Cylindrical heat exchanger trajectory planning and tracking using orthogonal functions

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Abstract

This paper proposes a trajectory planning and tracking approach for cylindrical heat exchanger process that is considered, under some assumptions, as a bilinear system. The proposed technique is based on orthogonal functions and especially the use of operational integration and product matrices. These operational tools allow the conversion of a bilinear differential state equation into an algebraic one depending on initial and final conditions. Arranging and solving the obtained algebraic equation lead to an open loop control law that allows the planning of a system trajectory. The parameters setting of the tracking state feedback closed loop control is yielded by considering a reference model characterizing the desired performances. A high gain observer is associated to heat exchanger process in the planning trajectory step and tracking one. A planned open loop control and a state feedback control that ensures tracking of reference trajectory were applied to the system exchanger associated to observer which is subject to noise and disturbance.

Keywords: *Feedback control, Nonlinear systems, Orthogonal functions and polynomials, Observability, heat and mass transfer.*

1. Introduction

Plan a trajectory is finding the open loop control which allows the system to reach a final state x_f set from a known initial state x_0 . Track a trajectory is to synthesize a closed loop control law that correct deviations between the real system trajectory and the planned one (so-called reference) $\Delta x = x - x_{ref} \to 0$.

Both problems were addressed in the literature in several ways: for linear systems are mentioned the technique of spline functions [1], for non linear systems notion of flatness was used [2]. However, we have proposed a new approach based on orthogonal functions as a tool of approximation to solve the planning and tracking trajectory problem for time variant linear systems [3] and bilinear system ([4],[5], [10],[11]).

In what follows we propose to apply this technique in order to plan and track a trajectory of a cylindrical heat exchanger, which is basically a six order non linear system which can be assimilated to a bilinear one by considering some practical assumptions. Moreover, a high gain observer is going to be associated with the system for state estimation.

2. Heat exchanger presentation and modeling

We present in this section the heat exchanger model with its parameters.

2.1 Process presentation and assumptions

We consider a cylindrical heat exchanger composed by three compartments shown in figure1.

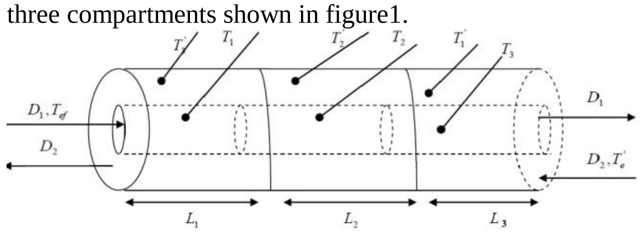

Fig. 1 Process: Three compartments heat exchanger

with:

D_i : volumetric flow

$T_e, T_e^{'}$: entry temperatures

S : section of inner cylinder

$S^{'}$: section of outer cylinder

S : heat exchange surface

$T_e, T_e^{'}$ temperatures are assumed to be homogeneous.

Also, the following assumptions are considered:

- There is no heat loss;
- There's is heat exchange through the shell of the inner cylinder under the law of Fourier: $h(T_{inner\ cylinder} - T_{outer\ cylinder}) = exchange\ flow\ (in\ watts)$ with h is the global heat transfer coefficient;
- C_p, $C_p^{'}$: constant specific heats;
- $m_i = \rho V_i = \rho S L_i$ and $m_i^{'} = \rho^{'} V_i^{'} = \rho^{'} S^{'} L_i$

2.2 Process Modeling

The modeling of the heat exchanger is based on the following thermal balance law:

$$\frac{d}{dt} m C_p T = Entry\ flow - Exit\ flow \qquad (1)$$

Applying equation of thermal balance (1) for three parts I, II and III of heat exchanger.

Part I:

$$\begin{cases} \frac{d}{dt} m_1 C_p T_1 = \rho D_1 C_p T_e + hs(T_3^{'} - T_1) - \rho D_1 C_p T_e \\ \frac{d}{dt} m_1^{'} C_p^{'} T_3^{'} = \rho^{'} D_1 C_p^{'} T_2^{'} + hs(T_1 - T_3^{'}) - \rho^{'} D_2 C_p^{'} T_3^{'} \end{cases}$$

Part II:

$$\begin{cases} \frac{d}{dt} m_2 C_p T_2 = \rho D_1 C_p T_1 + hs(T_2^{'} - T_2) - \rho D_1 C_p T_2 \\ \frac{d}{dt} m_2^{'} C_p^{'} T_2^{'} = \rho^{'} D_2 C_p^{'} T_1^{'} + hs(T_2 - T_2^{'}) - \rho^{'} D_2 C_p^{'} T_2^{'} \end{cases}$$

Part III:

$$\begin{cases} \frac{d}{dt} m_3 C_p T_3 = \rho D_1 C_p T_2 + hs(T_1^{'} - T_3) - \rho D_1 C_p T_3 \\ \frac{d}{dt} m_3^{'} C_p^{'} T_1^{'} = \rho^{'} D_2 C_p^{'} T_e^{'} + hs(T_3 - T_1^{'}) - \rho^{'} D_2 C_p^{'} T_1^{'} \end{cases}$$

Let us consider the state vector of the process as $x = \begin{bmatrix} T_1 & T_2 & T_3 & T_1^{'} & T_2^{'} & T_3^{'} \end{bmatrix}^T$, a control vector $u = \begin{bmatrix} D_1 & T_e & D_2 & T_e^{'} \end{bmatrix}$ and the output $y = \begin{bmatrix} T_1 & T_3^{'} \end{bmatrix}^T$.

Note that the heat exchanger is controlled in temperature and debit.

The system evolution is described by the following state equation:

$$\begin{cases} \dot{x} = A(u)x + B(u) \\ y = Cx \end{cases} \qquad (2)$$

with:

$$A(u) = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & a_{16} \\ a_{21} & a_{22} & 0 & 0 & a_{25} & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & a_{55} & 0 \\ a_{61} & 0 & 0 & 0 & a_{65} & a_{66} \end{pmatrix}$$

where:

$$a_{11} = \frac{-hs}{m_1 C_p} - \frac{\rho D_1}{m_1} \qquad a_{16} = \frac{hs}{m_1 C_p}$$

$$a_{21} = \frac{\rho D_1}{m_2} \qquad a_{22} = \frac{-hs}{m_2 C_p} - \frac{\rho D_1}{m_2}$$

$$a_{25} = \frac{hs}{m_2 C_p} \qquad a_{32} = \frac{\rho D_1}{m_3}$$

$$a_{33} = \frac{-hs}{m_3 C_p} - \frac{\rho D_1}{m_3} \qquad a_{34} = \frac{hs}{m_3 C_p}$$

$$a_{43} = \frac{hs}{m_3^{'} C_p^{'}} \qquad a_{44} = \frac{-hs}{m_3^{'} C_p^{'}} - \frac{\rho^{'} D_1}{m_3^{'}}$$

$$a_{52} = \frac{hs}{m_2^{'} C_p^{'}} \qquad a_{54} = \frac{\rho^{'} D_2}{m_2^{'}}$$

$$a_{55} = \frac{-hs}{m_2^{'} C_p^{'}} - \frac{\rho^{'} D_2}{m_2^{'}} \qquad a_{61} = \frac{hs}{m_1^{'} C_p^{'}}$$

$$a_{65} = \frac{\rho^{'} D_2}{m_1^{'}} \qquad a_{66} = \frac{-hs}{m_1^{'} C_p^{'}} - \frac{\rho^{'} D_2}{m_1^{'}}$$

$$B(u) = \begin{bmatrix} \frac{D_1 T_e \rho}{m_1} & 0 & 0 & \frac{D_2 T_e^{'} \rho^{'}}{m_3^{'}} & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By considering hot and cold entry temperatures $(T_e, T_e^{'})$ constant, and controlling the heat exchanger by debits D_1 and D_2 we obtain, then, a bilinear model of the exchanger; since the fourth input vector u is reduced

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to the second order vector $u = \begin{bmatrix} D_1 & D_2 \end{bmatrix}$ and the state matrix becomes: $A(u) = A_0 + A_1 u$.

It is proposed to apply the trajectory planning technique based on the use of orthogonal functions developed in ([4],[6]) to the bilinear model of the heat exchanger.

3. Trajectory planning

The proposed approach to solve the problem of trajectory planning for the heat exchanger based on the use of orthogonal functions that offer the possibility of representing various systems algebraically. We first give a brief overview of the orthogonal functions.

3.1 Orthogonal functions

We consider a complete set of orthogonal functions $\Phi = \{\varphi_i(t), i \in \mathbb{N}\}$, defined on an interval $L^2([a,b])$. A projection of any function $f(t)$ in a complete space of orthogonal functions Φ is given by:

$$f(t) = \sum_{i=0}^{\infty} f_i \varphi_i(t), \quad \forall t \in [a,b] \tag{3}$$

where f_i are constant coefficients given by: $f_i = \frac{1}{r_i} \int_a^b \omega(x) \varphi_i(x) f(x) dx, \quad \forall i \in \mathbb{N}$

To obtain a practical approximation of the function $f(t)$, development (3) is truncated to N order. We thus obtain:

$$f(t) \cong \sum_{i=0}^{N-1} f_i \varphi_i(t) = F_N^T \Phi_N(t) \tag{4}$$

Where: $F_N = \begin{bmatrix} f_0 & f_1 & \dots & f_{N-1} \end{bmatrix}$ is a constant coefficient vector and $\Phi_N = \begin{bmatrix} \varphi_0 & \varphi_1 & \dots & \varphi_{N-1} \end{bmatrix}^T$ is an orthogonal function composed vector.

An orthogonal functions development truncated at order N of a matrix function $A(t) = \begin{bmatrix} a_{ij}(t) \end{bmatrix}$ is given by:

$$A(t) \cong \sum_{i=0}^{N-1} A_{Ni} \varphi_i(t)$$

With $A_{Ni} \in \mathbb{R}^{n \times m}$, $i \in \{0, 1, \dots, N-1\}$ are constant coefficient matrix.

Besides the approximation (4), the orthogonal functions provide useful tools such as: the operational matrix of integration, the operational matrix of product and the operational matrix of derivation, for solving differential equations.

a) Operational matrix of integration: The integral of orthogonal functions basis vector $\Phi_N(t)$ can be approximated by a constant matrix $P_N \in \mathbb{R}^{N \times N}$ which verifies: $\int_0^t \Phi_N(\tau) d\tau \cong P_N \Phi_N(t)$. Form of the matrix P_N depends on the basis of orthogonal functions chosen.

b) Operational matrix of product: The approximation of product of orthogonal basis vectors is given by the operational matrix of product M_{iN} satisfying the following relation: $\varphi_i(t) \Phi_N(t) \cong M_{iN} \Phi_N(t)$

with $M_{iN} = \begin{bmatrix} K_{0,i} & \dots & K_{N-1,i} \end{bmatrix}$ and $\forall i \in \{0, 1, \dots, N-1\}$, $\varphi_i(t) \varphi_j(t) \cong K_{ij}{}^T \Phi_N(t)$

Orthogonal functions also provide product property [7] for any constant vector $V \in \mathbb{R}^n$:

$$\Phi_N(t) \Phi_N{}^T(t) V \cong M_N(V) \Phi_N(t) \tag{5}$$

Where

$$M_N(V) = \begin{bmatrix} M_{0N}(V) \vdots & M_{1N}(V) \vdots & \dots & \vdots M_{(N-1)N}(V) \end{bmatrix}$$

3.2 Proposed approach

Consider a bilinear system described by a state equation.

$$\dot{x} = Ax + \sum_{i=0}^{m} A_i u_i x + Bu \tag{6}$$

its state variables projection on an orthogonal functions basis $\Phi_N(t)$ at truncation order N gives:

$$x(t) = x_N \Phi_N(t)$$
$$u_i(t) = u_{iN} \Phi_N(t)$$
$$u(t) = u_N \Phi_N(t)$$

A state representation (6) approximation is given by:

$$\dot{x}(t) = Ax_N \Phi_N(t) + Bu_N \Phi_N(t) + \sum_{i=0}^{m} A_i x_N u_{iN} \Phi_N(t) \tag{7}$$

integration of equation (7) between initial $t_0 = 0$ instant and instant t:

$$x(t) - x(0) = Ax_N \int_0^t \Phi_N(\tau) d\tau + Bu_N \int_0^t \Phi_N(\tau) d\tau + \sum_{i=0}^{m} A_i x_N u_{iN} \int_0^t \Phi_N(\tau) d\tau \tag{8}$$

orthogonal basis projection of equation (8) and by replacing the initial state $x(0)$ (at the instant $t_0 = 0$) by its orthogonal basis projection: $x(0) = x_{N,0}\Phi_N(t)$ with $x_{N,0} = [x(0)\vdots \; 0\vdots \; \ldots\vdots \; 0]$ and the use of operational matrix of integration P_N and product proprieties (5), one obtains:

$$x_N - x_{N,0} = Ax_N P_N \Phi_N(t) + Bu_N P_N \Phi_N(t) + \sum_{i=0}^{m} A_i x_N M_N(u_{iN}) P_N \Phi_N(t) \quad (9)$$

Using Vec operator and its main property [6]: $Vec(ABC) = (C^T \otimes A)Vec(B)$ equation (9) gives the following relation:

$$Vec(x_N) - Vec(x_{N,0}) = (P_N{}^T \otimes A)Vec(x_N) + (P_N{}^T \otimes B)Vec(u_N) + \sum_{i=0}^{m} (P_N{}^T M_N{}^T(u_{iN}) \otimes A_i)Vec(x_N)$$

thus we have:

$$Vec(x_N) = (I_{nN} - \sum_{i=0}^{m} (P_N{}^T M_N{}^T(u_{iN}) \otimes A_i)Vec(x_N) - (P_N{}^T \otimes A))^{-1}((P_N{}^T \otimes B)Vec(u_N) + Vec(x_{N,0})) \quad (10)$$

integrating equation (7) between instant t and final time ($t_{fin} = T$):

$$x(T) - x(t) = Ax_N \int_t^T \Phi_N(\tau)d\tau + Bu_N \int_t^T \Phi_N(\tau)d\tau + \sum_{i=0}^{m} A_i x_N u_{iN} \int_t^T \Phi_N(\tau)d\tau \quad (11)$$

and replace $x(T)$ by its orthogonal functions projection: $x(T) = x_{N,T}\Phi_N(T)$ and $x_{N,T} = [x(T)\vdots \; 0\vdots \; \ldots\vdots \; 0]$ using the fact that the orthogonal basis vector at final time T verifies: $\Phi_N(T) = K_N \Phi_N(t)$

one obtains:

$$x_{N,T} - x_N = Ax_N P_N(K_N - I_N) + Bu_N P_N(K_N - I_N) + \sum_{i=0}^{m} A_i x_N M_N(u_{iN}) P_N(K_N - I_N)$$

let us pose $\Pi_N = P_N(K_N - I_N)$ and apply Vec operator, we obtain:

$$Vec(x_{N,T}) - Vec(x_N) = (\Pi_N{}^T \otimes A)Vec(x_N) + (\Pi_N{}^T \otimes B)Vec(u_N) + \sum_{i=0}^{m} (\Pi_N{}^T M_N{}^T(u_{iN}) \otimes A_i)Vec(x_N)$$

this equation yields to:

$$Vec(x_N) = (I_{nN} + \sum_{i=0}^{m} (\Pi_N{}^T M_N{}^T(u_{iN}) \otimes A_i)Vec(x_N) + (\Pi_N{}^T \otimes A))^{-1}(Vec(x_{N,T}) - (\Pi_N{}^T \otimes B)Vec(u_N)) \quad (12)$$

by equalizing (10) and (12) one obtains the following relation:

$$H_N^{-1}(Vec(x_{N,0}) + (P_N{}^T \otimes B)Vec(u_N)) = G_N^{-1}(Vec(x_{N,T}) - (\Pi_N{}^T \otimes B)Vec(u_N))$$

where:

$$H_N = H_N(u_N) = I_{nN} - R_u$$

$$R_u = (P_N{}^T \otimes A) + \sum_{i=0}^{m} (P_N{}^T M_N{}^T(u_{iN}) \otimes A_i)$$

$$G_N = I_{nN} + (\Pi_N{}^T \otimes A) + \sum_{i=0}^{m} (\Pi_N{}^T M_N{}^T(u_{iN}) \otimes A_i)$$

substituting Π_N by its expression $\Pi_N = P_N(K_N - I_N)$, thus we have: $G_N = H_N + (K_N^T \otimes I_N)R_u$

relation (12) becomes:

$$(K_N^T \otimes I_N)Z(u_N) = \Gamma(x_{N,0}, x_{N,T}) \quad (13)$$

with:

$$Z(u_N) = H_N^{-1}(u_N)((P_N{}^T \otimes B)Vec(u_N) + Vec(x_{N,0}))$$

$$\Gamma(x_{N,0}, x_{N,T}) = (K_N^T \otimes I_{Nn})Vec(x_{N,0}) + Vec(x_{N,T})$$

Open loop planning control is obtained by minimizing, with respect of u_N, the norm of the difference between the two parts of equality (13):

$$\varsigma = \left\| (K_N^T \otimes I_N)Z(u_N) - \Gamma(x_{N,0}, x_{N,T}) \right\| \quad (14)$$

The minimization can be led by using the tools provided by Matlab optimization toolbox as the "fmincon" function.

3.3 Trajectory planning of the heat exchanger

Applying the method previously developed for the bilinear model of the heat exchanger (6), using Legendre modified polynomials as a tool for the approximation [7] and a truncation order $N = 16$.

a) Numeric data :

are given below the parameter values of the heat exchanger.

$\rho = 1000\ kg/m^3$ volumetric density of the exchanger inner part.

$\rho^{'} = 1000\ kg/m^3$ volumetric density of the exchanger outer part.

$C_p = 4183\ J/kgK$ mass heat of inner part.

$C_p^{'} = 4183\ J/kgK$ mass heat of outer part.

$L = 1m$ length of heat exchanger.

$L_1 = L_2 = L_3 = L/3$ length of respectively exchanger part I, part II and part III.

$d_i = 0.03m$ inner diameter of the exchanger.

$d_e = 0.05m$ outer diameter of the exchanger.

$s_e = \dfrac{\pi d_e^2}{4}\ m^2$ section of the inner tube.

$s_e = \dfrac{\pi d_i^2}{4}\ m^2$ flow section of the fluid in the outer tube.

$m_1 = \rho s_i L_1$, $m_2 = \rho s_i L_2$, $m_3 = \rho s_i L_3$ masses of inner three parts of exchanger.

$m_1^{'} = \rho^{'} s_e L_1$, $m_2^{'} = \rho^{'} s_e L_2$, $m_3^{'} = \rho^{'} s_e L_3$ masses of outer three parts of exchanger.

$s = \pi d_i L$ exchange surface.

$T_f = T_e = 25^{\circ}C$ cold temperature.

$T_c = T_e^{'} = 45^{\circ}C$ hot temperature.

$D_f = \dfrac{4010^{-3}}{3600} m^3/s$ cold flow.

$D_c = \dfrac{16510^{-3}}{3600} m^3/s$ hot flow.

$h = 1150\ sw/m^2K$

b) Matlab simulation:

The proposed method was implemented in Matlab, for the exchanger model (2) based on previous numeric data. An open loop planning control u_p was calculated for the process reaches the final temperatures $x_f = \begin{bmatrix} 30^{\circ}C & 35^{\circ}C & 40^{\circ}C & 45^{\circ}C & 40^{\circ}C & 35^{\circ}C \end{bmatrix}^T$ with $t_{fin} = T = 20s$ from initial state $x_0 = \begin{bmatrix} 15^{\circ}C & 15^{\circ}C & 15^{\circ}C & 25^{\circ}C & 25^{\circ}C & 25^{\circ}C \end{bmatrix}^T$. The solution of equation (14) allowing the calculation of the control u_p was led by the Matlab function "fmincon" with an initial debit control $u_0 = \begin{bmatrix} D_c & 2D_c \end{bmatrix}$, a minimum level control $u_{\min} = \begin{bmatrix} D_f & D_f \end{bmatrix}$ and an upper level control $u_{\max} = 4u_0$. Modified Legendre polynomials have been used as an approximation tool with a truncation order $N = 16$.

The figure (Fig.2) show that the open loop generated trajectory allows the bilinear model of the heat exchanger to reach desired temperatures from chosen temperatures.

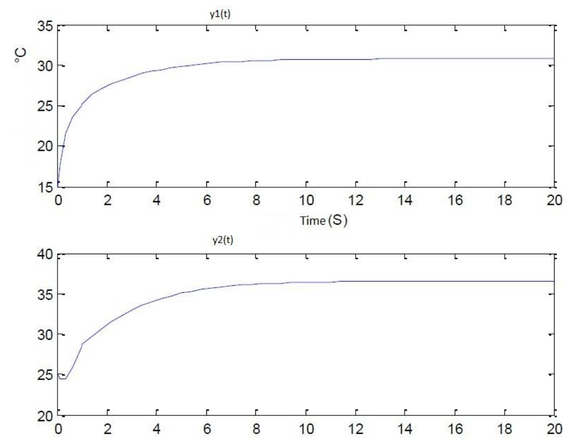

Fig. 2 Exchanger planned trajectory

4. Trajectory tracking

In this section we propose to present a method that permit to find a close loop control that ensures the planned trajectory.

4.1 Proposed approach

We consider the following difference variables between a system (6) trajectory $(x(t), u(t))$ and its planned one $(x_p(t), u_p(t))$:

$$\begin{cases} \delta x = x(t) - x_p(t) \\ \delta u = u(t) - u_p(t) \end{cases} \tag{15}$$

the planned trajectory variables verifies:

$$\dot{x}_p = Ax_p + \sum_{i=0}^{m} A_i u_{ip} x_p + Bu_p$$

the state equation of difference system is given by:

$$\delta \dot{x} = (A + \sum_{i=0}^{m} A_i u_{ip})\delta x(t) + (B + \sum_{i=0}^{m} A_i x_p)\delta u(t) + \sum_{i=0}^{m} A_i \delta x \delta u_i \tag{16}$$

neglecting the term $\delta x \delta u_i$ front δx and δu, the state equation (16) can be simplified to a state representation of a linear time variant system [3]:

$$\delta \dot{x} = A(t)\delta x(t) + B(t)\delta u(t) \tag{17}$$

with: $A(t) = A + \sum_{i=0}^{m} A_i u_{ip}$

$$B(t) = B + \sum_{i=0}^{m} A_i x_p$$

Our objective is to characterize a state feedback control law $\delta u(t) = -k\delta x(t)$ that gives the controlled time variant linear system (17) the desired performances which are defined by a suitably chosen linear model:

$$\delta \dot{x} = E\delta x(t) \tag{18}$$

the projection of time variant matrices $A(t)$, $B(t)$ and the vector $\delta x(t)$ in orthogonal functions basis is given by:

$$A(t) = \sum_{i=0}^{N-1} A_{Ni}\varphi_i(t) \qquad B(t) = \sum_{i=0}^{N-1} B_{Ni}\varphi_i(t)$$

$$\delta x(t) = \delta x_N \Phi_N(t)$$

yields the following differential equation:

$$\delta \dot{x} = \sum_{i=0}^{N-1} A_{Ni}\varphi_i(t) - k\sum_{i=0}^{N-1} B_{Ni}\varphi_i(t)\delta x_N \Phi_N(t) \tag{19}$$

integrating the previous relation and using operational matrix of product and of integration with the use of Vec operator, we obtain:

$$\begin{aligned} Vec(\delta x_N) - Vec(\delta x_{N,0}) &= \sum_{i=0}^{N-1} (M_{iN}P_N)^T \otimes A_{Ni} \\ &- k\Big(\sum_{i=0}^{N-1} (M_{iN}P_N)^T \otimes B_{Ni}\Big)Vec(\delta x_N) \end{aligned} \tag{20}$$

a similar development for the reference model (18) gives:

$$Vec(\delta x_{N,r}) - Vec(\delta x_{N,0}) = (P_N^T \otimes E)Vec(\delta x_N) \tag{21}$$

Equalization between $Vec(\delta x_N)$ obtained from (20) and $Vec(\delta x_{N,r})$ obtained from (21) provides the following linear algebraic equation whose unknown is the state feedback control gain k:

$$\phi k = \psi \tag{22}$$

with:

$$\phi = \sum_{i=0}^{N-1} (M_{iN}P_N)^T \otimes B_{Ni}$$

$$\psi = \sum_{i=0}^{N-1} (M_{iN}P_N)^T \otimes A_{Ni} - (P_N^T \otimes E)$$

Solving the equation (22) using the least squares method leads to a state feedback control law $\delta x(t) = -k\delta u(t)$ that tracks the bilinear system (6) trajectory.

It should be noted that the development (17) to (22) can be extended to synthesize a time variant state feedback control law in which the time variant control gain $k(t)$ can be determined by a projection in an orthogonal functions base: $k(t) = \sum_{i=0}^{N-1} K_{Ni}\varphi_i(t)$.

4.2 Heat exchanger trajectory tracking

The developed method has been applied to track the planned trajectory of the heat exchanger bilinear model (6), using as reference linear model: $\delta \dot{x} = E\delta x(t)$ with $E = -I_6$.

To simulate the system provided with the obtained control law, we have introduced a perturbation on the exchanger planned trajectory at instant $t = 8s$, then we applied the closed loop control law for the disturbed trajectory. The simulation results are presented in the figure (Fig.3).

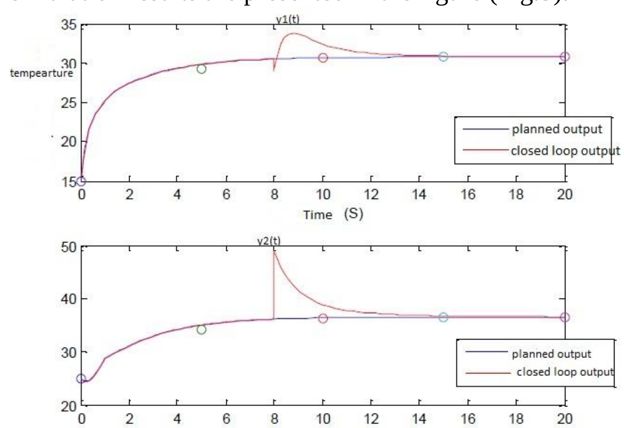

Fig. 3 Evolution of perturbed outputs of heat exchanger

Note that the closed loop system trajectory tracks the planned trajectory despite the disturbance injected into the system. The synthesized control allows the system to reach the reference trajectory (obtained in open loop) with performances defined by the choice of the linear reference model. These performances can be adjusted by changing the reference model.

5. State observer

5.1 State observer presentation

The implementation of the synthesized control and planned law needs the measure or the estimation of all the state variables. Since the temperatures $T_1^{'}$, T_2, $T_2^{'}$ and T_3 are not measurable, it is required to reconstruct them using a state observer. For this goal we propose to apply a high gain state observer ([8], [9]) described by the following equation:

$$\begin{cases} \dot{\hat{x}} = A(u)\hat{x}(t) + B(u) - S^{-1}C^{T}(Cx(t) - C\hat{x}(t)) \\ \dot{S}(t) = -\theta S(t) - A^{T}(u)S(t) - S(t)A(u) + C^{T}C \end{cases} \quad (23)$$

with:

θ: tuning parameter (observer gain)

$S = I$: at the first iteration.

In the next we propose to associate this state observer to the heat exchanger first in trajectory generation step, second in trajectory tracking phase.

5.2 Use of state observer in open loop step

a) Simulation without noise:

Simulating on MATLAB exchanger system (2) associated to the state observer (23) knowing that the initial state of the observer is

$$\bar{x}_0 = \begin{bmatrix} 10^{\circ}C & 10^{\circ}C & 10^{\circ}C & 20^{\circ}C & 20^{\circ}C & 20^{\circ}C \end{bmatrix}^{T}$$

and the observer gain $\theta = 4$, the heat exchanger is simulated under the same conditions chosen for trajectory planning. Results are shown in figure (Fig.4).

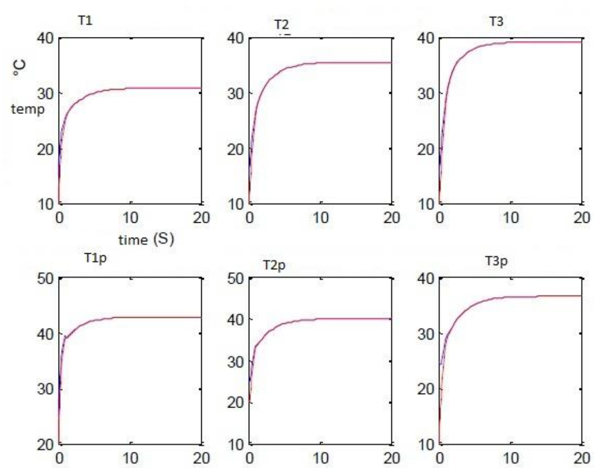

Fig. 4 Evolution of exchanger observed states without noise.

Note that starting from a state $\bar{x}_0 = \begin{bmatrix} 10^{\circ}C & 10^{\circ}C & 10^{\circ}C & 20^{\circ}C & 20^{\circ}C & 20^{\circ}C \end{bmatrix}^{T}$ different from the initial state $x_0 = \begin{bmatrix} 15^{\circ}C & 15^{\circ}C & 15^{\circ}C & 25^{\circ}C & 25^{\circ}C & 25^{\circ}C \end{bmatrix}^{T}$ chosen for the heat exchanger, the observer converges quickly to the measured output. The observer time convergence depends on the tuning parameter θ ; the higher this parameter more rapid the convergence is.

b) Simulation with noise:

Injecting a Gaussian white noise of amplitude 3% on the exchanger output and simulating the system exchanger associated with state high gain observer response, one obtains the results illustrated on the curves of figure (Fig.5).

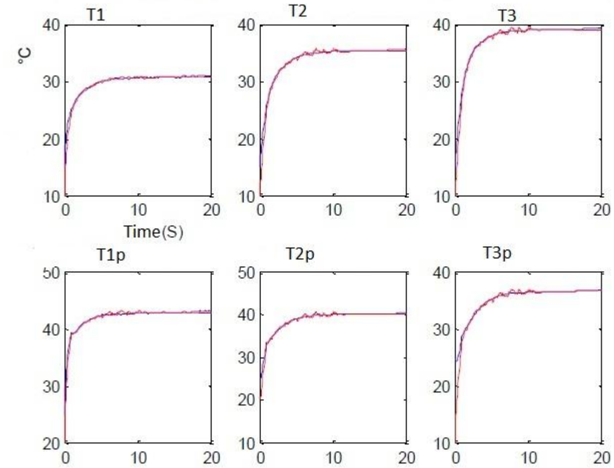

Fig. 5 Evolution of exchanger observed states with noise.

Note that despite the presence of noise on the output, the observer converges to the exchanger measured output.

5.3 Feedback control using state observer

We apply the trajectory tracking approach [11] for the system using the state observer (23) in the presence of noise by injecting at time $t = 8s$ a perturbation; the results obtained are given by figure (Fig.6).

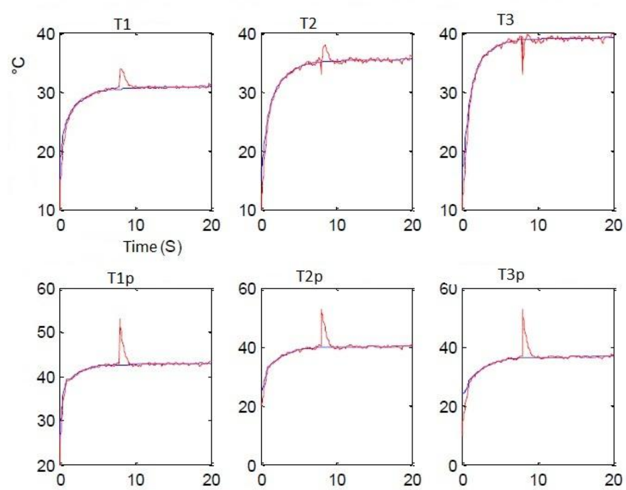

Fig. 6 Evolution of exchanger observed and controlled states with noise.

It appears that the system (exchanger+state observer + feedback control) succeed to track the planned trajectory in spite of perturbations injected.

5. Conclusion

A trajectory planning and tracking approach based on the use of orthogonal functions has been proposed and applied to a bilinear model of cylindrical heat exchanger.
This approach allowed finding an open loop planning control ensuring for system to evolve from a fixed initial state to a known final state. It should be noted that the considered control is a debit heat exchanger control that has achieved the desired temperatures for external and internal parts of the cylindrical tube.
The planned trajectory was generated in the first. Second, a tracking trajectory approach based on the use of orthogonal functions and a reference model was applied to the exchanger bilinear model. Then, a control state feedback law was synthesized allowing controlled system to stay in a neighborhood of the reference trajectory despite disturbances that may occur.
Moreover, a high gain state observer of the heat exchanger has been integrated in the control loop which has permitted the tracking of the planned trajectory despite of the non measurability of the system state variables and the noise which may occur on the process.

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