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D-centro dominating sets in graphs

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Abstract

In this paper a new concept D-centro dominating set in graphs is introduced and graphs are characterized with some results. A subset $S \subset V(G)$ of a connected graph G is said to be D-centro dominating set of G, if for every $v \in V - S$, there exists a vertex u in S such that $D(u,v) = Rad(G)$. The minimum cardinality of the D-centro dominating set is called D-centro domination number, denoted by $DC_\gamma(G)$. The D-centro dominating set with cardinality $DC_\gamma(G)$ is called DC_γ-set of G. Some bounds for the D-centro domination number are determined. An important realization result on D-centro domination number is proved that for any integers a and b with $2 \leq a \leq b$, there exists a connected graph G such that $DC_\gamma(G) = a$ and $DC(G) = b$.

Keywords

Detour distance, detour eccentricity, detour radius, D-centro sets.

AMS Subject Classification

05C12, 05C69.

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Contents

1. Introduction

By a graph $G = (V,E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by n and m respectively. For basic graph terminology we refer to Harary [7]. For vertices r and s in a connected graph G, the detour distance $D(r,s)$ is the length of the farthest r-s path in G. For any vertex r of G, the *detour eccentricity* of r is $e_D(r) = max\{D(r,s) : s \in V\}$. A vertex s of G such that $D(r,s) = e_D(r)$ is called an *detour eccentric vertex* of r. The *detour radius R* and *detour diameter D* of G are defined by $Rad(G) = min\{e_D(s) : s \in V\}$ and $Diam(G) = max\{e_D(s) : s \in V\}$ respectively. An r-s path of length $D(r,s)$ is called an $r-s$ detour path. These concepts were studied by *Chartrand* et al [6]. If $e_D(s) = Rad(G)$ then s is called a *detour central vertex* of G and the subgraph induced by all detour central vertices of G is called *detour center* of G and is denoted by $CD(G)$. Next we study the following definitions given in [1]. For any vertex p in G, a set S of vertices of V is an *p-D-centro set* if $D(p,s) = Rad(G)$ for every $s \in S$, that is, p and $s$are said to be *D-centro* to each other. It is denoted by $DC_p(G)$. Let p be a vertex of G and S be the *p-D-centro* set of G. Then p is said to be the *D-centro vertex* of G with respect to S if the cardinality of S is the maximum among all S. The maximum p-D-centro set is denoted as S_p. The set of all D-centro vertices of G is called *D-set* of G and the cardinality of D-set is said to be *D-centro number* of G and it is denoted by $Dn(G)$. A set S is said to be *D-centro set* of G if $D(r,s) = Rad(G)$ for every pair of vertices of S. That is, r and s are D-centro to each other in S. The maximum cardinality among all D-centro sets is called *DC-set*. It is denoted by $DC(G)$.

2. D-centro dominating set

Next we define and study the properties of D-centro dominating set.

Definition 2.1. *A subset $S \subset V(G)$ of a connected graph G is said to be D-centro dominating set of G if for every $v \in V - S$, there exists a vertex u in S such that $D(u,v) = Rad(G)$. The minimum cardinality of the D-centro dominating set is called D-centro domination number, denoted by $DC_\gamma(G)$. The D-centro dominating set with cardinality $DC_\gamma(G)$ is called DC_γ-*

set of G.
Sometimes, there exists no u-D-centro vertex in G for a vertex u. Next we study these types of vertices in G

Definition 2.2. *A vertex $u \in G$ has no u-D-centro vertex is called null D-centro vertex. The collection of null D-centro vertices is called as null D-centro set of G.*

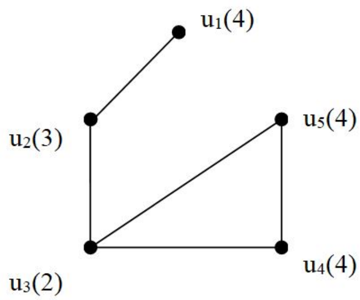

Figure 1 A graph G with detour eccentricities
$\{u_2, u_3\}$ is a DC_γ-set. Thus $DC_\gamma(G) = |u_2, u_3| = 2$. Here u_2 is the null *D*-centro vertex.

Vertex(x)	*x- D-centro set*
u_1	$\{u_3\}$
u_2	$\{\phi\}$
u_3	$\{u_1, u_4, u_5\}$
u_4	$\{u_3, u_5\}$
u_5	$\{u_3, u_4\}$

Theorem 2.3. *Every null D-centro vertices belongs to every D-centro dominating set.*

Proof. Let x be a null *D*-centro vertex and $S_x = \phi$. There exists no $w \in G$ such that $D(x,w) = Rad(G)$. Let S be a *D*-centro dominating set. For all $r \in V - S$, there exists $s \in S$ such that $D(r,s) = Rad(G)$. We claim that $x \in S$. By the definition of *D*-centro dominating set, each vertex r in $V - S$ has atleast one *D*-centro vertex in *S*. Since x is a null *D*-centro vertex, that is *x*-*D*-centro set = ϕ, it must belongs to the set *S*. Suppose x does not belongs to the set *S*. Then $x \in V - S$ and therefore there exists a vertex $s \in S$ such that $D(x,s) = Rad(G)$, which is a contradiction. Hence the theorem. □

Theorem 2.4. *A D-centro dominating set S without null D-centro vertices is a minimal D-centro dominating set if and only if for each vertex $r \in S$ one of the following two conditions hold: (a) r is an isolate of S (b) there exists a vertex $s \in V - S$ for which $DC_s(G) \cap S = \{r\}$*

Proof. Suppose that the set *S* be a minimal *D*-centro dominating set without null *D*-centro vertices. For every vertex $r \in S$, $S - \{r\}$ is not a *D*-centro dominating set. There exists some vertex s in $(V - S) \cup r$ such that s has no *D*-centro vertex in $S - \{u\}$.
Case(i): Suppose that $r = s$, then r is an isolate of *S* with respect to *D*-centro domination
Case(ii): Suppose that $s \in V - S$. If s has no *D*-centro vertex in $S - \{u\}$, but it has *D*-centro vertex in *S*, then r is the only *D*-centro vertex of s in *S*. Hence $DC_s(G) \cap S = \{r\}$.
For the converse part, we have to prove *S* is a minimal *D*-centro dominating set. Suppose that *S* is not a minimal *D*-centro dominating set. There exists a vertex $r \in S$ such that $S - \{r\}$ is a *D*-centro dominating set. Therefore r is *D*-centro to atleast one vertex s in $S - \{r\}$ and r has an *D*-centro vertex in $S - \{r\}$. Hence condition (a) does not hold. Further if $S - \{r\}$ is an *D*-centro dominating set, every element s in $V - S$ is *D*-centro to at least one vertex w in $S - \{r\}$ and the vertex r has a *D*-centro vertex in $S - \{r\}$. Hence, condition (b) does not hold. This contradicts to our assumption that for each $r \in S$, one of the following conditions hold. □

Theorem 2.5. *If $DC_\gamma(G) = p - 1$ where p is the order of G. Then G has $p - 2$ null D-centro vertices.*

Proof. Let *S* be a *D*-centro dominating set of *G* with order p. Since $|S| = p - 1$, there is only one vertex r in $V - S$. By the definition of $DC_\gamma(G)$, this vertex r is *D*-centro to any one of the vertex in *S* say s. Suppose that the vertex is *D*-centro to two or more vertices in *G*. Then $D(v_1, r) = D(v_2, s) = Rad(G)$ and $D(v_i, r) \neq Rad(G)$, where $i = 3, \ldots, p - 1$. Since, v_1, v_2 are the *D*-centro vertices of r, it is enough to take the vertex r instead of v_1, v_2 in *S* and $DC_\gamma(G) \leq p - 2$, which is a contradiction by our hypothesis. Therefore, there are only two vertices are null *D*-centro vertices. Hence, the cardinality of null *D*-centro vertices is $p - 2$. □

Theorem 2.6. *A graph G with no cycles does not contains the null D-centro vertices.*

Proof. Let *G* be a graph with no cycles. Suppose that *G* contains a null *D*-centro vertex w and so $DC_w(G) = \phi$. Clearly $D(w,r) < Rad(G)$ for all $r \in G$ is not possible since no pair of vertices have detour distance less than detour radius. Therefore $D(w,r) > Rad(G)$ for all $r \in G$. Now, consider $D(w,r) = Rad(G) + 1$ where $r \in G$. Since no vertex in the path *w*-*r* has detour length from w is equal to *R*, *G* contain a cycle. It is a contradiction and so it completes the proof. □

Next we develop a bound for $DC_\gamma(G)$.

Theorem 2.7. *Let G be a graph with k null D-centro vertices. Then $1 \leq DC_\gamma(G) - k \leq \frac{n}{2}$.*

Proof. Let *K* be the null *D*-centro set of *G* with k number of vertices. By theorem 2.3, the null *D*-centro vertices lie in *D*-centro dominating set. Therefore $DC_\gamma(G) \geq k$. But $Rad(G) \geq 1$. Then there exists atleast a path of detour length 2 such that the set $DC_\gamma(G)$ must contain atleast one non null *D*-centro vertex. Therefore $DC_\gamma(G) > k$ and so $DC_\gamma(G) \geq k + 1$. Obviously, the set $V - K$ contains atmost $\frac{n}{2}$ vertices. Hence $DC_\gamma(G) \leq \frac{n}{2} + k$. Thus, $k + 1 \leq DC_\gamma(G) \leq \frac{n}{2} + k$ and so $1 \leq DC_\gamma(G) - k \leq \frac{n}{2}$. □

Theorem 2.8. *Let P be a diametral path in a Tree T and S be the D-centro dominating set. Let $a,b \in P$ are the vertices D-centro to each other. Then $DC_\gamma(T) = R$ if and only if for each vertex in the set S satisfies one of the following condition:*
(i) $DC_a(G) \geq 1$ and $DC_b(G) = 1$ $\forall a,b \in P$.
(ii) If $1 \leq DC_b(G) \leq DC_a(G)$ and $DC_x(G) \bigcap V(P-\{a\}) \neq \emptyset$ for some $x \in DC_a(G)$, then there exists a vertex $r \in DC_x(G) \bigcap V(P-\{a\})$ and $DC_y(G) = \{r\}$ where y and r are D-centro to each other and $y \in V(P)$

Proof. Assume that $DC_\gamma(T) = R$. Let G be a tree T with radius R. We know that for any pair of distinct vertices of T, there exists a unique path between them. Now consider a diametral path P_{2n} of 2n vertices in T. Then $Rad(P_{2n}) = \lfloor \frac{2n}{2} \rfloor$. Partitioned P into two subsets $V_1 = \{a_1, a_2, \ldots.., a_n\}$ and $V_2 = \{b_1, b_2, \ldots.., b_n\}$ where a_n and b_1 are the central vertices. The vertices $a_2, \ldots b_{n-1}$ in the diametral path P_{2n} contains branches whose length from $V(P_{2n})$ is less than or equal to R. Further, the D-centro vertices of the remaining vertices of the branches must contains some D-centro vertices in P_{2n}. So that it is enough to take vertices in P_{2n} as the member of S. Now for each vertex a_i where $1 \leq i \leq n$ in V_1, either $a_i - D$-centro set contains singleton vertex or $DC_{a_i}(G) \geq DC_{b_i}(G)$. Therefore condition (i) holds. Suppose that $1 \leq DC_{b_i}(G) \leq DC_{a_i}(G)$ and $DC_x(G) \bigcap V(P-\{a_i\}) = \emptyset$ for some $x \in DC_{a_i}(G)$, then both a_i and b_i belongs to the set S. Therefore $DC_\gamma(T) > R$ which is a contradiction. Hence condition (ii) holds. Conversely, assume that for each vertex in S satisfies one of the stated conditions holds. It is notice that the cardinality of D-centro dominating set increases when $1 \leq DC_{b_i}(G) \leq DC_{a_i}(G)$ and $DC_x(G) \bigcap V(P-\{a_i\}) = \emptyset$ for some $x \in DC_{a_i}(G)$. Therefore by the hypothesis, it is obvious that $DC_\gamma(T) = R$. This proof is similar when the diametral path is odd. □

Theorem 2.9. *For a Tree T, $R \leq DC_\gamma(T) \leq D-1$, where R and D be radius and diameter of T.*

Proof. Let G be a tree T with radius R and diameter D. Suppose $DC_\gamma(T) < R$. Let P be any diametral path of even vertices in T. Let S be the DC_γ-set. Since P is a diametral path, the branches of T does not has length greater than R. So it is dominated by any one of the vertex in P with respect to D-centro domination. Therefore it is enough to choose S in $V(P)$. Each vertex of P has only one D-centro vertex in P and the central vertices of odd path contains two end vertices as D-centro vertex and viceversa. Suppose $DC_\gamma(T) \leq R-1$. Then there are atleast three or more vertices in the diametral path as D-centro vertices to any vertex in the diametral path which of them, two vertices forms a cycle with any vertices of T, which is a contradiction. Therefore $DC_\gamma(T) \geq R$. Now take the path (P_n) where n is even. Partitioned $V(P)$ into two subsets $V_1 = \{a_1, a_2, \ldots.a_n\}$ and $V_2 = \{b_1, b_2, \ldots.b_n\}$ where a_n and b_1 are the central vertices of P. Start with the vertex a_1 which is D-centro to b_1. For each vertex in S has the following condition. (i) If $DC_{a_i}(G) > 1$ and $DC_{b_i}(G) = 1$ where $1 \leq i \leq n$, then $a_i \in S$. (ii) If $1 \leq DC_{b_i}(G) \leq DC_{a_i}(G)$ and $DC_x(G) \bigcap V(P-\{a_i\}) = \emptyset$ for some $x \in DC_{a_i}(G)$, then both a_i and b_i belongs to the set S. (iii) If $1 \leq DC_{b_i}(G) \leq DC_{a_i}(G)$ and $DC_x(G) \bigcap V(P-\{a_i\}) \neq \emptyset$ for some $x \in DC_{a_i}(G)$, then there exists a vertex $b \in DC_x(G) \bigcap V(P-\{a_i\})$ and $DC_y(G) = \{b\}$ where y and b are D-centro to each other and $y \in V(P)$, then $a_i \in S$. From the above conditions, the set S contains $n-1$ maximum possible vertices. Now suppose $DC_\gamma(T) > D-1$. Then the graph G requires n number of vertices to dominate all other vertices with respect to D-centro domination where n is the total number of vertices in the diametral path P_n. Therefore, $DC_\gamma(G) \leq D-2$. This is a contradiction since S is not minimum.
This proof is similar when the diametral path is odd. □

Theorem 2.10. *Every vertex except end vertices in a diametral path P of a tree T is a support vertex. Then $DC_\gamma(T) = |S(T)|$ where S is the D-centro dominating set.*

Theorem 2.11. *(i) For a complete graph G = K_n, $DC_\gamma(G) = 1$.*
(ii) For a complete bipartite graph $G = K_{m,n}$, $DC(G) = DC_\gamma(G) = 2$, $m,n \geq 2$.

Proof. (i) Let $G = K_n$ and let $V(G) = v_i; 1 \leq i \leq n$. The detour length of any two vertices is $n-1$. Every singleton set v_i $(1 \leq i \leq n)$ forms a DC_γ-set and so $DC_\gamma(G) = 1$.
(ii) Let$G = K_{m,n}$ and be partitioned into two sets $V_1 = \{u_1, u_2, \ldots\ldots.u_n\}$ and $V_2 = \{v_1, v_2, \ldots\ldots..v_m\}$ such that every edge of G joins a vertex of V_1 with a vertex of V_2.
Case(i): If $m < n$, then the detour distance between two vertices from V_1 is $2m$ and that of two vertices from V_2 is $2m-1$. That is, $e_D(u) = 2m \forall u \in V_1$ and $e_D(v) = 2m-1 \forall v \in V_2$. Therefore $Rad(G) = 2m-1$. The D-centro vertices of each element of V_1 is V_2 and the set V_2 is V_1. Therefore the D-centro set contains only two elements. That is, an element from V_1 and an element from V_2. And also by the definition, it is enough to take one element from V_1 and one element from V_2 to satisfy the minimum D-centro dominating set. Hence $DC(G) = DC_\gamma(G) = 2$.
Case(ii): If $m = n$, the proof is same as case(i). □

Theorem 2.12. *(i) For a path P_n,*

$$DC_\gamma(P_n) = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n-1}{2}, & n \text{ is odd} \end{cases}$$

(ii) For a path P_n, $DC(P_n)$ = 2.

Proof. (i) Let $V(P_n) = \{v_1, v_2, \ldots\ldots..v_n\}$. In a path P_n, $Rad(P_n) = \lfloor \frac{n}{2} \rfloor$ and $Diam(P_n) = n-1$. Let S be the D-centro dom set.
Case(i): Suppose that n is odd. We take $n = 2k+1$, where k is a positive integer. Now take the vertex v_{k+1}. Then v_{k+1} has the minimum eccentricity R, where R is the eccentric radius of P_n. Since P_n is a path, the two end vertices v_1, v_n are the D-centro vertices of v_{k+1} and the detour distance

of these two vertices v_1, v_n from the vertex v_{k+1} is equal to detour radius and so $v_{k+1} \in S$. In the remaining vertices $v_2, \ldots\ldots, v_k, v_{k+2}, \ldots.., v_{k+i}, \ldots.., v_{n-i}$ for i from 2 to k, v_i and v_{k+i} are D-centro to each other. In the set there are $\frac{n-3}{2}$ vertices, which are also in D-centro dominating set. Hence, $DC_\gamma(G) = 1 + \frac{(n-3)}{2} = \frac{(n-1)}{2}$.
Case(ii): Suppose that n is even and so $n = 2k$ for every positive integer k. Each vertex has only one vertex as D-centro vertex. Therefore $DC_\gamma(G) = \frac{n}{2}$.
(ii) Let $V(P_n) = \{v_1, v_2, \ldots\ldots.., v_n\}$. In the path P_n, $Rad(P_n) = \lfloor \frac{n}{2} \rfloor$ and $Diam(P_n) = n-1$. Let S be the D-centro set. Since each vertex has only one vertex as D-centro vertex, the set S contains only two vertices. Hence, $DC(G) = 2$. □

Theorem 2.13. *For a cycle $G = C_n$ where $n \geq 3$, $DC_\gamma(G) = \lceil \frac{n}{3} \rceil$*

Proof. Consider this cycle, $G = C_n$. By Theorem 2.5 in [1], $N(x) = DC_x(G)$ for all x in C_n. That is, neighborhood vertices of every vertex of G are D-centro vertices. Therefore, by the definition of D-centro dominating set, $DC_\gamma(G) = \lceil \frac{n}{3} \rceil$ □

Theorem 2.14. *For any wheel graph W_n, $DC_p(W_n) = 1$ for $n \geq 3$.*

Proof. Let $V(W_n) = \{u, v_1, v_2, \ldots\ldots v_{n-1}\}$ with u as its central vertex. Since u is adjacent to all other vertices
$v_1, v_2, \ldots\ldots.. v_{n-1}$, the detour distance between any pair of vertices of $V(W_n)$ is $n-1$. Therefore any one vertex of $V(W_n)$ is a D-centro dom set. Since it is minimum, $DC_\gamma(Wn) = 1$. □

Theorem 2.15. *For a double star $G = S_{m,n}, DC_\gamma(G) = 2$ and $DC(G) = 1 + m$ where $m \geq n$.*

Proof. Consider the graph $G = S_{m,n}$ whose vertex set is
$\{r, s, u_1, u_2, \ldots, u_m, v_1, v_2, ..., v_n\}$. Now the eccentricity,
$e_D(x) = 2$ if $x \in r, s$ and $e_D(x) = 3$ if $x \in V(S_{m,n} - r, s)$ and $Rad(G) = 2$. Therefore r-D-centro set of G is $\{v_1, v_2, ..., v_n\}$ and s-D-centro set of G is $\{u_1, u_2, \ldots, u_m\}$. The u_i-D-centro set, $DC_{u_i}(G) = \{s, u_1, u_2, \ldots, u_{i-1}, u_{i+1}, ..., u_m\}$ and the v_i-D-centro set,
$DC_{v_i}(G) = \{r, v_1, v_2, ..., v_{i-1}, v_{i+1}, ..., v_n\}$. Now $S = r, s$. Then it is enough to take S as D-centro dominating set. Hence $DC_\gamma(G) = 2$. Now we see that every pair of vertices between the sets $\{r, v_1, v_2, ..., v_n\}$ and $\{s, u_1, u_2, ..., u_m\}$ are D-centro to each other. Therefore by the definition, $DC_\gamma(G) = 1 + m$ where $m \geq n$. □

3. Realization Results

Next we develop three realization results on $DC(G)$ and $DC_\gamma(G)$.

Theorem 3.1. *For every consecutive pair k, n of integers with $3 \leq k < n$, there exists a connected graph G of order n such that $DC(G) = k$.*

Proof. Suppose that $3 \leq k < n$.

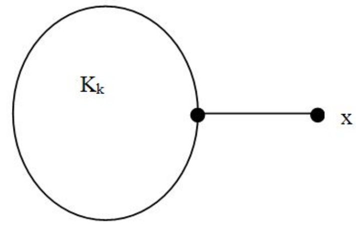

Figure 2 A graph K_k
Construct a complete graph K_k of vertices $\{u_1, u_2, \ldots\ldots, u_k\}$ of order k. By previous results, $DC(K_k) = k$ and $Rad(G) = k-1$. Now add a new vertex x to any one of $\{u_1, u_2, \ldots\ldots, u_k\}$. Now we join x to $u_i \forall (1 \leq i \leq n)$ for some i. It forms a new graph G of order n where $n = k+1$. Since x is an end vertex adjacent to u_i, it does not affect the radius. Hence the detour eccentricity of u_i is $k-1$ and $e_D(v) = \{k / v \neq u_i \forall v \in G\}$. Further since each vertex except x are adjacent to all other vertices, $D(u, v) = k-1$, for any pair of vertices u and v. Hence there exists a graph of order n such that $DC(G) = k$ and $3 \leq k \leq n$. □

Theorem 3.2. *For every pair r, s of positive integers with $2 \leq r \leq s$, there exists a connected graph G of order s such that $DC(G) = r$.*

Proof. Let r and s be positive integers such that $2 \leq r \leq s$.
Case (i): If $2 = r = s$. Then there exists a path of length 2 such that $DC(G) = 2$.
Case (ii): Let $s = 3$.
Subcase (i): If $s = 3$ and $2 = r < s$. Then there exists a path of length 3 such that $DC(G) = 2$.
Subcase (ii): If $s = 3$ and $2 < r = s$, that is $3 = r = s$. Then there exists a complete graph K_3 such that $DC(G) = 3$.
Case (iii): Let $s = 4$.
Subcase (i): If $s = 4$ and $2 = r < s$, then there exists a path of length 4 such that $DC(G) = 2$.
Subcase (ii): If $s = 4$ and $2 < r = s$, that is $4 = r = s$, then there exists a complete graph K_4 such that $DC(G) = 4$.
Subcase (iii): If $s = 4$ and $2 < r < s$, that is $r = 3$, then there exists a graph $G = K_3 \cup K_1$ such that $DC(G) = 3$ by previous theorem.
Case (iv): Take $2 \leq r \leq s$ where $s \geq 5$. The graph G has desired properties if $2 \leq r = s$ by the above cases. Now we have to prove $2 < r < s$ where $s \geq 5$.

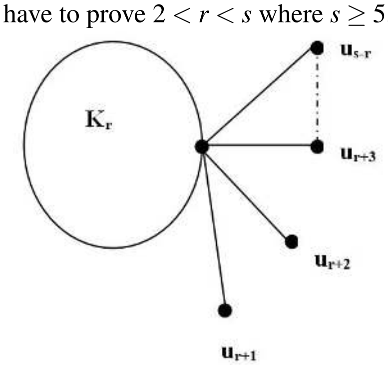

Figure 3 A graph G for case (iv)

Construct a complete graph $G = K_r$ where $V(K_r) = u_1, u_2, \ldots\ldots u_r$. Clearly $r \geq 4$, since $s > 4$. Now add new vertices $u_{r+1}, u_{r+2}, \ldots\ldots, u_{s-r}$ to u_1. It forms a new graph G of order s. Since the vertices $u_{r+1}, u_{r+2}, \ldots\ldots, u_{s-r}$ are end vertices adjacent to u_1, it does not alter the radius. That is, the eccentricity of u_1 is $r-1$ and $e_D(x) = \{r+1/x \neq u_1 \forall x \in G\}$. Further, each vertex of $\{u_1, u_2, \ldots\ldots, u_r\}$ is adjacent to all other vertices in $G - \{u_{r+1}, u_{r+2}, \ldots, u_{s-r}\}$ and $D(x,y) = r-1 \forall x, y \in \{u_1, u_2, \ldots, u_r\}$. Hence there exists a graph G of order s such that $DC(G) = r$. □

Theorem 3.3. *For any integers a and b with $2 \leq a \leq b$, there exists a connected graph G of order $n > 2$ such that $DC_\gamma(G) = a$ and $DC(G) = b$.*

Proof. Let a and b be any integers with $2 \leq a \leq b$. Then we can have the following cases.

Case(i): Assume that $2 = a = b$. Then there exists a complete bipartite graph $G = K_{m,n}$ for any integer m, n such that $DC_\gamma(G) = DC(G) = 2$.

Case(ii): Suppose that $2 = a < b$. Take G a double star $S_{m,n}$. Then G satisfies the desired properties.

Case(iii): Suppose that $2 < a < b$. Construct a complete graph $G = K_b$ of vertices with $b > 2$. Add a path P_{a-2} : $v_1, v_2, \ldots, v_{a-2}$ to u_i for any i, between i and b and $a - 2 < b$. Further add a new pendant vertex x to any of the vertices $v_1, v_2, \ldots, v_{a-2}$. It forms a new graph G of order $n = (a+b)-1$. The subgraph induced by the set of vertices $\{u_1, u_2, \ldots, u_b\}$ is complete and the path $v_1, v_2, \ldots, v_{a-2}$ joined to u_i and join x to v_2 as shown in the Figure 4. Hence the eccentricity of u_i does not exceed $b-1$. That is, $e_D(u_i) = b-1$. Therefore the new graph G does not alter its radius. Furthermore $e_D(u) = (a+b)-3/u \in \{u_1, \ldots\ldots, u_{i-1}, u_{i+1}, \ldots\ldots, u_b\}$. The vertices from $G - \{v_1, v_2, \ldots, v_{a-2}, x\}$ are D-centro to each other. Therefore $DC(G) = b$. Further since $e_D(v) > b-1$ for all $v \in \{v_1, v_2, \ldots, v_{a-2}, x\}$, $DC_x(G) = \phi$ for every $x \in \{v_1, v_2, \ldots\ldots, v_{a-2}, x\}$. By definition, $DC_\gamma(G) = 1 + (a-2) + 1$. That is, $DC_\gamma(G) = a$. Hence there exists a graph G such that $DC_\gamma(G) = a$ and $DC(G) = b$.

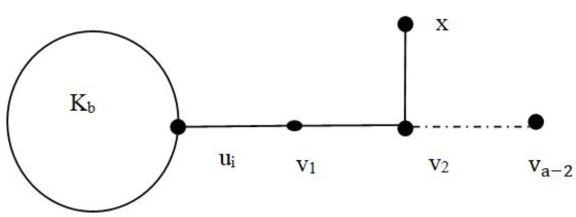

Figure 4 For case (iii)

Case(iv): Let $2 < a = b$. Construct a complete graph $G = K_b$ of vertices $\{u_1, u_2, \ldots\ldots, u_b\}$ with $n > 2$. Add a path $P_{a-1} : v_1, v_2, \ldots, v_{a-1}$ to u_i for any i, between i and b and $a-1 < b$. It forms a new graph G of order $n = (a+b)-1$. The subgraph induced by the set of vertices $\{u_1, u_2, \ldots, u_b\}$ is complete, the path $\{v_1, v_2, \ldots, v_{a-1}\}$ join to u_i as shown in the Figure 5. Hence the eccentricity of u_i does not exceed $b-1$. That is, $e_D(u_i) = b-1$ and so the new graph G does not alter its radius. Furthermore $e_D(u) = (a+b)-2$ for every u from the set $\{u_1, \ldots\ldots, u_{i-1}, u_{i+1}, \ldots, u_b\}$. The vertices from $G - \{v_1, v_2, \ldots, v_{a-1}\}$ are D-centro to each other. Therefore $DC(G) = b$. Further, since $e_D(v) > b-1$ for any v from the set $\{v_1, v_2, \ldots, v_{a-1}\}$ and $DC_x(G) = \phi$ for any x from the set $\{v_1, v_2, \ldots, v_{a-1}\}$. Hence $DC_\gamma(G) = 1 + (a-1)$ and so, $DC_\gamma(G) = a$. Thus there exists a graph G such that $DC_\gamma(G) = a$ and $DC(G) = b$.

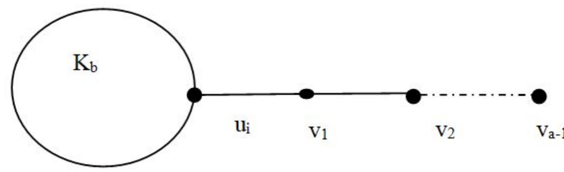

Figure 5 For case (iv) □

Theorem 3.4. *For positive integers R, D with $R < D \leq 2R$, there exists a connected graph G with $Rad(G) = R$, $Diam(G) = D$ and $DC(G) = R+1$ and $DC_\gamma(G) = R$.*

Proof. We prove this theorem by considering two cases relating this values of R and D.

Case (i): Assume that $R < D = 2R$. We construct a graph as shown in the Figure 6:

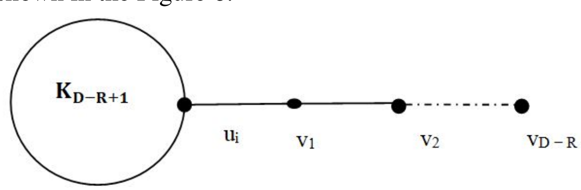

Figure 6 For case (i)

Consider two positive integers R and D such that $R < D = 2R$. Consider a complete graph K_{D-R+1} of vertices $u_1, u_2, \ldots, u_{D-R+1}$. Let P_{D-R} be a path having $v_1, v_2, \ldots, v_{D-R}$ as vertices. Construct a new graph G by joining P_{D-R} with a vertex u_i of K_{D-R+1}. The detour eccentricity of u_i is R and that of other vertices $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_{D-R+1}$ is $2R$. The detour eccentricity of v_1 is $R+1$, v_2 is $R+2$ and so on. The detour eccentricity of v_{D-R} is $2R (= D)$. Further, since K_{D-R+1} is complete and by the definition, the remaining vertices from $G - K_{D-R+1}$ are null D-centro vertices. Therefore by the definition of D-centro dom set $DC_\gamma(G) = 1 + (D-R) - 1 = R$ and every pair of vertices of K_{D-R+1} is D-centro to each other. Therefore, $DC(G) = R+1$ and $DC_\gamma(G) = R$.

Case (ii): Suppose that $R < D < 2R$, We construct a graph as follows:

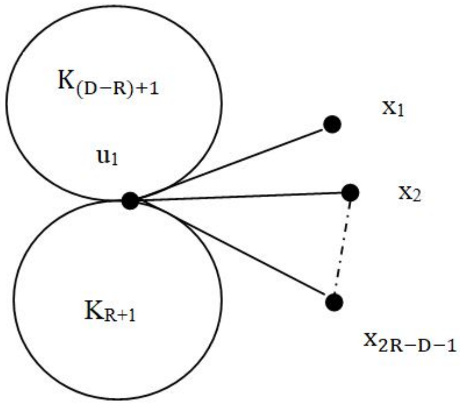

Figure 7 For case (ii)

Consider two positive integers R and D such that $R < D < 2R$. Consider a complete graph K_{R+1}, with the vertices $u_1, u_2, ..., u_{R+1}$. Let $K_{(D-R)+1}$ be another complete graph of order $(D-R)+1$ with $R+1 > (D-R)+1$. Let the vertices of $K_{(D-R)+1}$ be $u_i, v_1, ..., v_{(D-R)}$. Let H be a graph obtained from K_{R+1} and $K_{(D-R)+1}$ by identifying u_i as the common vertex in K_{R+1} and $K_{(D-R)+1}$. Now add the set S of new pendant vertices $\{x_1, x_2, ..., x_{2R-D-1}\}$ to H and join each vertex $x_i (1 \leq i \leq 2R-D-1)$ to the vertex u_i to obtain a new graph G as shown in the Figure 7. The detour eccentricity of u_i is R and that of other vertices $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_{R+1}$ are equal to $2R$. The detour eccentricity of $v_i (1 \leq i \leq (D-R)+1)$ is D and the detour eccentricity of $x_i (1 \leq i \leq 2R-D-1)$ is $R+1$. Further, K_{R+1} and $K_{(D-R)+1}$ are complete and the detour length of any vertex from K_{R+1} to a vertex u_i is R. Hence, by the definition $DC(G) = R+1$. Now, since $K_{(D-R)+1}$ is complete and S contains all pendant vertices, the remaining vertices from $G - K_{(D-R)+1} - \{u_i\} \cup S$ are the null D-centro vertices. Therefore, by the definition of D-centro dom set, $DC_\gamma(G) = 1 + D - R + 2R - D - 1 = R$. Hence $DC_\gamma(G) = R$. □

4. Conclusion

In this paper, the *D*-centro dominating sets in graphs has been studied, It is simply a dominating set of G with a detour distance $R(G)$. Also a special type of vertex, null *D*-centro vertex has been defined and the bounds for *D*-centro domination number interms of the number of null *D*-centro vertices have been found. The *D*-centro domination number for some special graphs like complete graph, cycle, wheel and star have been determined. Algorithms can be developed for finding the parameter, *D*-centro domination number for arbitrary graphs. This theory can be developed for finding *k*-center with respect domination based detour distance.

References

[1] A. Anto Kinsley and P. Siva Ananthi, D-centro sets in graphs, IJSRD - International Journal for Scientific Research and Development— Vol. 4, Issue 01, 2016 — ISSN (online): 2321-0613.

[2] F. Buckley and F. Harary, Distance in graphs, Addison-Wesley, Longman, 1990.

[3] G. Chartrand, T.W. Haynes, M.A. Henning, and Ping Zhang, Detour domination in graphs, Ars Combin. 71 (2004) 149 – 160.

[4] G. Chartrand, David Erwin, G. L. Johns and P. Zhang, On boundary vertices in graphs, J.Combin. Math. Combin.Comput. 48, (2004), 39-53.

[5] G. Chartrand, David Erwin, G. L. Johns and P. Zhang, Boundary vertices in graphs, Disc.Math. 263 (2003), 25-34.

[6] G. Chartrand and P. Zang, Introduction to Graph Theory, Tata McGraw-Hill, (2006).

[7] F. Harary, Graph theory, Addison- Wesley, 1969.

[8] T.N. Janakiraman, M. Bhanumathi and S. Muthammai, Eccentric domination in graphs, International Journal of Engineering Science, advanced Computing and Bio-Technology, Volume 1, No.2, pp 1-16, 2010.

[9] KM. Kathiresan, G. Marimuthu and M. Sivanandha Saraswathy, Boundary domination in graphs, Kragujevac J. Math. 33, (2010), 63-70.

[10] A. P. Santhakumaran, P. Titus, The vertex detour number of a graph, AKCE International J. Graphs. Combin., 4(2007), 99-112.

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