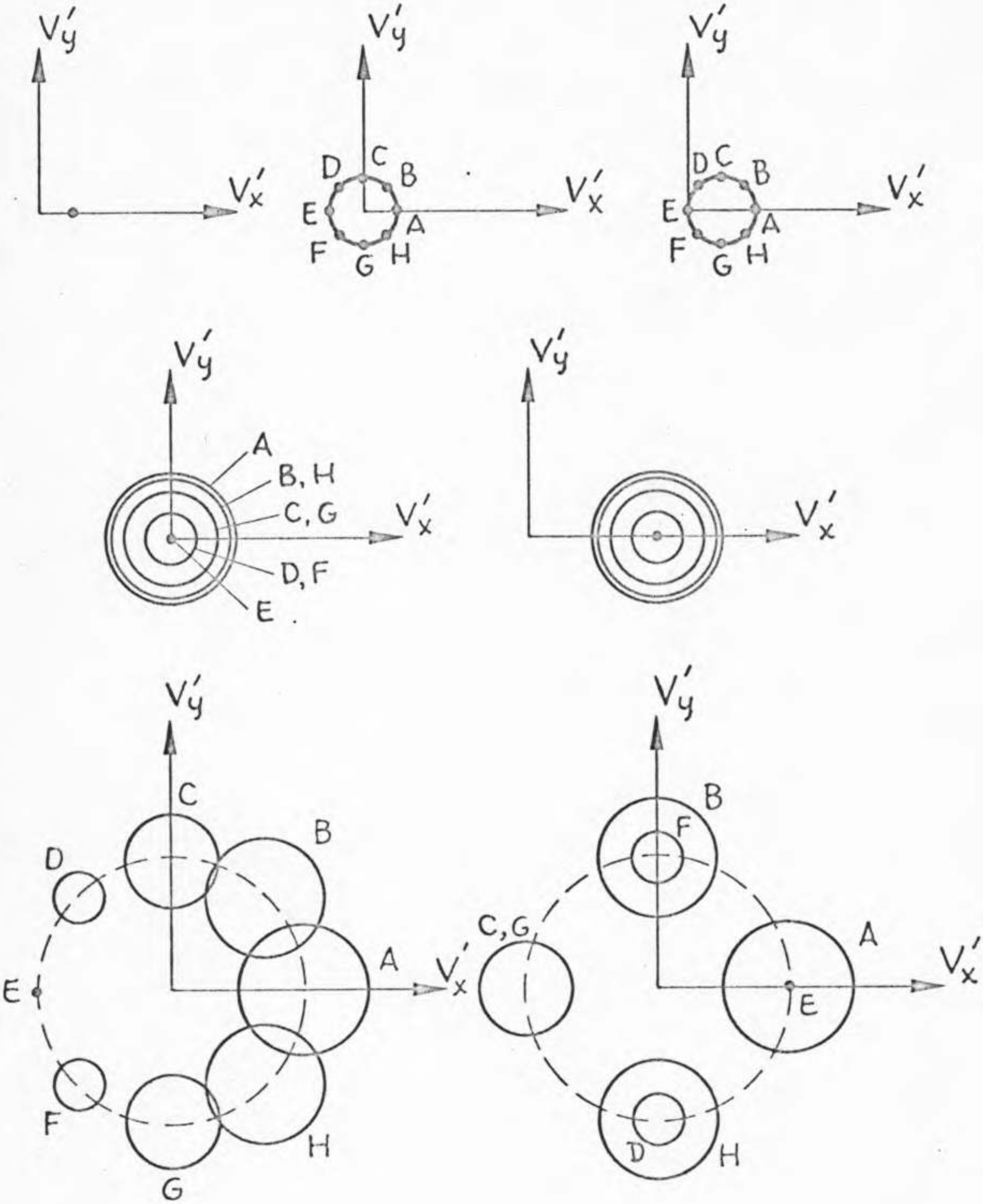
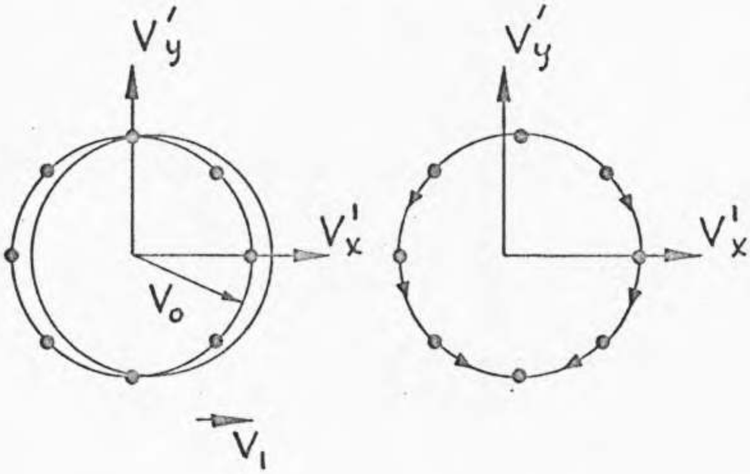

V'y
V'x
V'y
D C B
E A
F G H
V'x
V'y
D C B
E A
F G H
V'x
V'y
A
B, H
C, G
D, F
E
V'x
V'y
V'x
V'y
C
B
D
A
E
F
H
G
V'x
V'y
B
F
C, G
A
V'x
E
D H

Fig. 10

RESPONSE TO A SINGLE PULSE:



$$\phi \cong \phi_0 - \left(\frac{\partial \omega_c}{\partial V^2}\right)\left(V_0^2 + V_1^2 + 2V_1 V_0 \cos \phi_0\right) t$$

$$J_y' \cong - N e V_0 \; J_1 \left(\frac{\partial \omega_c}{\partial V^2} \, 2 V_1 V_0 t\right) \qquad V_1 << V_0$$

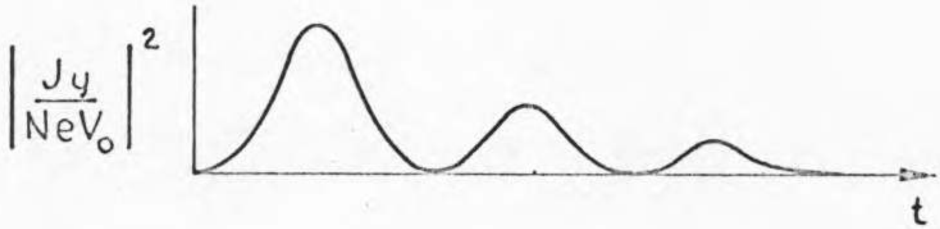


Fig. 11

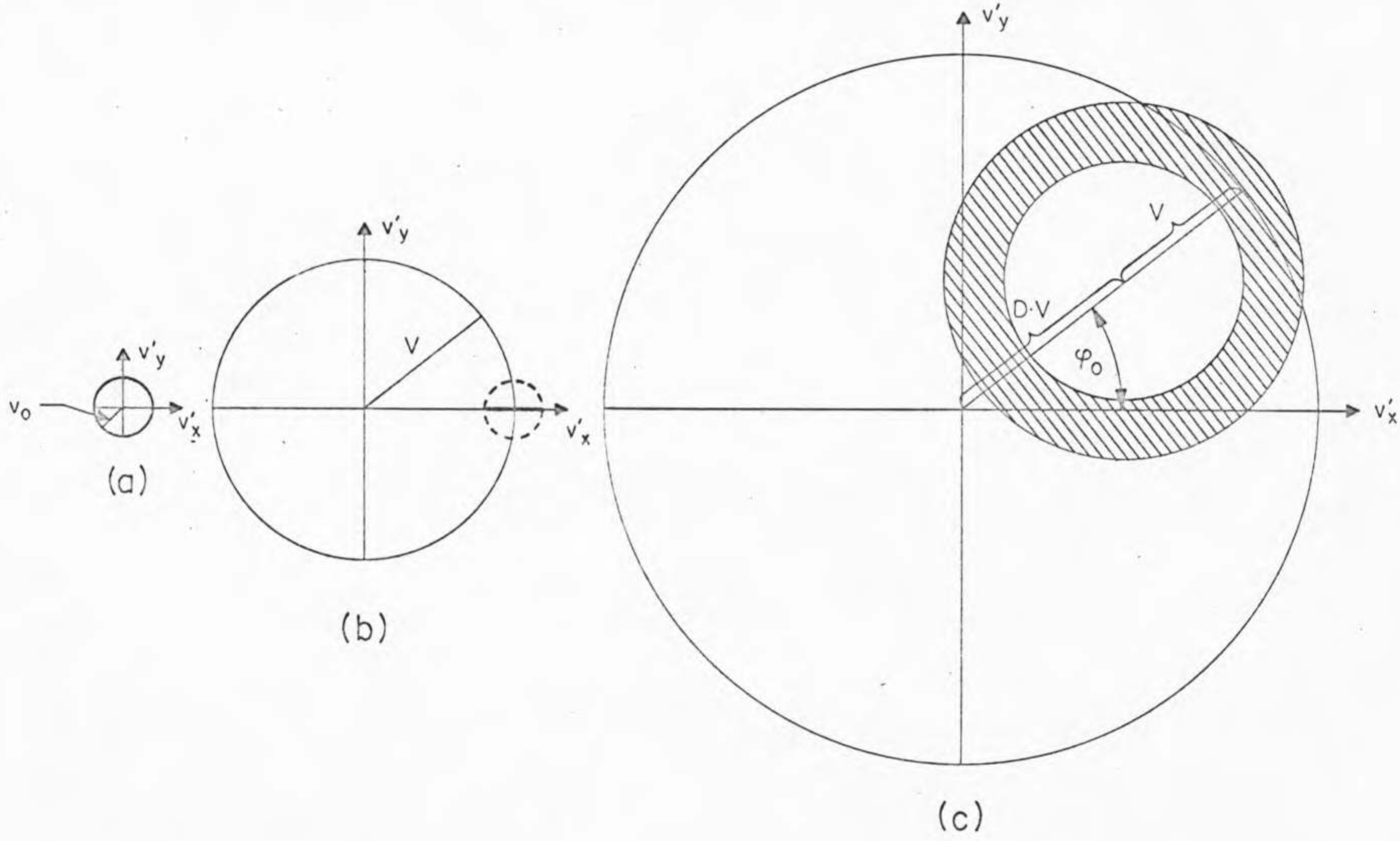
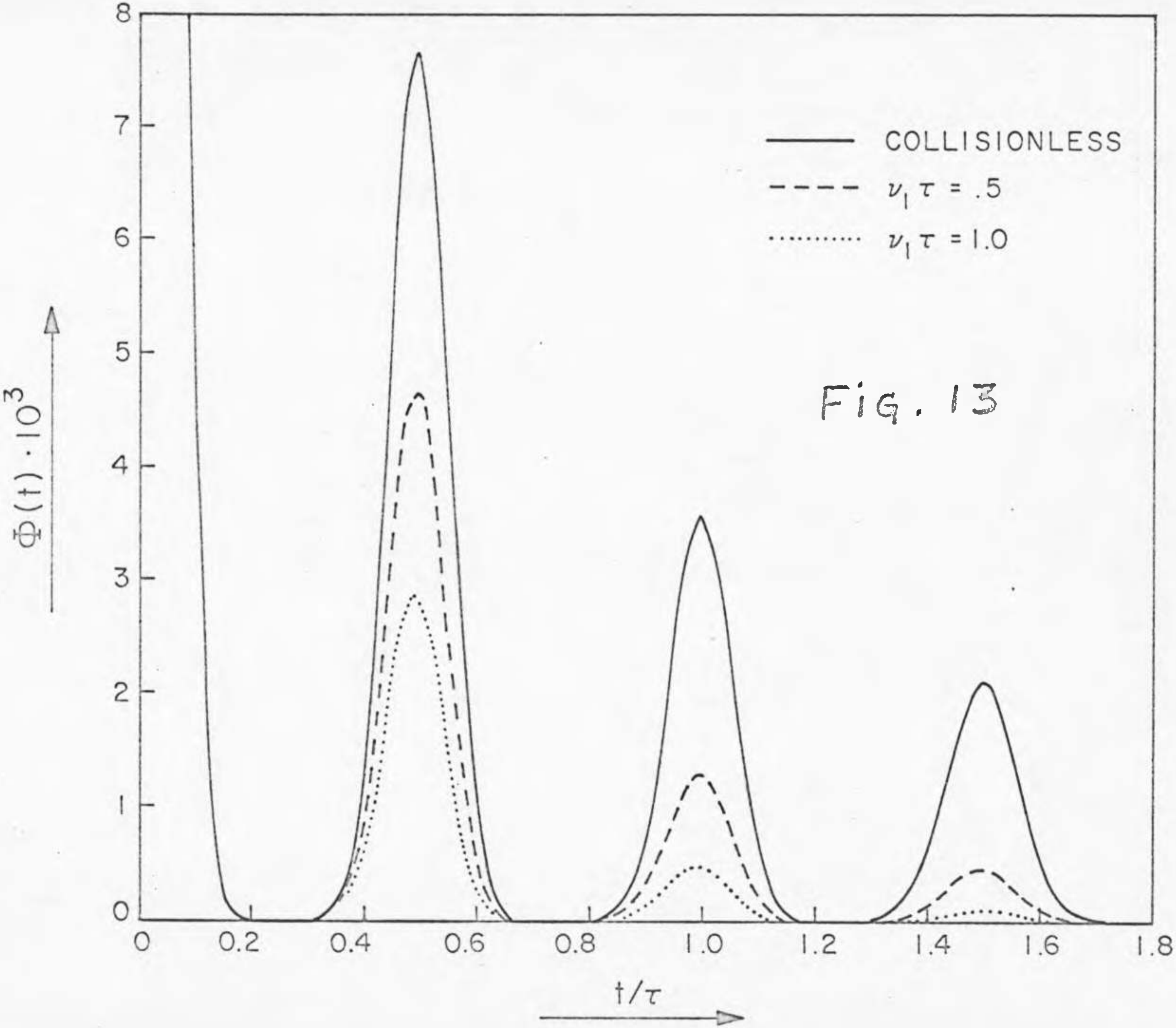
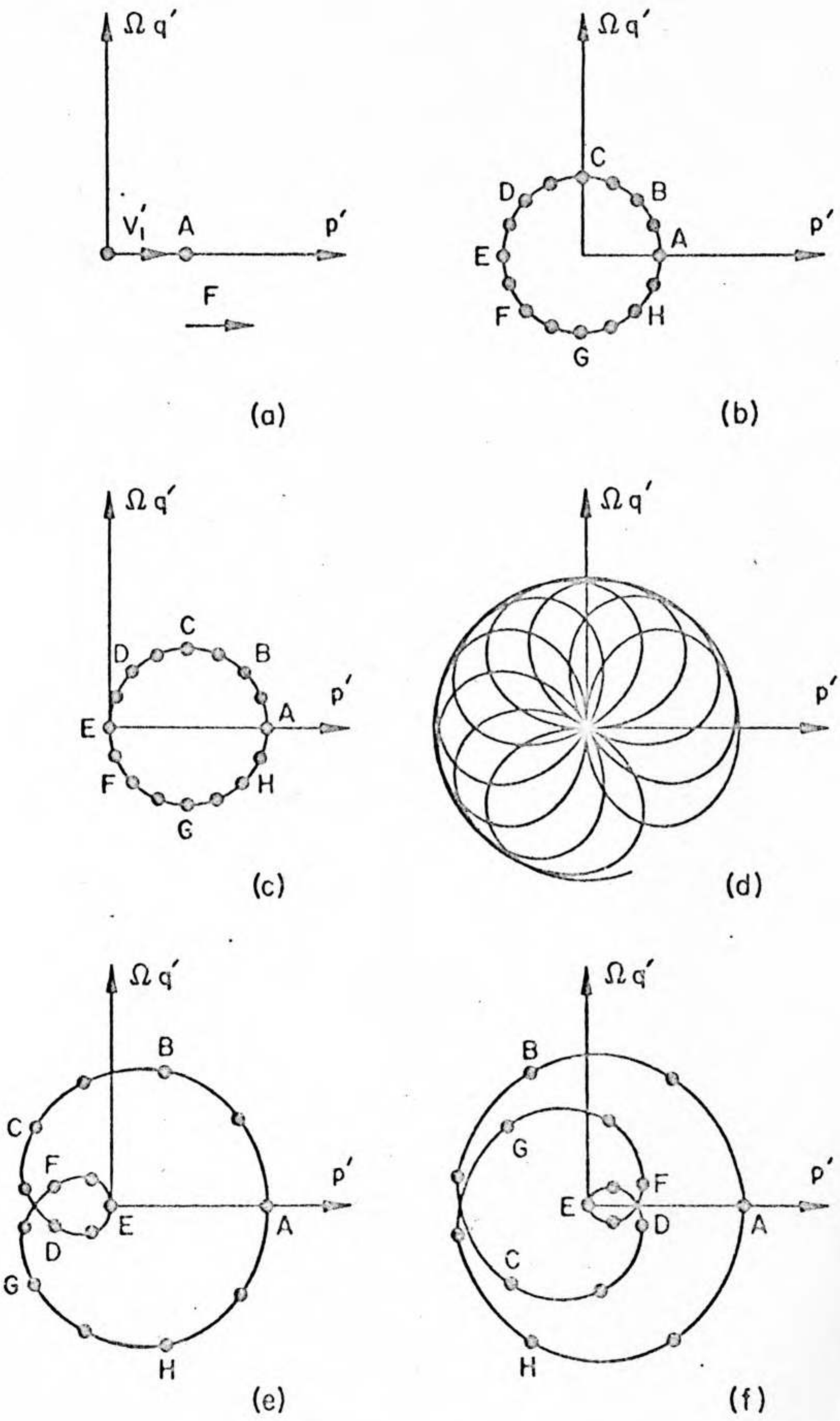


FIG. 12



8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
$\Phi(t)\cdot 10^3$
t/τ
COLLISIONLESS
$\nu_1\tau = .5$
$\nu_1\tau = 1.0$

Fig. 13



Ω q′
V′₁
A
p′
F
(a)
Ω q′
C
D
B
A
E
p′
F
H
G
(b)
Ω q′
C
D
B
A
p′
E
F
H
G
(c)
Ω q′
p′
(d)
Ω q′
B
C
F
p′
E
A
D
G
H
(e)
Ω q′
B
G
F
E
D
A
p′
C
H
(f)

Fig. 14