

**$D^*D\pi$  and  $B^*B\pi$  couplings in QCD**

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We calculate the  $D^*D\pi$  and  $B^*B\pi$  couplings using QCD sum rules on the light cone. In this approach, the large-distance dynamics is incorporated in a set of pion wave functions. We take into account two-particle and three-particle wave functions of twist 2, 3, and 4. The resulting values of the coupling constants are  $g_{D^*D\pi} = 12.5 \pm 1$  and  $g_{B^*B\pi} = 29 \pm 3$ . From this we predict the partial width  $\Gamma(D^{*+} \rightarrow D^0\pi^+) = 32 \pm 5$  keV. We also discuss the soft-pion limit of the sum rules which is equivalent to the external axial field approach employed in earlier calculations. Furthermore, using  $g_{B^*B\pi}$  and  $g_{D^*D\pi}$  the pole dominance model for the  $B \rightarrow \pi$  and  $D \rightarrow \pi$  semileptonic form factors is compared with the direct calculation of these form factors in the same framework of light-cone sum rules.

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**I. INTRODUCTION**

The extraction of fundamental parameters from data on heavy-flavored hadrons inevitably requires some information about the physics at large distances. Numerous theoretical studies have been devoted to making this extraction as reliable as possible. While the inclusive  $B$  and  $D$  decays appear to be the most clean reactions theoretically, exclusive decays are often much easier to measure experimentally. However, for their interpretation one needs accurate estimates of decay form factors and other hadronic matrix elements. In the exceptional case  $B \rightarrow D\ell\nu$ , the form factor at zero recoil can be calculated in the heavy quark limit [1, 2]. In most other important cases, one has to rely on less rigorous nonperturbative approaches. Among those, QCD sum rules [3] have proved to be particularly powerful.

In this paper we employ sum rule methods in order to calculate the  $D^*D\pi$  and  $B^*B\pi$  coupling constants. These couplings are interesting for several rea-

sons. In particular, they determine the normalization of the heavy-to-light form factors  $D \rightarrow \pi$  and  $B \rightarrow \pi$  near zero pionic recoil, where the  $D^*$  and  $B^*$  poles are believed to dominate. For further discussion one may consult Refs. [4, 5]. Recently, it has been argued that in the combined heavy quark and chiral limit vector meson dominance becomes even exact [6]. As noted in Refs. [7, 8],  $B^*$  dominance is also compatible with the dependence of the  $B \rightarrow \pi$  form factor on the momentum transfer  $p$  predicted by QCD sum rules at low values of  $p^2$ . A similar conclusion is reached in Ref. [9] concerning  $D^*$  dominance in the  $D \rightarrow \pi$  form factor. In addition, the  $D^*D\pi$  coupling can be directly measured in the decay  $D^* \rightarrow D\pi$  and thus provides one more independent test of the sum rule approach.

Calculations of couplings of heavy mesons to a pion have already been undertaken several times in the framework of QCD sum rules. Unfortunately, the sum rules obtained in Refs. [10–13] differ in nonleading terms and, to some extent, also in numerical results. Here, we suggest an alternative method known as QCD sum rules on the light cone. In this approach, the ideas of duality and matching between parton and hadron descriptions, intrinsic to the QCD sum rules, are combined with the specific operator product expansion (OPE) techniques used to study hard exclusive processes in QCD [14, 15]. In contrast to the conventional sum rules based on the Wilson OPE of the  $T$  product of currents at small distances, one considers expansions near the light cone in terms of nonlocal operators, the matrix elements of which define

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hadron wave functions of increasing twist. As one advantage, this formulation allows one to incorporate additional information about the Euclidean asymptotics of correlation functions in QCD for arbitrary external momenta. These new features are related to the (approximate) conformal invariance of QCD and are coded in the hadron wave functions. Many of the theoretical results obtained in the context of exclusive processes (see, e.g., Ref. [16]) are very useful in the present context as well. In turn, we will see that heavy-flavor decays can provide valuable constraints on the wave functions.

Previous applications of light-cone sum rules include calculations of the amplitude of the radiative decay  $\Sigma \rightarrow p\gamma$  [17], the nucleon magnetic moments [18], the strong couplings  $g_{\pi NN}$  and  $g_{\rho\omega\pi}$  [18], form factors of semileptonic and radiative  $B$ - and  $D$ -meson decays [8,19–21], the pion form factor at intermediate momentum transfers [22], and the  $\pi A\gamma^*$  form factors [23]. In all these cases the results are encouraging.

The light-cone sum rule for the coupling of heavy mesons to a pion is the principal result of the present paper which is organized as follows. In Sec. II we discuss possible strategies in constructing sum rules for coupling constants and explain the concept of light-cone sum rules. The derivation of the sum rule for the  $B^*B\pi$  and  $D^*D\pi$  couplings is then completed in Sec. III, taking into account the pion two- and three-particle wave functions up to twist 4. Section IV is devoted to a detailed numerical analysis. In Sec. V we show that in a simplified case, putting the external momenta in the correlation function equal to each other and performing a Borel transformation in one momentum instead of two, we obtain the sum rule proposed previously in Refs. [10–13]. We demonstrate that despite the slightly different terminology of these papers the sum rules must coincide with each other, and elaborate on possible subtleties in these earlier calculations. Furthermore, in Sec. VI using our results on the  $B^*B\pi$  and  $D^*D\pi$  coupling constants, we confront the pole model for the heavy-to-light form factors  $B \rightarrow \pi$  and  $D \rightarrow \pi$  with a direct calculation of these form factors in the same framework of light-cone sum rules following Ref. [8]. A comprehensive comparison of our results on the  $D^*D\pi$  and  $B^*B\pi$  couplings with other estimates and our conclusions are presented in Sec. VII.

Technical details are collected in two appendixes. Appendix A summarizes the relevant features of the pion wave functions and specifies the input in our numerical calculations. In Appendix B we derive a simple rule how to subtract the contribution from excited resonances and continuum states in the sum rule.

## II. LIGHT-CONE VERSUS SHORT-DISTANCE EXPANSION

For definiteness, we focus on the  $D^*D\pi$  coupling defined by the on-mass-shell matrix element

$$\langle D^{*+}(p)\pi^-(q) | D^0(p+q) \rangle = -g_{D^*D\pi} q_\mu \epsilon^\mu, \quad (1)$$

where the momentum assignment is specified in brackets and  $\epsilon_\mu$  is the polarization vector of the  $D^*$ . The couplings

for the different charge states are related by isospin symmetry:

$$\begin{aligned} g_{D^*D\pi} &\equiv g_{D^{*+}D^0\pi^+} = -\sqrt{2}g_{D^{*+}D^+\pi^0} = \sqrt{2}g_{D^{*0}D^0\pi^0} \\ &= -g_{D^{*0}D^+\pi^-}. \end{aligned} \quad (2)$$

Most of what is said below applies equally to  $B^*B\pi$  couplings. The corresponding relations are obtained by the obvious replacements  $c \rightarrow b$ ,  $D^* \rightarrow \bar{B}^*$ , and  $D \rightarrow \bar{B}$ .

Following the general strategy of QCD sum rules, we want to obtain quantitative estimates for  $g_{D^*D\pi}$  by matching the representations of a suitable correlation function in terms of hadronic and quark-gluon degrees of freedom. For this purpose, we choose

$$\begin{aligned} F_\mu(p, q) &= i \int d^4x e^{ipx} \\ &\times \langle \pi^-(q) | T \{ \bar{d}(x) \gamma_\mu c(x), \bar{c}(0) i \gamma_5 u(0) \} | 0 \rangle. \end{aligned} \quad (3)$$

To the best of our knowledge, the study of correlation functions with the  $T$  product of currents sandwiched between the vacuum and one-pion state was first suggested in Ref. [24].

With the pion being on mass shell,  $q^2 = m_\pi^2$ , the correlation function (3) depends on two invariants  $p^2$  and  $(p+q)^2$ . Throughout the paper we set  $m_\pi = 0$ . The contribution of interest is the one having poles in  $p^2$  and  $(p+q)^2$ :

$$\begin{aligned} F_\mu(p, q) &= \frac{m_D^2 m_{D^*} f_D f_{D^*} g_{D^*D\pi}}{m_c(p^2 - m_{D^*}^2)[(p+q)^2 - m_D^2]} \\ &\times \left[ q_\mu + \frac{1}{2} \left( 1 - \frac{m_D^2}{m_{D^*}^2} \right) p_\mu \right]. \end{aligned} \quad (4)$$

Obviously, this term stems from the ground states in the  $(\bar{d}c)$  and  $(\bar{c}u)$  channels. To derive Eq. (4) we have made use of Eq. (1) and the decay constants  $f_D$  and  $f_{D^*}$  defined by the matrix elements

$$\langle D | \bar{c} i \gamma_5 u | 0 \rangle = \frac{m_D^2 f_D}{m_c} \quad (5)$$

and

$$\langle 0 | \bar{d} \gamma_\mu c | D^* \rangle = m_{D^*} f_{D^*} \epsilon_\mu, \quad (6)$$

respectively.

The main theoretical task is the calculation of the correlation function (3) in QCD. This problem can be solved in the Euclidean region where both virtualities  $p^2$  and  $(p+q)^2$  are negative and large, so that the charm quark is sufficiently far off shell. Substituting, as a first approximation, the free  $c$ -quark propagator

$$\begin{aligned} \langle 0 | T \{ c(x) \bar{c}(0) \} | 0 \rangle &= i \hat{S}_c^0(x) \\ &= \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\not{k} + m_c}{m_c^2 - k^2} \end{aligned} \quad (7)$$

into Eq. (3), one readily obtains

$$\begin{aligned} F_\mu(p, q) &= i \int \frac{d^4x d^4k}{(2\pi)^4 (m_c^2 - k^2)} e^{i(p-k)x} \\ &\times [m_c \langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle \\ &+ k^\nu \langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_\nu \gamma_5 u(0) | 0 \rangle]. \end{aligned} \quad (8)$$

Diagrammatically, this contribution is depicted in Fig. 1(a). Applying the short-distance expansion (SDE) in terms of local operators to the first matrix element of Eq. (8),

$$\bar{d}(x)\gamma_\mu\gamma_5 u(0) = \sum_n \frac{1}{n!} \bar{d}(0) (\overleftarrow{D} \cdot x)^n \gamma_\mu\gamma_5 u(0), \quad (9)$$

one has, after integration over  $x$  and  $k$ ,

$$F_\mu(p, q) = i \frac{m_c}{m_c^2 - p^2} \sum_{n=0}^{\infty} \frac{(2p \cdot q)^n}{(m_c^2 - p^2)^n} M_n q_\mu, \quad (10)$$

where

$$\begin{aligned} \langle \pi(q) | \bar{d} \overleftarrow{D}_{\alpha_1} \overleftarrow{D}_{\alpha_2} \cdots \overleftarrow{D}_{\alpha_n} \gamma_\mu \gamma_5 u | 0 \rangle \\ = (i)^n q_\mu q_{\alpha_1} q_{\alpha_2} \cdots q_{\alpha_n} M_n + \cdots, \end{aligned}$$

$D$  being the covariant derivative, has been used. One immediately encounters the following problem. If the ratio

$$\tilde{\xi} = 2(p \cdot q) / (m_c^2 - p^2) = [(p+q)^2 - p^2] / (m_c^2 - p^2) \quad (11)$$

is finite, one must keep an *infinite* series of local operators in Eq. (10). All these operators give contributions of the same order in the heavy quark propagator  $1/(m_c^2 - p^2)$ , differing only by powers of the dimensionless parameter<sup>1</sup>  $\tilde{\xi}$ . Therefore, SDE of Eq. (8) is useful *only* if  $\tilde{\xi} \rightarrow 0$ , i.e., if  $p^2 \simeq (p+q)^2$  or, equivalently,  $q \simeq 0$ . Under this condition, the series in Eq. (10) can be truncated after a few terms involving only a small number of unknown matrix elements  $M_n$ . However, for general momenta with  $p^2 \neq (p+q)^2$  one has to sum up the infinite series of matrix elements of local operators in some way.

This formidable task is solved by using the techniques developed for hard exclusive processes in QCD [15, 16]. We illustrate the solution for the correlation function

$$\begin{aligned} i \int d^4x e^{ipx} \langle \pi^0(q) | T \{ \bar{q}(x) \gamma_\mu \mathcal{Q} q(x), \bar{q}(0) \gamma_\nu \mathcal{Q} q(0) \} | 0 \rangle \\ = \varepsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F^*(p^2, (p+q)^2), \quad (12) \end{aligned}$$

which is similar to Eq. (3) and defines the form factor of the coupling of a pion to a pair of virtual photons [16, 25]. In Eq. (12),  $\mathcal{Q}$  is a matrix of electromagnetic charges, and  $q$  is a row vector composed of the up and down quark flavors. As is well known [14], for sufficiently virtual photons,  $p^2 \rightarrow -\infty$  and  $(p+q)^2 \rightarrow -\infty$ , this form factor can be calculated in perturbative QCD. The principal result reads

<sup>1</sup>This feature is also observed in deep inelastic scattering, with the variables  $\{Q^2, \nu, x\}$  playing the role of  $\{-p^2, p \cdot q, \tilde{\xi}\}$ . As is well known, there one applies an expansion near the light cone in terms of operators of increasing twist, rather than of increasing dimension.

$$F^*(p^2, (p+q)^2) = F_0 \int_0^1 \frac{du \varphi_\pi(u)}{(p+uq)^2}, \quad (13)$$

$$F_0 = \frac{4\pi\alpha\sqrt{2}f_\pi}{3},$$

with calculable radiative corrections, and with power corrections suppressed by the photon virtualities. Here,  $\varphi_\pi(u)$  is the pion wave function of leading twist, defined by the following matrix element of a nonlocal operator on the light cone  $x^2 = 0$ :

$$\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle = -iq_\mu f_\pi \int_0^1 du e^{iuqx} \varphi_\pi(u). \quad (14)$$

Physically,  $\varphi_\pi$  represents the distribution in the fraction of the light-cone momentum  $q_0 + q_3$  of the pion carried by a constituent quark. Note the normalization of  $\varphi_\pi$  to unity following from Eq. (14) for  $x = 0$ .

Let us first concentrate on the form factor (13) at (almost) equal photon virtualities, i.e., at  $\tilde{\xi} = (2p \cdot q)/(-p^2) \ll 1$ . Expanding the denominator in Eq. (13) around  $\tilde{\xi} = 0$  one obtains a sum over moments of the pion wave function:

$$F^*(p^2, (p+q)^2) = \frac{F_0}{p^2} \sum_n \xi^n \int_0^1 du u^n \varphi_\pi(u). \quad (15)$$

From the definition (14) it is easy to see that these moments are given by vacuum-to-pion transition matrix elements involving increasing powers of the covariant derivative. For  $p^2 = (p+q)^2$ , i.e.,  $q = 0$ , only the lowest moment  $n = 0$  contributes in Eq. (15), and the form factor reduces to  $F_0/p^2$  which is the classical result. In contrast, if the photon virtualities differ strongly from each other, then many moments contribute to Eq. (15). In this case, the calculation of the form factor requires knowledge of the shape of the pion wave function.

Returning to the correlation function (3) one realizes that the same technique may be used to obtain a representation analogous to Eq. (13). The only new element in the correlation function (3) is the virtual heavy quark propagating between the points  $x$  and 0 instead of the light quarks present in Eq. (12). This gives rise to important differences which, however, do not change the formalism substantially. For the present discussion it is sufficient to stick to the approximation (8) and confine ourselves to the first term proportional to  $m_c$ . The complete analysis of this expression and the calculation of further corrections will be carried out in the next section. Furthermore, writing  $F_\mu$  in terms of invariant amplitudes,

$$F_\mu(p, q) = F(p^2, (p+q)^2) q_\mu + \tilde{F}(p^2, (p+q)^2) p_\mu, \quad (16)$$

we focus on the function  $F$ . Using the definition Eq. (14) of the leading twist wave function and integrating over  $x$

and  $k$  one finds

$$F(p^2, (p+q)^2) = m_c f_\pi \int_0^1 \frac{du \varphi_\pi(u)}{m_c^2 - (p+uq)^2}. \quad (17)$$

Thus, the infinite series of matrix elements of local operators encountered before in Eq. (10) is effectively replaced by an unknown wave function. The expression (17) is rather similar to the one quoted in Eq. (13) for the  $\pi^0 \gamma^* \gamma^*$  form factor. Most noteworthy is the fact that the large-distance dynamics is described by one and the same pion wave function. This universal property is essential for the whole approach.

Next we indicate how the relation (17) can be turned into a sum rule for the coupling constant  $g_{D^* D \pi}$ . The key idea is to write a hadronic representation of  $F$  by means of a double dispersion integral:

$$\mathcal{B}_{M^2} f(Q^2) = \lim_{Q^2, n \rightarrow \infty, Q^2/n = M^2} \frac{(Q^2)^{(n+1)}}{n!} \left( -\frac{d}{dQ^2} \right)^n f(Q^2) \equiv f(M^2) \quad (19)$$

to Eq. (18) with respect to  $p^2$  and  $(p+q)^2$ , we obtain

$$F(M_1^2, M_2^2) \equiv \mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} F(p^2, (p+q)^2) = \frac{m_D^2 m_{D^*} f_D f_{D^*} g_{D^* D \pi}}{m_c} e^{-\frac{m_{D^*}^2}{M_1^2} - \frac{m_D^2}{M_2^2}} + \int e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \rho^h(s_1, s_2) ds_1 ds_2, \quad (20)$$

where  $M_1^2$  and  $M_2^2$  are the Borel parameters associated with  $p^2$  and  $(p+q)^2$ , respectively. Note that contributions from heavier states are now exponentially suppressed by factors  $\exp\{-\frac{s_{1,2} - m_{D^*,D}^2}{M_{1,2}^2}\}$  as desired, while the subtraction terms depending only on one of the variables,  $p^2$  or  $(p+q)^2$ , vanish.

The same transformation has to be applied to the expression (17). To this end we rewrite  $(p+uq)^2 = (1-u)p^2 + u(p+q)^2$ , and use

$$\begin{aligned} \mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} \frac{(l-1)!}{[m_c^2 - (1-u)p^2 - u(p+q)^2]^l} \\ = (M^2)^{2-l} e^{-m_c^2/M^2} \delta(u - u_0), \quad (21) \end{aligned}$$

where the Borel parameters  $M_1^2$  and  $M_2^2$  have been replaced by

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \quad (22)$$

Finally, equating the quark-gluon and the hadronic representations of  $F(M_1^2, M_2^2)$  and discarding for a moment contributions of higher states, we end up with the sum rule

$$\begin{aligned} \frac{m_D^2 m_{D^*} f_D f_{D^*}}{m_c} g_{D^* D \pi} \\ = m_c f_\pi \varphi_\pi(u_0) M^2 \\ \times \exp \left[ \frac{m_{D^*}^2 - m_c^2}{M_1^2} + \frac{m_D^2 - m_c^2}{M_2^2} \right] + \dots \quad (23) \end{aligned}$$

$$\begin{aligned} F(p^2, (p+q)^2) = \frac{m_D^2 m_{D^*} f_D f_{D^*} g_{D^* D \pi}}{m_c (p^2 - m_{D^*}^2) [(p+q)^2 - m_D^2]} \\ + \int \frac{\rho^h(s_1, s_2) ds_1 ds_2}{(s_1 - p^2) [s_2 - (p+q)^2]} \\ + \int \frac{\rho_1^h(s_1) ds_1}{s_1 - p^2} + \int \frac{\rho_2^h(s_2) ds_2}{s_2 - (p+q)^2}. \quad (18) \end{aligned}$$

The first term arises from the ground state contribution already indicated in Eq. (4), while the spectral function  $\rho^h(s_1, s_2)$  is supposed to take into account higher resonances and continuum states in the  $D^*$  and  $D$  channels. The additional single dispersion integrals originate in subtractions which are generally necessary to make the double dispersion integral finite. Then, considering  $p^2$  and  $(p+q)^2$  as independent variables one can perform the usual Borel improvement in both channels. Applying the Borel operator

The ellipsis refers to higher-twist contributions which we discuss in detail later. Since  $M_1^2$  and  $M_2^2$  are expected to be quite similar in magnitude, the coupling constant  $g_{D^* D \pi}$  is determined by the value of the pion wave function at  $u \simeq 1/2$ , that is, by the probability for the quark and the antiquark to carry equal momentum fractions in the pion [17]. This interesting feature is shared by the sum rules for many other important hadronic couplings involving the pion.

The quantity  $\varphi_\pi(1/2)$  is considered to be a nonperturbative parameter, similar to quark and gluon condensates in the standard approach. It may be determined from suitable sum rules in which the phenomenological part is known experimentally. We use the value

$$\varphi_\pi(1/2) = 1.2 \pm 0.2 \quad (24)$$

obtained in Ref. [18] and comment on this choice in Sec. IV.

The dependence on the pion wave function disappears in the kinematical limit  $q \rightarrow 0$  as can be seen from Eq. (17). This is just the limit where the correlation function (3) can be treated in the SDE. The condition  $q \simeq 0$  is implicitly assumed in Refs. [10, 12] where the correlation function (3) is calculated using the external field method. This technique is equivalent to the soft-pion approximation used in Refs. [11, 13] as will become clear later. For comparison, we present the sum rule following from Eqs. (17) and (18) by putting  $q = 0$  or, equivalently,  $(p+q)^2 = p^2$ . Since  $p^2$  is the only variable left, one now can only perform a single Borel transformation and, hence, the subtraction terms in the double disper-

sion relation (18) are no longer eliminated. Moreover, the contributions to Eq. (18) of transitions from excited states to ground states are not suppressed after Borel transformation [26]. This point will be explained in more detail later in Sec. V. In the approximation considered in Eq. (23) one obtains

$$\begin{aligned} & \frac{m_D^2 m_{D^*} f_D f_{D^*}}{m_c} g_{D^*D\pi} + M^2 A \\ &= m_c f_\pi M^2 \exp \left[ \frac{m_{D^*}^2 - m_c^2}{2M^2} + \frac{m_D^2 - m_c^2}{2M^2} \right] + \dots, \end{aligned} \quad (25)$$

where  $A$  is an unknown constant corresponding to the contributions of unwanted transitions and subtraction terms.

From Eqs. (23) and (25) one can clearly see the advantages and disadvantages of the two approaches. In the light-cone sum rule (23) the hadronic input is simple, whereas the theoretical expression involves a new *universal* nonperturbative parameter, namely,  $\varphi_\pi(1/2)$ .

$$\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle = -i q_\mu f_\pi \int_0^1 du e^{iuqx} [\varphi_\pi(u) + x^2 g_1(u) + O(x^4)] + f_\pi \left( x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u). \quad (26)$$

On the right-hand side (RHS) of this relation one sees the first few terms of the light-cone expansion in  $x^2$  of the matrix element on the LHS. While  $\varphi_\pi$  parametrizes the leading twist-2 contribution,  $g_1$  and  $g_2$  are associated with twist-4 operators. In the second matrix element of Eq. (8) we substitute

$$\gamma_\mu \gamma_\nu = -i \sigma_{\mu\nu} + g_{\mu\nu} \quad (27)$$

and express the result in terms of the twist-3 wave functions  $\varphi_p$  and  $\varphi_\sigma$  defined by the matrix elements

$$\langle \pi(q) | \bar{d}(x) i \gamma_5 u(0) | 0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{iuqx} \varphi_p(u) \quad (28)$$

and

$$\begin{aligned} F^{(a)}(p^2, (p+q)^2) &= \int_0^1 \frac{du}{m_c^2 - (p+uq)^2} \left\{ m_c f_\pi \varphi_\pi(u) + \frac{f_\pi m_\pi^2}{m_u + m_d} \left[ u \varphi_p(u) + \frac{1}{6} \left( 2 + \frac{p^2 + m_c^2}{m_c^2 - (p+uq)^2} \right) \varphi_\sigma(u) \right] \right. \\ &\quad \left. + m_c f_\pi \left[ \frac{2u g_2(u)}{m_c^2 - (p+uq)^2} - \frac{8m_c^2 [g_1(u) + G_2(u)]}{[m_c^2 - (p+uq)^2]^2} \right] \right\}, \end{aligned} \quad (31)$$

where

$$G_2(u) = - \int_0^u g_2(v) dv.$$

The superscript (a) refers to the diagram in Fig. 1(a) which represents the leading twist term in the light-cone

Just the opposite is the case for the sum rule (25) at  $q = 0$ . Here the QCD part is straightforward, while the hadronic representation now involves an additional unknown quantity, which is *nonuniversal* and specific for this particular sum rule. A comparison of the results obtained in these two approaches should allow one to check the reliability and improve the accuracy of the predictions.

### III. LIGHT-CONE SUM RULE FOR $g_{D^*D\pi}$ AND $g_{B^*B\pi}$

In this section we systematically derive the light-cone sum rule for the  $D^*D\pi$  and  $B^*B\pi$  couplings taking into account the two- and three-particle pion wave functions up to twist 4. First, we complete the calculation of the diagram Fig. 1(a) which represents the contribution from quark-antiquark wave functions. To this end we return to the expression (8). In the first matrix element we include the twist-4 corrections in addition to the leading twist term already given in Eq. (14):

$$\begin{aligned} \langle \pi(q) | \bar{d}(x) \sigma_{\mu\nu} \gamma_5 u(0) | 0 \rangle &= i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \\ &\quad \times \int_0^1 du e^{iuqx} \varphi_\sigma(u). \end{aligned} \quad (29)$$

It should be noted that in Eqs. (26), (28), and (29) the path-ordered gauge factors

$$P \exp \left\{ i g_s \int_0^1 d\alpha x_\mu A^\mu(\alpha x) \right\}, \quad (30)$$

appearing in between the quark fields and assuring gauge invariance, are not shown for brevity since they formally disappear in the light-cone gauge  $x_\mu A^\mu = 0$  assumed throughout this paper. More details on these wave functions can be found in Refs. [18, 27, 28] and in Appendix A.

Collecting all terms, we obtain the following result for the invariant function  $F$  as defined in Eq. (16):

expansion of the  $c$ -quark propagator given in Eq. (7).

In addition, to the accuracy of Eq. (31) we must also take into account higher-twist terms in the propagator up to twist 4 which are numerous, in general. The complete expansion is given in Ref. [27]. One has contributions from  $\bar{q}Gq$ ,  $\bar{q}GGq$ , and  $\bar{q}q\bar{q}q$  nonlocal operators,  $G$  denoting the gluon field strength. Here we only consider operators with one gluon field, corresponding to quark-antiquark-gluon components in the pion, and neglect components with two extra gluons, or with an additional  $\bar{q}q$  pair. This is consistent with the approximation of the twist-4 two-particle wave functions derived in Ref. [28] and used here. Taking into account higher Fock-space components would demand corresponding modifications in the two-particle functions via the equations of motion. Formally, the neglect of the  $\bar{q}GGq$  and  $\bar{q}q\bar{q}q$  terms can be justified on the basis of an expansion in conformal spin [28]. In this approximation the  $c$ -quark propagator reads

$$\langle 0|T\{c(x)\bar{c}(0)\}|0\rangle = i\hat{S}_c^0(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[ \frac{1}{2} \frac{\not{k} + m_c}{(m_c^2 - k^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} + \frac{1}{m_c^2 - k^2} vx_\mu G^{\mu\nu}(vx) \gamma_\nu \right], \quad (32)$$

where  $G_{\mu\nu} = G_{\mu\nu}^c \frac{\lambda^c}{2}$  with  $\text{tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ , and  $g_s$  is the strong coupling constant.

Substituting Eq. (32) into Eq. (3) and using Eq. (16) one obtains the contribution to the invariant function  $F$  represented by the diagram in Fig. 1(b):

$$F_\mu^{(b)}(p^2, (p+q)^2) = ig_s \int \frac{d^4k d^4x dv}{(2\pi)^4 (m_c^2 - k^2)} e^{i(p-k)x} \langle \pi | \bar{d}(x) \gamma_\mu \left[ vx_\rho G^{\rho\lambda}(vx) \gamma_\lambda + \frac{\not{k} + m_c}{m_c^2 - k^2} \frac{1}{2} G^{\rho\lambda}(vx) \sigma_{\rho\lambda} \right] \gamma_5 u(0) | 0 \rangle. \quad (33)$$

With Eq. (27) and the identities

$$\gamma_\mu \sigma_{\rho\lambda} = i(g_{\mu\rho} \gamma_\lambda - g_{\mu\lambda} \gamma_\rho) + \varepsilon_{\mu\rho\lambda\delta} \gamma^\delta \gamma_5 \quad (34)$$

and

$$\gamma_\mu \gamma_\nu \sigma_{\rho\lambda} = (\sigma_{\mu\lambda} g_{\nu\rho} - \sigma_{\mu\rho} g_{\nu\lambda}) + i(g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) - \varepsilon_{\mu\nu\rho\lambda} \gamma_5 - i\varepsilon_{\nu\rho\lambda\alpha} g^{\alpha\beta} \sigma_{\mu\beta} \gamma_5, \quad (35)$$

one is led to the three-particle pion wave functions [25, 28] defined by

$$\langle \pi | \bar{d}(x) g_s G_{\mu\nu}(vx) \sigma_{\alpha\beta} \gamma_5 u(0) | 0 \rangle = if_{3\pi} [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \int \mathcal{D}\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)}, \quad (36)$$

$$\begin{aligned} \langle \pi | \bar{d}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) u(0) | 0 \rangle &= f_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)} \\ &+ f_\pi \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)}, \end{aligned} \quad (37)$$

$$\begin{aligned} \langle \pi | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 \rangle &= if_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)} \\ &+ if_\pi \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)}, \end{aligned} \quad (38)$$

where  $\tilde{G}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\sigma\tau} G^{\sigma\tau}$  and  $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ . While  $\varphi_{3\pi}(\alpha_i)$  is a twist-3 wave function, the remaining functions  $\varphi_\perp$ ,  $\varphi_\parallel$ ,  $\tilde{\varphi}_\perp$ , and  $\tilde{\varphi}_\parallel$  are all of twist 4. Substitution of these expressions in Eq. (33), finally, yields

$$F^{(b)}(p^2, (p+q)^2) = \int_0^1 dv \int \mathcal{D}\alpha_i \left\{ \frac{4f_{3\pi} \varphi_{3\pi}(\alpha_i) v(pq)}{\{m_c^2 - [p + (\alpha_1 + v\alpha_3)q]^2\}^2} + m_c f_\pi \frac{2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)}{\{m_c^2 - [p + (\alpha_1 + v\alpha_3)q]^2\}^2} \right\}. \quad (39)$$

In addition to Fig. 1(b) there are further gluonic diagrams such as the ones depicted in Figs. 1(c) and 1(d). Note, however, that it is not necessary to take the diagram in Fig. 1(c) into account separately, since its contribution (to twist-4 accuracy) is already included in the two-particle wave functions. In contrast, the two-loop perturbative corrections exemplified in Fig. 1(d) should be included in a systematic way, but their calculation lies beyond the scope of this paper.

Putting together Eqs. (31) and (39) and applying the double Borel transformation (21) with respect to  $p^2$  and  $(p+q)^2$ , we end up with the following expression for the invariant amplitude  $F$ :

$$\begin{aligned}
F(M_1^2, M_2^2) &\equiv F(M^2, u_0) \equiv \mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} F(p^2, (p+q)^2) \\
&= e^{-\frac{m_c^2}{M^2}} M^2 \left\{ m_c f_\pi \varphi_\pi(u_0) + \frac{f_\pi m_\pi^2}{m_u + m_d} \left( u_0 \varphi_p(u_0) + \frac{1}{3} \varphi_\sigma(u_0) \right. \right. \\
&\quad \left. \left. + \frac{1}{6} u_0 \frac{d\varphi_\sigma}{du}(u_0) + \frac{m_c^2}{3M^2} \varphi_\sigma(u_0) \right) \right. \\
&\quad \left. + \frac{2f_\pi m_c}{M^2} u_0 g_2(u_0) - \frac{4f_\pi m_c^3}{M^4} [g_1(u_0) + G_2(u_0)] + 2f_{3\pi} I_3^G(u_0) + m_c f_\pi \frac{I_4^G(u_0)}{M^2} \right\}. \quad (40)
\end{aligned}$$

Here,  $I_3^G$  and  $I_4^G$  involve the three-particle wave functions of twist 3 and 4, respectively:

$$I_3^G(u_0) = \int_0^{u_0} d\alpha_1 \left[ \frac{\varphi_{3\pi}(\alpha_1, 1-u_0, u_0-\alpha_1)}{u_0-\alpha_1} - \int_{u_0-\alpha_1}^{1-\alpha_1} d\alpha_3 \frac{\varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)}{\alpha_3^2} \right], \quad (41)$$

$$I_4^G(u_0) = \int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3} [2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)]. \quad (42)$$

The Borel parameters  $M^2$  and  $u_0$  are given in Eq. (22). The above is the desired quark-gluon representation of the invariant amplitude  $F$  in the correlation function (3).

The remaining task now is to match Eq. (40) with the corresponding hadronic representation (20) and to extract the coupling  $g_{D^*D\pi}$ . As usual, invoking duality, we assume that above certain thresholds in  $s_1$  and  $s_2$  the double spectral density  $\rho^h(s_1, s_2)$  associated with higher resonances and continuum states coincides with the spectral density derived from the diagrams in Fig. 1. The procedure is explained in detail in Appendix B. For  $M_1^2 = M_2^2 = 2M^2$  and  $u_0 = \frac{1}{2}$ , and for standard polynomial wave functions, the effect of the continuum subtraction is remarkably simple [17, 18]. It amounts to the following replacement of the exponential factor multiplying the twist-2 and -3 terms proportional to  $M^2$  in Eq. (40):

$$e^{-\frac{m_c^2}{M^2}} \rightarrow \left( e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}} \right), \quad (43)$$

$s_0$  being the threshold parameter defined in Eq. (B10). The higher-twist terms which are suppressed in Eq. (40) by inverse powers of  $M^2$  with respect to the leading one remain unaffected. With Eqs. (20), (40), and (43) it is then easy to derive the following QCD sum rule for the  $DD^*\pi$  coupling:

$$\begin{aligned}
f_D f_{D^*} g_{D^*D\pi} &= \frac{m_c^2 f_\pi}{m_D^2 m_{D^*}^2} e^{-\frac{m_D^2 + m_{D^*}^2}{2M^2}} \left\{ M^2 [e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}] \left[ \varphi_\pi(u_0) \right. \right. \\
&\quad \left. \left. + \frac{\mu_\pi}{m_c} \left( u_0 \varphi_p(u_0) + \frac{1}{3} \varphi_\sigma(u_0) + \frac{1}{6} u_0 \frac{d\varphi_\sigma}{du}(u_0) \right) + \frac{2f_{3\pi}}{m_c f_\pi} I_3^G(u_0) \right] \right. \\
&\quad \left. + e^{-\frac{m_c^2}{M^2}} \left[ \frac{\mu_\pi m_c}{3} \varphi_\sigma(u_0) + 2u_0 g_2(u_0) - \frac{4m_c^2}{M^2} [g_1(u_0) + G_2(u_0)] + I_4^G(u_0) \right] \right\}_{u_0=1/2}, \quad (44)
\end{aligned}$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = \frac{-2\langle \bar{q}q \rangle}{f_\pi^2}. \quad (45)$$

In Eq. (45) we have used the familiar PCAC (partial conservation of axial vector current) relation between  $m_\pi$ ,  $f_\pi$ , the quark masses, and the quark condensate density  $\langle \bar{q}q \rangle$ . Note that the twist-2 and -3 and the twist-4 wave functions have different dimensions (see Appendix A).  $G$  parity implies  $g_2(1/2) = d\varphi_\sigma/du(1/2) = 0$ , so that these terms vanish in the sum rule (44).

For completeness, we also repeat the standard two-point sum rules for the decay constants  $f_D$  and  $f_{D^*}$ :

$$\begin{aligned}
\frac{f_D^2 m_D^4}{m_c^2} &= \frac{3}{8\pi^2} \int_{m_c^2}^{s_0} ds e^{-\frac{m_{D^*}^2 - s}{M^2}} \frac{(s - m_c^2)^2}{s} - m_c \langle \bar{q}q \rangle e^{-\frac{m_D^2 - m_c^2}{M^2}} \\
&\quad \times \left[ 1 + \frac{m_0^2}{2M^2} \left( 1 - \frac{m_c^2}{2M^2} \right) \right] \quad (46)
\end{aligned}$$

and

$$\begin{aligned}
f_{D^*}^2 m_{D^*}^2 &= \frac{1}{8\pi^2} \int_{m_c^2}^{s_0} ds e^{-\frac{m_{D^*}^2 - s}{M^2}} \frac{(s - m_c^2)^2}{s} \left( 2 + \frac{m_c^2}{s} \right) \\
&\quad - m_c \langle \bar{q}q \rangle e^{-\frac{m_{D^*}^2 - m_c^2}{M^2}} \left( 1 - \frac{m_0^2 m_c^2}{4M^4} \right), \quad (47)
\end{aligned}$$

where  $m_0^2 = \langle \bar{q}\sigma_{\alpha\beta} G^{\alpha\beta} q \rangle / \langle \bar{q}q \rangle$  is a conventional

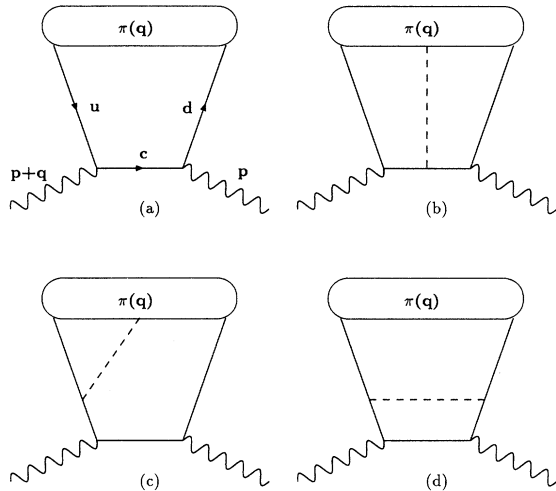


FIG. 1. Diagrams contributing to the correlation function (3). Solid lines represent quarks, dashed lines gluons, wavy lines are external currents, and the ovals denote  $\pi$  meson wave functions on the light cone.

parametrization for the quark-gluon condensate. In the above, we have omitted numerically insignificant contributions of the gluon and four-quark condensates. For consistency, we do not take into account perturbative  $O(\alpha_s)$  corrections to these sum rules, since they are also not included in the sum rule (44).

#### IV. NUMERICAL ANALYSIS

The principal nonperturbative inputs in the sum rule (44) are pion wave functions on the light cone. In Ref. [14] a theoretical framework has been developed to study these functions. In particular, it has been shown that the wave functions can be expanded in terms of matrix elements of conformal operators which in a leading logarithmic approximation do not mix under renormalization. For example, for the leading twist pion wave function one finds an expansion in Gegenbauer polynomials,

$$\varphi_\pi(u, \mu) = 6u(1-u) \left[ 1 + a_2(\mu) C_2^{3/2}(2u-1) + a_4(\mu) C_4^{3/2}(2u-1) + \dots \right], \quad (48)$$

where all the nonperturbative information is included in the set of multiplicatively renormalizable coefficients  $a_n$ ,  $n = 2, 4, \dots$ . The corresponding anomalous dimensions are such that the coefficients  $a_n$  vanish for  $\mu \rightarrow \infty$ , and the wave function is uniquely determined by the first term in the expansion. Therefore, this term is called the asymptotic wave function. Similar expansions also exist for the wave functions of nonleading twist [28].

For practical applications it is important that the expansion in conformal spin converges sufficiently fast. How fast the wave functions approach their well-known asymptotic form is still under debate. However, there are indications [29] that nonasymptotic deviations have been

overestimated previously. Corrections to the asymptotic expressions in next-to-leading (and in some cases also next-to-next-to-leading) order in conformal spin are known for all wave functions which appear in the sum rule (44). For details, we must refer the reader to the original literature [16, 28].

However, we should comment on the choice of the parameter  $\varphi_\pi(1/2)$  specified in Eq. (24). Current knowledge of the coefficients in the expansion (48) is not sufficient to derive a reliable value for  $\varphi_\pi(u)$  at a particular point, e.g., at  $u = 1/2$ . Presently, the only reasonable way is to determine  $\varphi_\pi(1/2)$  directly from a light-cone sum rule for a known physical parameter and to apply the result in order to predict other physical quantities. This is exactly the standard procedure of the QCD sum rule approach. In this spirit, we use the value (24) as determined in Ref. [18] from the light-cone sum rule for the pion-nucleon coupling. If we instead calculated  $\varphi_\pi(1/2)$  from the model wave function suggested in Ref. [16] which satisfies the constraints on the lowest moments, the resulting value would be considerably lower than in Eq. (24) and would therefore disagree with the light-cone sum rule for the pion-nucleon coupling. Note that the wave function  $\varphi_\pi(u)$  used in Sec. VI to calculate the  $D \rightarrow \pi$  and  $B \rightarrow \pi$  form factors is consistent with both Eq. (24) and the lowest moments considered in Ref. [16].

In our numerical analysis we use the set of nonleading twist wave functions proposed in Ref. [28]. The explicit expressions and the values of the various parameters are collected in Appendix A. Furthermore, we take  $f_\pi = 132$  MeV,  $\mu_\pi(1 \text{ GeV}) = 1.65$  GeV, corresponding to  $\langle \bar{q}q \rangle = -(243 \text{ MeV})^3$ ,  $m_0^2 = 0.8 \text{ GeV}^2$ , and, in the charmed meson channels,  $m_c = 1.3$  GeV,  $s_0 = 6 \text{ GeV}^2$ ,  $m_D = 1.87$  GeV, and  $m_{D^*} = 2.01$  GeV. The same parameters are also used to determine the decay constants  $f_D$  and  $f_{D^*}$  from the sum rules (46) and (47). One obtains

$$f_D = 170 \pm 10 \text{ MeV}, \quad f_{D^*} = 240 \pm 20 \text{ MeV}. \quad (49)$$

The uncertainty quoted characterizes the variation with the Borel parameter  $M^2$  in the interval  $1 \text{ GeV}^2 < M^2 < 2 \text{ GeV}^2$ .

Having fixed the input parameters, one must find the range of values of  $M^2$  for which the sum rule (44) is reliable. The lowest possible value of  $M^2$  is usually determined by the requirement that the terms proportional to the highest inverse power of the Borel parameter stay reasonably small. The upper limit is determined by demanding that the continuum contribution does not get too large. In the  $D^*D\pi$  sum rule we take the interval  $2 \text{ GeV}^2 < M^2 < 4 \text{ GeV}^2$ . In this interval the twist-4 term proportional to  $M^{-2}$  does not exceed 5%. Simultaneously, higher states contribute less than 30%. The dependence of the RHS of Eq. (44) on the Borel parameter is shown in Fig. 2(a). As can be seen, in the fiducial range of  $M^2$  given above, the sum rule is quite stable. From Fig. 2(a) one can directly read off the prediction

$$f_D f_{D^*} g_{D^*D\pi} = 0.51 \pm 0.05 \text{ GeV}^2. \quad (50)$$

Dividing this product of couplings by the decay constants (49) finally yields, for the  $D^*D\pi$  coupling constant,



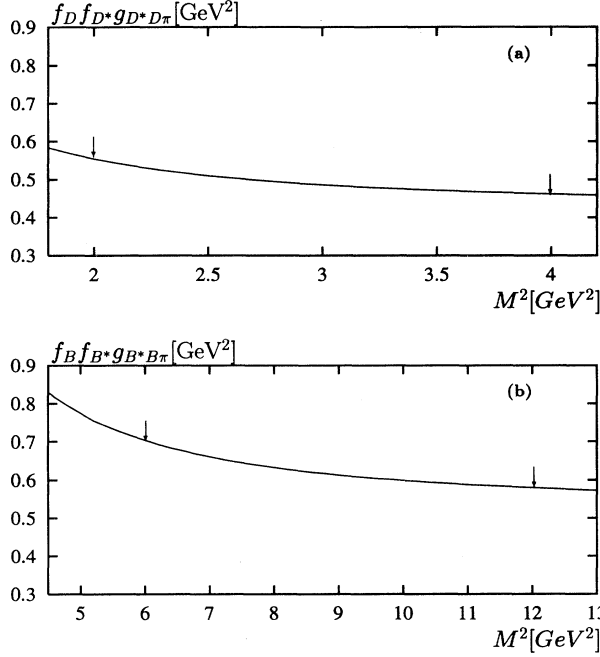


FIG. 2. The RHS of the sum rule (44) for the coupling constants (a)  $g_{D^*D\pi}$  and (b)  $g_{B^*B\pi}$  as a function of the Borel mass squared. The arrows indicate the interval in  $M^2$  allowed by the reliability criteria specified in the text.

$$g_{D^*D\pi} = 12.5 \pm 1.0, \quad (51)$$

where the error is understood to indicate the range of values corresponding to the correlated variation of the results (49) and (50) within the fiducial intervals. While the combination of couplings (50) is not affected by uncertainties in the decay constants, it is more sensitive to the charm quark mass and threshold  $s_0$  than the coupling  $g_{D^*D\pi}$  itself. For example, if we allow the threshold  $s_0$  to vary between 5 and 7  $\text{GeV}^2$  and fix all other parameters, the product  $f_D f_{D^*} g_{D^*D\pi}$  varies by (7–10)% around the central value (50). Similarly, the product of decay constants  $f_D f_{D^*}$  varies by (10–14)% around the central value implied by Eq. (49). However, the total variation of the coupling  $g_{D^*D\pi}$  turns out to be less than 5%.

A few comments are in order. The twist-3 terms contribute to the sum rule at the level of (50–60)% and are therefore as important as the twist-2 contribution. On the other hand, the impact of twist 4 is small, amounting to about 5%. Two sources of uncertainties not included in Eq. (50) are the nonasymptotic corrections to the leading twist wave function  $\varphi_\pi$  and to the three-particle (twist-3) wave function  $\varphi_{3\pi}$ . The latter in turn induces modifications in the two-particle (twist-3) wave functions  $\varphi_p$  and  $\varphi_\sigma$ , apart from the corrections generated by the asymptotic  $\varphi_{3\pi}$  wave function. In order to estimate the sensitivity of our results to the nonasymptotic effects in  $\varphi_\pi$  and  $\varphi_{3\pi}$  we drop these corrections altogether and recalculate the product  $f_D f_{D^*} g_{D^*D\pi}$ . Remarkably, the result changes by only 5%. One can thus be confident that the total uncertainty in Eq. (50) does not exceed 20%.

We should emphasize that the above prediction can be

directly tested experimentally in the decay  $D^* \rightarrow D\pi$ . With the value of  $g_{D^*D\pi}$  given in Eq. (51) one predicts the decay width

$$\Gamma(D^{*+} \rightarrow D^0\pi^+) = \frac{g_{D^*D\pi}^2}{24\pi m_{D^*}^2} |\vec{q}|^3 = 32 \pm 5 \text{ keV}. \quad (52)$$

Predictions for other charge combinations are easily obtained from Eq. (52) taking into account the isospin relations (2) as well as small differences in the phase space volumes:

$$\begin{aligned} \Gamma(D^{*+} \rightarrow D^0\pi^+) &= 2 \times 1.1 \Gamma(D^{*+} \rightarrow D^+\pi^0) \\ &= 2 \times 0.72 \Gamma(D^{*0} \rightarrow D^0\pi^0). \end{aligned} \quad (53)$$

The current experimental upper limit

$$\Gamma(D^{*+} \rightarrow D^0\pi^+) < 89 \text{ keV} \quad (54)$$

is obtained by combining the limit  $\Gamma_{\text{tot}}(D^{*+}) < 131 \text{ keV}$  [30] with the branching ratio  $\mathcal{B}(D^{*+} \rightarrow D^0\pi^+) = (68.1 \pm 1.0 \pm 1.3)\%$  [31]. Our prediction is well below this upper limit.

The sum rule for  $g_{D^*D\pi}$  given in Eq. (44) is easily converted into a sum rule for the coupling  $g_{B^*B\pi} = g_{\bar{B}^0 B^- \pi^+}$  by replacing  $c$  with  $b$ ,  $D$  with  $\bar{B}$ , and  $D^*$  with  $\bar{B}^*$ . The corresponding parameters are  $m_B = 5.279 \text{ GeV}$ ,  $m_{B^*} = 5.325 \text{ GeV}$ ,  $m_b = 4.7 \text{ GeV}$ , and  $s_0 = 35 \text{ GeV}^2$ . In addition, one has to evolve the wave function parameters to a higher scale  $\mu_b$  (see Appendix A). With these changes the two-point sum rules (46) and (47) yield

$$f_B = 140 \text{ MeV}, \quad f_{B^*} = 160 \text{ MeV}, \quad (55)$$

with negligible uncertainties due to variation of  $M^2$ . Using the same criteria as for the  $g_{D^*D\pi}$  sum rule the fiducial range in  $M^2$  turns out to be  $6 \text{ GeV}^2 < M^2 < 12 \text{ GeV}^2$ . The stability of the sum rule for the product of couplings  $f_B f_{B^*} g_{B^*B\pi}$  is illustrated in Fig. 2(b). We obtain

$$f_B f_{B^*} g_{B^*B\pi} = 0.64 \pm 0.06 \text{ GeV}^2 \quad (56)$$

and, with Eq. (55),

$$g_{B^*B\pi} = 29 \pm 3. \quad (57)$$

Similarly to the  $D^*D\pi$  case, if the threshold  $s_0$  is varied between 33 and 37  $\text{GeV}^2$  while all other parameters are kept fixed, the product (56) changes by  $\pm 5\%$  and  $f_B f_{B^*}$  by roughly  $\pm 10\%$ . The coupling  $g_{B^*B\pi}$  itself changes by  $-6\%$  to  $+8\%$  relative to the central value given in Eq. (57). The hierarchy of various twists as well as the uncertainty due to the nonasymptotic corrections is found to be similar as in the case of  $g_{D^*D\pi}$ . Contrary to the latter, the coupling constant  $g_{B^*B\pi}$  cannot be measured directly, since the corresponding decay  $B^* \rightarrow B\pi$  is kinematically forbidden. However, the  $B^*B\pi$  on-shell vertex is of a great importance for understanding of the behavior of heavy-to-light form factors as will be discussed in Sec. VI.

### V. SUM RULES FROM THE SHORT-DISTANCE EXPANSION

With the results of Sec. IV at hand we are now also in a position to study in more detail the soft-pion limit (25) of the sum rule (44) which is obtained from the correlation function (3) at  $p^2 = (p+q)^2$  or, equivalently, for  $q \rightarrow 0$ . As discussed in Sec. II, in this limit one can apply a short-distance expansion in terms of local operators with increasing dimensions in contrast to the light cone expansion involving nonlocal operators with increasing twist. In technical terms, at  $q \rightarrow 0$  only the lowest moments of the wave functions contribute. Thus the integrals reduce to overall normalization factors. The explicit expression for the invariant function  $F$  in this limit is directly obtained from Eqs. (31) and (39):

$$F(p^2) = \frac{m_c f_\pi}{m_c^2 - p^2} \left[ 1 + \frac{\mu_\pi}{3m_c} \left( 2 + \frac{m_c^2}{m_c^2 - p^2} \right) + \frac{10\delta^2}{9(m_c^2 - p^2)} \left( 1 - \frac{m_c^2}{m_c^2 - p^2} \right) \right], \quad (58)$$

where the parameter  $\delta^2$  is specified in Appendix A. After Borel transformation in  $p^2$  one has

$$F(M^2) = m_c f_\pi e^{-\frac{m_c^2}{M^2}} \left[ 1 + \frac{2\mu_\pi}{3m_c} + \frac{1}{M^2} \left( \frac{\mu_\pi m_c}{3} + \frac{10}{9} \delta^2 \right) - \frac{5m_c^2 \delta^2}{9M^4} \right]. \quad (59)$$

As indicated in Eq. (25), the price for simplifying the QCD representation of the correlation function is a more complicated hadronic representation. This in turn makes it more difficult to extract the ground state contribution containing the  $D^*D\pi$  coupling. For illustration, we consider the contribution in the dispersion representation of the correlation function (3) from the transition of a given excited state in the  $D^*$  channel with mass  $m_* > m_{D^*}$  to the ground state  $D$  meson at  $q \rightarrow 0$ . This contribution is proportional to

$$\frac{1}{(m_*^2 - p^2)(m_D^2 - p^2)} \quad (60)$$

and, after Borel transformation, to

$$\frac{1}{m_*^2 - m_D^2} \left( e^{-\frac{m_D^2}{M^2}} - e^{-\frac{m_*^2}{M^2}} \right). \quad (61)$$

Similar expressions hold for the ground state transition  $D^* \rightarrow D$  with  $m_* = m_{D^*}$ . In the limit  $m_D = m_{D^*}$ , one has a double pole instead of Eq. (60) and

$$\frac{1}{M^2} e^{-\frac{m_D^2}{M^2}} \quad (62)$$

instead of Eq. (61). Clearly, the contribution (61) is not exponentially suppressed relative to the ground state contribution (62), and can therefore not be subtracted assuming duality. On the other hand, transitions involving excited states in both the initial and the final states are suppressed by Borel transformation with respect to the ground state transitions and cause no problems. Schematically, the complete hadronic part of the

invariant amplitude  $F(M^2)$  can be written as

$$F(M^2) \simeq \frac{1}{M^2} \left\{ \frac{m_D^2 m_{D^*} f_D f_{D^*}}{m_c} g_{D^*D\pi} + AM^2 \right\} \times e^{-\frac{m_{D^*}^2 + m_D^2}{2M^2}} + Ce^{-\frac{m_c^2}{M^2}}, \quad (63)$$

where the constant  $A$  incorporates all unsuppressed contributions of the type (61), while the term proportional to  $C$  contains all exponentially suppressed contributions.

To get rid of the contaminating term  $AM^2$  we follow Ref. [26] and apply the operator

$$\left( 1 - M^2 \frac{d}{dM^2} \right) M^2 e^{-\frac{m_{D^*}^2 + m_D^2}{2M^2}} \quad (64)$$

to both representations (59) and (63). Now it is possible to subtract the continuum contribution by the same substitution (43) which we have already employed to get the light-cone sum rule (44). In this way we obtain a new sum rule for  $g_{D^*D\pi}$ :

$$f_D f_{D^*} g_{D^*D\pi} = \frac{m_c^2 f_\pi}{m_D^2 m_{D^*}} \left( 1 - M^2 \frac{d}{dM^2} \right) e^{-\frac{m_D^2 + m_{D^*}^2 - 2m_c^2}{2M^2}} \times \left[ M^2 \left( 1 - e^{-\frac{s_0 - m_c^2}{M^2}} \right) \left( 1 + \frac{2\mu_\pi}{3m_c} \right) + \frac{\mu_\pi m_c}{3} + \frac{10}{9} \delta^2 - \frac{5m_c^2 \delta^2}{9M^2} \right]. \quad (65)$$

For the same input parameters and the range of  $M^2$  leading to the prediction (51) this sum rule yields

$$g_{D^*D\pi} = 11 \pm 2. \quad (66)$$

Similarly, replacing in Eq. (65) the charmed meson parameters by the corresponding quantities in the beauty channel, one finds

$$g_{B^*B\pi} = 28 \pm 6. \quad (67)$$

As compared with the predictions (51) and (57) the uncertainties are larger by a factor of 2 due to the worse stability of the sum rule (65) against variation of  $M^2$ . The agreement of the results indicates self-consistency of the sum rule approach and gives support to the approximations used in the pion wave functions.

Furthermore, one can show that the sum rule (65) is actually equivalent to the sum rule obtained in Ref. [10] using external field techniques. Indeed, applying the usual reduction formalism to the pion, one can rewrite the correlation function (3) in the form

$$F_\mu(p, q) = (q^2 - m_\pi^2) \int d^4x d^4y e^{i(px - qy)} \times \langle 0 | T \{ \bar{d}(x) \gamma_\mu c(x), \phi_\pi(y), \bar{c}(0) \gamma_5 u(0) \} | 0 \rangle, \quad (68)$$

where  $\phi_\pi(y)$  is the interpolating pion field. According to PCAC

$$\phi_\pi(y) = \frac{\partial^\mu j_\mu^5(y)}{f_\pi m_\pi^2}, \quad (69)$$

$j_\mu^5(y) = \bar{u}(y) \gamma_\mu \gamma_5 d(y)$  being the axial vector current.

Substituting Eq. (69) into Eq. (68) and integrating by parts, one gets<sup>2</sup>

$$F_\mu(p, q) = i \frac{q^2 - m_\pi^2}{f_\pi m_\pi^2} T_{\mu\rho}(p, q) q^\rho = -\frac{i}{f_\pi} T_{\mu\rho}(p, q) q^\rho, \quad (70)$$

with

$$T_{\mu\rho}(p, q) = \int d^4x d^4y e^{i(p x - q y)} \times \langle 0 | T \{ \bar{d}(x) \gamma_\mu c(x), j_\rho^5(y), \bar{c}(0) \gamma_5 u(0) \} | 0 \rangle. \quad (71)$$

Instead of dealing with the three-point correlation function  $T_{\mu\rho}(p, q)$  directly, it is more convenient to consider the following two-point correlation function in the constant external axial vector field  $A_\mu^5$ :

$$T_\mu^A(p, q) = \int d^4x e^{i p x} \times \langle 0 | T \{ \bar{d}(x) \gamma_\mu c(x), \bar{c}(0) \gamma_5 u(0) \} | 0 \rangle_A. \quad (72)$$

It is assumed that a term  $A^{5\mu} j_\mu^5$  corresponding to the interaction of the external field with the light quarks is added to the QCD Lagrangian. To first order in the external field, this correlation function is given by

$$T_\mu^A(p, q) = T_{\mu\rho}(p, q) A^{5\rho},$$

with  $T_{\mu\rho}(p, q)$  as defined in Eq. (71). In the above sense, the two-point correlation function (72) in the constant axial vector field is equivalent to the three-point function (71) and, via the PCAC relation (69), also to the two-point correlation function (3) at<sup>3</sup>  $q \rightarrow 0$ . Consequently, the sum rules obtained in Refs. [10] and [13] should coincide with each other and with the sum rule (65) derived in this paper.

In particular, the expression (59) for  $F(M^2)$  can be compared with the result given in Eq. (19) of Ref. [10] and in Eq. (2.15) of Ref. [13], after normalization and kinematical structures are adjusted properly. In Refs. [10, 13] the correlation function (3) is separated as

$$F_\mu = A q_\mu + B(2p_\mu + q_\mu), \quad (73)$$

and the sum rule is obtained by evaluation of the invariant function  $A$  for  $q \rightarrow 0$ . In terms of the invariant amplitudes defined in Eq. (16) one has

$$A = F - \frac{\tilde{F}}{2}, \quad B = \frac{\tilde{F}}{2}. \quad (74)$$

<sup>2</sup>In addition, one obtains two-point correlation functions because of contact terms. These do not lead to double poles in  $p^2$  in the relevant dispersion relation and are therefore eliminated by applying the differentiation operator (64).

<sup>3</sup>As a side remark, the light-cone approach leading to Eq. (44) corresponds to a calculation in the background of a *variable* external axial vector field [17, 18].

Hence, for comparison we need also the second invariant function  $\tilde{F}$ , which we have not discussed so far, but which can be calculated along the same lines. Adding this contribution to Eq. (59), we have checked that our result for the invariant amplitude  $A$  coincides with the one presented in Ref. [13], apart from terms proportional to  $m_0^2$  which, being associated with twist-5 contributions in the light-cone sum rules, are neglected in our approximation. Numerically, these terms are not important. On the other hand, we disagree with Ref. [10] in the non-leading terms proportional to  $\delta^2$ .

Although it is legitimate to use different Lorentz decompositions of the correlation function (3) in order to derive the desired sum rule, we think that the choice adopted in the present paper is more adequate for the following reason. Since the vector current  $\bar{q}\gamma_\mu c$  is not conserved, it not only couples to  $J^P = 1^-$  vector mesons, but also to  $J^P = 0^+$  scalar mesons ( $D_0$ ). The corresponding transition matrix element is proportional to the momentum  $p_\mu$ :

$$\langle 0 | \bar{q}\gamma_\mu c | D_0 \rangle = f_{D_0} m_{D_0} p_\mu. \quad (75)$$

The mass of the ground state  $D_0$  meson is expected to be in the vicinity of 2400 MeV which is not far from the mass of the  $D^*$  and below the accepted continuum threshold in the  $D^*$  channel. For this reason, the  $D_0$  contribution should be added to the sum rule. Unfortunately, this introduces additional uncertainties in the hadronic representation as is the case, for example, in Ref. [13]. In contrast, our sum rules based on the invariant function  $F$  defined in Eq. (16) are not affected by scalar meson contributions, which is a clear advantage.

A calculation rather similar to Ref. [13], but with particular emphasis on the heavy quark limit, has been carried out in Ref. [11]. Very recently, another calculation in the heavy quark limit appeared in Ref. [12] using the external field technique. In this paper, a wrong expression for the induced quark condensate in the external field is used, as can be seen by consulting Refs. [10, 32]. The error can be traced back to the equation of motion for the quark field in presence of the external field which is modified from  $i\not{D}q = 0$  to  $i\not{D}q = \not{A}q$ . By this modification the axial vector current insertions into the vacuum quark legs are properly taken into account. The numerical comparison of these different calculations is left for the concluding section.

## VI. POLE MODEL FOR $D \rightarrow \pi$ AND $B \rightarrow \pi$ FORM FACTORS AND QCD SUM RULES

The couplings  $g_{D^*D\pi}$  and  $g_{B^*B\pi}$  fix the normalization of the form factors of the heavy-to-light transitions  $D \rightarrow \pi$  and  $B \rightarrow \pi$ , respectively, in the pole model description [5,6]. This model is based on the vector dominance idea, suggesting a momentum dependence dominated by the  $D^*$  and  $B^*$  poles, respectively. More definitely, the form factor  $f_D^+(p^2)$  defined by the matrix element

$$\langle \pi(q) | \bar{d}\gamma_\mu c | D(p+q) \rangle = 2f_D^+(p^2)q_\mu + [f_D^+(p^2) + f_D^-(p^2)]p_\mu \quad (76)$$

is predicted to be given by

$$f_D^+(p^2) = \frac{f_{D^*} g_{D^* D \pi}}{2m_{D^*}(1 - p^2/m_{D^*}^2)}. \quad (77)$$

An analogous expression holds for the form factor  $f_B^+(p^2)$ .

It is difficult to justify the pole model from first principles. Generally, it is believed that the vector dominance approximation is valid at zero recoil, that is, at  $p^2 \rightarrow m_D^2$ . Arguments based on heavy quark symmetry suggest a somewhat larger region of validity characterized by  $(m_D^2 - p^2)/m_c \sim 1$  GeV. However, there are no convincing arguments in favor of this model to be valid also at small values of  $p^2$  which are the most interesting from a practical point of view. Therefore, the finding [7, 9] that the pole behavior is consistent with the  $p^2$  dependence at  $p^2 \rightarrow 0$  predicted by sum rules is very remarkable. Meanwhile, this claim has been confirmed by independent calculations within the framework of the light-cone sum rules [8].

In this section we want to demonstrate that not only the shape but also the absolute normalization of the above form factors appears to be comparable with the pole model description. This assertion is nontrivial, since contributions of several low-lying resonances in the  $D^*$  or  $B^*$  channel could still mimic the  $p^2$  dependence of a single pole, but the relation to the coupling  $g_{D^* D \pi}$  or

$g_{B^* B \pi}$  should then be lost [4]. However, despite the overall agreement in the mass range of  $D$  and  $B$  mesons, there is a clear disagreement on the asymptotic dependence of the form factors on the heavy mass. The QCD sum rules on the light cone provide a unique framework to examine these issues, since both the form factors  $f_{D,B}^+(p^2)$  at  $m_{c,b}^2 - p^2 \gtrsim 1$  GeV<sup>2</sup> and the couplings  $g_{D^* D \pi}$  and  $g_{B^* B \pi}$  can be calculated from the same correlation function (3) using the same technique. In addition, contrary to conventional sum rules [7], this approach leads to consistent results in the heavy quark limit [21].

The detailed derivation of the light-cone sum rules for the  $D \rightarrow \pi$  and  $B \rightarrow \pi$  form factors is discussed in Ref. [8] (see also Refs. [19–21]). Here we just mention that the sum rule for  $f_D^+(p^2)$  is obtained by matching the expressions (31) and (39) for the invariant amplitude  $F(p^2, (p+q)^2)$  in terms of the pion wave functions with the hadronic representation

$$F(p^2, (p+q)^2) = \frac{2m_D^2 f_D f_D^+(p^2)}{m_c [m_D^2 - (p+q)^2]} + \int_{s_0}^{\infty} \frac{\rho^h(p^2, s) ds}{s - (p+q)^2}. \quad (78)$$

In the above, the pole term is due to the ground state in the heavy channel, while the excited and continuum states are taken into account by the dispersion integral above the threshold  $s_0$ . Invoking duality, the latter contributions are canceled against the corresponding pieces in Eqs. (31) and (39). After Borel transformation in the variable  $(p+q)^2$ , the resulting sum rule takes the form

$$f_D^+(p^2) = \frac{f_\pi m_c^2}{2f_D m_D^2} \left\{ \int_{\Delta}^1 \frac{du}{u} \exp \left[ \frac{m_D^2}{M^2} - \frac{m_c^2 - p^2(1-u)}{uM^2} \right] \Phi_2(u, M^2, p^2) - \int_0^1 u du \int \frac{D\alpha_i \Theta(\alpha_1 + u\alpha_3 - \Delta)}{(\alpha_1 + u\alpha_3)^2} \exp \left[ \frac{m_D^2}{M^2} - \frac{m_c^2 - p^2(1 - \alpha_1 - u\alpha_3)}{(\alpha_1 + u\alpha_3)M^2} \right] \Phi_3(u, \alpha_i, M^2, p^2) \right\}, \quad (79)$$

where

$$\begin{aligned} \Phi_2 &= \varphi_\pi(u) + \frac{\mu_\pi}{m_c} \left[ u\varphi_p(u) + \frac{1}{6}\varphi_\sigma(u) \left( 2 + \frac{m_c^2 + p^2}{uM^2} \right) \right] \\ &\quad - \frac{4m_c^2 g_1(u)}{u^2 M^4} - \frac{2G_2(u)}{uM^2} \left( 1 + \frac{m_c^2 + p^2}{uM^2} \right), \quad (80) \\ \Phi_3 &= \frac{2f_{3\pi}}{f_\pi m_c} \varphi_{3\pi}(\alpha_i) \left[ 1 - \frac{m_c^2 - p^2}{(\alpha_1 + u\alpha_3)M^2} \right] \\ &\quad - \frac{1}{uM^2} \left[ 2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i) \right], \quad (81) \end{aligned}$$

and  $\Delta = (m_c^2 - p^2)/(s_0 - p^2)$ . The notation is as in Eq. (44). Improving the approximation given in Ref. [8] we have added the contributions of three-particle wave functions of twist 4. The analogous sum rule for the  $B \rightarrow \pi$  form factor follows from the above by replacing  $c \rightarrow b$  and  $D \rightarrow \bar{B}$ , and by rescaling  $\mu_\pi$  and the wave function parameters from  $\mu_c$  to  $\mu_b$ .

The maximum momentum transfer  $p^2$  at which these

sum rules are applicable is estimated to be about 15 GeV<sup>2</sup> for  $B$  mesons and 1 GeV<sup>2</sup> for  $D$  mesons. For numerical evaluation we use the approximations of the wave functions given in Appendix A. We emphasize that the input here is exactly the same as in the calculation of the couplings  $g_{D^* D \pi}$  and  $g_{B^* B \pi}$ . The form factor  $f_D^+(p^2)$  resulting from the sum rule (79) is plotted in Fig. 3(a), together with the corresponding prediction (77) of the pole model. We see that in the region of overlap both calculations approximately agree with each other. To a lesser extent, this also applies to the form factor  $f_B^+(p^2)$  illustrated<sup>4</sup> in Fig. 3(b). Quantitatively, at  $p^2 = 0$  we find

$$f_D^+(0)_{\text{SR}} = 0.66, \quad f_D^+(0)_{\text{PM}} = 0.75 \quad (82)$$

and

<sup>4</sup>The dependence of Eq. (79) on the Borel parameter is weak [8]. For definiteness, we take here  $M^2 = 4$  GeV<sup>2</sup> for the  $D \rightarrow \pi$  form factor and  $M^2 = 10$  GeV<sup>2</sup> for the  $B \rightarrow \pi$  form factor.

$$f_B^+(0)_{\text{SR}} = 0.29, \quad f_B^+(0)_{\text{PM}} = 0.44. \quad (83)$$

Thus, in the regions  $m_Q^2 - p^2 \gtrsim 1 \text{ GeV}^2$  with  $Q = c$  and  $b$ , respectively, the numerical agreement between the light-cone sum rule and the pole model is better than 15% for  $f_D^+$ , but only within 50% for  $f_B^+$ .

At this point, we must add a word of caution concerning the applicability of the pole model too far away from the zero recoil point, in particular at  $p^2 = 0$ . The two descriptions differ markedly in the asymptotic dependence of the form factors on the heavy mass. Focusing on  $B$  mesons and using the familiar scaling laws

$$f_B \sqrt{m_B} = \hat{f}_B, \quad f_{B^*} \sqrt{m_B} = \hat{f}_{B^*}, \quad (84)$$

and

$$g_{B^*B\pi} = \frac{2m_B}{f_\pi} \hat{g}, \quad (85)$$

which are expected to be valid at  $m_b \rightarrow \infty$  modulo logarithmic corrections, one obtains

$$f_B^+(0)_{\text{PM}} \sim 1/\sqrt{m_B}, \quad (86)$$

whereas the light-cone sum rule (79) yields [19]

$$f_B^+(0)_{\text{SR}} \sim 1/m_B^{3/2}. \quad (87)$$

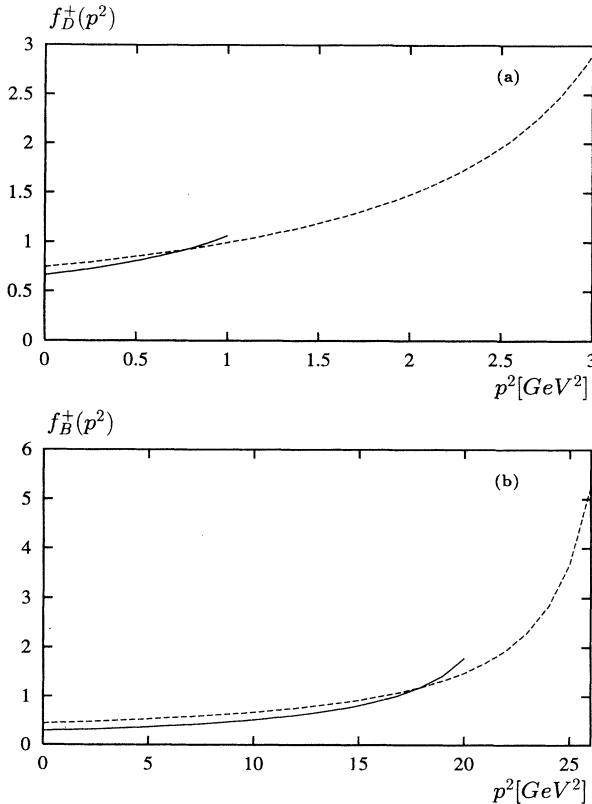


FIG. 3. The form factors for the transitions (a)  $D \rightarrow \pi$  and (b)  $B \rightarrow \pi$  as predicted by the light-cone sum rule (solid lines) in comparison to the single-pole approximation (dashed lines) with the normalization fixed by the coupling constants  $g_{D^*D\pi}$  and  $g_{B^*B\pi}$ , respectively.

This result rests on the QCD prediction [14] of the behavior of the leading twist pion wave function near the end point, that is,  $\varphi_\pi(u) \sim 1 - u$  at  $u \rightarrow 1$ . It should be noted that the contribution estimated by the sum rules corresponds to the so-called Feynman mechanism. In the case of heavy-to-light transitions it leads to the same asymptotic behavior as the hard rescattering mechanism [19, 33]. Recently it has been shown [34] that the power behavior (87) of hard rescattering is not modified by the Sudakov-type double logarithmic corrections. We believe that the light cone sum rules reproduce the true asymptotic behavior, although a rigorous proof in QCD is still lacking. On the other hand, we see no theoretical justification for extrapolating the pole model to the region  $p^2 = 0$ . The solution suggested by Fig. 3 is to match the two descriptions in the region of intermediate momentum transfer  $p^2 \simeq m_Q^2 - 1 \text{ GeV}^2$ .

Referring for a detailed discussion to Refs. [21] and [35] we want to emphasize that the light-cone sum rules seem to be generally consistent with the heavy quark expansion. In particular, the light-cone sum rule (44) correctly reproduces the heavy quark mass dependence of the coupling  $g_{B^*B\pi}$ . Fitting our predictions for  $g_{B^*B\pi}$  and  $g_{D^*D\pi}$  to the form

$$g_{B^*B\pi} = \frac{2m_B}{f_\pi} \hat{g} \left[ 1 + \frac{\Delta}{m_B} \right] \quad (88)$$

and the analogous expression for  $g_{D^*D\pi}$ , we find for the coupling  $\hat{g}$  and the strength  $\Delta$  of the  $1/m_Q$  correction:

$$\hat{g} = 0.32 \pm 0.02, \quad \Delta = (0.7 \pm 0.1) \text{ GeV}. \quad (89)$$

Furthermore, we are able to make a numerical prediction for the theoretically interesting ratio

$$\frac{g_{B^*B\pi} f_{B^*} \sqrt{m_D}}{g_{D^*D\pi} f_{D^*} \sqrt{m_B}} \simeq 0.92. \quad (90)$$

This ratio is unity in the heavy quark limit and is shown to be subject to  $1/m_Q$  corrections only in the next-to-leading order [5]. Our result (90) is nicely consistent with this expectation. The deviation from unity also agrees in magnitude with the estimate in Ref. [13], but has a different sign. This is due to a sizable difference in the ratio  $f_{B^*}/f_{D^*}$ . While the values of the decay constants given in Eqs. (47) and (55) yield

$$\frac{f_{B^*} \sqrt{m_B}}{f_{D^*} \sqrt{m_D}} = 1.12, \quad (91)$$

in agreement with the expectation quoted in Ref. [5], the latter ratio turns out to be larger by 30% if calculated from  $f_{B^*}$  and  $f_{D^*}$  as assumed in Ref. [13].

## VII. SUMMARY AND CONCLUSIONS

We have presented a comprehensive analysis of the pion couplings to heavy mesons in the framework of QCD sum rules. The main new result of this paper is the light-cone sum rule (44) providing the numerical estimates for  $g_{D^*D\pi}$  and  $g_{B^*B\pi}$  given in Eqs. (51) and (57), respec-

tively. The decay width  $\Gamma(D^* \rightarrow D\pi)$  predicted in Eq. (52) turns out to be 3 times smaller than the present experimental upper limit. We have compared our results to earlier QCD sum rule calculations [10–13], and resolved the existing discrepancies.

Clearly, our predictions for  $g_{D^*D\pi}$  and  $g_{B^*B\pi}$  de-

pend crucially on the value of the pion wave function  $\varphi_\pi(u, \mu_{c,b})$  at  $u \simeq 1/2$ . We have used the value extracted from a light-cone sum rule for the pion-nucleon coupling. Conversely, a measurement of the decay width for  $D^* \rightarrow D\pi$  could be used to fix this important non-perturbative parameter. The  $B^*B\pi$  coupling can then

TABLE I. Summary of theoretical estimates.

Reference	$\hat{g}$	$g_{B^*B\pi}$	$g_{D^*D\pi}$	$\Gamma(D^{*+} \rightarrow D^0\pi^+)$ (keV)
This paper	$0.32 \pm 0.02$	$29 \pm 3$	$12.5 \pm 1.0$	$32 \pm 5$
This paper <sup>a</sup>	–	$28 \pm 6$	$11 \pm 2$	–
[11] <sup>a</sup>	–	$32 \pm 6$	–	–
[12] <sup>a</sup>	0.2–0.7	–	–	–
[13] <sup>a</sup>	$0.39 \pm 0.16$	$20 \pm 4$	$9 \pm 1$	–
[13] <sup>a,b</sup>	$0.21 \pm 0.06$	$15 \pm 4$	$7 \pm 1$	$10 \pm 3$
[37] <sup>c</sup>	0.7	–	–	–
[4] <sup>c</sup>	–	64	–	–
[38] <sup>c</sup>	0.75–1.0	–	–	100–180
[39] <sup>d</sup>	0.6–0.7	–	–	61–78
[40] <sup>d</sup>	0.4–0.7	–	–	–
[41] <sup>e</sup>	0.3	–	–	–
[42] <sup>f</sup>	–	–	16.2	53.4
[43] <sup>g</sup>	–	–	$19.5 \pm 1.0$	$76 \pm 7$
[44] <sup>h</sup>	–	–	$16.2 \pm 0.3$	$53.3 \pm 2.0$
[45] <sup>i</sup>	–	–	8.9	16
[46] <sup>j</sup>	–	–	8.2	13.8
Experiment <sup>k</sup>	–	–	$< 21$	$< 89$

<sup>a</sup>QCD sum rules in external axial vector field or soft-pion limit.

<sup>b</sup>Including perturbative correction to the heavy meson decay constants.

<sup>c</sup>Quark model + chiral HQET.

<sup>d</sup>Chiral HQET with experimental constraints on  $D^*$  decays.

<sup>e</sup>Extended NJL model + chiral HQET.

<sup>f</sup>Quark model + scaling relation.

<sup>g</sup>Relativistic quark model.

<sup>h</sup>Bag model.

<sup>i</sup>SU(4) symmetry.

<sup>j</sup>Reggeon quark-gluon string model.

<sup>k</sup>Combination of ACCMOR [33] and CLEO [34] measurements.

be predicted using this result.

A rather complete compilation of estimates<sup>5</sup> on the pion couplings to heavy mesons is given in Table I. In the first row we show predictions on the reduced coupling  $\hat{g}$  defined in Eq. (85). As one can see, the values obtained by combining the nonrelativistic constituent quark model with PCAC [4, 37, 38] are roughly 2 times larger than the values favored by our sum rule. However, more recent analyses [39, 40] combining chiral heavy quark effective theory (HQET) with experimental constraints on  $D^*$  decays tend to give somewhat smaller values of  $\hat{g}$ . Moreover, another recent calculation [41] based on the extended Nambu–Jona-Lasinio model and chiral HQET is in perfect agreement with our estimate.

The next two rows list the estimates of the couplings  $g_{B^*B\pi}$  and  $g_{D^*D\pi}$ . These predictions are even more widely spread. Quark models [42, 43] as well as the bag model [44] seem to give the strongest couplings, whereas SU(4) symmetry [45] and the Reggeon quark-gluon string model [46] predict a relatively small coupling. Two comments are in order concerning the analysis of Ref. [13]. First, these predictions are consistently lower than ours. There are several reasons for that: the different Lorentz decompositions (16) and (73) of the correlation function (3), the differences between the sum rule (44) and the soft-pion limit (65) of it, the different regions of the Borel parameter  $M^2$ , and the different values used for the decay constants  $f_{D^{(*)}}$  and  $f_{B^{(*)}}$ . In fact, as can be seen in Fig. 2, the couplings shrink with  $M^2$ . However, given the reliability criteria, generally accepted for sum rules, we see no possibility to shift  $M^2$  to larger values beyond the regions considered in this paper, in contrast with Ref. [13]. Second, we find it inconsistent to include the perturbative gluon correction in the estimates of  $f_{D,D^*}$  and  $f_{B,B^*}$ , since they are not included in the sum rule for the combination of couplings  $f_D f_{D^*} g_{D^*D\pi}$  and  $f_B f_{B^*} g_{B^*B\pi}$ . At least, we see no convincing argument in favor of such a procedure. For these two reasons we believe that the couplings are underestimated in Ref. [13].

For convenience and direct comparison with future measurements the decay width  $\Gamma(D^{*+} \rightarrow D^0\pi^+)$  as calculated from  $g_{D^*D\pi}$  or  $\hat{g}$  is quoted in the last row of Table I. The widths in the channels  $D^{*+} \rightarrow D^+\pi^0$  and  $D^{*0} \rightarrow D^0\pi^0$  are related to the above by coefficients which can be read off from Eq. (53). Note that in contrast with the evaluation of  $\Gamma(D^* \rightarrow D\pi)$  from  $g_{D^*D\pi}$  in this paper and in Ref. [13] the estimates in Refs. [38, 39] using the reduced coupling  $\hat{g}$  do not include  $1/m$  corrections. However, the latter are important as can be seen from Eq. (89).

In addition, we have examined the pole model for the  $B \rightarrow \pi$  and  $D \rightarrow \pi$  form factors. Using our results on the  $g_{B^*B\pi}$  and  $g_{D^*D\pi}$  coupling constants, we have found approximate numerical agreement between the pole model description and the direct sum rule calculation. However, the dependence on the heavy quark mass is found to be completely different in the region of small momen-

tum transfers. We have argued in favor of the sum rule approach. Moreover, writing a heavy quark expansion for the couplings  $g_{B^*B\pi}$  and  $g_{D^*D\pi}$  we have determined the expansion coefficients, in particular, the leading  $1/m$  correction.

Last but not least, we have discussed in some detail the theoretical foundations and advantages of the light-cone sum rules, complementing the work of Refs. [14–19]. We believe that this approach is especially suitable for the study of heavy-to-light decay form factors, and coupling constants of the type considered in this paper. Further obvious applications include the radiative decays  $D^* \rightarrow D\gamma$  and  $B^* \rightarrow B\gamma$ . Since the photon wave functions are expected to deviate less from their asymptotic forms than the pion wave functions [17], these decays should provide a rather conclusive consistency check of the light cone approach.

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### APPENDIX A

For convenience, we collect here the explicit expressions for the pion wave functions used in our numerical calculations and specify the values of the parameters involved.

For the leading twist-2 wave function we take [18]

$$\varphi_\pi(u, \mu) = 6u(1-u) \left[ 1 + a_2(\mu) C_2^{3/2}(2u-1) + a_4(\mu) C_4^{3/2}(2u-1) \right], \quad (\text{A1})$$

with the Gegenbauer polynomials

$$C_2^{3/2}(2u-1) = \frac{3}{2} [5(2u-1)^2 - 1],$$

$$C_4^{3/2}(2u-1) = \frac{15}{8} [21(2u-1)^4 - 14(2u-1)^2 + 1], \quad (\text{A2})$$

and the coefficients  $a_2 = \frac{2}{3}$ ,  $a_4 = 0.43$  corresponding to the normalization point  $\mu = 0.5$  GeV. In the present applications the appropriate scales are set by the typical virtuality of the heavy quark. We choose

$$\mu_c = \sqrt{m_D^2 - m_c^2} \simeq 1.3 \text{ GeV},$$

$$\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4 \text{ GeV}. \quad (\text{A3})$$

Renormalization group evolution of the coefficients  $a_2$  and  $a_4$  to these higher scales yields

$$a_2(\mu_c) = 0.41, \quad a_4(\mu_c) = 0.23,$$

$$a_2(\mu_b) = 0.35, \quad a_4(\mu_b) = 0.18. \quad (\text{A4})$$

We stress that the value of  $\varphi_\pi$  at  $u = 1/2$  varies by only 2% when the scale  $\mu$  is increased from 0.5 GeV to 2.4 GeV. Obviously, one can neglect this effect given the

<sup>5</sup>We have not included the results of Ref. [10] since to our knowledge this analysis is being reconsidered [36].

15% uncertainty in the value of  $\varphi_\pi(u = 1/2, \mu = 0.5 \text{ GeV})$  quoted in Eq. (24).

According to the analysis in Refs. [18, 28] the set of wave functions of twist 3 is uniquely specified by the choice of the three-particle wave function  $\varphi_{3\pi}$ . Taking into account the contributions to  $\varphi_{3\pi}$  up to next-to-next-to-leading order in conformal spin, one has

$$\begin{aligned} \varphi_{3\pi}(\alpha_i) = & 360\alpha_1\alpha_2\alpha_3^2[1 + \omega_{1,0}\frac{1}{2}(7\alpha_3 - 3) \\ & + \omega_{2,0}(2 - 4\alpha_1\alpha_2 - 8\alpha_3 + 8\alpha_3^2) \\ & + \omega_{1,1}(3\alpha_1\alpha_2 - 2\alpha_3 + 3\alpha_3^2)] . \end{aligned} \quad (\text{A5})$$

This implies, for the two-particle wave functions of twist 3 [28],

$$\begin{aligned} \varphi_p(u) = & 1 + B_2\frac{1}{2}[3(u - \bar{u})^2 - 1] \\ & + B_4\frac{1}{8}[35(u - \bar{u})^4 - 30(u - \bar{u})^2 + 3] \end{aligned} \quad (\text{A6})$$

and

$$\begin{aligned} \varphi_\sigma(u) = & 6u\bar{u} \left[ 1 + C_2\frac{3}{2}[5(u - \bar{u})^2 - 1] \right. \\ & \left. + C_4\frac{15}{8}[21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right] , \end{aligned} \quad (\text{A7})$$

where  $\bar{u} = 1 - u$  and

$$\begin{aligned} B_2 &= 30R , \\ B_4 &= \frac{3}{2}R(4\omega_{2,0} - \omega_{1,1} - 2\omega_{1,0}) , \\ C_2 &= R \left( 5 - \frac{1}{2}\omega_{1,0} \right) , \\ C_4 &= \frac{1}{10}R(4\omega_{2,0} - \omega_{1,1}) , \end{aligned} \quad (\text{A8})$$

with

$$R = \frac{f_{3\pi}}{\mu_\pi f_\pi} . \quad (\text{A9})$$

The coefficients  $f_{3\pi}$  and  $\omega_{i,k}$  have been determined at the normalization point  $\mu = 1 \text{ GeV}$  from QCD sum rules [16]:

$$f_{3\pi} = 0.0035 \text{ GeV}^2, \quad \omega_{1,0} = -2.88, \quad (\text{A10})$$

$$\omega_{2,0} = 10.5, \quad \omega_{1,1} = 0 .$$

After renormalization [27] to the relevant scales (A3), we get

$$\begin{aligned} f_{3\pi}(\mu_c) &= 0.0032 \text{ GeV}^2, \quad \omega_{1,0}(\mu_c) = -2.63, \quad \omega_{2,0}(\mu_c) = 9.62, \quad \omega_{1,1}(\mu_c) = -1.05 , \\ f_{3\pi}(\mu_b) &= 0.0026 \text{ GeV}^2, \quad \omega_{1,0}(\mu_b) = -2.18, \quad \omega_{2,0}(\mu_b) = 8.12, \quad \omega_{1,1}(\mu_b) = -2.59 . \end{aligned} \quad (\text{A11})$$

The corresponding numerical values of the coefficients (A8) are

$$\begin{aligned} B_2(\mu_c) &= 0.41, \quad B_4(\mu_c) = 0.925, \quad C_2(\mu_c) = 0.087, \quad C_4(\mu_c) = 0.054 , \\ B_2(\mu_b) &= 0.29, \quad B_4(\mu_b) = 0.58, \quad C_2(\mu_b) = 0.059, \quad C_4(\mu_b) = 0.034 . \end{aligned} \quad (\text{A12})$$

In addition, the running of light quark masses induces a scale dependence of the parameter  $\mu_\pi = m_\pi^2/(m_u + m_d)$ :

$$\begin{aligned} \mu_\pi(1 \text{ GeV}) &= 1.65 \text{ GeV}, \quad \mu_\pi(\mu_c) = 1.76 \text{ GeV}, \\ \mu_\pi(\mu_b) &= 2.02 \text{ GeV} . \end{aligned} \quad (\text{A13})$$

The wave functions of twist 4 are more numerous. The complete set given in Ref. [28] (see also Ref. [25]) includes four three-particle wave functions. However, in leading and next-to-leading order in conformal spin, these are specified by only two parameters:

$$\begin{aligned} \varphi_\perp(\alpha_i) &= 30\delta^2(\alpha_1 - \alpha_2)\alpha_3^2 \left[ \frac{1}{3} + 2\varepsilon(1 - 2\alpha_3) \right] , \\ \varphi_\parallel(\alpha_i) &= 120\delta^2\varepsilon(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3 , \\ \tilde{\varphi}_\perp(\alpha_i) &= 30\delta^2\alpha_3^2(1 - \alpha_3) \left[ \frac{1}{3} + 2\varepsilon(1 - 2\alpha_3) \right] , \\ \tilde{\varphi}_\parallel(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3 \left[ \frac{1}{3} + \varepsilon(1 - 3\alpha_3) \right] . \end{aligned} \quad (\text{A14})$$

The two-particle twist-4 wave functions are related to these by equations of motion. To the above order in

conformal spin they involve no new parameters and are given by

$$\begin{aligned} g_1(u) &= \frac{5}{2}\delta^2\bar{u}^2u^2 + \frac{1}{2}\varepsilon\delta^2 \\ &\times \left[ \bar{u}u(2 + 13\bar{u}u) + 10u^3 \ln u \left( 2 - 3u + \frac{6}{5}u^2 \right) \right. \\ &\left. + 10\bar{u}^3 \ln \bar{u} \left( 2 - 3\bar{u} + \frac{6}{5}\bar{u}^2 \right) \right] , \\ g_2(u) &= \frac{10}{3}\delta^2\bar{u}u(u - \bar{u}) , \\ G_2(u) &= \frac{5}{3}\delta^2\bar{u}^2u^2 . \end{aligned} \quad (\text{A15})$$

One of the parameters is defined by the matrix element

$$\langle \pi | g_s \bar{d} \tilde{G}_{\alpha\mu} \gamma^\alpha u | 0 \rangle = i\delta^2 f_\pi q_\mu . \quad (\text{A16})$$

The QCD sum rule estimate of Ref. [47] yields  $\delta^2 = 0.2 \text{ GeV}^2$  at  $\mu = 1 \text{ GeV}$ . The remaining parameter is associated with the deviation of twist-4 wave functions



from their asymptotic form. At  $\mu = 1$  GeV it takes the value  $\varepsilon = 0.5$  [28]. Renormalization to the relevant scales (A3) gives

$$\begin{aligned} \delta^2(\mu_c) &= 0.19 \text{ GeV}^2, \quad \varepsilon(\mu_c) = 0.45, \\ \delta^2(\mu_b) &= 0.17 \text{ GeV}^2, \quad \varepsilon(\mu_b) = 0.36. \end{aligned} \quad (\text{A17})$$

This completes the description of the pion wave functions, as far as is needed for the applications in this paper.

## APPENDIX B

Here we derive the substitution (43) used in the sum rule (44) in order to subtract the continuum contribution. For this purpose we have to write the invariant amplitude  $F$  given by Eqs. (31) and (39) in the form of a double dispersion integral:

$$\begin{aligned} F(p^2, (p+q)^2) &= \int_{m_c^2}^{\infty} \frac{ds_1}{s_1 - p^2} \int_{m_c^2}^{\infty} \frac{ds_2}{s_2 - (p+q)^2} \\ &\quad \times \rho^{\text{QCD}}(s_1, s_2). \end{aligned} \quad (\text{B1})$$

Focusing first on the leading contribution (17),

$$\begin{aligned} F(p^2, (p+q)^2) &= m_c f_\pi \int_0^1 \frac{du \varphi_\pi(u)}{m_c^2 - (p+uq)^2} \\ &= m_c f_\pi \int_0^1 \frac{du \varphi_\pi(u)}{m_c^2 - (p+q)^2 u - p^2(1-u)}, \end{aligned} \quad (\text{B2})$$

and changing variable from  $u$  to  $(m_c^2 - p^2)/(s - p^2)$ , one obtains

$$F(p^2, (p+q)^2) = m_c f_\pi \int_{m_c^2}^{\infty} \frac{ds \varphi_\pi(u(s))}{[s - (p+q)^2](s - p^2)}. \quad (\text{B3})$$

In general, the wave function  $\varphi_\pi(u)$  can be expressed as a power series in  $(1-u)$ :

$$\varphi_\pi(u) = \sum_k a_k (1-u)^k = \sum_k a_k \left( \frac{s - m_c^2}{s - p^2} \right)^k. \quad (\text{B4})$$

Substituting this representation into Eq. (B3) and introducing formally two variables  $s_1$  and  $s_2$  instead of  $s$ , it is easy to rewrite this expression in the form (B1) with the double spectral density

$$\rho^{\text{QCD}}(s_1, s_2) = m_c f_\pi \sum_k \frac{(-1)^k a_k}{\Gamma(k+1)} (s_1 - m_c^2)^k \delta^{(k)}(s_1 - s_2). \quad (\text{B5})$$

The validity of Eq. (B5) can easily be checked by direct calculation. The above derivation may seem tricky. However, there is a convenient general method [48] to find the appropriate double spectral densities. One takes the Borel-transformed invariant amplitude  $F(M_1^2, M_2^2)$  and performs two more Borel transformations in the variables  $\tau_1 = 1/M_1^2$  and  $\tau_2 = 1/M_2^2$ , to get

$$\mathcal{B}_{\sigma_1} \mathcal{B}_{\sigma_2} F(1/\tau_1, 1/\tau_2) = \rho^{\text{QCD}}(1/\sigma_1, 1/\sigma_2). \quad (\text{B6})$$

Details and useful examples can be found in Ref. [49].

To proceed, we apply a double Borel transformation to the dispersion integral (B1) with  $\rho^{\text{QCD}}$  given by Eq. (B5):

$$\begin{aligned} \mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} F &= m_c f_\pi \sum_k \int_{m_c^2}^{\infty} ds_1 \int_{m_c^2}^{\infty} ds_2 \frac{(-1)^k a_k}{\Gamma(k+1)} \\ &\quad \times (s_1 - m_c^2)^k \delta^{(k)}(s_1 - s_2) e^{-s_1/M_1^2} e^{-s_2/M_2^2}. \end{aligned} \quad (\text{B7})$$

Introducing again new variables  $s = s_1 + s_2$  and  $v = s_1/s$  we can use the  $\delta$  function to evaluate the integral over  $v$ . The result is

$$F(M_1^2, M_2^2) = m_c f_\pi \sum_k \frac{a_k}{2^{k+1} k!} \int_{2m_c^2}^{\infty} ds \left( \frac{d}{dv} \right)^k \left[ \left( v - \frac{m_c^2}{s} \right)^k \exp \left( -\frac{svM_2^2 + s(1-v)M_1^2}{M_1^2 M_2^2} \right) \right]_{v=1/2}. \quad (\text{B8})$$

At  $M_1^2 = M_2^2 = 2M^2$  the  $v$  dependence of the exponent disappears and the differentiation of the bracket gives a factor  $k!$ . We then get

$$\begin{aligned} F(M^2) &= m_c f_\pi \sum_k \frac{a_k}{2^{k+1}} \int_{2m_c^2}^{\infty} ds e^{-\frac{s}{2M^2}} \\ &= m_c f_\pi \varphi_\pi(1/2) M^2 e^{-\frac{m_c^2}{M^2}}, \end{aligned} \quad (\text{B9})$$

which is the leading contribution in the sum rule (44). For arbitrary values of  $M_1^2$  and  $M_2^2$  a similar expression is obtained, with the argument of the wave function and

the Borel parameter in Eq. (B9) generalized to  $u_0$  and  $M^2$ , respectively, as defined in Eq. (22).

We now turn to the problem of subtracting the contributions from excited and continuum states in sum rules. In the usual approximation based on duality, one identifies the spectral functions  $\rho^{\text{QCD}}$  and  $\rho^h$  beyond a given boundary in the  $(s_1, s_2)$  plane. Then, the subtraction effectively amounts to restricting the dispersion integrals in Eq. (B1) to the region below this boundary. Ideally, the result should not depend on the precise shape of this region. To be specific, one may take

$$s_1^a + s_2^a \leq s_0^a, \quad (\text{B10})$$

where  $s_0$  plays the role of an effective threshold. Popular choices of the duality region are triangles in the  $(s_1, s_2)$  plane corresponding to  $a = 1$  and squares corresponding to  $a \rightarrow \infty$ . Since the spectral density (B5) vanishes everywhere except at  $s_1 = s_2$ , it is actually irrelevant which form of the boundary we adopt.<sup>6</sup>

Using duality as outlined above we have to evaluate the integral in Eq. (B7) with the integration region restricted by Eq. (B10). Changing variables and integrating over  $v$  one obtains an expression similar to Eq. (B8), but with the upper limit of integration in  $s$  lowered to  $2s_0$  and with the addition of surface terms. The latter disappear for  $M_1^2 = M_2^2$ . Hence, one is again led to Eq. (B9) with a simple modification of the integration limit:

$$F(M^2) = m_c f_\pi \sum_k \binom{a_k}{2^{k+1}} \int_{2m_c^2}^{2s_0} ds e^{-\frac{s}{2M^2}} \\ = m_c f_\pi \varphi_\pi(1/2) M^2 \left[ e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}} \right]. \quad (\text{B11})$$

This proves the substitution rule quoted in Eq. (43). It is important to note that the proportionality of the Borel transform  $F(M_1^2, M_2^2)$  given in Eq. (B8) to the wave function  $\varphi_\pi$  at the point  $u_0 = M_1^2/(M_1^2 + M_2^2)$  is generally destroyed by the continuum subtraction. It is only retained in the symmetric point  $M_1^2 = M_2^2$ , implying  $u_0 = 1/2$ .

The above procedure is not possible for higher-twist contributions which are proportional to zero or negative powers of the Borel parameters. The reason is that the corresponding spectral densities are not concentrated near the diagonal  $s_1 = s_2$ . In fact, the continuum subtraction is rather complicated in these cases. For further discussion we refer the reader to the second paper of Ref. [49]. Here, we neglect the continuum subtraction in higher-twist terms altogether. This is justified to a good approximation since the corresponding spectral densities decrease fast with  $s_1$  and  $s_2$  as a consequence of ultraviolet convergence and, hence, the continuum contribution is expected to be small anyway.

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<sup>6</sup>This is literally true only if the power series defining the wave function (B4) is truncated at some finite order, or if it converges rapidly. However, this condition is always met at a sufficiently high normalization point where the wave function deviates little from the asymptotic form.

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