

$D = 4$ Chiral String Compactifications from Intersecting Branes

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Abstract

Intersecting Dp -branes often give rise to chiral fermions living on their intersections. We study the construction of four-dimensional chiral gauge theories by considering configurations of type II $D(3+n)$ -branes wrapped on non-trivial n -cycles on $\mathbf{T}^{2n} \times (\mathbf{R}^{2(3-n)}/\mathbf{Z}_N)$, for $n = 1, 2, 3$. The gauge theories on the four non-compact dimensions of the brane world-volume are generically chiral and non-supersymmetric. We analyze consistency conditions (RR tadpole cancellation) for these models, and their relation to four-dimensional anomaly cancellation. Cancellation of $U(1)$ gauge anomalies involves a Green-Schwarz mechanism mediated by RR partners of untwisted and/or twisted moduli. This class of models is of potential phenomenological interest, and we construct explicit examples of $SU(3) \times SU(2) \times U(1)$ three-generation models. The models are non-supersymmetric, but the string scale may be lowered close to the weak scale so that the standard hierarchy problem is avoided. We also comment on the presence of scalar tachyons and possible ways to avoid the associated instabilities. We discuss the existence of (meta)stable configurations of D-branes on 3-cycles in $(\mathbf{T}^2)^3$, free of tachyons for certain ranges of the six-torus moduli.

1 Introduction

D-branes have turned out to be a key ingredient in our present understanding of the structure of string theory. Interestingly, the fact that D-branes contain gauge fields localized on their world-volume has also suggested new scenarios for string phenomenology and phenomenology beyond the standard model in general (see e.g.[1, 2, 3, 4, 5]). From this point of view, it is important to explore different configurations of branes which can lead to interesting features for phenomenological model building.

An important observation [6] is that intersecting D-branes in flat space may give rise to chiral fermions propagating on the intersection of their world-volumes, arising from open strings stretching between the D-branes. Hence, it is natural to consider the construction of four-dimensional chiral models by compactifications including intersecting D-branes. In this framework, the compactification space can be essentially flat, like a six-torus, since chirality arises from fermions at the intersection of D-brane world-volumes, and does not depend so much on the holonomy group of the ambient space. This is in contrast with more familiar compactifications with D-branes, like type IIB orientifolds [7, 8, 9] or heterotic string compactifications [10, 11], where chirality arises due to the ambient space being Calabi-Yau threefold.

In this paper we perform a systematic exploration of configurations of $D(3+n)$ -branes wrapped on n -cycles in an $2n$ -dimensional torus \mathbf{T}^{2n} , and sitting at a point in a transverse $6 - 2n$ -dimensional space \mathbf{B} . We are interested in configurations leading to chiral four-dimensional gauge theories after reduction on the torus. Chirality is automatically achieved for D6-branes on 3-cycles on \mathbf{T}^6 . However, for models with D4- or D5-branes, if the point in \mathbf{B} at which the D-branes sit is smooth, the resulting intersection lead to vector-like matter. Chiral matter at the intersections can be obtained by considering branes sitting at singular points in \mathbf{B} . We will center on abelian orbifold singularities, whose local geometry can be modeled as $\mathbf{R}^{6-2n}/\mathbf{Z}_N$.

Hence we consider configurations of stacks of $D(3+n)$ -branes wrapped on n -cycles in $\mathbf{T}^{2n} \times \mathbf{R}^{6-2n}/\mathbf{Z}_N$. Each stack of D-branes gives rise to gauge factors, while open strings stretched between them give rise to chiral fermions propagating on the intersections. The resulting four-dimensional gauge theories are of potential phenomenological interest. When the singularity is embedded in a globally compact $(6 - 2n)$ -dimensional variety, one obtains a full-fledged compactification, where gravity is also four-dimensional.

Notice that the case of $n = 0$ corresponds to configurations of D3-branes at $\mathbf{R}^6/\mathbf{Z}_N$ singularities, which were employed in [5] to build realistic gauge theories (see [12] for more formal applications of these systems). Full-fledged compactifications were subse-

quently obtained by embedding the singularities in global compact geometries. Our approach here is similar in spirit to the bottom-up approach introduced in [5], although we mainly center on local features in the present paper. The models also present a number of interesting new properties.

The opposite extreme case, $n = 3$, corresponds to D6-branes wrapped on 3-cycles in \mathbf{T}^6 . Configurations of this type have appeared in [13], but in the presence of an additional orientifold projection¹. This projection is not a necessary ingredient, and it does not improve the phenomenological or theoretical features of the model, hence we choose not to include it. In particular, this allows to get around the orientifold symmetry constraints in [13], which prevented the appearance of three-generation models. Without orientifold action, three-family models are easy to build, and we present a specific example.

In this paper we perform a detailed analysis of the configurations for $n = 1, 2, 3$, their construction with explicit examples, and the main features of the resulting four-dimensional theories. We determine the tadpole cancellation conditions, their geometrical interpretation, and their interplay with the cancellation of chiral four-dimensional anomalies. Interestingly, we find that the theories contain several anomalous $U(1)$'s, and that their anomalies are cancelled by a generalized Green-Schwarz mechanism. The fields that mediate this mechanism correspond, for D6-branes wrapped on 3-cycles on \mathbf{T}^6 , to *untwisted* closed string modes, in contrast with the situation in other string constructions. For $n = 1, 2$ the exchanged fields correspond to reduction on \mathbf{T}^{2n} of fields in twisted sectors of $\mathbf{R}^{6-2n}/\mathbf{Z}_N$.

The models are generically non-supersymmetric, even if the orbifold twist is chosen to preserve some bulk supersymmetry. This leads to two important issues. First, although the discussion of more phenomenological aspects in these constructions will appear elsewhere [15], we would like to mention here the question of scales. Even though models are non-supersymmetric, it is possible to avoid a hierarchy problem in any realistic application, by lowering the string scale down to a TeV. This is possible, i.e. consistent with a large four-dimensional Planck mass, for models with D4- or D5-branes, in the usual way, by simply taking the transverse $(6 - 2n)$ -dimensional space \mathbf{B} large enough. Notice that the $2n$ -torus should remain small (with compactification scale \approx TeV) to avoid too light KK resonances of gauge bosons. Hence, as observed in [13], for models with D6-branes solving the hierarchy problem by large volume compactification is not possible. It is interesting to consider them, however, in case

¹Other models with branes at angles and orientifold and orbifold projections have appeared in [14].

another mechanism is eventually devised. On the other hand, let us emphasize again that a low string scale is consistent with low-energy physics in models with D4- or D5-branes, with a large transverse volume.

The second comment concerns the generic presence of tachyons at brane intersections, which signal an instability against recombining intersecting branes into a single smooth one. Interestingly, for the case of D6-branes on 3-cycles, there exist brane orientations such that the brane recombination process is not energetically favoured, since it implies an increase of the wrapped volume. The corresponding intersection hence does not lead to tachyonic states. Hence it is in principle possible to construct compact models of D6-branes on \mathbf{T}^6 where all intersections have this property, and the resulting model is (meta)stable, as we discuss in some detail. In models with D4- or D5-branes, it is possible to construct models where most of the tachyons at intersections are projected out by the \mathbf{Z}_N orbifold twist in the quotient singularity.

In any event, we think it is also interesting to consider models with a small set of tachyons. Recent developments (see [16] for a review) have suggested that much can be learnt by considering unstable configurations in string theory and their decay. On the speculative side, a possible phenomenological application of these ideas would be to interpret the tachyon condensation process as a Higgs mechanism, in which the two gauge factors associated to the intersecting branes break to a smaller subgroup carried by the recombined brane. In fact, it is possible to construct explicit semirealistic models of D4-branes, where the only tachyons have the quantum numbers of Standard Model Higgs multiplets. It is tempting to speculate that the effect of the instability is Higgs breaking of electroweak symmetry (see [15] for further details).

The paper is organized as follows. In Section 2 we discuss generalities about the configuration of intersecting branes, and the spectrum arising on their world-volume and on their intersections. In Section 3 we discuss the construction of models of D6-branes wrapped on 3-cycles in \mathbf{T}^6 . We analyze their spectrum, the tadpole cancellation conditions and their interpretation, and cancellation of non-abelian and mixed $U(1)$ anomalies. We also present several explicit examples, e.g. leading to three-generation Standard Model gauge sectors. We also comment on the possibility of understanding tachyon condensation as symmetry breaking by a Higgs mechanism. A similar analysis is carried out for configurations of D4-branes in Section 4, and of D5-branes in Section 5. Section 6 contains our final remarks.

2 Intersecting Dp-branes

Let us start by considering some generic properties of the spectrum for branes at angles. We start considering D-branes wrapped on d -cycles \mathbf{T}^{2d} . We choose a factorizable \mathbf{T}^{2d} , product of d two-dimensional rectangular tori \mathbf{T}_I^2 parameterized by compact coordinates X_1^I, X_2^I , with radii R_1^I, R_2^I , with $I = 1, \dots, d$. We introduce K different sets of N_a coincident Dp_a -branes, labeled by an index a , $a = 1 \dots, K$. Each set wraps around a 1-cycle Π_a^I , of type (n_a^I, m_a^I) , on each of the d two-tori. Namely, it wraps n_a^I times around the X_1^I direction and m_a^I times around the X_2^I direction. The angle of these branes with the X_1^I axis is hence given by

$$\tan \vartheta_a^I = \frac{m_a^I R_2^I}{n_a^I R_1^I} \quad (2.1)$$

with an obvious modification for skewed two-tori.

The compactification preserves all 32 supersymmetries of type II theory in the closed string sector. The sector of open strings stretching between Dp_a -branes within the same set, preserves 16 supersymmetries, hence giving rise to the corresponding gauge supermultiplet with gauge group $U(N_a)$ ². This piece of the spectrum is non-chiral, so the only source of chiral fields is the sector of open strings stretched between different sets of branes.

The spectrum of such sectors has been studied in [6]. World-sheet bosonic fields for open strings stretching between Dp_a - and Dp_b -branes, at a relative angle $\vartheta_{ba}^I = (\vartheta_b^I - \vartheta_a^I)/\pi$, (given in ‘units of π ’ for convenience), in the I^{th} two-torus, satisfy the boundary conditions

$$\begin{aligned} \sin \vartheta_a^I \partial_\sigma X_1^I - \cos \vartheta_a^I \partial_\sigma X_2^I &= 0 \\ \sin \vartheta_a^I \partial_t X_2^I - \cos \vartheta_a^I \partial_t X_1^I &= 0 \end{aligned} \quad (2.2)$$

at $\sigma = 0$, and a similar equation for $\sigma = \pi$ with $a \rightarrow b$. Corresponding equations are satisfied by fermionic coordinates. Such boundary conditions lead to *twisted* mode expansions, with twist given by the relative angle ϑ_{ba}^I between branes. For instance, one obtains worldsheet fermionic modes $\psi_{r_\pm}^I$, with modes $r_\pm = n \pm \vartheta_{ba}^I + \nu$, where n is integer and $\nu = 0, 1/2$ for R and NS boundary conditions respectively. No windings

²If the wrapping numbers (n, m) are not coprime, $r = \text{gcd}(n, m) \neq 1$, the D-brane is multiwrapped r times over the cycle $(n/r, m/r)$. This state can be equivalently described as r D-branes on $(n/r, m/r)$ with an order r permutation wilson line turned on. For N such multiwrapped branes, the world-volume gauge group is $U(N)^r$. We thank R. Blumenhagen, B. K ors and D. L ust for discussion on multiwrapped branes.

or KK momenta are allowed for non trivial angles. Antiparticles of states in the ab sector appear in the ba sector.

We are mainly interested in four-dimensional intersections, hence we consider the cases of $D(3+n)$ -branes wrapped on n -cycles on \mathbf{T}^{2n} , for $n = 1, 2, 3$. As mentioned in the introduction, configurations $n < 3$ would lead to non-chiral intersections, hence we will eventually turn to configurations with singular transverse spaces, namely $D(3+n)$ -branes on n -cycles in $\mathbf{T}^{2n} \times \mathbf{R}^{6-2n}/\mathbf{Z}_N$. Before that, it is convenient to discuss the simpler case of $\mathbf{T}^{2n} \times \mathbf{R}^{6-2n}$ in this section, namely D6-branes on \mathbf{T}^6 , D5-branes on $\mathbf{T}^4 \times \mathbf{C}$, D4-branes on $\mathbf{T}^2 \times \mathbf{C}^2$.

The mass operator for strings stretching between branes in the a^{th} and b^{th} set, which make an angle $\vartheta^I \equiv \vartheta_{ab}^I$ on the I^{th} two-torus is [17, 6]

$$\alpha' M_{ab}^2 = \frac{Y^2}{4\pi^2\alpha'} + N_\nu + \nu \sum_{I=1}^d \vartheta_{ab}^I - \nu \quad (2.3)$$

where Y^2 measures the length of the stretched string (minimal distance between branes for minimum winding states), and N_ν is the number operator given by

$$\begin{aligned} N_0 &= \sum_{n>0} (\alpha_{-n} \cdot \alpha_n + r \psi_{-r} \psi_r) + \sum_{I=1}^d [\sum_{n>0} (\alpha_{-n_+}^I \cdot \alpha_{n_+}^I + \alpha_{-n_-}^I \cdot \alpha_{n_-}^I) + \alpha_{-\vartheta^I}^I \alpha_{\vartheta^I}^I] \\ &+ \sum_{I=1}^d [\sum_{n>0} (r_- \psi_{-r_+}^I \cdot \psi_{r_+}^I + r_- \psi_{-r_-}^I \cdot \psi_{r_-}^I) + \vartheta^I \psi_{-\vartheta^I}^I \cdot \psi_{\vartheta^I}^I] \end{aligned} \quad (2.4)$$

for the R sector ($\nu = 0$) and by

$$\begin{aligned} N_{\frac{1}{2}} &= \sum_{n>0} (\alpha_{-n} \cdot \alpha_n + r \psi_{-r} \psi_r) + \sum_{I=1}^d [\sum_{n>0} (\alpha_{-n_+}^I \cdot \alpha_{n_+}^I + \alpha_{-n_-}^I \cdot \alpha_{n_-}^I) + \alpha_{-\vartheta^I}^I \alpha_{\vartheta^I}^I] \\ &+ \sum_{I=1}^d \sum_{n=0} (r_+ \psi_{-r_+}^I \cdot \psi_{r_+}^I + r_- \psi_{-r_-}^I \cdot \psi_{r_-}^I) \end{aligned} \quad (2.5)$$

in the NS($\nu = \frac{1}{2}$) sector.

The ϑ_{ab}^I in the mass equation (2.3) arises from normal ordering of twisted zero modes and it cancels out in the R sector. In the derivation of this expression we have assumed that $0 \leq \vartheta_{ab}^I \leq \frac{1}{2}$, so oscillators modes as above are correctly normal ordered. For negative angles one should replace $\vartheta_I \rightarrow |\vartheta_I|$.

The spectrum can be described in bosonic language as follows. We introduce a four-dimensional twist vector v_ϑ , whose I^{th} entry is given by ϑ_{ab}^I . The GSO projected states are labeled by a four-dimensional vector $r + v_\vartheta$, where $r_I \in \mathbf{Z}, \mathbf{Z} + \frac{1}{2}$ for NS,

R sectors respectively, and $\sum_I r_I = \text{odd}$. The last entry provides four-dimensional Lorentz quantum numbers. The mass of the states is then given by

$$\alpha' M_{ab}^2 = \frac{Y^2}{4\pi^2\alpha'} + N_{bos}(\vartheta) + \frac{(r+v)^2}{2} - \frac{1}{2} + E_{ab} \quad (2.6)$$

with

$$E_{ab} = \sum_I \frac{1}{2} |\vartheta_I| (1 - |\vartheta_I|) \quad (2.7)$$

and $N_{bos}(\vartheta)$ is a contribution from bosonic oscillators.

Let us discuss the computation of lowest lying states in the different models. As mentioned above, models for $n < 3$ have a non-chiral spectrum, as is easily seen from the fact that all massless states can be made massive in a continuous way by increasing the separation Y^2 in transverse space.

We first consider the case of D4-branes on $\mathbf{T}^2 \times \mathbf{C}^2$. In the NS sector, the lowest mass state allowed by GSO projection [6] corresponds to $\psi_{(\vartheta^1-1/2)}^I |0\rangle_{NS}$, or $r+v = (-1 + \vartheta^1, 0, 0, 0)$ in bosonic language. Its mass is given by

$$\alpha' M_1^2 = \frac{Y}{4\pi^2\alpha'} - \frac{1}{2} |\vartheta^1| \quad (2.8)$$

Thus, a tachyon is generated when D4 branes come closer than the critical distance $Y^2 = 2\pi^2\alpha' |\vartheta^1|$. The tachyon signals an instability against joining the intersecting branes into a single one, which then wraps a one-cycle in the homology class $[\Pi_a] + [\Pi_b]$, namely a $(n_a + n_b, m_a + m_b)$ cycle on \mathbf{T}^2 . In Section 4 we discuss how to use a \mathbf{Z}_N orbifold twist to project out some of these tachyons.

The R groundstate contains four fermions that become massless at zero transverse distance. They are given by

$$\begin{aligned} \left(-\frac{1}{2} + \vartheta^1, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\right) & \quad ; \quad \left(-\frac{1}{2} + \vartheta^1, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}\right) \\ \left(-\frac{1}{2} + \vartheta^1, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) & \quad ; \quad \left(-\frac{1}{2} + \vartheta^1, +\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}\right) \end{aligned} \quad (2.9)$$

There are two pairs of opposite chirality spinors, so the spectrum is non-chiral. A possibility to obtain a chiral spectrum is to project out some of the above fermions, for instance by locating the D4-branes at $\mathbf{C}^2/\mathbf{Z}_N$ singularities in transverse space, see Section 4.

In the case of configurations of D5-branes on $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{C}$, open strings at intersections have a twist vector $(\vartheta_1, \vartheta_2, 0, 0)$. In the NS sector, assuming $0 < \vartheta_I < 1$ the lowest mass NS states correspond to $\psi_{-1/2+\vartheta_1} |0\rangle$, $\psi_{-1/2+\vartheta_2} |0\rangle$, or in bosonic language $(-1 + \vartheta_1, 0, 0, 0)$, $(0, -1 + \vartheta_2, 0, 0)$. Their masses are $M_1^2 = \frac{1}{2}(\vartheta_2 - \vartheta_1)$ and

$M_2^2 = -\frac{1}{2}(\vartheta_2 - \vartheta_1) = -M_1^2$, respectively. Thus, unless $|\vartheta_2| = |\vartheta_1|$, in which case the intersection preserves some supersymmetry, there is always a tachyonic state. The R spectrum contains a set of non-chiral massless fermions, corresponding to the states $(-\frac{1}{2} + \vartheta^1, -\frac{1}{2} + \vartheta^2, -\frac{1}{2}, \frac{1}{2})$ and $(-\frac{1}{2} + \vartheta^1, -\frac{1}{2} + \vartheta^2, \frac{1}{2}, -\frac{1}{2})$. Again, tachyon elimination and chirality may be obtained by imposing an orbifold projection, namely by considering D5-branes wrapped on \mathbf{T}^4 and located at the origin of a \mathbf{C}/\mathbf{Z}_N singularity, as we do in section 5.

As mentioned, a chiral spectrum is obtained for D6-brane intersections on \mathbf{T}^6 . The twist vector is now given by $(\vartheta_1, \vartheta_2, \vartheta_3, 0)$. In the NS sector, the lowest lying states, for $0 \leq \vartheta^I \leq 1$, are given by $(-1 + \vartheta^1, \vartheta^2, \vartheta^3, 0)$, $(\vartheta^1, -1 + \vartheta^2, \vartheta^3, 0)$, $(\vartheta^1, \vartheta^2, -1 + \vartheta^3, 0)$, and $(-1 + \vartheta^1, -1 + \vartheta^2, -1 + \vartheta^3, 0)$. As discussed in more detail in Section 3, some of them may be tachyonic, but not necessarily. In the R sector, we obtain a single chiral fermion, given by $(-\frac{1}{2} + \vartheta^1, -\frac{1}{2} + \vartheta_2, -\frac{1}{2} + \vartheta_3, +\frac{1}{2})$.

We conclude by emphasizing an important point. Branes wrapped on cycles generically intersect at multiple points, hence the above states in mixed ab sectors appear in several copies, this multiplicity being given by the intersection number of the corresponding wrapped cycles. (If e.g. one of the branes, say the b^{th} has non-coprime (n, m) , the multiplets in the ab sector transform as $\sum_{l=1}^r \tilde{I}_{ab}(N_a, N_{b,r})$ under the gauge group $U(N_a) \times U(N_b)^r$, and $\tilde{I}_{ab} = I_{ab}/r$).

3 D6-branes wrapping at angles on $(\mathbf{T}^2)^3$

3.1 Construction

In this section we consider type IIA theory compactified on a factorizable \mathbf{T}^6 . We consider a configuration containing K stacks of N_a D6-branes, $a = 1, \dots, K$, wrapped on three-cycles Π_a obtained as the product of one-cycles (n_a^I, m_a^I) on each of the three two-tori $I = 1, 2, 3$ ³. In [13] this kind of D6-brane configurations were considered in the presence of an orientifold projection. Since the projection is not required for consistency, we prefer not to impose this restriction and keep our analysis general.

The models admit a T-dual description [6, 13] in terms of type IIB compactified on a T-dual torus $\tilde{\mathbf{T}}^6$ (with the Kahler and complex structure on each two-tori exchanged with respect to the original one), with a set of D9-branes (and anti-D9-branes), with wrapping numbers n_a^I and world-volume magnetic flux with charge m_a^I along the I^{th}

³This type of configuration is a particular case of configurations of D6-branes wrapped on special lagrangian cycles in a Calabi-Yau threefold (see [18] for recent discussions).

two-torus. (Models with such fluxes and orbifold and orientifold projections have appeared in [19]). Even though we phrase our discussion in D6-brane language, we will find it useful to occasionally turn to this T-dual picture.

The configuration can be described by a free world-sheet CFT, and the consistency conditions (tadpole cancellation conditions) can be analyzed by usual factorization of one-loop amplitudes. They read

$$\begin{aligned}
\sum_a N_a n_a^1 n_a^2 n_a^3 &= 0 & \sum_a N_a n_a^1 m_a^2 m_a^3 &= 0 \\
\sum_a N_a m_a^1 n_a^2 n_a^3 &= 0 & \sum_a N_a m_a^1 n_a^2 m_a^3 &= 0 \\
\sum_a N_a n_a^1 m_a^2 n_a^3 &= 0 & \sum_a N_a m_a^1 m_a^2 n_a^3 &= 0 \\
\sum_a N_a n_a^1 n_a^2 m_a^3 &= 0 & \sum_a N_a m_a^1 m_a^2 m_a^3 &= 0
\end{aligned} \tag{3.1}$$

In the D6-brane picture, they are equivalent to the condition that the homology classes $[\Pi_a]$ of the cycles Π_a wrapped by the D6-branes, counted with multiplicity N_a , add up to zero. Denoting by $[a_I]$, $[b_I]$ the homology classes of the $(1, 0)$ and $(0, 1)$ basis cycles in the I^{th} two-torus, we have

$$[\Pi_a] = (n_a^1 [a_1] + m_a^1 [b_1]) \otimes (n_a^2 [a_2] + m_a^2 [b_2]) \otimes (n_a^3 [a_3] + m_a^3 [b_3]) \tag{3.2}$$

and (3.1) can be recast as

$$\sum_a N_a [\Pi_a] = 0 \tag{3.3}$$

The vanishing of the total homology class is required by consistency with the equations of motion for the RR 7-form, under which the D6-branes are electrically charged

$$d * H_8 = \sum_a N_a \delta(\Pi_a) \tag{3.4}$$

where H_8 is the field strength of the 7-form, and $\delta(\Pi_a)$ is a three-form supported at the location of the $D6_a$ -branes, the Poincare dual of $[\Pi_a]$. Since $d * H_8$ is exact, the above equation in homology becomes (3.3).

In the language of D9-branes with fluxes, conditions (3.1) receive the following interpretation. In the presence of background magnetic fluxes, D9-branes carry charges under RR forms of all even degrees, due to the WZ world-volume couplings [20]. The above tadpole conditions amount to the cancellation of overall D9-, D7 $_I$ -, D5 $_I$ - and D3-brane charges, (where D5 $_I$ - and D7 $_I$ -branes, are wrapped on, or transverse to, the I^{th} two-torus, respectively). This is required for consistency of the equations of motion of the corresponding RR forms, i.e. the T-dual statement to our argument in the D6-brane picture.

From our discussion in Section 2, the four-dimensional field theory arising after compactification of the D6-branes on the torus contains chiral fermions arising from brane intersections, hence a priori have phenomenological interest. They are also non-supersymmetric, but in principle the existence of tachyon-free stable configuration is not excluded, see section 3.4.

Let us obtain the massless (and tachyonic) four-dimensional spectrum. The $6_a 6_a$ sector has unbroken $\mathcal{N} = 4$ supersymmetry, and leads, in component fields, to $U(N_a)$ gauge bosons, six real scalars in the adjoint representation and four Majorana fermions in the adjoint as well. In the mixed $6_a 6_b$ and $6_b 6_a$ sectors, the field content appears in general in several replicas ⁴, due to the multiple intersection number I_{ab} of the cycles Π_a and Π_b , given by

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_i (n_a^i m_b^i - m_a^i n_b^i) \quad (3.5)$$

In the R sector, we obtain I_{ab} chiral left-handed fermions in the bifundamental representation (N_a, \overline{N}_b) , with the understanding that a negative multiplicity corresponds to a positive multiplicity of right-handed fermions. In the NS sector, we obtain a set of I_{ab} bifundamental scalars, whose masses are controlled by the angles ϑ_I between the D6_a and the D6_b-branes, which depend on the six-torus moduli. Their masses are given by (assuming $0 \leq \vartheta_i \leq 1$)

State	Mass	
$(-1 + \vartheta_1, \vartheta_2, \vartheta_3, 0)$	$\alpha' M^2 = \frac{1}{2}(-\vartheta_1 + \vartheta_2 + \vartheta_3)$	
$(\vartheta_1, -1 + \vartheta_2, \vartheta_3, 0)$	$\alpha' M^2 = \frac{1}{2}(\vartheta_1 - \vartheta_2 + \vartheta_3)$	(3.6)
$(\vartheta_1, \vartheta_2, -1 + \vartheta_3, 0)$	$\alpha' M^2 = \frac{1}{2}(\vartheta_1 + \vartheta_2 - \vartheta_3)$	
$(-1 + \vartheta_1, -1 + \vartheta_2, -1 + \vartheta_3, 0)$	$\alpha' M^2 = 1 - \frac{1}{2}(\vartheta_1 + \vartheta_2 + \vartheta_3)$	

Hence certain intersections may lead to the appearance of tachyons. If present, they signal an instability against joining the intersecting branes into a single smooth one. As observed in [21], tachyon modes arise precisely in the range of ϑ_I 's for which the joining process is energetically favoured, namely decreases the 3-cycle volume. In Section 3.4 we discuss the construction of models which, for a range of six-torus moduli, do not contain tachyons at brane intersections. Hence the corresponding configurations are protected against recombination by a energy barrier.

In next section we center of robust aspects of the theory, such us the chiral fermion content, and potential gauge anomalies. Hence recall that the gauge group and chiral

⁴In the T-dual picture in terms of D9-branes with magnetic fluxes, the multiplicities arise from the Landau level multiplicities.

fermions in the models are ⁵

$$\prod_{a=1}^K U(N_a) \sum_{a<b} I_{ab}(N_a, \bar{N}_b) \quad (3.7)$$

The spectrum is generically chiral, leading to an interesting set of four-dimensional field theories.

3.2 Anomaly cancellation

3.2.1 Non-abelian anomalies

Following [6, 22], the gauge anomaly induced by the chiral fermions living on each intersection is cancelled by an anomaly inflow mechanism associated to the intersecting branes (see [23] for string computations of the relevant couplings). Namely, the violation of charge induced by the anomaly is compensated by a charge inflow from the bulk of the intersecting branes. This explanation is sufficient in situations where the branes are infinitely extended. In the compact context, however, within a single brane the charge ‘inflowing’ into an intersection must be compensated by charge ‘outflowing’ from other intersections ⁶. Consistency of anomaly inflow in a compact manifold imposes global constraints on the configuration.

From the point of view of the compactified four-dimensional effective field theory, which does not resolve the localization of the different chiral fermions, these global constraints correspond to cancellation of triangle gauge anomalies in the usual sense. In fact, the cancellation of cubic non-abelian anomalies for the gauge factor $SU(N_a)$ in (3.7) reads

$$\sum_{b=1}^K I_{ab} N_b = 0 \quad (3.8)$$

From the ten-dimensional viewpoint, (3.8) expresses the cancellation of inflows from different intersections in the $D6_a$ -branes.

By replacing (3.5) in (3.8), one can see that tadpole cancellation conditions imply the cancellation of cubic non-abelian anomalies. Thus, as usual, string theory consistency conditions imply consistency of the low-energy effective theory. However, tadpole

⁵In fact, the chiral piece of the spectrum of a set of D6-branes wrapped on 3-cycles in a threefold (not necessarily Calabi-Yau) has this form, and our arguments about cancellation of four-dimensional anomalies are valid (with some obvious modifications) in this general case.

⁶This anomaly flow picture is analogous to that in [24].

cancellation conditions are in general much stronger than anomaly cancellation conditions (see also [25]), a feature also found in the context of standard type IIB orientifolds [26] (see also [27, 25]).

3.2.2 Mixed $U(1)$ anomaly cancellation

Let us turn to mixed $U(1)$ anomalies. Again, anomalies at each intersections are cancelled by the inflow mechanism [6, 22]. However, the global consistency of the inflow, or equivalently, cancellation of anomalies from the perspective of the compactified four-dimensional theory, is in this case more intricate, and involves a Green-Schwarz mechanism ⁷. Using the fermion spectrum in (3.7), the mixed $U(1)_a - SU(N_b)$ triangle anomaly reads

$$\mathcal{A}_{ab} = \frac{1}{2} \delta_{ab} \sum_c N_c I_{bc} + \frac{1}{2} N_b I_{ab} \quad (3.9)$$

The first piece is proportional to the non-abelian anomaly, and vanishes, while the last piece is generically non-vanishing even after imposing tadpole conditions.

We now show that the residual anomaly is cancelled by a generalized Green-Schwarz mechanism mediated by RR partners of closed string *untwisted* geometric moduli. This situation contrasts with that in type IIB orientifolds, where $U(1)$ anomalies are cancelled through exchange of closed string *twisted* moduli [29] (see [30] for the six-dimensional case, and e.g.[31, 32] for subsequent work). It also differs from that in heterotic compactifications, in not involving the dilaton multiplet, and in allowing the existence of several anomalous $U(1)$'s.

Let us consider a $D6_a$ -brane wrapped on a 3-cycle $[\Pi_a]$. It has several relevant world-volume couplings [20] to the RR 3-form C_3 and its ten-dimensional Hodge dual, the 5-form C_5

$$\int_{D6_a} C_3 \wedge F_a \wedge F_a \quad ; \quad \int_{D6_a} C_5 \wedge F_a \quad (3.10)$$

In order to obtain the couplings after Kaluza-Klein reduction to four dimensions, it is convenient to introduce two basis of homology 3-cycles, $\{[\Sigma_i]\}$, $\{[\Lambda_i]\}$, dual to each other, namely $[\Lambda_i] \cdot [\Sigma_j] = \delta_{ij}$. On these two basis, the $D6_a$ -brane 3-cycle $[\Pi_a]$ has the expansions

$$[\Pi_a] = \sum_i r_{ai} [\Sigma_i] \quad ; \quad [\Pi_a] = \sum_i p_{ai} [\Lambda_i] \quad (3.11)$$

⁷The interplay between the inflow and Green-Schwarz anomaly cancellation mechanisms has been studied in [28] in a different context.

Defining the untwisted RR fields $\Phi_i = \int_{[\Lambda_i]} C_3$; $B_2^i = \int_{[\Sigma_i]} C_5$, which are Hodge duals in the four-dimensional sense, the couplings (3.10) read

$$\sum_i p_{ai} \int_{M_4} \Phi_i F_a \wedge F_a \quad ; \quad N_a \sum_i r_{ai} \int_{M_4} B_2^i \wedge F_a \quad (3.12)$$

where the prefactor N_a arises from normalization of the $U(1)$ generator, as in [29]. These couplings can be combined in a GS diagram where $U(1)_a$ couples to the i^{th} untwisted field, which then couples to F_b^2 . The coefficient of this amplitude is (modulo an a, b independent numerical factor)

$$N_a \sum_i r_{ai} p_{bi} = N_a \sum_{i,j} r_{ai} p_{bj} [\Sigma_i] \cdot [\Lambda_j] = N_a [\Pi_a] \cdot [\Pi_b] = N_a I_{ab} \quad (3.13)$$

precisely of the form required to cancel the residual $U(1)_a$ - $SU(N_b)^2$ anomaly in (3.9).

The same mechanism may be described in the T-dual picture of D9-branes with magnetic fluxes. The couplings on the world-volume of D9-branes to bulk RR fields⁸ are of the form (wedge products implied)

$$\begin{aligned} \int_{D9_a} C_0 F_a^5 \quad ; \quad \int_{D9_a} C_2 F_a^4 \quad ; \quad \int_{D9_a} C_4 F_a^3 \\ \int_{D9_a} C_6 F_a^2 \quad ; \quad \int_{D9_a} C_8 F_a \quad ; \quad \int_{D9_a} C_{10} \end{aligned} \quad (3.14)$$

In order to obtain the four-dimensional version of these couplings, we define

$$\begin{aligned} C_2^I &= \int_{(\mathbf{T}^2)_I} C_4 & ; & \quad C_0^I = \int_{(\mathbf{T}^2)_I} C_2 \\ B_2^I &= \int_{(\mathbf{T}^2)_J \times (\mathbf{T}^2)_K} C_6 & ; & \quad B_0^I = \int_{(\mathbf{T}^2)_J \times (\mathbf{T}^2)_K} C_4 \\ B_2 &= \int_{(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_2} C_8 & ; & \quad B_0 = \int_{(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2 \times (\mathbf{T}^2)_3} C_6 \end{aligned}$$

where $I \neq J \neq K \neq I$ in second row. The fields C_2 and C_6 , and also C_0 and C_8 are Hodge duals, while C_4 is self-dual. In four dimensions, the duality relations are

$$\begin{aligned} dC_0 &= *dB_2 & ; & \quad dB_0^I = *dC_2^I \\ dC_2^I &= -*dB_2^I & ; & \quad dB_0 = -*dC_2 \end{aligned}$$

In the dimensional reduction, one should take into account that integration of F_a along the I^{th} two-torus yields a factor m_a^I . Also, integrating the pullback of the RR forms on the (multiply wrapped) D9_a-brane over the I^{th} two-torus yields a factor n_a^I . We obtain the couplings

$$\begin{aligned} N_a m_a^1 m_a^2 m_a^3 \int_{M_4} C_2 \wedge F_a & \quad ; & \quad n_b^1 n_b^2 n_b^3 \int_{M_4} B_0 \wedge F_b \wedge F_b \\ N_a n_a^I m_a^J m_a^K \int_{M_4} C_2^I \wedge F_a & \quad ; & \quad n_b^J n_b^K m_b^I \int_{M_4} B_0^I \wedge F_b \wedge F_b \\ N_a n_a^J n_a^K m_a^I \int_{M_4} B_2^I \wedge F_a & \quad ; & \quad n_b^I m_b^J m_b^K \int_{M_4} C_0^I \wedge F_b \wedge F_b \\ N_a n_a^1 n_a^2 n_a^3 \int_{M_4} B_2 \wedge F_a & \quad ; & \quad m_b^1 m_b^2 m_b^3 \int_{M_4} C_0 \wedge F_b \wedge F_b \end{aligned}$$

⁸The role of these couplings in anomaly cancellation in a different class of models has been suggested in [19].

As usual, the N_a prefactors arise from $U(1)_a$ normalization.

The GS amplitude where $U(1)_a$ couples to one untwisted field which propagates and couples to two $SU(N_b)$ gauge bosons is proportional to

$$\begin{aligned}
& -N_a m_a^1 m_a^2 m_a^3 n_b^1 n_b^2 n_b^3 + N_a \sum_I n_a^I m_a^J m_a^K n_b^J n_b^K m_b^I - N_a \sum_I n_a^I n_a^J m_a^K n_b^K m_b^I m_b^J + \\
& N_a n_a^1 n_a^2 m_a^3 m_b^1 m_b^2 m_b^3 = N_a \prod_I (n_a^I m_b^I - m_a^I n_b^I) = N_a I_{ab}
\end{aligned} \tag{3.15}$$

as required to cancel the residual mixed $U(1)$ anomaly in (3.9).

Finally, it is straightforward to check that these theories do not produce mixed $U(1)$ gravitational anomalies.

Due to the linear couplings between the $U(1)$'s and the closed string moduli, anomalous $U(1)$'s become massive with a mass of the order of the string scale. Therefore it is important, for any (phenomenological or not) application of these models, to determine the precise linear combinations becoming massive and those staying massless.

One can advance that since there are eight fields mediating the anomaly cancellation, at most eight $U(1)$ linear combinations can gain mass. Denoting Q_a the generator of the a^{th} $U(1)$, and writing a general linear combination as

$$Q = \sum_{a=1}^K \frac{c_a}{N_a} Q_a \tag{3.16}$$

non-anomalous $U(1)$ s correspond to zero modes of the intersection matrix $\sum_a c_a I_{ab} = 0$.

We conclude with a brief discussion of Fayet-Iliopoulos terms for the anomalous $U(1)$'s. An important observation is that the standard low-energy field theory arguments relating GS mechanism with FI terms in [33] are based on supersymmetry, hence do not directly apply to our models. Notice however that the string theory diagram giving rise to linear couplings between anomalous $U(1)$ and closed string modes is a disk, with boundary on the relevant D6-brane and a closed string mode insertion. This diagram does not notice the breaking of supersymmetries by other branes, hence yields superpartner interactions, and in particular a FI term, proportional to the NS-NS part of untwisted moduli. As opposed to supersymmetric cases, where the FI terms are not renormalized, in the present non-supersymmetric situations higher loop contributions are expected.

3.3 Explicit models

Here we construct an example with Standard Model gauge group and three quark-lepton families, in order to illustrate how our more general starting point overcomes

the difficulty found in [13] to obtain three generations. Notice that the model, just like the examples in [13], contains tachyons, but we prefer not to list them since they are moduli dependent (and might even disappear for certain regions in parameter space).

We consider six stacks of D6-branes, $K = 6$, with multiplicities and wrapping numbers given by

$$\begin{array}{cccc}
\mathbf{N}_a & (\mathbf{n}_a^1, \mathbf{m}_a^1) & (\mathbf{n}_a^2, \mathbf{m}_a^2) & (\mathbf{n}_a^3, \mathbf{m}_a^3) \\
N_1 = 3 & (1, 2) & (1, -1) & (1, -2) \\
N_2 = 2 & (1, 1) & (1, -2) & (-1, 5) \\
N_3 = 1 & (1, 1) & (1, 0) & (-1, 5) \\
N_4 = 1 & (1, 2) & (-1, 1) & (1, 1) \\
N_5 = 1 & (1, 2) & (-1, 1) & (2, -7) \\
N_6 = 1 & (1, 1) & (3, -4) & (1, -5)
\end{array} \tag{3.17}$$

This choice satisfies the tadpole conditions. The intersection numbers are

$$\begin{array}{cccccc}
I_{12} = 3 & I_{13} = -3 & I_{14} = 0 & I_{15} = 0 & I_{16} = -3 \\
I_{23} = 0 & I_{24} = 6 & I_{25} = 3 & I_{26} = 0 & I_{34} = -6 \\
I_{35} = -3 & I_{36} = 0 & I_{45} = 0 & I_{46} = 6 & I_{56} = 3
\end{array} \tag{3.18}$$

The spectrum under $U(3) \times U(2) \times U(1)^4$ is

$$\begin{aligned}
& 3(3, 2)_{[1, -1, 0, 0, 0, 0]} + 3(\bar{3}, 1)_{[-1, 0, 1, 0, 0, 0]} + 3(\bar{3}, 1)_{[-1, 0, 0, 0, 0, 1]} + \\
& + 6(1, 2)_{[0, 1, 0, -1, 0, 0]} + 3(1, 2)_{[0, 1, 0, 0, -1, 0]} + 6(1, 1)_{[0, 0, -1, 1, 0, 0]} + \\
& + 3(1, 1)_{[0, 0, -1, 0, 1, 0]} + 6(1, 1)_{[0, 0, 0, 1, 0, -1]} + 3(1, 1)_{[0, 0, 0, 0, 1, -1]}
\end{aligned} \tag{3.19}$$

where subindices give $U(1)$ charges. Out the six $U(1)$'s the diagonal linear combination decouples, and two of the remaining are anomalous. A basis of non-anomalous linear combinations (3.16) is provided by the coefficient vectors

$$\vec{c} = (1, 0, 0, 0, 1, 0) \ ; \ \vec{c} = (0, 1, 1, 0, 0, 0) \ ; \ \vec{c} = (0, 1, 0, 0, 0, 1) \tag{3.20}$$

One can check that the non-anomalous linear combination

$$Q_Y = -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_3 - Q_5 \tag{3.21}$$

can play the role of hypercharge. Indeed, the spectrum, showing only charges under this $U(1)$, is

$$\begin{aligned}
& SU(3) \times SU(2) \times U(1)_Y \\
& 3(3, 2)_{1/6} + 3(\bar{3}, 1)_{-2/3} + 3(\bar{3}, 1)_{1/3} + 6(1, 2)_{-1/2} + 3(1, 2)_{1/2} + \\
& + 6(1, 1)_1 + 3(1, 1)_0 + 6(1, 1)_0 + 3(1, 1)_{-1}
\end{aligned} \tag{3.22}$$

giving the chiral fermion content of a three-generation standard model (up to charges under additional $U(1)$ symmetries).

This example illustrates it is relatively easy to do model building in this framework. Unfortunately, as explained in the introduction, these models suffer a hierarchy problem, since they are not supersymmetric, and it is not possible to lower the string scale by making the six-torus volume large, since this would give rise to too light KK resonances for gauge bosons. However, we cannot exclude that further modifications of the setup improve this aspect. It is conceivable to consider spaces with a small volume region similar to \mathbf{T}^6 , where D6-branes wrap leading to heavy KK excitation, while the volume of the complete space is much larger. A simple example can be obtained by surgery, taking a small \mathbf{T}^6 , removing a ball in a region away from the branes, and gluing a throat connecting it to a large volume manifold. Of course, a concrete realization of this would require much more careful analysis, and our comment is just intended for illustration. In any event, the problem in lowering the string scale is not present in the models of D4- and D5-branes, to be studied in next sections.

3.4 Stability and Tachyons

The lowest lying states in the NS sector of an open string stretched between intersecting D6-branes are given in (3.6), along with their masses. These can be tachyonic or not, depending on the angles between the D-branes. For instance, for $\vartheta_1 \leq \vartheta_2 + \vartheta_3$, $\vartheta_2 \leq \vartheta_3 + \vartheta_1$, $\vartheta_3 \leq \vartheta_1 + \vartheta_2$, $\vartheta_1 + \vartheta_2 + \vartheta_3 \leq 2$, all states at the intersections have non-negative mass square. In fact, these are the conditions for the two intersecting 3-cycles be stable against recombination into a single smooth 3-cycle [21].

In principle there seems to be no obstruction to the existence of compactifications on \mathbf{T}^6 with D6-branes wrapped on 3-cycles, such that every intersection fulfills the above conditions, yielding a four-dimensional non-supersymmetric chiral theory free of tachyons. Such configurations would be stable against small perturbations, but, carrying no net charges, may decay to the vacuum by tunneling through a potential barrier. Such metastable (rather than absolutely stable) non-BPS configurations could however lead to perfectly sensible phenomenological models if their lifetime, exponentially suppressed by the barrier height, is long enough for cosmological standards.

3.5 Explicit examples of tachyon elimination

In this section we construct specific models where all intersections are tachyon-free for certain regions in the six-torus parameter space, namely the complex structure of the two-tori.

It turns out that it is easier to build such models if the construction includes an orientifold projection ΩR (where $R : z_i \rightarrow \bar{z}_i$) as in [13]. The only differences with respect to our configurations above is that the angle between the tori axis is projected out, the $D6_a$ -branes wrapped on cycles (n_a^I, m_a^I) must have ΩR orientifold images (denoted $D6a'$ -branes) wrapped on cycles (n_a^I, m_a^I) , and the first tadpole condition in (3.1) becomes $\sum_a N_a n_a^1 n_a^2 n_a^3 = 16$ (not counting images). The potential tachyon masses are however obtained as above.

The model under consideration is one of the four-dimensional constructions presented in [13]. The sets of D6-branes are given by

$$\begin{array}{cccc}
 N_a & (\mathbf{n}_a^1, \mathbf{m}_a^1) & (\mathbf{n}_a^2, \mathbf{m}_a^2) & (\mathbf{n}_a^3, \mathbf{m}_a^3) \\
 N_1 = 3 & (1, 0) & (1, 0) & (1, 1) \\
 N_2 = 3 & (1, 2) & (1, 1) & (1, 0) \\
 N_3 = 1 & (1, 2) & (1, -2) & (1, 0) \\
 N_4 = 1 & (1, 0) & (1, 0) & (10, 1)
 \end{array} \tag{3.23}$$

plus their ΩR images. The main advantage in searching tachyon-free models by using constructions with an ΩR orientifold projection, is that, as can be appreciated in (3.23), it allows all integers n_a^I to be positive. This simplifies the search for tachyon-free regions, since ensures that taking large ratios R_1/R_2 all angles between branes become small, and states become less tachyonic. For instance, choosing

$$R_2^1/R_1^1 = 1 \quad ; \quad R_2^2/R_1^2 = 3/2 \quad ; \quad R_2^3/R_1^3 = 2 \tag{3.24}$$

the masses for the scalars (3.6) at the different intersections are

$$\begin{array}{cccccc}
 \mathbf{Intersection} & \alpha' \mathbf{m}_1^2 & \alpha' \mathbf{m}_2^2 & \alpha' \mathbf{m}_3^2 & \alpha' \mathbf{m}_4^2 & \\
 12, 12', 21', 1'2' & 0.16 & 0.20 & 0.16 & 0.49 & \\
 13, 13', 31', 1'3' & 0.20 & 0.15 & 0.20 & 0.45 & \\
 24, 24', 42', 2'4' & 0.01 & 0.05 & 0.30 & 0.64 & \\
 34, 34', 43', 3'4' & 0.05 & 0.01 & 0.34 & 0.59 &
 \end{array} \tag{3.25}$$

We can see that they are all positive, hence the intersections are free of tachyons, and the system is stable against recombination of the corresponding cycles.

Pairs of branes with zero intersection number are parallel in some two-torus. In this model, in the non-generic case that the branes overlap in this two-torus, open strings stretched between them would lead to additional tachyons

Intersection	$\alpha' \mathbf{m}_1^2$	$\alpha' \mathbf{m}_2^2$	$\alpha' \mathbf{m}_3^2$	$\alpha' \mathbf{m}_4^2$	
11'	0.35	0.35	-0.35	0.65	
22'	-0.04	0.04	0.67	0.33	
33'	0.05	-0.05	0.75	0.25	
44'	0.06	0.06	-0.06	0.94	(3.26)
14, 1'4'	0.14	0.14	-0.14	0.86	
14', 41'	0.21	0.21	-0.21	0.79	
23, 2'3'	0.36	-0.36	0.36	0.64	
23', 32'	-0.31	0.31	0.39	0.61	

However, these states are not tachyonic if the branes are separated beyond a critical distance in the corresponding two-torus. It is possible that higher effects, due to brane interactions (one loop in the open string channel), induce a non-zero attractive force between such non-intersecting branes, pushing them to the tachyonic region. In any event, this would be a higher order effect which might be avoided in more complicated models. Our point here is that tachyons and intersections, which appear at tree level and are therefore more dangerous, can be eliminated in some models by a suitable choice of background geometry.

In principle it is possible that this kind of tachyon-free configurations exist in models without the orientifold projection, even though a systematic exploration of parameter space is more difficult. We would like to conclude by pointing out that, since the main difficulty arising from satisfying the tadpole conditions, the above ideas may have a much simpler implementation in other contexts, where such conditions are not relevant. For instance, one may construct a large class of (meta)stable non-BPS states in type IIB theory on \mathbf{T}^6 , by considering D3-branes wrapped on 3-cycles with tachyon-free intersections.

3.6 Tachyons and Higgs mechanism

Even if tachyons are present, we would like to point out a quite different perspective on them, which is actually applicable to more general examples (among others, those of D4- and D5-branes in coming sections). As in the more familiar example of brane-antibrane systems (see e.g. [16, 34]), condensation of open string tachyons may in

certain situations be interpreted as a Higgs mechanism. In our present context, the tachyon is charged under the gauge groups on the intersecting branes, and its condensation reduces the gauge symmetry to that of the recombined brane. From the spacetime viewpoint, it is physically clear that the tachyon has a potential with a minimum, at which the energy of the condensate compensates the difference of tensions between the final and initial states, and at which the tachyon vev breaks the initial gauge symmetry. Adapting Sen's ideas [16], the intersecting branes with the tachyon condensed to its minimum is *exactly* the final configuration of the recombined brane (stretched along a minimum volume cycle in its homology class)⁹

This idea has an important and interesting caveat, in the interpretation of the inverse process as un-Higgsing. Basically, the final state does not keep track of what initial state it came from. Hence if the system is given energy, it will nucleate not only the W bosons corresponding to the original initial state, but also W bosons of enhanced symmetries associated to all other possible initial states in the same energy range. However, there may be situations where one possible initial state is substantially lighter than the rest. In this situation, a low energy observer, with a limited range of available energies, would systematically find a single pattern of gauge symmetry enhancement. This situation is close enough to a standard Higgs mechanism, so that tachyons may be interpreted as standard Higgs fields (at least for processes in the appropriate range of energies), even for electroweak symmetry breaking. A more detailed understanding of the tachyon potential and dynamics [34] would help in determining if such scenario is indeed viable for electroweak or other phenomenological Higgs mechanisms. For the moment, we just point out the tantalizing existence of tachyon fields with the quantum numbers of standard model Higgs fields in some of the models we have explored (see section 4.3 and [15] for further details).

4 D4-branes wrapping at angles on $\mathbf{T}^2 \times \mathbf{C}^2/\mathbf{Z}_N$

4.1 Construction

As discussed in Section 2, configurations of D4-branes wrapped on 1-cycles in $\mathbf{T}^2 \times \mathbf{C}^2$ lead necessarily to non-chiral spectra. In this section we study a simple modification of this basic framework, which leads to generically chiral four-dimensional gauge field

⁹This is particularly clear in the T-dual picture of D9-branes with magnetic fluxes, where the above process often amounts to annihilation of topological defects on the D9-brane world-volume. Some remarks on tachyon condensation as a Higgs mechanism in this picture have appeared in [35].

theories on the D-brane world-volume.

We consider configurations of D4-branes on $\mathbf{T}^2 \times (\mathbf{C}^2/\mathbf{Z}_N)$, where the D4-branes are distributed in stacks of multiplicity N_a , wrapped along one-cycles Π_a defined by wrapping numbers (n_a, m_a) , on \mathbf{T}^2 , and sitting at the origin in $\mathbf{C}^2/\mathbf{Z}_N$. The models admit a T-dual description in terms of type IIB D5-branes on $\mathbf{T}^2 \times (\mathbf{C}^2/\mathbf{Z}_N)$, with non-trivial wrapping numbers and fluxes on \mathbf{T}^2 . We usually phrase our results in the D4-brane picture, but translation to the D5-brane picture is straightforward, as in the models in Section 3.

The twist \mathbf{Z}_N is generated by a geometric action θ with twist vector given by $v = \frac{1}{N}(b_1, b_2, 0, 0)$, where $b_1 = b_2 \pmod{2}$ for the variety to be spin. The supersymmetric case is recovered when $b_2 = -b_1 \pmod{N}$, hence with twist $v = \frac{1}{N}(b_1, -b_1, 0, 0)$. In this case, since b_1 and N must be coprime for a \mathbf{Z}_N action, the orbifold group can be equivalently generated by the twist θ^k , with $kb_1 = 1 \pmod{N}$, which has the more familiar twist vector $v = \frac{1}{N}(1, -1, 0, 0)$.

We would like to emphasize that we imagine this framework as a local description of the configuration near the location of the branes. Globally, the local configuration above may be embedded in a spacetime of the form $\mathbf{T}^2 \times \mathbf{B}$, where \mathbf{B} is a four-dimensional space (not necessarily Calabi-Yau) with a $\mathbf{C}^2/\mathbf{Z}_N$ singularity at which the D-branes sit. More generally, the complete space may not be globally a product, but rather a torus bundle over \mathbf{B} , or even a torus fibration, as long as singular fibers are away from the D-brane location. Our configuration is a good local description in these cases, and completely controls the structure of the D-brane world-volume gauge theory.

Let us briefly mention another interesting aspect. These configurations admit a seemingly simple lift to M-theory, as a set of M-theory fivebranes sitting at a $\mathbf{C}^2/\mathbf{Z}_N$ singularity, and wrapped on a two-cycle in $\mathbf{T}^2 \times \mathbf{S}^1$. Obviously, a detailed description of the model in M-theory will involve a number of interesting subtleties, on which our analysis may shed some light. Note that the existence of this six-dimensional parent theory, which reduces to the four-dimensional field theory after compactification, is not in contradiction with chirality in the latter. The higher dimensional theory is not a conventional field theory, and in fact four dimensional chiral states arise from membranes stretched between M5-branes and wrapped on \mathbf{S}^1 , i.e. they do not descend by KK reduction from any six-dimensional field.

Let us describe the computation of the spectrum in our configuration. The closed string sector is computed using standard orbifold techniques. In the supersymmetric case, it gives rise to an $D = 4$ $\mathcal{N} = 4$ $U(1)^{N-1}$ gauge multiplet. In the non-

supersymmetric case, the main feature is that it leads to tachyons in the NS-NS sector. Their interpretation is, as usual with closed string tachyons, not understood, and we will have nothing new to say about them. Nevertheless, we choose to study these models and in particular their open string spectrum even for non-supersymmetric singularities.

The \mathbf{Z}_N action may be embedded in the $U(N_a)$ gauge degrees of freedom of the a^{th} stack of D4-branes, through a unitary matrix of the form

$$\gamma_{\theta,a} = \text{diag}(\mathbf{1}_{N_a^0}, e^{2\pi i \frac{1}{N}} \mathbf{1}_{N_a^1}, \dots, e^{2\pi i \frac{N-1}{N}} \mathbf{1}_{N_a^{N-1}}) \quad (4.1)$$

with $\sum_i N_a^i = N_a$.

Let us compute the spectrum in the open string sector. In the $4_a 4_a$ sector, the massless states surviving the GSO projection, along with their behaviour under the \mathbf{Z}_N twist, are

NS State	\mathbf{Z}_N phase	R State	\mathbf{Z}_N phase	
$(\pm 1, 0, 0, 0)$	$e^{\pm 2\pi i \frac{b_1}{N}}$	$\pm \frac{1}{2}(-, +, +, +)$	$e^{\mp \pi i \frac{b_1 - b_2}{N}}$	(4.2)
$(0, \pm 1, 0, 0)$	$e^{\mp 2\pi i \frac{b_2}{N}}$	$\pm \frac{1}{2}(+, -, +, +)$	$e^{\pm \pi i \frac{b_1 - b_2}{N}}$	
$(0, 0, \pm 1, 0)$	1	$\pm \frac{1}{2}(+, +, -, +)$	$e^{\pm \pi i \frac{b_1 + b_2}{N}}$	
$(0, 0, 0, \pm 1)$	1	$\pm \frac{1}{2}(+, +, +, -)$	$e^{\pm \pi i \frac{b_1 + b_2}{N}}$	

The open string spectrum is obtained by keeping states invariant under the combined geometric plus Chan-Paton \mathbf{Z}_N action [36]. After the \mathbf{Z}_N projections, the resulting gauge group and matter fields are

Gauge Bosons	$\prod_{a=1}^K \prod_{i=1}^N U(N_a^i)$	
Cmplx. Scalars	$\sum_{a=1}^K \sum_{i=1}^N [(N_a^i, \overline{N}_a^{i+b_1}) + (N_a^i, \overline{N}_a^{i+b_2})]$	
Left Fermion	$\sum_{a=1}^K \sum_{i=1}^N [(N_a^i, \overline{N}_a^{i+(b_1-b_2)/2}) + (N_a^i, \overline{N}_a^{i-(b_1-b_2)/2})]$	
Right Fermion	$\sum_{a=1}^K \sum_{i=1}^N [(N_a^i, \overline{N}_a^{i+(b_1+b_2)/2}) + (N_a^i, \overline{N}_a^{i-(b_1+b_2)/2})]$	(4.3)

Notice that this piece of the spectrum is generically non-supersymmetric, and always non-chiral. In the supersymmetric case, $v = (1, -1, 0, 0)/N$, this sector preserves $\mathcal{N} = 2$ supersymmetry in the four-dimensional field theory. The above fields form the multiplets

$$\begin{aligned} \mathcal{N} = 2 \text{ Vector} & \quad \prod_{a=1}^K \prod_{i=1}^N U(N_a^i) \\ \mathcal{N} = 2 \text{ Hyper} & \quad \sum_{a=1}^K \sum_{i=1}^N (N_a^i, \overline{N}_a^{i+1}) \end{aligned} \quad (4.4)$$

In the $4_a 4_b$ sector, open strings are twisted by the angle formed by the branes, denoted ϑ , resulting in a sector twisted by the shift $(\vartheta, 0, 0, 0)$. Assuming $0 \leq \vartheta \leq 1$, tachyonic

and massless states, along with their \mathbf{Z}_N phases, are

Sector	State	\mathbf{Z}_N phase	
NS	$(-1 + \vartheta, 0, 0, 0)$	1	
R	$(-\frac{1}{2} + \vartheta, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2})$	$e^{-2\pi i \frac{(b_1+b_2)}{2N}}$	(4.5)
	$(-\frac{1}{2} + \vartheta, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2})$	$e^{-2\pi i \frac{(b_1-b_2)}{2N}}$	
	$(-\frac{1}{2} + \vartheta, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$	$e^{2\pi i \frac{(b_1-b_2)}{2N}}$	
	$(-\frac{1}{2} + \vartheta, +\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2})$	$e^{2\pi i \frac{(b_1+b_2)}{2N}}$	

This piece of the spectrum is non-supersymmetric, even for supersymmetric \mathbf{Z}_N twists. The NS states are tachyonic, with $\alpha' M^2$ equal to $-\frac{1}{2}|\vartheta|$, and signal an instability against recombining intersecting D4-branes with same Chan-Paton eigenvalue. Hence, they may be avoided by suitable choices of the \mathbf{Z}_N actions $\gamma_{\theta, 4_a}$. A different possibility is to interpret these tachyons as triggering breaking of gauge symmetries by a Higgs mechanism, as mentioned in section 3.4. The R states are massless, and provide a set of chiral fermions in the model. Notice that the antiparticles of these states appear in the $4_b 4_a$ sector, which is twisted by $(-\vartheta, 0, 0, 0)$.

In these sectors the spectrum generically appears in several replicas, whose number is given by the intersection number I_{ab} of the one-cycles Π_a and Π_b in \mathbf{T}^2 ,

$$I_{ab} = n_a m_b - m_a n_b \quad (4.6)$$

The spectrum after the Chan-Paton projections is given by

Cmplx. Tachyons	$\sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, \overline{N}_b^i)$
Left Fermion	$\sum_{a<b} \sum_{i=1}^N I_{ab} \times [(N_a^i, \overline{N}_b^{i+(b_1+b_2)/2}) + (N_a^i, \overline{N}_b^{i-(b_1+b_2)/2})]$
Right Fermion	$\sum_{a<b} \sum_{i=1}^N I_{ab} \times [(N_a^i, \overline{N}_b^{i+(b_1-b_2)/2}) + (N_a^i, \overline{N}_b^{i-(b_1-b_2)/2})]$ (4.7)

which is non-supersymmetric and generically chiral. Therefore the resulting field theories may lead to phenomenologically interesting models. In fact, in Section 4.3 we construct an explicit example with Standard Model group and three quark lepton generations.

4.2 Tadpoles and anomalies

4.2.1 Tadpole cancellation conditions

The consistency conditions (RR tadpole cancellation conditions) for these configurations are easily computed, and read

$$\begin{aligned} \prod_{r=1,2} \sin(\pi k b_r / N) \sum_{a=1}^K n_a \text{Tr} \gamma_{\theta^k, 4_a} &= 0 \\ \prod_{r=1,2} \sin(\pi k b_r / N) \sum_{a=1}^K m_a \text{Tr} \gamma_{\theta^k, 4_a} &= 0 \end{aligned} \quad \text{for } k = 1, \dots, N-1$$

There is no constraint associated to $k = 0$, since the untwisted tadpole is associated to a flux that can escape along the non-compact dimensions of $\mathbf{C}^2/\mathbf{Z}_N$.

These conditions can be interpreted geometrically, at least for supersymmetric singularities, by regarding the fractional [37] $D4_a^s$ -branes (i.e. the set of $D4_a$ branes associated to the phase $e^{2\pi i \frac{s}{N}}$ in $\gamma_{\theta,4_a}$) as D6-branes wrapped on the 1-cycle $[\Pi_a] = n_a[a] + m_a[b]$ in \mathbf{T}^2 times the s^{th} collapsed two-cycle $[\Sigma_s]$ in the singularity. The conditions above amount to the vanishing of the total homology class

$$\sum_{a=1}^K \sum_{s=0}^{N-1} N_a^s [\Pi_a] \otimes [\Sigma_s] = 0 \quad (4.8)$$

Since $\sum_{s=0}^{N-1} [\Sigma_s] = 0$, one can increase the N_a^s by an s -independent (but possibly a -dependent) amount and still satisfy the homological condition. Hence, the Chan-Paton matrices for $k = 0$ are unconstrained. Note that regarding branes at singularities as branes wrapped on collapsed cycles, our models of D4-branes become a degenerate version of D6-branes wrapped on 3-cycles in a curved ambient space, and following footnote 4, our results here are reminiscent of those in section 3.2.

4.2.2 Anomaly cancellation

The spectrum of the model is generically chiral, and has potential gauge anomalies. In analogy with the case of D6-branes on \mathbf{T}^6 , cancellation of the anomalies due to chiral fermions at each intersection would be achieved by an inflow mechanism. Since the intersections sit at the singularity in transverse space, this inflow mechanism would be more involved and interesting, but still tractable. Leaving aside its study, we prefer to center on the compactified effective four-dimensional description of anomaly cancellation.

The cancellation of cubic non-abelian anomalies for $SU(N_a^i)$ gives the conditions

$$\sum_{b=1}^K I_{ab} (-N_b^{i+(b_1+b_2)/2} - N_b^{i-(b_1+b_2)/2} + N_b^{i+(b_1-b_2)/2} + N_b^{i-(b_1-b_2)/2}) = 0 \quad (4.9)$$

These conditions should follow from the tadpole cancellation conditions. In fact, using (4.1) we can rewrite

$$N_b^i = \frac{1}{N} \sum_{k=0}^{N-1} e^{-2\pi i \frac{ki}{N}} \text{Tr } \gamma_{\theta^k, 4_b} \quad (4.10)$$

as in [38], and the anomaly cancellation conditions read

$$\prod_{r=1,2} \sin(\pi k b_r / N) \sum_{b=1}^K I_{ab} \text{Tr } \gamma_{\theta^k, 4_b} = 0 \quad (4.11)$$

These conditions are indeed guaranteed by the tadpole conditions (4.8), but, as usual, are much milder than the latter.

Let us consider cancellation of mixed $U(1)$ anomalies, which involves a generalized Green-Schwarz mechanism, mediated by $2(N - 1)$ fields, corresponding to the integration of $N - 1$ twisted RR-fields along the two independent 1-cycles in the \mathbf{T}^2 .

In fact, one can compute the mixed anomaly between the $U(1)_{ai}$ and $SU(N_b^j)$ using the chiral piece of the spectrum (4.7). After removing a vanishing piece proportional to the non-abelian anomaly, there remains

$$\mathcal{A}_{ai,bj} = \frac{1}{2} N_a^i I_{ab} (\delta_{j,i+(b_1+b_2)/2} + \delta_{j,i-(b_1+b_2)/2} - \delta_{j,i+(b_1-b_2)/2} - \delta_{j,i-(b_1-b_2)/2}) \quad (4.12)$$

Substituting the discrete Fourier transform representation of the Kronecker deltas, as in [29], the anomaly acquires the nice factorized form

$$A_{ai,bj} = i N_a^i I_{ab} \frac{1}{N} \sum_{k=1}^{N-1} 4 \prod_{r=1,2} \sin(\pi k b_r / N) e^{2\pi i \frac{k i}{N}} e^{-2\pi i \frac{k j}{N}} \quad (4.13)$$

The anomaly may therefore be cancelled by exchange of the four dimensional fields obtained by integrating over the two one-cycles in \mathbf{T}^2 the RR twisted forms, which give the four-dimensional couplings

$$\begin{aligned} c_k N_a^i n_a \int_{M_4} \text{Tr}(\gamma_{\theta^k, 4_a} \lambda_i) C_2^{(k)} \wedge \text{tr} F_{a,i} & \quad ; \quad c_k m_b \int_{M_4} \text{Tr}(\gamma_{\theta^k, 4_b} \lambda_j^2) C_0^{(k)} \wedge \text{Tr} F_{b,j}^2 \\ -c_k N_a^i m_a \int_{M_4} \text{Tr}(\gamma_{\theta^k, 4_a} \lambda_i) B_2^{(k)} \wedge \text{tr} F_{a,i} & \quad ; \quad c_k n_b \int_{M_4} \text{Tr}(\gamma_{\theta^k, 4_b} \lambda_j^2) B_0^{(k)} \wedge \text{Tr} F_{b,j}^2 \end{aligned}$$

where λ denotes the CP wavefunction of the gauge boson state. The prefactors $c_k = (\prod_r \sin(\pi k b_r / N))^{1/2}$ can be thought of as arising from $\hat{A}^{1/2}$ in D-brane couplings [20], and have been explicitly computed in string theory in e.g. [31, 32]. Since B_2 and B_0 , and C_2 and C_0 are Hodge dual in four dimensions, the sum over GS diagrams has the structure (4.13). The GS mechanism is analogous to that for D6-branes on \mathbf{T}^6 , as is manifest from the appearance of the intersection number, the main difference being that the exchanged fields belong to twisted sectors of the $\mathbf{C}^2/\mathbf{Z}_N$ factor.

The above results can be interpreted geometrically by regarding the fractional $D4_a$ -branes as D6-branes wrapped on collapsed cycles Σ_s of the singularity and the one-cycle Π_a in \mathbf{T}^2 . This is simplest in the more familiar supersymmetric case where the anomaly is given by

$$A_{ai,bj} = \frac{1}{2} N_b^j I_{ab} (2\delta_{j,i} - \delta_{j,i+1} - \delta_{j,i-1}) \quad (4.14)$$

The collapsed two-cycles have intersections given by (minus) the Cartan matrix of the (affine) \hat{A}_{N-1} algebra, $C_{ij} = [\Sigma_i] \cdot [\Sigma_j] = -2\delta_{ji} + \delta_{j,i+1} + \delta_{j,i-1}$. Hence, the intersection

number of D6-branes wrapped on cycles $[\Pi_a] \otimes [\Sigma_i]$ and $[\Pi_b] \otimes [\Sigma_j]$ is $I_{ab} C_{ij}$. Introducing a composite index I grouping together indices a and i , we can express the mixed anomaly (4.14) as

$$A_{IJ} = \frac{1}{2} N_I \mathcal{I}_{IJ} \quad (4.15)$$

where \mathcal{I}_{IJ} denotes the 3-cycle intersection form. The situation is hence analogous to that in section 3.2. As suggested in footnote 4, the GS cancellation mechanism in section 3.2 can be directly translated, with the obvious modifications, reproducing the cancellation of anomalies in the present context. Since here the wrapped 3-cycles are exceptional divisors of the singularity, the forms mediating the GS mechanism arise as twisted states in string theory. The above geometric interpretation follows also for non-supersymmetric \mathbf{Z}_N twist, by using the corresponding intersection matrix, obtained from (4.12).

As in section 3.2, anomalous $U(1)$'s get a mass of the order of the string scale. To find non-anomalous $U(1)$'s, we consider linear combinations

$$Q = \sum_{a=1}^K \sum_{j=0}^{N-1} \frac{c_{a,j}}{N_a^j} Q_{a,j} \quad (4.16)$$

(we choose $c_{b,i} = 0$ if the corresponding group is not present, namely if $N_a^i = 0$). Non-anomalous $U(1)$ s can again be found as zero modes of the (generalized) intersection matrix. In fact, we can be slightly more explicit. Taking the supersymmetric singularity case for concreteness, and using the expression (4.13), anomaly-free linear combinations satisfy

$$\frac{1}{N} \sum_{k=1}^{N-1} e^{-2\pi i \frac{kj}{N}} \sin^2(\pi k/N) \sum_{a=1}^K I_{ab} \sum_{i=0}^{N-1} e^{2\pi i \frac{ki}{N}} c_{a,i} = 0 \quad (4.17)$$

for each $b = 1, \dots, K$ and $j = 0, \dots, N-1$. A useful trick is to perform the change of coordinates [5] $r_{a,k} = \sum_{i=0}^{N-1} e^{2\pi i \frac{ki}{N}} c_{a,i}$, and obtain the conditions

$$\sin^2(\pi k/N) \sum_{a=1}^K I_{ab} r_{a,k} = 0 \quad (4.18)$$

A set of solutions is given by choosing, for a fixed a , $r_{a,k} = \delta_{k,0}$, and $r_{b,k} = 0$ for $b \neq a$. The resulting generator is

$$Q_a = \sum_{i=0}^{N-1} \frac{Q_{a,i}}{N_a^i} \quad (4.19)$$

Another combination is obtained by choosing $c_{a,i} = N_a^i$, or equivalently by $r_{a,k} = \text{Tr } \gamma_{\theta^k, 4_a}$, namely

$$Q = \sum_{a=1}^K \sum_{i=0}^{N-1} Q_{a,i} \quad (4.20)$$

Depending on the details of the orbifold group, there may be additional non-anomalous $U(1)$'s. These are most easily determined by directly computing the zero modes of the anomaly matrix in each case.

4.3 Explicit models

In the present context it is not possible to get rid of all the tachyons while maintaining a chiral fermion spectrum. A general argument goes as follows. Since tachyons arise in $4_a 4_b + 4_b 4_a$ sectors from strings stretching between D4-branes with same Chan-Paton phase, to avoid tachyons we must consider models where any two intersecting branes have no common Chan-Paton eigenvalues. Consider models with N stacks of $D4_a$ -branes, hence $K = N$, at a $\mathbf{C}^2/\mathbf{Z}_N$ singularity, with twist e.g. $v = (1, -1)/N$, and wrapped on arbitrary 1-cycles Π_a on \mathbf{T}^2 , and choose the Chan-Paton embeddings

$$\gamma_{\theta, 4_a} = e^{2\pi i \frac{a}{N}} \mathbf{1}_{N_a} \quad (4.21)$$

hence $N_a^i = N_a \delta_{ai}$ (more general choices can be treated analogously). The spectrum one naively obtains seems chiral, but the tadpole conditions

$$\sum_{a=1}^N e^{2\pi i \frac{ka}{N}} N_a [\Pi_a] = 0 \text{ for } k = 1, \dots, N-1 \quad (4.22)$$

turn out to be very constraining. By discrete Fourier transforming, they imply that all $[\Pi_a]$ are actually identical, so all D4-branes are parallel, leading to non-chiral spectra.

Allowing a non-supersymmetric singularity may relax the tadpole conditions, but introduces (closed string) tadpoles. Also, the case $K < N$ reduces to the above with some $N_a = 0$, while additional branes ($K > N$) necessarily repeat eigenvalues and must be non-intersecting, i.e. parallel, to the existing ones to avoid tachyons.

Allowing for some tachyons in the model, however, one can obtain large classes of models with chiral spectrum, which moreover can be quite close to realistic models. Let us discuss a simple explicit model, which illustrates a possible model building strategy.

Since \mathbf{Z}_2 leads to vector-like models, let us consider sets of D4-branes at $\mathbf{T}^2 \times \mathbf{C}^2/\mathbf{Z}_3$, with twist $v = (1, -1, 0, 0)/3$. A typical tachyon-free and hence non-chiral model would have three stacks of D4-branes, with $\gamma_{\theta, 4_a} = e^{2\pi i a/3} \mathbf{1}$, and parallel cycles. Consider e.g. the stack with $\gamma_{\theta} = \mathbf{1}$ with multiplicity 3 and cycle $(1, 0)$, and the remaining two with multiplicity 1 and cycle $(3, 0)$, yielding a gauge group $U(3) \times U(1) \times U(1)$ with vector-like matter. To get chirality, we must allow one of these sets to split into several intersecting stacks, a process which also implies the appearance of tachyons

(which would trigger the recombination to the original configuration). Let us build this enlarged model with Standard Model gauge group, namely including a stack with group $U(2)$, and with triplicated intersections. A possible choice is to split the brane with $\gamma_\theta = e^{2\pi i/3}$ wrapped on $(3, 0)$, into two branes wrapped on $(1, 3)$, one brane on $(0, -3)$ and one brane on $(1, -3)$. Hence we end up with

Multiplicity	Cycle	CP phase	
$N_1 = 3$	$(1, 0)$	1	
$N_2 = 2$	$(1, 3)$	$e^{2\pi i/3}$	(4.23)
$N_3 = 1$	$(0, -3)$	$e^{2\pi i/3}$	
$N_4 = 1$	$(1, -3)$	$e^{2\pi i/3}$	
$N_5 = 1$	$(3, 0)$	$e^{2\pi i 2/3}$	

The chiral spectrum contains left-handed fermions transforming under $U(3) \times U(2) \times U(1)_3^3 \times U(1)_4 \times U(1)_5^3$ as

$$\begin{aligned}
& 3(3, 2)_{[1, -1, (0^3), 0, (0^3)]} + (\bar{3}, 1)_{[-1, 0, \underline{1, 0, 0, 0}, (0^3)]} + 3(\bar{3}, 1)_{[-1, 0, (0^3), 1, (0^3)]} + \\
& 2(1, 2)_{[0, 1, \underline{-1, 0, 0, 0}, (0^3)]} + 12(1, 2)_{[0, 1, (0^3), -1, (0^3)]} + 3(1, 2)_{[0, -1, (0^3), 0, \underline{1, 0, 0}]} \\
& + 2(1, 1)_{[0, 0, \underline{-1, 0, 0, 1}, (0^3)]} + (1, 1)_{[0, 0, \underline{1, 0, 0, 0}, \underline{-1, 0, 0}]} + 3(1, 1)_{[0, 0, (0^3), 1, \underline{-1, 0, 0}]} \quad (4.24)
\end{aligned}$$

where underlining means permutation. Besides the diagonal combination, which decouples, there are six non-anomalous $U(1)$ linear combinations. One of them, given by

$$Q_Y = -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_4 - (Q_5^{(1)} + Q_5^{(2)} + Q_5^{(3)}) \quad (4.25)$$

provides correct hypercharge assignments for the above theory, which therefore has the chiral content of a three-generation standard model. Indeed, highlighting the charges under this $U(1)$, the fermion spectrum is

$$\begin{aligned}
& 3(3, 2)_{1/6} + 3(\bar{3}, 1)_{1/3} + 3(\bar{3}, 1)_{-2/3} + 15(1, 2)_{-1/2} + \\
& 12(1, 2)_{1/2} + 6(1, 1)_{-1} + 9(1, 1)_1 + 9(1, 1)_0 \quad (4.26)
\end{aligned}$$

Further properties of these models will be discussed in [15]. Here let us simply point out that, in models constructed using the above strategy, the tachyons trigger the recombination of branes involving the $U(2)$ factor, and therefore have the gauge quantum numbers of standard model Higgs fields. These models therefore illustrate that tachyonic modes may be phenomenologically interesting (to trigger electroweak or

other extended symmetry breakings), and that in this class of models they are linked to the existence of chiral fermions.

As mentioned in the introduction, even though the models are non-supersymmetric, the hierarchy problem is avoided by considering a low string scale and a compactification with large volume for the space transverse to the two-torus.

5 D5-branes wrapping at angles on $(\mathbf{T}^2)^2 \times \mathbf{C}/\mathbf{Z}_N$

5.1 Construction

For completeness, in this section we center on a last type of configuration, similar to those in the preceding section, and also leading to four-dimensional chiral theories. We consider configurations of D5-branes in $\mathbf{T}^4 \times (\mathbf{C}/\mathbf{Z}_N)$, where the D5-branes sit at the origin in \mathbf{C}/\mathbf{Z}_N , and are grouped in stacks of multiplicity N_a wrapped on 2-cycles defined by (n_a^I, m_a^I) , with $I = 1, 2$, in a factorizable \mathbf{T}^4 .

The \mathbf{Z}_N action on the third dimension is encoded in the twist vector of the form $v = \frac{1}{N}(0, 0, 2, 0)$ for the variety to be spin. The closed string sector necessarily contains tachyons in its twisted sector, whose interpretation is unclear. Nevertheless, we proceed studying these models. The Chan-Paton twist matrices have the general form

$$\gamma_{\theta,a} = \text{diag}(\mathbf{1}_{N_a^0}, e^{2\pi i \frac{1}{N}} \mathbf{1}_{N_a^1}, \dots, e^{2\pi i \frac{N-1}{N}} \mathbf{1}_{N_a^{N-1}}) \quad (5.1)$$

with $\sum_i N_a^i = N_a$. The lowest lying states in the $5_a 5_a$ open string NS and R sectors, along with their \mathbf{Z}_N phases, are

NS State	\mathbf{Z}_N phase	;	R State	\mathbf{Z}_N phase	
$(\pm 1, 0, 0, 0)$	1		$\pm \frac{1}{2}(-, +, +, +)$	$e^{\pm 2\pi i \frac{1}{N}}$	
$(0, \pm 1, 0, 0)$	1		$\pm \frac{1}{2}(+, -, +, +)$	$e^{\pm 2\pi i \frac{1}{N}}$	(5.2)
$(0, 0, \pm 1, 0)$	$e^{\pm 4\pi i \frac{1}{N}}$		$\pm \frac{1}{2}(+, +, -, +)$	$e^{\mp 2\pi i \frac{1}{N}}$	
$(0, 0, 0, \pm 1)$	1		$\pm \frac{1}{2}(+, +, +, -)$	$e^{\pm 2\pi i \frac{1}{N}}$	

The spectrum is non-supersymmetric. The fourth NS state leads to $\prod_{a=1}^K \prod_{i=1}^N U(N_a^i)$ gauge bosons, while the remaining give a set of scalars in bifundamental or adjoint representations. In the R sector, no state is invariant under \mathbf{Z}_N , and the model contains no gauginos. On the other hand, it contains a non-chiral set of fermions in diverse bifundamental representations. Summarizing, the spectrum contains the following fields

$$\text{Gauge Bosons} \quad \prod_{a=1}^K \prod_{i=1}^N U(N_a^i)$$

$$\begin{aligned}
\text{Real Scalars} & \quad \sum_{a=1}^K \sum_{i=1}^N [(N_a^i, \overline{N}_a^{i+2}) + 2 \times \text{Adj}_{a,i}] \\
\text{Right Fermion} & \quad \sum_{a=1}^K \sum_{i=1}^N (N_a^i, \overline{N}_a^{i+1}) \\
\text{Left Fermion} & \quad \sum_{a=1}^K \sum_{i=1}^N (\overline{N}_a^{i+1}, N_a^i)
\end{aligned} \tag{5.3}$$

In the $5_a 5_b$ sector, open strings are twisted by the angle formed by the branes in the two-tori, encoded in a twist vector $(\vartheta_1, \vartheta_2, 0, 0)$. The lowest lying states, along with their behaviour under \mathbf{Z}_N , are (assuming $0 \leq \vartheta_I \leq 1$)

Sector	State	\mathbf{Z}_N phase
NS	$(-1 + \vartheta_1, 0, 0, 0)$	1
	$(0, -1 + \vartheta_2, 0, 0)$	1
R	$(-\frac{1}{2} + \vartheta_1, -\frac{1}{2} + \vartheta_2, +\frac{1}{2}, -\frac{1}{2})$	$e^{2\pi i \frac{1}{N}}$
	$(-\frac{1}{2} + \vartheta_1, -\frac{1}{2} + \vartheta_2, -\frac{1}{2}, +\frac{1}{2})$	$e^{-2\pi i \frac{1}{N}}$

Recall that at most one of the two NS states is tachyonic (both are massless for $|\vartheta_1| = |\vartheta_2|$), while fermions are massless.

The spectrum of tachyonic and massless states, after the Chan-Paton projections, and taking into account the multiplicity due to the intersection numbers

$$I_{ab} = I_{ab}^1 I_{ab}^2 = (n_a^1 m_b^1 - m_a^1 n_b^2)(n_a^2 m_b^2 - m_a^2 n_b^2) \tag{5.4}$$

is given by

$$\begin{aligned}
\text{Cmplx. Tachyons} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, \overline{N}_b^i) \\
\text{Left Fermion} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, \overline{N}_b^{i+1}) \\
\text{Right Fermion} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, \overline{N}_b^{i-1})
\end{aligned} \tag{5.5}$$

In the case $|\vartheta_2| = |\vartheta_1|$ we would have two bosonic massless states instead of the above tachyon.

5.2 Tadpoles and anomalies

The analysis of tadpole and anomaly cancellation is similar to that for configurations in section 4.2, hence our discussion is more sketchy.

Tadpole cancellation conditions read

$$\begin{aligned}
\sin(4\pi k/N) n_a^1 n_a^2 \text{Tr } \gamma_{\theta^k, 4_b} &= 0 \quad ; \quad \sin(4\pi k/N) m_a^1 n_a^2 \text{Tr } \gamma_{\theta^k, 4_b} = 0 \\
\sin(4\pi k/N) n_a^1 m_a^2 \text{Tr } \gamma_{\theta^k, 4_b} &= 0 \quad ; \quad \sin(4\pi k/N) m_a^1 m_a^2 \text{Tr } \gamma_{\theta^k, 4_b} = 0
\end{aligned} \tag{5.6}$$

and clearly have the interpretation of cancellation of charges analogous to that for equations (4.8).

These conditions must ensure the consistency of the low-energy four-dimensional field theory on the D-brane world-volume. In particular, the cancellation of cubic non-abelian chiral anomalies for $SU(N_a^i)$ reads

$$\sum_{b=1}^K I_{ab}(N_b^{i+1} - N_b^{i-1}) = 0 \quad (5.7)$$

or, equivalently, by performing the discrete Fourier transform (4.10),

$$\sin(4\pi k/N) I_{ab} \text{Tr} \gamma_{\theta^k, 5_b} = 0 \quad (5.8)$$

By substituting (5.4) in this equation, we see it is implied by the tadpole constraints.

Using the spectrum (5.3) it is easy to compute the mixed anomalies between $U(1)_{a_i}$ and $SU(N_{b_j})$. We obtain

$$A_{a_i, b_j} = \frac{1}{2} N_a^j I_{ab} (\delta_{j, i+1} - \delta_{j, i-1}) = i N_a^i I_{ab} \frac{1}{N} \sum_{k=1}^{N-1} \sin(2\pi k/N) e^{2\pi i \frac{k i}{N}} e^{-2\pi i \frac{k j}{N}} \quad (5.9)$$

where, again, the second equality shows the residual anomaly has a factorized structure, which can be cancelled by a GS mechanism mediated by four-dimensional fields obtained by integrating twisted RR fields on diverse two-cycles in \mathbf{T}^4 .

The existence and form of non-anomalous (and therefore massless) $U(1)$ linear combinations can be carried out in complete analogy with that in section 4.2.2

6 Final comments and outlook

In this paper we have studied the construction of four-dimensional chiral string compactifications with gauge sector localized on D-branes wrapped on non-trivial cycles in the internal space. Specifically, we have studied configurations of D(3 + n)-branes wrapped on n -cycles in $\mathbf{T}^{2n} \times \mathbf{C}^{3-n}/\mathbf{Z}_N$, where the last factor should be understood as a local model of a singularity within a compact $(6 - 2n)$ -dimensional variety, so that correct four-dimensional gravity is recovered. Several properties (like the anomaly cancellation mechanisms) however hold in more general setups.

The configurations allow a bottom-up approach to embedding realistic gauge sectors in string theory models, in the sense explained in [5]. In fact, the configurations are a natural extension of the work on D3-branes at threefold singularities (e.g. $\mathbf{C}^3/\mathbf{Z}_N$) in [5]. However, we have found a number of interesting differences, and original features in the configurations considered in this paper.

Our results in this paper extend the early results in [6] on intersecting branes to the context of compact models, leading to a large class of non-supersymmetric chiral four-dimensional models. We have provided a simple set of rules to construct explicit models, and studied their general features. One amusing feature is that, as observed in [13], compact models of intersecting branes lead naturally to replication of the chiral fermion content, due to the multiple intersections between different wrapped branes. In fact, we have used this property to construct explicit three generation models with realistic gauge groups.

In analogy with other string compactifications, we have found a rich structure of mixed $U(1)$ anomalies. We have shown that they are cancelled by a generalized GS mechanism mediated by untwisted or twisted RR fields. While the GS mediation by the latter is familiar from type IIB orientifolds [30, 29], the former (valid for D6-brane models) is rather unusual and interesting. We expect it has relevant phenomenological applications.

Finally, we have discussed that although the models are non-supersymmetric, they can be used for phenomenological purposes without a hierarchy problem, by simply lowering the string scale, and enlarging the volume transverse to the D-branes. Lack of supersymmetry also induces the appearance of tachyons, for which we have suggested an elimination mechanism, and a tantalizing phenomenological application in certain regimes.

Leaving further phenomenological properties of these configurations for discussion in [15], we conclude hoping these results are helpful in the construction of new open string vacua, and in their phenomenological application in the brane-world scenario.

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