Cylindrical invisibility cloak based on photonic crystal layers that permits communication with the outside

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Invisibility cloaks designed by transformation optics include a perfect shield, which exclude electromagnetic fields from the cloaked region. Due to the shield, observers inside the cloak cannot see the outside. We propose a cloak that permits communication with the outside, based on a layered photonic crystal (PC) structure. The PC acts as an effective shield in the reflection bandgap, leaving the transmission band available for communication with the outside. A procedure to design an infinitely long cylindrical cloak consisting of concentric layers of dielectric and metal is given. For the proposed structure, the performance of cloaking in the reflection band and of communication in the transmission band is computed.

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1. INTRODUCTION

An invisibility cloak guides light around an object and conceals it from electromagnetic detection. Currently, there is increasing interest in achieving theoretically and practically invisibility using nonmagnetic homogeneous isotropic materials. The original design of the cloak via coordinate transformation was proposed by Pendry *et al.* [1 and Leonhardt and co-workers [,3]. Some practical cloaks were successfully demonstrated both by experiments [4] and by numerical simulations [5,6]. However, the design involves inhomogeneous anisotropic permittivities and permeabilities that cannot be easily realized. A nonmagnetic cloak was proposed by Cai *et al.* [7,8], but the design still has inhomogeneous anisotropic permittivities. A nonmagnetic cloak consisting of homogeneous isotropic materials was proposed by Huang *et al.* [9], which is realized by a subwavelength concentric layered structure. Recently, an alternating five-layered cloak consisting of dielectric and metal was demonstrated by optimizing layer thicknesses [10].

However, these conventional cloaks lead in principle to a perfect shield, excluding electromagnetic fields from the cloaked region [1]. A perfect electric conductor (PEC) shell effectively plays a role of the perfect shield because it also gives zero electromagnetic fields inside the cloaked region, and is numerically more convenient than the infinite permittivity values of a perfect shield [6,11–13]. Due to the shield, observers inside the cloaks cannot see the outside as shown in Fig. 1(a). Although cloaking can work only for a single wavelength because of the causality limit [1–3,14], communication using other wavelengths has many potential applications. A photonic crystal (PC) is suitable for the purpose, because the PC acts as an effective shield for an invisible wavelength in the reflection bandgap, leaving the transmission band available for communication [15]. As shown in Fig. 1(b), we can use the PC as a substitute for the PEC in the conventional cloak (Fig. 1(a)).

The open-cloak [16] and anticloak [17] have been proposed to see the outside. While the open-cloak has an open window and permits both cloaking and communication, its cloaking performance is degraded by the window. The anticloak is joined to the inner cloak and prevents the cloaking, but does not permit simultaneous cloaking and communication with the outside. On the other hand, the frequency-tunable cloaks by moderating input intensities in nonlinear structures [18] and cloaks employing PC structures [19–22] potentially can see the outside by using the transmission band. In particular, Farhat and co-workers have proposed the cylindrical cloak consisting of small infinitely conducting sectors [23–25], and have described the dispersive properties. However, it is difficult to simultaneously optimize the cloaking and communication performance in such structures, and communication with the outside has not been addressed.

In this paper, we design an infinitely long cylindrical cloak consisting of alternating layered structure of dielectric and metal. A procedure to optimize the cloaking and communication performances is illustrated. The proposed cloak successfully functions without disturbing the light path for an invisible wavelength in the reflection bandgap of the PC. For a wavelength in the transmission band, about 63% transmittance is demonstrated using the finite-difference time-domain (FDTD) method.

2. DESIGN OF A LAYERED CLOAK WITH THE PHOTONIC CRYSTAL

For the sake of simplicity, we restrict the problem to the two-dimensional transverse magnetic (TM) mode. In the TM mode, the nonmagnetic cylindrical cloak can be expressed by the following parameter set [7]:

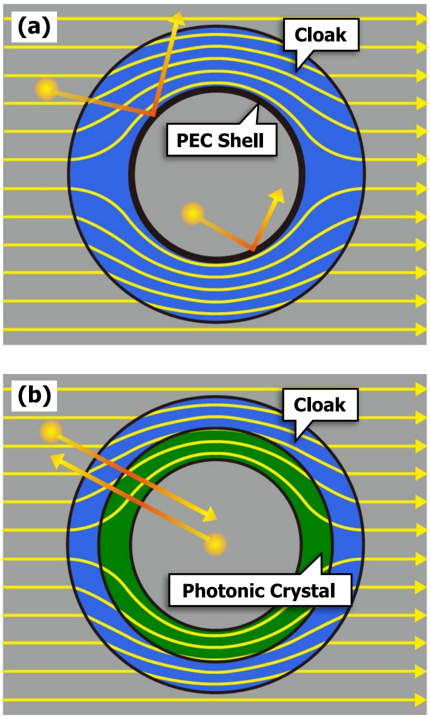

Fig. 1. (Color online) Invisibility cloaks with (a) PEC shield and (b) photonic crystal (PC). Whereas the PEC reflects light of all wavelengths, the PC permits communication between the inside and outside in the transmission band.

$$\varepsilon_\theta = \left(\frac{b}{b-a}\right)^2, \qquad \varepsilon_r = \left(\frac{b}{b-a}\right)^2\left(\frac{r-a}{r}\right)^2, \qquad \mu_z = 1, \quad (1)$$

where ε is the relative permittivity, μ is the relative permeability, and a, b are the inner and outer radii of the cloak, respectively. When the layers are sufficiently thin compared with electromagnetic field wavelengths, the single anisotropic medium can be constructed by concentric layers [26]. Equation (1) is then related to two different layer permittivities (ε_M, ε_D) as follows [9,10]:

$$\varepsilon_\theta = \frac{\varepsilon_D + \eta\varepsilon_M}{1+\eta}, \qquad \frac{1}{\varepsilon_r} = \frac{1}{1+\eta}\left(\frac{1}{\varepsilon_D} + \frac{\eta}{\varepsilon_M}\right), \quad (2)$$

where η is the ratio of ε_M and ε_D thicknesses.

Based on these fundamentals, we design an infinitely long cylindrical cloak consisting of 12 alternating layers of dielectric (relative permittivity $= \varepsilon_D$) and metal (relative permittivity $= \varepsilon_M$). Figure 2(a) shows the proposed structure: six inside and six outside layers act as the PC and cloak, respectively; the inside and outside of the structure is vacuum; R is the radius of the cloaked region; t_0 is the thickness of the six PC layers; $t_1, t_2, \ldots, t_6$ are the thicknesses of the six cloak layers; and $m = 1, 2, \ldots, 12$ is the layer number. Relations of layers, materials and thicknesses in the designed structure are listed in Table 1. As an example, we take the invisible wavelength to be $\lambda_0 = 700$ nm (angular frequency $\omega = 2.691$ PHz), $R = 500$ nm, $t_0 = 20$ nm, $\varepsilon_D = 100$, and ε_M is given by the nonabsorbing Drude model,

$$\varepsilon_M = 1 - \frac{\omega_p^2}{\omega^2}, \quad (3)$$

where the plasma frequency is $\omega_p = 2.677$ PHz ($\varepsilon_M = 0.01$ for $\lambda_0 = 700$ nm).

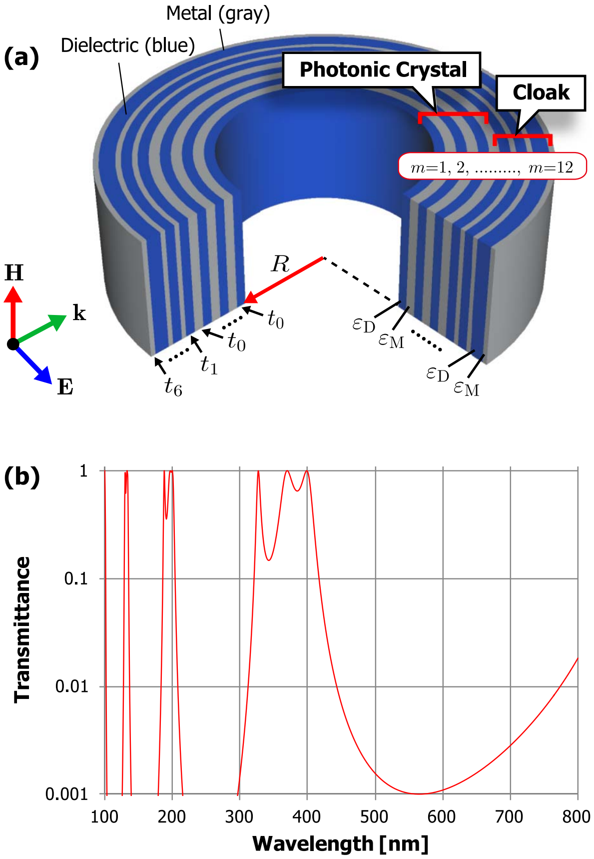

Fig. 2. (Color online) Infinitely-long cylindrical cloak with PC for TM plane wave incidence. (a) Alternating layered structure consisting of dielectric (relative permittivity $= \varepsilon_D$) and metal (relative permittivity $= \varepsilon_M$). Six inside and six outside layers act as the PC and cloak, respectively. Thickness of the six PC layers is t_0. Thicknesses of the six cloak layers are $t_1, t_2, \ldots, t_6$, respectively. Electromagnetic fields **E**, **H**, and the wave vector **k** are depicted. (b) Transmission spectrum for the six-layered 1D PC with $t_0 = 20$ nm.

We first confirm that the PC layers functions as an effective shield for the invisible wavelength $\lambda_0 = 700$ nm. The concentric layers form a one-dimensional (1D) PC with respect to radial propagation. Figure 2(b) shows the transmission spectrum for a six-layered 1D PC with $t_0 = 20$ nm. The PC gives less than 1% transmission at $\lambda_0 = 700$ nm in the reflection bandgap. Even though the plane wave impinging on the infinitely long cylinder is not radially 1D, the light entering into the cloaked region is significantly reduced as demonstrated later.

Next we optimize the thicknesses of the cloak layers, $t_1, t_2, \ldots, t_6$, by minimizing the total scattering width. Whereas, mapping the ideal parameters of Eq. (1) to layer parameters in Eq. (2) requires many layers [9], the optimization permits one to construct the cloak with just a few layers and furthermore improves the invisibility [10]. The total scattering width can be analytically calculated by Mie theory [27] as the following. The incident (H_z^0) and scattered (H_z^s) magnetic fields for TM incidence along the x-direction are given by

$$H_z^0 = \sum_{n=-\infty}^{\infty} i^n J_n(k_0 r) e^{in\phi}, \quad (4)$$

Table 1. Relations of Layers, Materials and Thicknesses in Designed Structure, with *D* and *M* Indicating the Dielectric and Metal, Respectively

	Inside						Outside					
Layer No.	1	2	3	4	5	6	7	8	9	10	11	12
Material	*D*	*M*	*D*	*M*	*D*	*M*	*D*	*M*	*D*	*M*	*D*	*M*
Thickness	t_0	t_0	t_0	t_0	t_0	t_0	t_1	t_2	t_3	t_4	t_5	t_6

$$H_z^s = \sum_{n=-\infty}^{\infty} i^n c_n H_n^{(1)}(k_0 r) e^{in\phi}, \tag{5}$$

where J_n is the Bessel function of the first kind, $H_n^{(1)}$ is the Hankel function of the first kind, k_0 is the wavenumber outside the structure, and c_n is the expansion coefficient. The total magnetic fields in the cloaked region (H_z^i) and in the mth layer (H_z^m) are expressed by

$$H_z^i = \sum_{n=-\infty}^{\infty} i^n d_n J_n(k_0 r) e^{in\phi}, \tag{6}$$

$$H_z^m = \sum_{n=-\infty}^{\infty} i^n (a_n^m J_n(k_m r) + b_n^m H_n^{(1)}(k_m r)) e^{in\phi}, \tag{7}$$

where k_m is the wavenumber in the mth layer ($m = 1, 2, \ldots, 12$), and a_n^m, b_n^m, d_n are expansion coefficients. The a_n^m, b_n^m, c_n, and d_n are determined by enforcing the boundary conditions. In the TM mode, the continuity of H_z and E_ϕ parallel to the interface are enforced, where E_ϕ is given by

$$E_\phi = \frac{1}{i\omega\varepsilon} \frac{\partial H_z}{\partial r}. \tag{8}$$

The strength of the electromagnetic scattering by an infinitely long cylinder can be measured by computing the scattering width,

$$\sigma(\phi) = \lim_{r\to\infty} 2\pi r \frac{|H_z^s|^2}{|H_z^0|^2}. \tag{9}$$

The cloak successfully functions when the σ is minimized in all directions. We optimize the thicknesses of the cloak layer, $t_1, t_2, \ldots, t_6$, using the quasi-Newton method. To minimize the σ in all directions, the total scattering width is chosen as the optimization function,

$$\sigma_t = \int \sigma \mathrm{d}\phi. \tag{10}$$

By the optimization, we obtained a set of thicknesses for the cloak layer as follows: $t_1 = 22.66$ nm, $t_2 = 4.53$ nm, $t_3 = 18.12$ nm, $t_4 = 2.249$ nm, $t_5 = 32.63$ nm, and $t_6 = 1.206$ nm. We note that the thicknesses are scalable to any size increasing the wavelength, and several decimal places are retained in order to permit scaling up. On the other hand, the number of layers and their thicknesses are functions of permittivity. In order to simplify the problem, we chose a high value of dielectric permittivity. Using more layers, one could choose a lower value of permittivity and thicker layers to achieve cloaking [9,10].

To demonstrate the performance of the designed cloak with the PC, we calculate the magnetic field distribution given by Eqs. (4)–(7). Figures 3(a) and 3(b) show the calculated H_z field and intensity distributions for the incident wavelength of 700 nm, respectively. The maximum intensity inside the cloaked region is about a thirtieth that of the incident intensity, and is much lower in many places. Therefore, even when an object is placed in the cloaked region, the external scattering increases very little. In figure 3(c), scattering widths for the designed structure and an uncloaked PEC cylinder with $R = 500$ nm are compared. The designed cloak has a scattering width less than about 1% of the PEC in all directions.

3. COMMUNICATION BETWEEN THE INSIDE AND OUTSIDE OF THE CLOAK

Communication through the designed cloak is realized in the transmission band. Figure 4 shows the transmission spectrum for the 12-layer radially 1D structure, which includes designed thicknesses, $t_0, t_1, \ldots, t_6$. For communication, we choose the wavelength $\lambda_0 = 198$ nm in the transmission band which has the highest transmittance ≅0.97. However, the practical ratio of the transmitted energy to the incidence is lower than in the 1D case, because only a small part of the incident field is

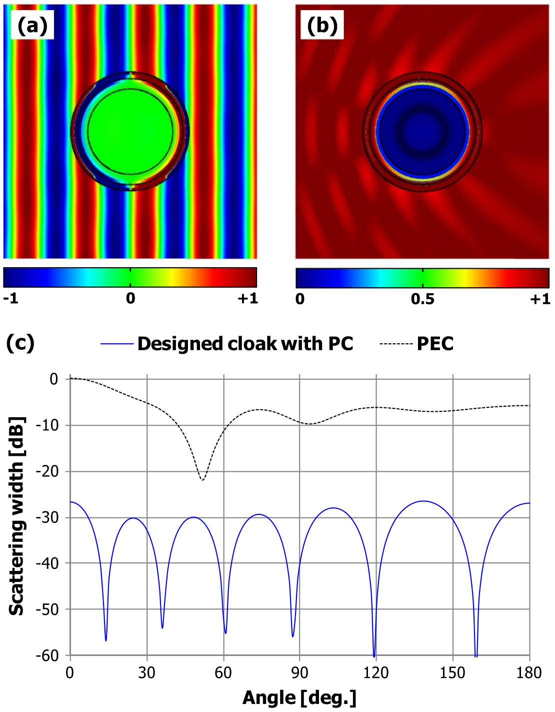

Fig. 3. (Color online) Calculated magnetic field distribution for the designed cloak with a PC (incident wavelength = 700 nm). (a) H_z field (Media 1). (b) H_z intensity. (c) Scattering widths for the designed cloak and an uncloaked PEC cylinder with $R = 500$ nm.

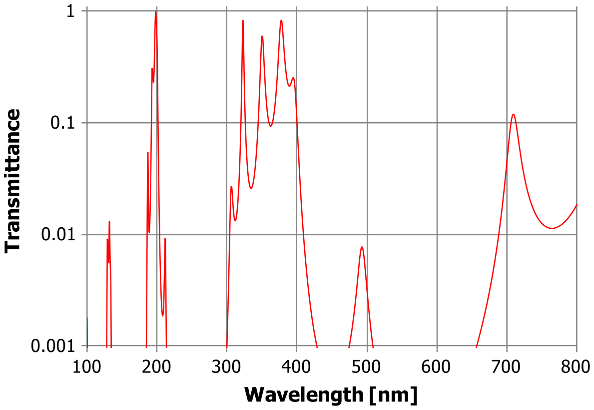

Fig. 4. (Color online) Transmission spectrum for the 12-layer radially 1D structure, which includes designed thicknesses, $t_0, t_1, \ldots, t_6$. We set 198 nm to be the communication wavelength.

normally incident for the cylindrical geometry. At the invisible wavelength of 700 nm, the transmittance for 12 layers (Fig. 4) rises to about 10% relative to those of six layers (Fig. 2(b)), but the light entering into the cloaked region is sufficiently reduced as shown in the previous section (Fig. 3).

In order to verify the transmission through the designed structure, we calculate electromagnetic propagation from the inside to the outside using the FDTD method [28]. Figure 5(a) shows the simulation setup: a TM wave source of $\lambda_0 = 198$ nm is inside the structure; the grid spacing is $h = 0.5$ nm; the time step is given by the Courant limit; and the computational domain of 1.8 μm × 1.5 μm is terminated by the perfectly matched layer (PML) absorbing boundaries. Figure 5(b) shows the calculated H_z field distribution at the steady state (about 120 wave periods). The energy ratio of the transmitted light through the layer structure to the incident is about 63%, and thus the designed structure allows practical communication between the inside and outside.

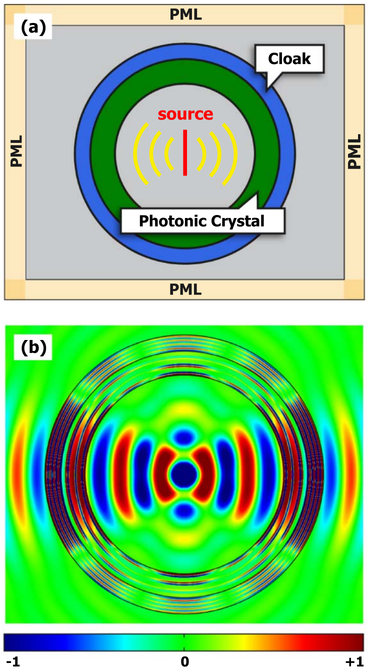

Fig. 5. (Color online) FDTD simulation of the designed cloak with a PC. (a) Simulation setup. (b) Calculated H_z field distribution at the steady state using the wavelength of 198 nm (Media 2).

4. CONCLUSION

We have designed an infinitely long cylindrical cloak with a PC consisting of 12 alternating layers of dielectric and metal. Six inside and six outside layers of the designed structure act as the PC and cloak, respectively. The PC prevents light from entering into the cloaked region in the reflection bandgap, leaving the transmission band available for communication. Designing the cloak with the PC requires three steps: (1) choose an invisible wavelength in the PC reflection bandgap; (2) optimize the thicknesses of cloak layers by minimizing the total scattering width; and (3) choose the optimal communication wavelength in the transmission band. In this example, $\lambda_0 = 198$ nm is the optimal wavelength for communication, but as Fig. 4 shows, other wavelengths (for example in the 360–400 nm band) are also available.

The proposed cloak successfully functions in the reflection band of the PC, and permits communication between the outside in transmission band. The performance of the designed cloak has been demonstrated in analytic calculations of the magnetic field, intensity, and scattering width. Communication through the designed structure has been verified by the FDTD simulation.

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REFERENCES

1. J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” Science **312**, 1780–1782 (2006).
2. U. Leonhardt, “Optical conformal mapping,” Science **312**, 1777–1780 (2006).
3. U. Leonhardt and T. G. Philbin, “Transformation optics and the geometry of light,” Prog. Opt. **53**, 69–152 (2009).
4. D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, “Metamaterial electromagnetic cloak at microwave frequencies,” Science **314**, 977–980 (2006).
5. S. A. Cummer, B. L. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, “Full-wave simulations of electromagnetic cloaking structures,” Phys. Rev. E **74**, 036621 (2006).
6. Y. Zhao, C. Argyropoulos, and Y. Hao, “Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures,” Opt. Express **16**, 6717–6730 (2008).
7. W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, “Optical cloaking with metamaterials,” Nat. Photonics **1**, 224–227 (2007).
8. W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, “Nonmagnetic cloak with minimized scattering,” Appl. Phys. Lett. **91**, 111105 (2007).
9. Y. Huang, Y. Feng, and T. Jiang, “Electromagnetic cloaking by layered structure of homogeneous isotropic materials,” Opt. Express **15**, 11133–11141 (2007).
10. Z. Yu, Y. Feng, X. Xu, J. Zhao, and T. Jiang, “Optimized cylindrical invisibility cloak with minimum layers of non-magnetic isotropic materials,” J. Phys. D **44**, 185102 (2011).
11. T. J. Cui, D. R. Smith, and R. Liu, *Metamaterials: Theory, Design, and Applications*, 1st ed. (Springer-Verlag, 2009).
12. W. Yan, M. Yan, Z. Ruan, and M. Qiu, “Influence of geometrical perturbation at inner boundaries of invisibility cloaks,” Nat. Photonics **25**, 968–973 (2008).
13. C. Argyropoulos, E. Kallos, Y. Zhao, and Y. Hao, “Manipulating the loss in electromagnetic cloaks for perfect wave absorption,” Opt. Express **17**, 8467–8475 (2009).

14. P. Yao, Z. Liang, and X. Jiang, “Limitation of the electromagnetic cloak with dispersive material,” Appl. Phys. Lett. **92**, 031111 (2008).
15. M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 7th ed. (Cambridge University, 1999).
16. H. Ma, S. Qu, Z. Xu, and J. Wang, “The open cloak,” Appl. Phys. Lett. **94**, 103501 (2009).
17. H. Chen, X. Luo, H. Ma, and C. T. Chan, “The anti-cloak,” Opt. Express **16**, 14603–14608 (2008).
18. C. Argyropoulos, P. Y. Chen, F. Monticone, G. D’Aguanno, and A. Alu, “Nonlinear plasmonic cloaks to realize giant all-optical scattering switching,” Phys. Rev. Lett. **108**, 263905 (2012).
19. J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, “An optical cloak made of dielectrics,” Nat. Mater. **8**, 568–571 (2009).
20. L. H. Gabrielli, J. Cardenas, C. B. Poitras, and M. Lipson, “Silicon nanostructure cloak operating at optical frequencies,” Nat. Photonics **3**, 461–463 (2009).
21. A. Yaroslav and D. R. Smith, “Transformation optics with photonic band gap media,” Phys. Rev. Lett. **105**, 163901 (2010).
22. H. Gao, B. Zhang, and G. Barbastathis, “Photonic cloak made of subwavelength dielectric elliptical rod arrays,” Opt. Commun. **284**, 4820–4823 (2011).
23. M. Farhat, S. Guenneau, A. B. Movchan, and S. Enoch, “Achieving invisibility over a finite range of frequencies,” Opt. Express **16**, 5656–5661 (2008).
24. S. Guenneau, R. C. McPhedran, S. Enoch, A. B. Movchan, M. Farhat, and N. A. P. Nicorovici, “The colours of cloaks,” J. Opt. **13**, 024014 (2011).
25. V. N. Smolyaninova, I. I. Smolyaninov, and H. K. Ermer, “Experimental demonstration of a broadband array of invisibility cloaks in the visible frequency range,” New J. Phys. **14**, 053029 (2012).
26. B. Wood and J. B. Pendry, “Directed subwavelength imaging using a layered metaldielectric system,” Phys. Rev. B **74**, 115116 (2006).
27. P. W. Barber and S. C. Hill, *Light Scattering by Particles: Computational Methods* (World Scientific, 1990).
28. A. Taflove, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, 2nd ed. (Artech House, 2000).