

The $D_{s0}^*(2317)$ and DK scattering from lattice QCD

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The scalar meson $D_{s0}^*(2317)$ is found 37(17)MeV below DK threshold in a lattice simulation of the $J^P = 0^+$ channel using, for the first time, both DK as well as $\bar{s}c$ interpolating fields. The simulation is done on $N_f = 2+1$ gauge configurations with $m_\pi \simeq 156$ MeV, and the resulting $M_{D_{s0}^*} - \frac{1}{4}(M_{D_s} + 3M_{D_s^*}) = 266(16)$ MeV is close to the experimental value 241.5(0.8) MeV. The energy level related to the scalar meson is accompanied by additional discrete levels due to DK scattering states. The levels near threshold lead to the negative DK scattering length $a_0 = -1.33(20)$ fm that indicates the presence of a state below threshold.

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The experimentally observed meson spectrum contains a number of states close to an s -wave threshold. Such structures are seen in the spectrum of light-quark mesons, in heavy-light mesons and in the spectra of charmonium and bottomonium, and many of them do not fit well with expectations based on a simple quark-antiquark picture. These states are frequently interpreted as either shallow-bound molecules, tightly-bound tetraquarks or mesons containing gluonic excitations and are sometimes referred to collectively as “exotics”.

In this paper we focus on the spectrum of charmed-strange $J^P = 0^+$ mesons. Prior to the discovery of the $D_{s0}^*(2317)$, quark models predicted a resonance above the D-meson kaon (DK) threshold. However, results obtained by various experiments [1] show a very narrow state which is well below the DK threshold. The coupling of $J^P = 0^+$ $\bar{s}c$ to the DK threshold was suggested as a mechanism for lowering the mass of the physical state [2]. The dependence on m_π of the mass differences between the scalar and pseudoscalar heavy-light mesons was investigated in [3]. Interestingly, the mass of the $D_{s0}^*(2317)$ is very close to the mass of its non-strange partner, which has given rise to a tetraquark interpretation [4].

Lattice QCD (LQCD) provides the possibility of calculating the spectrum of QCD without resorting to model assumptions. Nevertheless addressing states close to s -wave thresholds turns out to be a formidable task. In the channel of interest, $D_{s0}^*(2317)$ needs to be distinguished from the nearby level at $E \simeq m_D + m_K$ (corresponding to the infinite volume DK threshold) which is expected to be shifted on the lattice due to interactions [5]. Indeed, all physical states with $J^P = 0^+$ and $I = 0$ appear as discrete energy levels due to the finite volume. The energies of the s -wave discrete scattering states $D(\mathbf{p})K(-\mathbf{p})$, where $\mathbf{p} = \frac{2\pi}{L}\mathbf{n}$ in the non-interacting limit, are shifted in the the finite volume due the interaction. The energy

shifts are related to the infinite volume scattering phase shifts in the elastic region via Lüscher’s formula [6].

It was noticed in previous LQCD calculations, that many expected scattering levels are absent [7–10] when simulating mesons with quark-antiquark interpolating fields or baryons with three-quark interpolators, although there might be hints of these states in the case of close-by s -wave thresholds [11–13]. To ameliorate this problem better overlap with multi-particle states is needed which may be obtained by including them explicitly in the basis of lattice interpolators.

Previous lattice studies [14–23] considered $D_{s0}^*(2317)$ using only quark-antiquark interpolators. Early (quenched) lattice QCD calculations found energy levels substantially above the physical DK threshold [14–17]. Recent dynamical LQCD simulations [18–23] are also not definitive due to closeness of the DK threshold.

A novel feature of the present study is that quark-antiquark and meson-meson interpolators are combined in the charmed-strange $J^P = 0^+$ channel. The simulation is performed on two lattice ensembles with parameters listed in Table I. Further details about the ensembles may be found in [24] and [25–27] respectively. To minimize heavy-quark discretization effects at finite lattice spacing the Fermilab method [28, 29] is used for the charm quarks. Details of the procedure along with the relevant parameters for ensemble (1) can be found in [30]. In particular we use the dispersion relations (3) and (18) of [30] for our tuning procedure and the determination of p . The same procedure has been used to tune the charm quark mass on ensemble (2) and the resulting parameters are $c_{sw} = 1.64978$ and $\kappa_c = 0.12686$. Notice that at finite lattice spacing the mass splittings between states involving a heavy quark are expected to be close to physical while the rest masses are affected by large

ID	$N_L^3 \times N_T$	N_f	a [fm]	L [fm]	#configs	m_π [MeV]	m_K [MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

TABLE I. Details of gauge configurations used. N_L and N_T denote the number of lattice points in spatial and time directions, N_f the number of dynamical flavors and a the lattice spacing. The pion mass for ensemble (2) is taken from [24], while the kaon mass results from our calculation with partially quenched strange quarks.

discretization effects.¹ Consequently, mass splittings will be quoted with respect to the mass of the spin-averaged 1S state $M_{\overline{1S}} = (M_{D_s} + 3M_{D_s^*})/4$ in our final results and in all figures. For ensemble (2) the strange quark mass used in [24] differs significantly from the physical value. We therefore use a partially quenched strange quark $m_s^{val} \neq m_s^{sea}$ and determine the hopping parameter κ_s^{val} by minimizing the difference of the ϕ meson mass from the experimental mass and the difference of the unphysical η_s meson from the value expected from a high-precision lattice determination [31]. The determinations agree to high precision and $\kappa_s = 0.13666$ is obtained.

To handle the backtracking quark loops appearing in the Wick contractions, the powerful distillation method [32] is used. This can be seen as a smearing prescription producing quark sources and sinks that are approximately Gaussian. The method allows for a large freedom in the choice of interpolators and for momentum projection at source and sink. The exact Laplacian-Heaviside version is used for ensemble (1) and the stochastic extension of distillation [9] for ensemble (2). Within this approach we calculate the correlation matrix

$$\begin{aligned}
C_{ij}(t) &= \sum_{t_i} \langle 0 | O_i(t_i + t) O_j^\dagger(t_i) | 0 \rangle \\
&= \sum_n e^{-tE_n} \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle,
\end{aligned} \quad (1)$$

using interpolating fields O_i with $J^P = 0^+$ (irrep A_1^+), isospin $I = 0$ and total momentum zero. Four quark-antiquark interpolators $O_{1-4}^{\bar{s}c}$ = $\bar{s}A_{1-4}c$ taken to be the entries 1-4 of irrep A_1^+ in Table XII of [30] are used. There are also three meson-meson interpolators

$$\begin{aligned}
O_1^{DK} &= [\bar{s}\gamma_5 u](p=0) [\bar{u}\gamma_5 c](p=0) + \{u \rightarrow d\}, \\
O_2^{DK} &= [\bar{s}\gamma_t\gamma_5 u](p=0) [\bar{u}\gamma_t\gamma_5 c](p=0) + \{u \rightarrow d\}, \\
O_3^{DK} &= \sum_{p=\pm e_{x,y,z} 2\pi/L} [\bar{s}\gamma_5 u](p) [\bar{u}\gamma_5 c](-p) + \{u \rightarrow d\}.
\end{aligned} \quad (2)$$

The discrete energy levels E_n are extracted from the correlators (1) using the variational method [33–35]. Figure 1 illustrates the results for the spectrum obtained

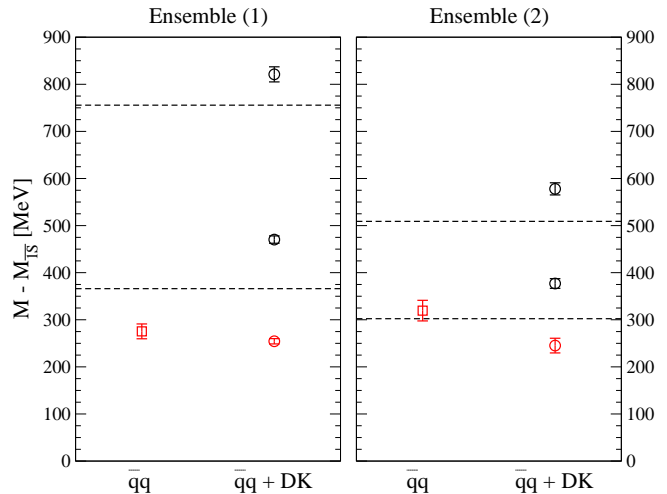


FIG. 1. Energy levels from ensemble (1) (left pane) and ensemble (2) (right pane). For each case results with just quark-antiquark ($\bar{q}q$) and with a combined basis of $\bar{q}q$ and DK interpolating fields are shown. The lower dashed lines indicate $M_D + M_K$ on both ensembles, while upper dashed lines show the energies of non-interacting $D(1)K(-1)$. The error bars include statistical and scale setting corrections.

from both lattices. In each pane the left set of points indicates the ground state level with just a quark-antiquark basis² while the right set of points indicates the energies using our full basis. The lower dashed lines denotes the $m_D + m_K$ threshold on both lattices, while the upper dashed line corresponds to the energy of the non-interacting $D(1)K(-1)$ scattering state. Note that two low-lying states are observed when using the combined basis. Their signal is unambiguous upon variation of the basis, as long as at least one of $O_{1,2}^{DK}$ and at least two $O^{\bar{s}c}$ interpolators, or if both of $O_{1,2}^{DK}$ and one or more of the $O^{\bar{s}c}$ interpolators are used. The interpolator O_3^{DK} is needed to render the $D(1)K(-1)$ state. This level will not be used in the analysis but for our conclusions it is important that it can indeed be identified with the interacting $D(1)K(-1)$.

¹ For the rest and kinetic mass of the D meson on ensemble (1) please refer to the caption of Figure (3) in [30]. For ensemble (2) we obtain $M_1 \approx 0.7534$, $M_2 \approx 0.828$ and $M_4 \approx 0.889$

² The second level from the $\bar{q}q$ basis is of poor statistical quality and it appears above the second level obtained from the full basis. It is away from the energy region of interest, it does not influence the conclusions and it is not plotted for clarity.

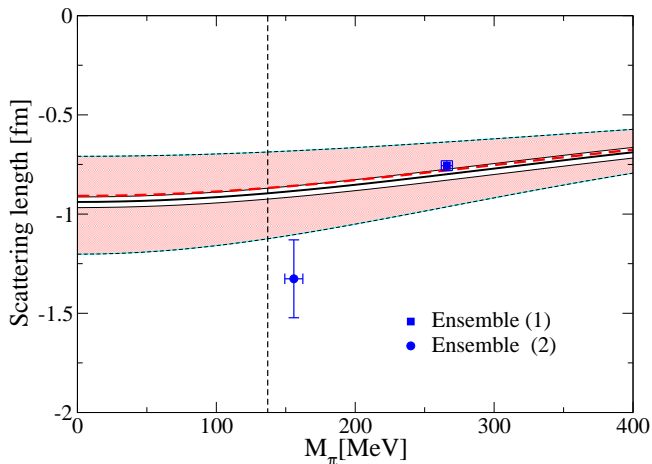


FIG. 2. Scattering length a_0 for s -wave DK scattering with $I = 0$. Our result is compared to the expectation from the indirect calculation in [42]. The vertical line corresponds to the physical pion mass. For an explanation of the curves please refer to the main text.

Taking a look at the lowest two energy levels on each ensemble there are two possible interpretations:

1. A sub-threshold state which is stable under the strong interaction (in the Isospin limit). We will refer to such a state as a “bound state” but stress that this choice of words makes no statement about a possible $\bar{q}q$ or meson-meson nature of the state. In this case a negative scattering length is expected and the up-shifted second level would be related to the interacting scattering threshold on the lattice. Such a scenario was discussed in the context of a simple model in [36], and was confirmed for a deuteron bound state pn [37, 38] and for a $D\bar{D}^*$ bound state $X(3872)$ [39]. The expected behavior of the energy levels in various scenarios was discussed in model studies in Ref. [40].
2. A QCD resonance (above threshold). In this case the attraction is not strong enough to form a bound state and the level associated with the finite volume scattering state will be found below threshold. A positive scattering length is expected and the additional level above threshold occurs due to the presence of a resonance in this channel. This is the situation encountered for $D\pi$ scattering in the $J^P = 0^+$ channel with resonance $D_0^*(2400)$ [30] or in $N\pi$ scattering in the negative parity sector [41].

The crucial insight is that the plausibility of these scenarios can be tested by determining the real number $p \cot \delta$ from energy levels using Lüscher’s formula [6], which applies above and below threshold. Here δ is the scattering phase shift for the elastic DK scattering in s -wave, while p is the D and K momentum related to

	level	Ensemble (1)	Ensemble (2)
$(pa)^2$	1	-0.0347(14)	-0.0092(24)
	2	0.0364(14)	0.0130(16)
$(pa) \cot \delta$	1	-0.1560(59)	-0.082(19)
	2	-0.1722(74)	-0.049(15)
$(p_b a)^2$	-	-0.0250(17)	-0.0060(26)

TABLE II. Values for p^2 and $p \cot \delta$ obtained from the two lowest energy levels for both ensembles. In addition the values for the binding momentum p_b are tabulated in the last row.

the energy via $E_n(L) = E_D(p) + E_K(p)$. We perform an effective range approximation

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi}L} Z_{00}(1, (\frac{pL}{2\pi})^2) \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2, \quad (3)$$

which seems well justified for the momenta at hand (given in Table II along with the values for $p \cot \delta$). The lowest two levels give two equations for two unknowns.

For ensemble (1) we obtain³

$$a_0 = -0.756(25) \text{ fm} \quad r_0 = -0.056(31) \text{ fm}, \quad (4)$$

while ensemble (2) yields

$$a_0 = -1.33(20) \text{ fm} \quad r_0 = 0.27(17) \text{ fm}. \quad (5)$$

In both cases the extracted scattering length is negative with a reasonably small statistical and systematic uncertainty, and the effective range is small. This is compatible with scenario (1) above, where the lower energy level is associated with a bound state up to corrections related to the finite volume of the simulation.

Figure 2 compares our results for the scattering length to the prediction from [42] where the authors performed a lattice calculation in a variety of channels and extracted the relevant low-energy constants of the chiral effective field theory. These low energy constants were then used to predict the DK $I = 0$ scattering length. Two distinct determinations of low energy constants were performed. For the first set of results only the lattice data was used as input (larger error band in the figure, central value thick dashed curve) while for the second set the $D_{s0}^*(2317)$ mass from experiment was used as further input leading to much smaller uncertainties (narrower error band in Fig. 2, central value thick solid curve). As can be seen, our results from the direct simulation agree qualitatively, although the uncertainty close to physical pion and kaon mass is large.

³ The effective range value for ensemble (2) has sizeable systematic uncertainty allowing even small negative values but has little influence on the final value of the binding energy.

	$E_1(L) - M_{\overline{1S}}$	$M_{L \rightarrow \infty}^{D_s^*(2317)} - M_{\overline{1S}}$
ensemble (1)	254.4(4.3)(2.3)	287.2(5.0)(3.0)
ensemble (2)	245(15)(4)	266(16)(4)
experiment		241.45(0.60)

TABLE III. The final result for the $D_{s0}^*(2317)$ mass $M_{L \rightarrow \infty}^{D_s^*(2317)}$, as obtained from the pole condition, compared to the experimental value [1] (right column). The energy levels in the finite-volume lattice are shown in the left column. The errors are statistical (1st) and due to scale setting (2nd).

In infinite volume a bound state would correspond to a pole of the S-matrix which translates to the pole condition $\cot \delta(p_b) = i$, where $p_b = i|p_b|$ denotes the binding momentum of the bound state. Taking the values for a_0 and r_0 extracted within the effective range approximation, we determine the binding momentum, which translates to our estimate of the bound state energy $M_{L \rightarrow \infty} = E_D(p_b) + E_K(p_b)$ using the dispersion relations $E_K(p) = (M_K^2 + p^2)^{1/2}$ and $E_D(p)$ given by Eq. (3) of [30] with $W_4 = 0$. It is this bound state energy and its value with respect to the DK threshold that should be compared to experiment. Analogous extraction of the deuteron binding energy from two lowest levels on a single volume was proposed for future simulations in [43].

The systematic uncertainties come from fitting the dispersion relation for the D -meson [30] and from determining the kaon mass. For both, the scattering length and the binding energy, we estimate those to be 30% of the statistical errors.

Our final result is given alongside the experimental $D_{s0}^*(2317)$ mass in Table III and Figure 3, together with DK thresholds on the lattice and in experiment. Notice that with a pion mass of 156 MeV and at finite lattice spacing we neither expect the thresholds to agree perfectly, nor do we expect the position of the sub-threshold state to agree exactly with the $D_{s0}^*(2317)$. In particular heavy quark discretization effects of an order of a few percent of the mass splittings are expected and their influence should be addressed in future simulations.

In summary, we have performed a simulation of the D_s ($J^P = 0^+$) spectrum with the novel feature of a combined basis of quark-antiquark and DK operators. The combination of both types of lattice interpolating fields was crucial to obtain energy levels with small statistical uncertainties and the variational analysis shows that both types of operators have non-vanishing overlap with the physical state. Further notable features of the simulation are the use of an improved heavy-quark action, distillation methods to deal with operator contractions, and almost physical pions, kaons and D mesons. Unlike previous lattice simulations, we observe a state below DK threshold whose mass is compatible with the experimental $D_{s0}^*(2317)$ within the remaining uncertain-

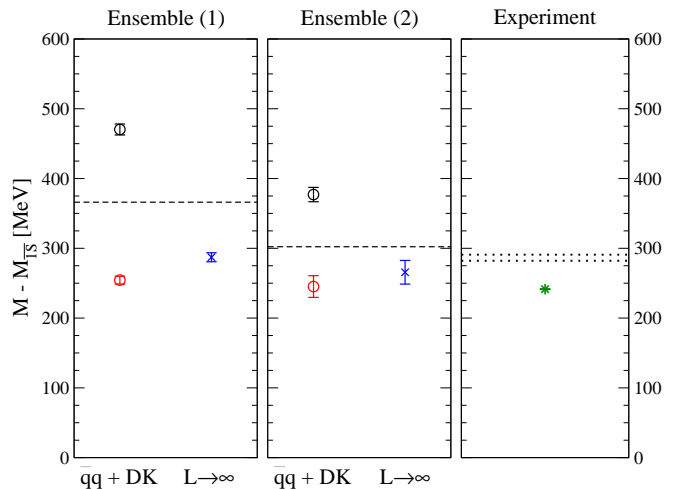


FIG. 3. The final result for $D_{s0}^*(2317)$ mass is given by the crosses in the left and middle panes, while the experimental value is given in the right pane. Instead of the mass itself, we compare the values of $M_{L \rightarrow \infty}^{D_s^*(2317)} - M_{\overline{1S}}$, where $M_{\overline{1S}}^{exp} = \frac{1}{4}(m_{D_s} + 3m_{D_s^*}) \simeq 2076$ MeV. The value of the bound state position in the infinite volume limit, $M_{L \rightarrow \infty}^{D_s^*(2317)}$ is obtained from the pole condition $\cot \delta = i$. The two lowest energy levels from our simulation in the finite volume are given by the circles in the left and middle panes. Dashed lines represent the threshold for DK in our simulation ($m_u = m_d$), and dotted lines the thresholds for $D^0 K^+$, $K^0 D^+$ in experiment.

ties. To obtain precision results, simulations at multiple lattice spacings and with multiple lattice volumes will be needed.

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