# Cylindrical and spherical dust ion–acoustic solitary waves

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The properties of cylindrical and spherical dust ion–acoustic solitary waves (DIASWs) in an unmagnetized dusty plasma, whose constituents are inertial ions, Boltzmann electrons, and stationary dust particles, are investigated by employing the reductive perturbation method. The modified Korteweg–de Vries equation is derived and its numerical solutions are obtained. It has been found that the properties of the DIASWs in a nonplanar cylindrical or spherical geometry differ from those in a planar one-dimensional geometry. 
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Shukla and Silin[1] have first theoretically shown that, due to the conservation of equilibrium charge density $n_{e0}e+n_{d0}Z_d e-n_{i0}e=0$ and the strong inequality $n_{e0}\ll n_{i0}$ [where $n_{s0}$ is the particle number density of the species $s$ with $s=e$ ($i$) $d$ for electrons (ions) dust particles, $Z_d$ is the number of electrons residing onto the dust grain surface, and $e$ is the magnitude of the electronic charge], a dusty plasma (with negatively charged static dust grains) supports low-frequency dust ion–acoustic (DIA) waves with phase speed much smaller (larger) than electron (ion) thermal speed. The dispersion relation (a relation between the wave frequency $\omega$ and the wave number $k$) of the linear DIA waves is[1] $\omega^2=(n_{i0}/n_{e0})k^2C_i^2/(1+k^2\lambda_{De}^2)$, where $C_i=(k_BT_e/m_i)^{1/2}$ is the ion–acoustic speed (with $T_e$ being the electron temperature and $m_i$ being the ion mass) and $\lambda_{De}=(k_BT_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye radius. When we consider a long wavelength limit (viz. $k\lambda_{De}\ll 1$), the dispersion relation for the DIA waves becomes $\omega=(n_{i0}/n_{e0})^{1/2}kC_i$. This form of spectrum is similar to the usual ion–acoustic wave spectrum[2–4] for a plasma with $n_{i0}=n_{e0}$ and $T_i\ll T_e$ (where $T_i$ is the ion temperature). However, in dusty plasmas we usually have $n_{i0}\gg n_{e0}$ and $T_i\simeq T_e$. Therefore, a dusty plasma cannot support the usual ion–acoustic waves, but can support the DIA waves of Shukla and Silin.[1] The phase speed ($\omega/k$) of the DIA waves is larger than $C_i$ because of $n_{i0}\gg n_{e0}$ for negatively charged dust grains. The increases in the phase velocity is attributed to the electron density depletion in the background plasma, so that the electron Debye radius becomes larger. As a result, there appears to be a stronger space charge electric field which is responsible for the enhanced phase velocity of the DIA waves. The DIA waves have been observed in laboratory experiments.[5,6] The linear properties of the DIA waves in dusty plasmas are now well understood from both theoretical and experimental points of view.[1,5–9]

It is important to note that the DIA waves (in which the ion dynamics is important, i.e., the inertia is provided by the ion mass, the restoring force comes from the pressure of inertialess electrons, and the equilibrium charge neutrality condition is maintained by the stationary dust particles) significantly differ from the dust–acoustic (DA) waves[7,10,11] in which the dust dynamics must be taken into account, i.e., the inertia is provided by the dust particle mass and the restoring force comes from the pressures of inertialess electrons and ions.[7,10,11] The linear and nonlinear properties of the DA waves in planar (one dimensional)[10,12–14] and nonplanar (spherical or cylindrical)[15] geometries have been rigorously investigated during the last few years.

Recently, nonlinear waves associated with the DIA waves, particularly the DIA solitary waves,[16] have also received a great deal of interest in understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas.[1,5–9] The DIA solitary waves (DIASWs) have been investigated by several authors.[16–18] However, all these investigations[16–18] are limited to one-dimensional geometry which may not be a realistic situation in laboratory devices, since the waves observed in laboratory devices are certainly not bounded in one dimension. Therefore, in this Communication, we analyze the DIASWs in nonplanar cylindrical and spherical geometries. We show here how the DIASWs in cylindrical and spherical geometries differ from those in one-dimensional geometry.

We consider a nonplanar cylindrical or spherical geometry and study the nonlinear propagation of the DIA waves in an unmagnetized dusty plasma whose constituents are inertial ions, Boltzmann electrons, and negatively charged immobile dust particles. The nonlinear dynamics of the DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed, in nonplanar cylindrical and spherical geometries is governed by

$$\frac{\partial n_i}{\partial t}+\frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i)=0, \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial r}=-\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial \phi}{\partial r}\right)=\mu\exp(\phi)-n_i+(1-\mu), \tag{3}$$

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where $\nu=0$ for a one-dimensional geometry and $\nu=1$ (2) for a nonplanar cylindrical (spherical) geometry, $n_i$ is the ion number density normalized by its equilibrium value $n_{i0}$, $u_i$ is the ion fluid speed normalized by $C_i$, and $\phi$ is the electrostatic wave potential normalized by $k_BT_e/e$. The time and space variables are in units of the ion plasma period $\omega_{pi}{}^{-1}=(m_i/4\pi n_{i0}e^2)^{1/2}$ and the Debye radius $\lambda_{Dm}=(k_BT_e/4\pi n_{i0}e^2)^{1/2}$, respectively. We have denoted $\mu=n_{e0}/n_{i0}$.

To investigate ingoing solutions of (1)–(3), we introduce the stretched coordinates[19] $\zeta=-\epsilon^{1/2}(r+v_0t)$ and $\tau=\epsilon^{3/2}t$, expand $n_i$, $u_i$, and $\phi$ in a power series of $\epsilon$

$$n_i=1+\epsilon n_i^{(1)}+\epsilon^2 n_i^{(2)}+\cdots, \tag{4a}$$

$$u_i=\epsilon u_i^{(1)}+\epsilon^2 u_i^{(2)}+\cdots, \tag{4b}$$

$$\phi=\epsilon\phi^{(1)}+\epsilon^2\phi^{(2)}+\cdots, \tag{4c}$$

and develop equations in various powers of $\epsilon$. To lowest order in $\epsilon$, Eqs. (1)–(4) give $n_i^{(1)}=-u_i^{(1)}/v_0$, $u_i^{(1)}=-\phi^{(1)}/v_0$, and $v_0=1/\sqrt{\mu}$. To next higher order in $\epsilon$, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial\tau}-v_0\frac{\partial n_i^{(2)}}{\partial\zeta}-\frac{\partial u_i^{(2)}}{\partial\zeta}-\frac{\partial}{\partial\zeta}[n_i^{(1)}u_i^{(1)}]-\frac{\nu u_i^{(1)}}{v_0\tau}=0, \tag{5a}$$

$$\frac{\partial u_i^{(1)}}{\partial\tau}-v_0\frac{\partial u_i^{(2)}}{\partial\zeta}-u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial\zeta}-\frac{\partial\phi^{(2)}}{\partial\zeta}=0, \tag{5b}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}-\mu\phi^{(2)}+n_i^{(2)}-\frac{1}{2}\mu[\phi^{(1)}]^2=0. \tag{5c}$$

Combining Eqs. (5a)–(5c) we deduce a modified Korteweg–de Vries equation

$$\frac{\partial\phi^{(1)}}{\partial\tau}+\frac{\nu}{2\tau}\phi^{(1)}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta}+B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3}=0, \tag{6}$$

where

$$A=\frac{\sqrt{\mu}}{2}\left(3-\frac{1}{\mu}\right), \tag{7a}$$

$$B=\frac{1}{2\mu^{3/2}}. \tag{7b}$$

We have already mentioned that $\nu=0$ corresponds to a one-dimensional geometry. Thus, for a one-dimensional geometry ($\nu=0$) and for a moving frame moving with a speed $u_0$, the stationary solitary wave solution of (6) is

$$\phi^{(1)}(\nu=0)=\left(\frac{3u_0}{A}\right)\mathrm{sech}^2\left[\sqrt{\frac{u_0}{4B}}(\zeta-u_0\tau)\right]. \tag{8}$$

It is obvious from (7) and (8) that for $\mu>(<)1/3$, a dusty plasma supports compressive (rarefactive) DIASWs which are associated with a positive (negative) potential. Since in most space and laboratory dusty plasmas $\mu<1/3$, unlike the usual ion–acoustic solitary waves (associated with a positive potential) in a two component electron–ion plasma,[2–4] DIASWs have a new feature in that these are associated with a negative potential which is due to the presence of negatively charged dust grains.

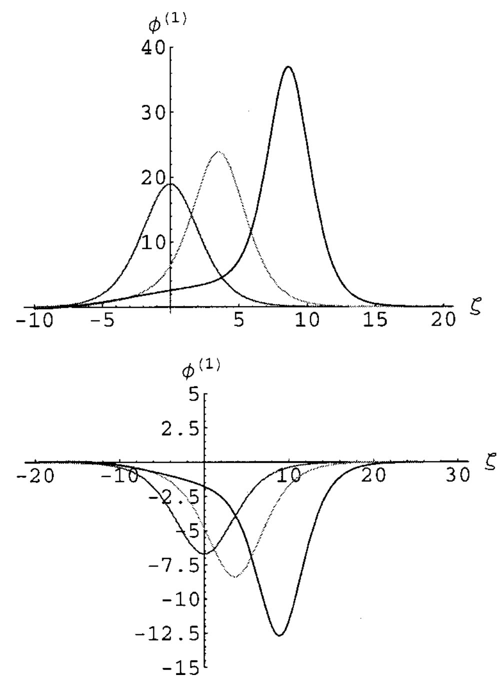


FIG. 1. Time evolution of cylindrical ($\nu=1$) DIASWs: $\phi^{(1)}$ versus spatial coordinate $\zeta$ at times $\tau=-9$ (left), $\tau=-6$ (middle), and $\tau=-3$ (right) for $\mu=0.4$ (upper plot) and $\mu=0.2$ (lower plot). We choose our initial pulse at $\tau=-9$ and show how its amplitude increases with decreasing the value of $\tau$.

We now turn to (6) with the term $(\nu/2\tau)\phi^{(1)}$ which is due to the effect of the nonplanar (cylindrical or spherical) geometry. An exact analytic solution of (6) is not possible. Therefore, we have numerically solved (6) and have studied the effects of cylindrical and spherical geometries on time-dependent DIASWs. The results are depicted in Figs. 1 and 2. The initial condition that we have used in our numerical analysis is in the form of the stationary solution of Eq. (6) without the term $(\nu/2\tau)\phi$, i.e., in the form $\phi^{(1)}=(3/A)\mathrm{sech}^2(\zeta/\sqrt{4B})$. Figure 1 shows how the effect of a cylindrical geometry modifies the DIASWs associated with both the positive ($\mu=0.4>1/3$) and negative ($\mu=0.2<1/3$) potentials. Figure 2 shows how the effect of a spherical geometry modifies the DIASWs associated with both the positive ($\mu=0.4>1/3$) and negative ($\mu=0.2<1/3$) potentials.

The numerical solutions of (6) (displayed in Figs. 1 and 2) reveal that for a large value of $\tau$ (e.g., $\tau=-9$) the spherical and cylindrical solitary waves are similar to one-dimensional solitary waves. This is because for a large value of $\tau$ the term $(\nu/2\tau)\phi^{(1)}$, which is due to the effect of the cylindrical or spherical geometry, is no longer dominant. However, as the value of $\tau$ decreases, the term $(\nu/2\tau)\phi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves differ from one-dimensional solitary waves. It is found that as the value of $\tau$ decreases, the amplitude of these localized pulses increases. It is also found that the amplitude of cylindrical solitary waves is larger than that of the one-dimensional solitary waves but smaller than that of the spherical ones.

To compare the properties of DIA cylindrical/spherical

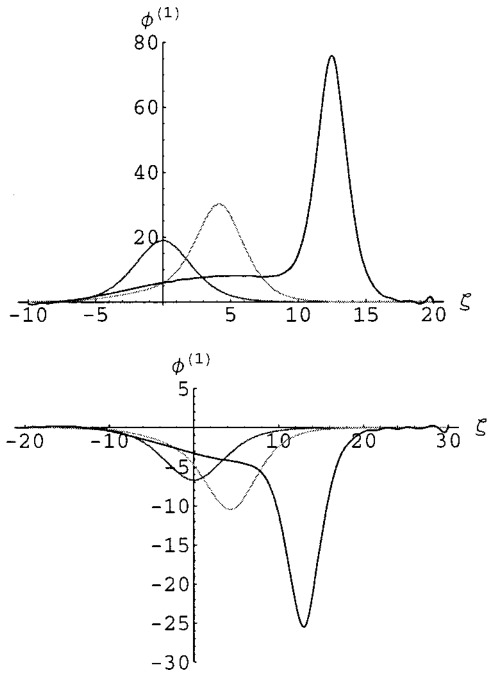


FIG. 2. Time evolution of spherical ($\nu=2$) DIASWs: $\phi^{(1)}$ versus spatial coordinate $\zeta$ at times $\tau=-9$ (left), $\tau=-6$ (middle), and $\tau=-3$ (right) for $\mu=0.4$ (upper plot) and $\mu=0.2$ (lower plot). We choose our initial pulse at $\tau=-9$ and show how its amplitude increases with decreasing the value of $\tau$.

solitary structures with those of DA cylindrical/spherical solitary structures,[15] we note that (i) the solitary potential associated with the DIA waves can be both positive and negative, but the solitary potential associated with the DA waves can be negative only,[15] and (ii) the other properties (such as amplitude, width, etc.) of the DIA waves also significantly differ from those of the DA waves.[15] It is also important to mention that the DIA solitary waves are more suitable than the DA waves to observe in laboratory dusty plasma conditions. We, therefore, propose to perform a laboratory experiment which can study such special new features of DIA solitary waves. We also stress that the present results may help to understand the salient features of the cylindrical/spherical DIA solitary waves when data for laboratory observations become available.

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[1] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[2] K. E. Lonngren, Plasma Phys. **25**, 943 (1983).
[3] K. E. Lonngren, Opt. Quantum Electron. **30**, 615 (1998).
[4] T. E. Sheridan, S. Yi, and K. E. Lonngren, Phys. Plasmas **5**, 3165 (1998).
[5] A. Barkan, N. D'Angelo, and R. L. Merlino, Planet. Space Sci. **44**, 239 (1996).
[6] Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999).
[7] P. K. Shukla and M. Rosenberg, Phys. Plasmas **6**, 1038 (1999).
[8] F. Verheest, *Waves in Dusty Plasmas* (Kluwer Academic, Dordrecht, 2000).
[9] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002).
[10] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[11] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[12] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 702 (1996).
[13] J. X. Ma and J. Liu, Phys. Plasmas **4**, 253 (1997).
[14] A. A. Mamun, Astrophys. Space Sci. **268**, 443 (1999).
[15] A. A. Mamun and P. K. Shukla, Phys. Lett. A **290**, 173 (2001).
[16] R. Bharuthram and P. K. Shukla, Planet. Space Sci. **40**, 973 (1992).
[17] S. I. Popel and M. Y. Yu, Contrib. Plasma Phys. **35**, 103 (1995).
[18] Y. Nakamura and A. Sharma, Phys. Plasmas **8**, 3921 (2001).
[19] S. Maxon and J. Viecelli, Phys. Rev. Lett. **32**, 4 (1974).