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Cyclotron emission from accreting magnetic white dwarfs

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Summary. A corrected version of the estimates of cyclotron emission from accreting magnetic white dwarfs by Fabian, Pringle & Rees, is presented, which takes into account harmonics higher than the fundamental.

1 Introduction

Fabian, Pringle & Rees (1976) have considered accretion onto magnetic white dwarfs, and in particular have made estimates of the relative efficiencies of bremsstrahlung (which yields X-rays) and cyclotron (infrared, optical, ultraviolet) emission at radiating away the accretion energy released in a standoff shock near the stellar surface. Fabian *et al.* considered only emission at the fundamental cyclotron frequency, ω_c, and so underestimated the efficiency of the cyclotron process. They also made an error in their calculations – their equation (14) – which, as it happens, roughly compensates for this omission. Here we present a better, but still somewhat uncertain, treatment of cyclotron emission, including the radiation from harmonics above the fundamental.

2 Estimate of cyclotron emission

The temperature of the emission region is approximately the shock temperature

$$T_s = 64(M/M_\odot)(R/5 \times 10^8\,\mathrm{cm})^{-1}\,\mathrm{keV} \tag{1}$$

where M and R are the dwarf mass and radius. The plasma is mildly relativistic at this temperature, and an electron would typically emit more than 90 per cent of its cyclotron radiation above the fundamental (Trubnikov 1958; see also Bekefi 1966, for a summary). When the plasma is optically thin to cyclotron radiation, the emission from an electron decreases approximately exponentially at successive harmonics. The spread of electron energies in (for example) a Maxwellian distribution, smears the higher harmonics into a continuum and causes the emissivity to fall off with frequency less steeply than an exponential.

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The optical depth of a mildly relativistic plasma to cyclotron absorption at the fundamental cyclotron frequency, ω_c, is approximately

$$\Lambda(m_e c^2/kT) \equiv (3/2\, \tau_{es} m_e c^2/a\hbar\omega_c)(m_e c^2/kT) \tag{2}$$

where τ_{es} is the electron scattering optical depth through the region, and a is the fine structure constant. In our case the dimensionless size parameter $\Lambda = 10^{10}\tau_{es}B_6^{-1} \gg 1$, where B_6 is the dwarf's surface field in units of 10^6 G, and $\tau_{es} \sim 1$. Consequently, cyclotron emission is self-absorbed up to a frequency $\omega^*(\gg \omega_c)$ where the optical depth is unity.

Various workers have provided calculations and approcimations with which to estimate ω^*/ω_c in plasma fusion devices (Trubnikov 1973, and references therein; Hirschfield, Baldwin & Brown 1961). Their formulae apply directly to values of Λ between $\sim 10^3$ and $\sim 10^6$, and we do not expect them to provide accurate extrapolations to $\Lambda \sim 10^{10}$. The yield expression of Trubnikov (1973) implies $(\omega^*/\omega_c) \propto \Lambda^{1/6}$ and probably overestimates (ω^*/ω_c) when Λ is large. The fits of Hirschfield *et al.* (1961), $\omega^*/\omega_c \propto \ln\Lambda$, are not reassuring when applied to their own graphs, and probably underestimate ω^*/ω_c when it is large (large Λ). Thus we might expect that these expressions bracket the 'true' value of ω^*/ω_c, and that the dependence of ω^*/ω_c on B_6 is weak. We find $120 \gtrsim \omega^*/\omega_c \gtrsim 25$, for $T_s = 64$ keV and $\Lambda = 10^{10}$.

These estimates assume a Maxwellian (i.e. collision dominated) velocity distribution for the electrons. This is reasonable if bremsstrahlung is important or if cyclotron emission is heavily self-absorbed. The assumption may fail at high temperature when the optically thin cyclotron emission at $\omega > \omega^*$ becomes important, or just behind the shock before the Maxwellian tail is established. If the electron distribution were monoenergetic and spherically symmetric, ω^*/ω_c would be of order 10 for our T_s and Λ.

Cyclotron radiation is directional, especially at high harmonics $(\omega/\omega_c > m_e c^2/kT)$. The extraordinary mode is emitted in a fan beam of FWHM

$$\Psi \simeq 90°(\omega/\omega_c)^{-1/6}(kT/64\ \text{keV})^{1/3} \tag{3}$$

(Trubnikov 1961). The ordinary mode is negligible (Drummond & Rosenbluth 1961). For our present purposes we ignore this angular dependence of the cyclotron emission, noting that other uncertainties – e.g. variations in density, temperature and magnetic field within the emitting region – can be just as important.

To estimate the rate at which cyclotron radiation is emitted, we approximate the spectrum as a Rayleigh–Jeans spectrum $(h\omega^* \ll kT)$ at the electron (shock) temperature, truncated above ω^* (Trubnikov 1958; Hirschfield *et al.* 1961; Drummond & Rosenbluth 1963). The cyclotron luminosity from a region of area A is then

$$L_{cyc} = A\ kT_s(\omega^*)^3/12\pi^2c^2, \tag{4}$$

where $A = f.4\pi R^2, f < 1$, and R is the stellar radius. The condition that the cyclotron and bremsstrahlung luminosities be equal, $L_{cyc} = L_{br} = 1/2\ GM\dot{M}/R$, is

$$F_{20} = 3.28 \times 10^{-13}(\omega^*/\omega_c)^{21/5}\, B_6^{17/5}\, R_9^{9/5}\, (M/M_\odot)^{1/5}, \tag{5}$$

where F_{20} is the mass accretion rate in units of 10^{20} g/s and R_9 the stellar radius in units of 10^9 cm. This condition corresponds to $\gamma_1 = 1$ in the work of Fabian *et al.* (1976). We have used a value of f appropriate to spherical accretion outside the white dwarf magnetosphere, making the standard assumptions (*cf.* for example Section 4 of Fabian *et al.*). These assumptions are incorrect if the Alfvén radius, R_A, is so small as to prevent cusp formation above the magnetic poles (Lamb, Pethick & Pines 1973; Arons & Lea 1976; Elsner & Lamb 1977a) or if the magnetospheric boundary layer is very thin or very thick (Elsner & Lamb 1977b; Basko & Sunyaev 1976).

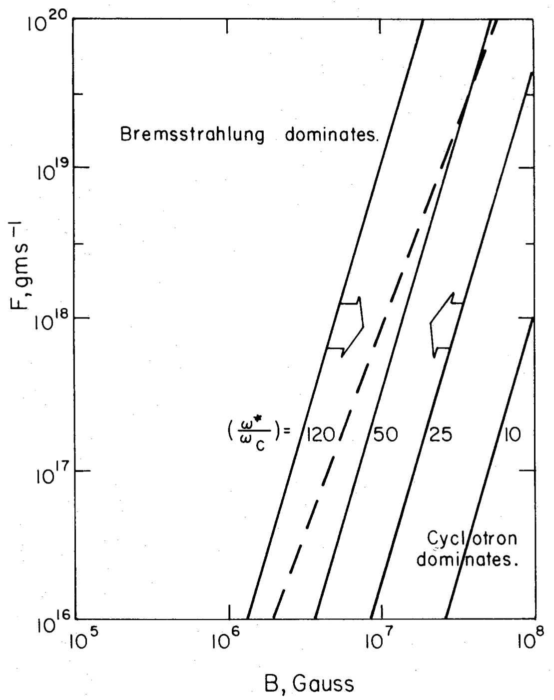

Figure 1. This diagram is relevant to spherically symmetric accretion at rate F g/s onto a magnetic white dwarf of mass 1 $M_\odot$, radius 5×10^8 cm and surface field B gauss (*cf.* Fig. 2 of Fabian *et al.* 1976). The line on which bremsstrahlung and cyclotron emission are comparable is drawn for various values of ω^*/ω_c, the cyclotron harmonic above which the emitting region is optically thin. We expect $120 > \omega^*/\omega_c > 25$ (see text). The dashed line derived (erroneously) by Fabian *et al.* is also shown. In practice, ω^*/ω_c will be a slowly decreasing function of B (for a given value of F). We would therefore expect the true curve to be (like that in Fabian *et al.*) slightly less steep than the lines drawn here for constant ω^*/ω_c.

Fig. 1 shows the behaviour of equation (5) in the (F, B) plane for a 1 $M_\odot$, 5×10^8 cm radius white dwarf. As the above discussion emphasizes, the actual line along which the cyclotron and bremsstrahlung luminosities are equal is expected to lie between the lines for $(\omega^*/\omega_c) = 25$ and $(\omega^*/\omega_c) = 120$ in Fig. 1. This result is similar to that obtained (erroneously) by Fabian *et al.*

3 Discussion

Although we have accounted for the principal qualitative features of cyclotron processes in the mildly relativistic plasma expected in shocked gas settling onto a white dwarf, we wish to stress that our quantitative results are very uncertain. This is partly because we lack an accurate approximation for ω^*/ω_c appropriate to the field geometry and large Λ of the white dwarf problem, even for a uniform plasma. A reliable quantitative calculation would treat radiative transfer through the (non-uniform) emission region, and include the directionality of the cyclotron radiation. Our general conclusions with regard to hard X-ray and cyclotron emission are similar to those presented in Fabian *et al.* We stress that the old nova DQ Herculis is a suitable object on which the above calculations can be tested (Bath, Evans & Pringle 1974; Lamb 1974). Observations of cyclotron emission (polarized ultraviolet, optical and infrared) and X-radiation are especially important. Optical polarization and soft X-ray observations of the nova-like variable AM Herculis suggest it to be another candidate (S. Tapia, private communication).

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