D3-brane potentials from fluxes in AdS/CFT

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ABSTRACT: We give a comprehensive treatment of the scalar potential for a D3-brane in a warped conifold region of a compactification with stabilized moduli. By studying general ultraviolet perturbations in supergravity, we systematically incorporate 'compactification effects' sourced by supersymmetry breaking in the compact space. Significant contributions to the D3-brane potential, including the leading term in the infrared, arise from imaginary anti-self-dual (IASD) fluxes. For an arbitrary Calabi-Yau cone, we determine the most general IASD fluxes in terms of scalar harmonics, then compute the resulting D3-brane potential. Specializing to the conifold, we identify the operator dual to each mode of flux, and for chiral operators we confirm that the potential computed in the gauge theory matches the gravity result. The effects of four-dimensional curvature, including the leading D3-brane mass term, arise directly from the ten-dimensional equations of motion. Furthermore, we show that gaugino condensation on D7-branes provides a local source for IASD flux. This flux automatically and precisely encodes the nonperturbative contributions to the D3-brane potential, yielding a promising ten-dimensional representation of four-dimensional nonperturbative effects. Our result encompasses all significant contributions to the D3-brane potential discussed in the literature, and does so in the single

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[1]On leave of absence from Stanford University and SLAC.

coherent framework of ten-dimensional supergravity. Moreover, we identify new terms with irrational scaling dimensions that were inaccessible in prior works. By decoupling gravity in a noncompact configuration, then systematically reincorporating compactification effects as ultraviolet perturbations, we have provided an approach in which Planck-suppressed contributions to the D3-brane effective action can be computed. This is the companion paper to [1].

Contents

1 Introduction

Since the dawn of time, humankind has wondered, "what is the potential on the Coulomb branch of the conifold gauge theory, and what are the consequences for models of D-brane inflation?" In this paper, we continue this quest.

Recent observations [2] give striking support to the idea that there was a period of inflation in the very early universe [3–5]. In a rather economical way, inflation explains both the large-scale homogeneity of the universe and the small-scale inhomogeneities required for the formation of galaxies [6, 7]. As a phenomenon in quantum field theory coupled to general relativity, inflation is sensitive to ultraviolet physics: the inflationary dynamics is controlled by Planck-suppressed contributions to the effective action (see e.g. [6–8]). This strongly motivates pursuing realizations of inflation in an ultraviolet-complete theory, such as string theory, and then computing these contributions in detail.

In practice, determining all of the significant Planck-suppressed contributions to the effective action is highly nontrivial and requires a detailed understanding of the stabilization of compactification moduli. Nevertheless, this undertaking is an essential prerequisite for any explicit realization of inflation in string theory. It is therefore critical to identify scenarios for inflation in string theory that enjoy a high degree of computability, so that one can hope to compute all relevant Planck-suppressed contributions to the inflaton action.

D3-brane inflation in a warped throat geometry [9] has been the subject of considerable research (for recent reviews, see e.g. refs. [7, 8, 10–12]). Much of the interest is due not to any intrinsic elegance of the scenario, but rather to the prospect of explicit computations of the inflaton action: the warped deformed conifold solution [13] provides a concrete arena with a known metric and known background fluxes, and the effects of moduli stabilization, e.g. in the scenario of [14], can be incorporated in detail [15, 16]. The essential simplification is that a finite warped region may be approximated by the *noncompact* warped deformed conifold solution, for which the supergravity solution can be written explicitly; similarly, the divisors responsible for Kähler moduli stabilization may be approximated by noncompact divisors.

Corrections to the noncompact approximation, i.e. contributions to the D3-brane effective action induced by objects and fields in the compact bulk, constitute the remaining Planck-suppressed contributions to the inflaton action [17]. On general grounds, one expects that these 'compactification effects' can make order-unity corrections to the inflationary slow-roll parameters, and must therefore be incorporated, or shown to be suppressed, in any explicit realization of D3-brane inflation. Specifically, D-branes, orientifold planes, fluxes, and quantum effects in the bulk may not preserve the same supersymmetry as a D3-brane, and their effects on the D3-brane potential must then be included. The relevant effects can be both perturbative and nonperturbative.

In this work, we provide a systematic treatment of these compactification corrections to the D3-brane Lagrangian. We take the compact bulk to be a rather general solution of type IIB supergravity, and then determine how the form of this solution in the ultraviolet region of a finite throat affects the potential for a D3-brane well inside the throat.

Our strategy is to study a noncompact solution subject to general non-normalizable deformations, as a computable proxy for a finite throat attached to a compact space. The critical simplification is that a completely general solution in the ultraviolet is well-approximated in the infrared by a solution parameterized by the handful of modes that diminish least rapidly under radial rescaling. In the dual field theory, this is the familiar statement that renormalization group flow filters out highly irrelevant perturbations, so that the dominant effect in the infrared is controlled by the coefficients of the most relevant modes. By careful study of these modes, one can determine the leading structure of the D3-brane potential.

We identify the nonlinear interactions of imaginary anti-self-dual (IASD) three-form fluxes as an important contribution to the D3-brane potential. In fact, under certain conditions this can be the dominant effect in the infrared. To incorporate this effect, we provide a general solution for IASD fluxes in the conifold. Our method extends to any Calabi-Yau cone, yielding the three-form fluxes in terms of the scalar harmonics on the angular manifold. Taking general IASD fluxes as sources, we compute the corresponding flux-induced potential. We then include an additional contribution sourced by four-dimensional scalar curvature, showing that the leading curvature correction gives rise to the well-known 'eta problem' mass term of [9] (see also [18, 19]). The inclusion of the nonlinear effects of fluxes and curvature is a substantial step beyond the linear treatment in our previous work [17].

Our analysis is simplified by a special property of D3-branes: a D3-brane couples only to a particular scalar mode, which we denote by Φ_-, and IASD flux G_- is the dominant source in the equation of motion for Φ_-. We systematically expand around solutions in which $\Phi_- = G_- = 0$. Crucially, metric and dilaton fluctuations do not couple to D3-branes at the order to which we work. Therefore, although the metric is distorted away from the conformally Calabi-Yau metric of the leading order solution, we do *not* need to determine its form in order to specify the D3-brane potential. Thus, although our solutions are genuinely nonlinear in fluxes, they are far simpler than the most general solutions nonlinear in all supergravity fields. Let us stress that our analysis rests on a double expansion: in small fluctuations around solutions in which $\Phi_- = G_- = 0$, and in the ratio of energy scales between the ultraviolet, where the throat is perturbed by effects from the compact bulk, and the infrared, where the D3-brane probes the supergravity background.

The compactification effects studied here are most efficiently described in supergravity as perturbations to the ultraviolet region of the warped deformed conifold solution. However, a nontrivial consistency check comes from representing these non-normalizable perturbations, through the AdS/CFT correspondence [20–22], as perturbations to the Lagrangian of the dual conformal field theory (CFT). Building on the work of Ceresole *et al.* [23], we provide the operator in the conifold CFT dual to each mode of flux. In our earlier work [17], we considered corrections to the D3-brane potential from linearized perturbations of the CFT Kähler potential; here we incorporate contributions up to quadratic order in the perturbations of the superpotential. In the case of chiral operators perturbing the superpotential, we compute the potential on the CFT side and find agreement with the supergravity result. For non-chiral operators, there is no reason to expect computability in the strongly-coupled CFT, and we do not attempt to match the corresponding potentials.

Ultimately, our approach is strongly reminiscent of a four-dimensional effective field theory analysis of the inflaton action. However, the field theory that governs the D3-brane potential is strongly coupled. Several of the contributing operators have irrational dimensions and cannot be studied effectively on the field theory side. Our method, which is to consider the most general perturbations of the ultraviolet region of the supergravity solution, effectively uses AdS/CFT to provide a tractable problem that realizes the spirit of the four-dimensional effective field theory approach.

An additional goal of this work is a better understanding of nonperturbative contributions to the D3-brane potential. When the Kähler moduli are stabilized nonperturbatively [14], a critical contribution to the D3-brane potential arises from nonperturbative effects on branes wrapping suitable four-cycles [9, 24, 25]. In the special case in which the dominant effect comes from a divisor that protrudes into the warped throat region, the nonperturbative corrections to the D3-brane potential can be computed [15, 16]. However, for general compactifications there will be non-negligible contributions from a variety of divisors, not all of which enter the throat region. An outstanding open problem is how to characterize these contributions.

We make progress in this direction by demonstrating that for any specified superpotential W for a D3-brane in a noncompact conifold geometry, there exists a ten-dimensional supergravity solution in which the scalar potential for a probe D3-brane precisely matches the scalar potential computed in the four-dimensional supersymmetric gauge theory with superpotential W. This solution contains IASD three-form flux of Hodge type $(1, 2)$, $G_{(1,2)}$, and amounts to an explicit example of the general result [26] that a $G_{(1,2)}$ background induces superpotential interactions for a probe D3-brane. We show that this relation persists even in the presence of large distortions of the metric and dilaton sourced by the classical backreaction of D7-branes. Perhaps more surprisingly, we demonstrate that for any specified superpotential for a D3-brane in a finite conifold region, the F-term potential computed in four-dimensional super*gravity* can be geometrized by a particular ten-dimensional background of IASD fluxes.

Finally, we establish that gaugino condensation on D7-branes wrapping a four-cycle Σ provides a source term, localized to Σ, for IASD flux. The ten-dimensional equation of motion for the flux is corrected by a term proportional to the expectation value $\langle \lambda \lambda \rangle$ of
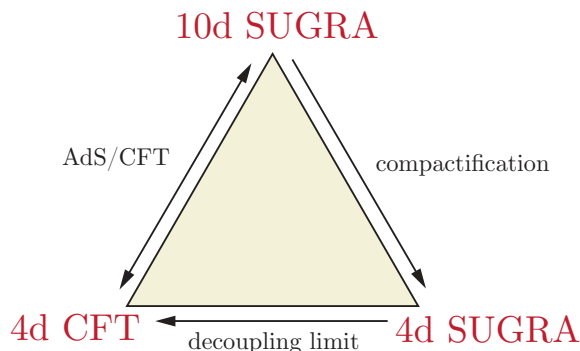
Figure 1. Three descriptions of D3-branes in warped throats: the ten-dimensional supergravity perspective is explored in section 2, section 3 and section 4, the dual four-dimensional conformal field theory is discussed in section 5, and connections to four-dimensional supergravity are made in section 6 and section 7.

the gaugino bilinear, and the corresponding solution necessarily involves $G_{(1,2)}$ flux proportional to $\langle \lambda\lambda \rangle$. We demonstrate that a probe D3-brane in this flux background experiences precisely the scalar potential computed in four dimensions with the gaugino condensate superpotential. In this sense, the induced flux encodes four-dimensional nonperturbative effects in the ten-dimensional supergravity solution. This result constitutes progress towards a geometric transition for D7-branes, in that it replaces four-dimensional nonperturbative effects on D7-branes by certain bulk fluxes. However, our methods serve only to identify terms in the supergravity solution to which a D3-brane is sensitive, and as noted above, at leading order a D3-brane does not couple to perturbations of the metric.[1] We leave for the future the very interesting problem of identifying further probes of the geometry that could guide the formulation of a complete geometric transition for D7-branes [28].

In summary, in this paper we present three distinct, but complementary descriptions of the system of interest (see figure 1): *i*) ten-dimensional supergravity, *ii*) four-dimensional gauge theory, and *iii*) four-dimensional supergravity. For an inflationary model, dynamical four-dimensional gravity is of course crucial, so that only the last description, which arises for a D3-brane probing a finite throat contained in a compact space, seems of direct interest. However, we show that one can usefully consider a decompactification limit in which a D3-brane probes a noncompact warped throat subject to suitable non-normalizable perturbations. This theory is then connected by the AdS/CFT correspondence to an approximately-conformal four-dimensional gauge theory subject to ultraviolet perturbations of the Lagrangian. Furthermore, the sourcing of fluxes by gaugino condensation provides a map from four dimensions to ten dimensions. We use the noncompact ten-dimensional supergravity solution in order to determine the structure of the inflaton potential in the compact case of interest. The internal consistency of these different approaches to computing the D3-brane potential, and the many cross-checks provided by relating them, give us confidence that we capture the leading contributions.

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[1]See [27] for an interesting related proposal that represents the nonperturbative superpotential in terms of generalized complex geometry.

For readers familiar with our prior work [15–17], we will now outline how the present analysis incorporates and extends those results. In [15, 16] the D3-brane potential induced by nonperturbative effects on a stack of D7-branes was studied in four-dimensional supergravity. The D7-branes were assumed to descend far into the throat region and to have limited support in the bulk, so that the effects computed explicitly in [15] would dominate over additional contributions from the compact bulk. In [17] we began to relax this assumption, studying more general contributions to the D3-brane action in the framework of ten-dimensional supergravity. In the present work we study very general compactification contributions to the D3-brane action and determine the full structure of the D3-brane potential.

To compare these results, we write the D3-brane potential as

$$V = \sum_i c_i \phi^{\Delta_i} h_i(\Psi) \,, \tag{1.1}$$

where c_i are constants, ϕ is the canonically-normalized field describing radial motion of the D3-brane, and $h_i(\Psi)$ are functions of the angular directions on the conical geometry (see section 3 for our conventions). In this notation, the four-dimensional supergravity result of [16] implies that

$$\Delta = 1 \,, \ \frac{3}{2} \,, \ 2_s \,, \ 2 \,, \ \frac{5}{2} \,, 3 \,, \ \ldots \tag{1.2}$$

where 2_s denotes a singlet term, and all other terms have nontrivial angular dependence. In [17] we showed that a subset of these terms correspond to perturbations of the supergravity solution by certain scalar field harmonics on the conifold, with

$$\Delta_{\mathcal{H}} \ = \ \frac{3}{2} \,, \ 2 \,, \ 3 \,, \ \ldots \tag{1.3}$$

In this paper, we will show that the remaining terms correspond to nonlinear perturbations sourced by fluxes,

$$\Delta_{G_-} \ = \ 1 \,, \ 2_s \,, \ \frac{5}{2} \,, \ \ldots \tag{1.4}$$

and by four-dimensional curvature $\mathcal{R}_4$,

$$\Delta_{\mathcal{R}} \ = \ 2_s \,, \ 3 \,, \ \ldots \tag{1.5}$$

Moreover, we will identify the leading compactification effect that could not be captured by the analysis of [16]: this is a flux perturbation dual to a non-chiral operator,

$$\Delta_{n\chi} \ = \ \sqrt{28} - \frac{5}{2} \ \approx \ 2.79 \,. \tag{1.6}$$

Such a contribution is generically present, but could not be found in [16], which incorporated only perturbations of the superpotential.

The outline of this paper is as follows: in section 2 we recall the basic fields and equations of motion of type IIB supergravity. We define our approximation scheme for studying non-normalizable perturbations of backgrounds with imaginary self-dual (ISD)

fluxes. As a prerequisite for analyzing the flux-induced potential, we classify in section 3 all closed, IASD three-form perturbations on general Calabi-Yau cones. In section 4 we then discuss the spectrum of contributions to the D3-brane potential sourced by UV deformations of $AdS_5 \times T^{1,1}$. In section 5 we explain how these results can be reflected in the dual conformal field theory studied in [29]. First, we carefully identify the operators dual to the perturbations of fluxes, building on [23]. We then perturb the CFT Lagrangian by these operators, allowing for explicit breaking of supersymmetry, and show that in the case of perturbations by chiral operators, the results agree with the gravity analysis. In section 6 we relate our results to the nonperturbatively-generated D3-brane potential in four-dimensional supergravity, showing that the effects of gaugino condensation on D7-branes can be represented in ten dimensions by suitably-chosen IASD fluxes. Finally, in section 7 we show that gaugino condensation on D7-branes actually sources IASD flux, providing an intriguing link between ten-dimensional supergravity and four-dimensional nonperturbative effects. We present our conclusions in section 8.

Two appendices contain computations supporting the results presented in the main text: in appendix A we give the details of the classification of all closed, IASD three-form flux perturbations on arbitrary Calabi-Yau cones, while in appendix B we extend some of these results to cases with significant dilaton variations.

Throughout we use units where $c = \hbar = 1$ and $M_{\rm pl}^2 = 1/8\pi G = 1$.

A condensed presentation of some of the key results of this paper has appeared in [1].

2   Ten-dimensional supergravity

Our goal is to understand the effective action for D3-branes in a flux compactification. Suppose that the entire compactification contains only ISD fluxes and that all local sources saturate the inequalities described in [30]; we will refer to such a solution as an *ISD compactification*. A D3-brane in an ISD compactification is known to feel no force at leading order in g_s and α'. However, the no-scale structure that forbids a D3-brane potential at leading order simultaneously forbids a potential, again at leading order, for the breathing mode of the compactification. Therefore, when the no-scale structure is ultimately broken to achieve Kähler moduli stabilization, e.g. by nonperturbative effects, the D3-brane also experiences a potential. We would like to describe the most general such potential in ten-dimensional supergravity by systematically expanding around an ISD solution.

We will also make use of an expansion around the noncompact warped deformed conifold solution [13]. Gluing a finite throat into a compact space requires suitable distortions of the supergravity fields in the ultraviolet region of the throat. Some of these distortions may involve only supergravity fields to which a D3-brane does not couple at leading order, such as the dilaton and the unwarped metric. Examples of ISD compactifications of precisely this sort are well-understood [30]. However, as explained above, in general one expects that some of the distortions associated with attaching a throat into a compact space *with all moduli stabilized* will require violations of the ISD conditions described in [30]. Therefore, in this section we consider general non-normalizable perturbations of ISD compactifications, allowing these perturbations to violate the ISD conditions.

In section 2.1 we review the basic degrees of freedom of type IIB supergravity and the equations of motions that the fields satisfy. We derive the equation of motion of the D3-brane potential and show that it is sourced by IASD fluxes and by four-dimensional Ricci curvature. In section 2.2 we explain that a filtering effect of the warped background allows us to focus on a handful of corrections, those with the smallest scaling dimensions. Then, in section 2.3, we describe the scheme by which we study perturbations around ISD compactifications. (We present these corrections in more detail in the following sections, section 3 and section 4.) Finally, in section 2.4, we describe some constraints on these backgrounds that arise when one wishes to consider a compact model; the most basic conditions arise from avoiding destabilization of the volume modulus of the compact space, and from satisfying global tadpole constraints.

2.1 Equations of motion

Our starting point is the bosonic low-energy action for type IIB supergravity in Einstein frame,

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g|} \left[\mathcal{R}_{10} - \frac{\partial_M \tau \partial^M \bar{\tau}}{2\,\mathrm{Im}(\tau)^2} - \frac{G_3 \cdot \bar{G}_3}{12\,\mathrm{Im}(\tau)} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right]$$
$$+ \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}(\tau)} + S_{\mathsf{local}} \,, \qquad (2.1)$$

where $\kappa_{10}^2 \equiv \frac{1}{2}(2\pi)^7 \alpha'^4$ is the ten-dimensional gravitational coupling (in the conventions of [30]). Here, $\tau \equiv C_0 + ie^{-\phi}$ is the axio-dilaton field and $G_3 \equiv F_3 - \tau H_3$ is a combination of the R-R and NS-NS three-form fluxes $F_3 \equiv \mathrm{d}C_2$ and $H_3 \equiv \mathrm{d}B_2$. The five-form $\tilde{F}_5 \equiv F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$ is self-dual, $\tilde{F}_5 = \star_{10}\tilde{F}_5$, where $\star_{10}$ is the ten-dimensional Hodge star operator. Finally, $\mathcal{R}_{10}$ is the ten-dimensional Ricci scalar and S_{local} denotes localized contributions from D-branes and orientifold planes.

For the warped line element, we take the ansatz[2]

$$\mathrm{d}s^2 = e^{2A(y)}g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)}g_{mn}\mathrm{d}y^m \mathrm{d}y^n \,. \qquad (2.2)$$

The metric g_{mn} of the internal space will not be Calabi-Yau in the configurations of interest, but it is useful to think of this metric as being Calabi-Yau at leading order in a certain perturbative expansion, as we shall explain in section 2.3. For the five-form flux, we assume

$$\tilde{F}_5 = (1 + \star_{10})\,\mathrm{d}\alpha(y) \wedge \sqrt{-\det g_{\mu\nu}}\ \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \,. \qquad (2.3)$$

The warp factor $e^{4A(y)}$ and the scalar function $\alpha(y)$ in eqs. (2.2) and (2.3) will play crucial roles in the potential felt by probe D-branes. The Einstein-frame action for a Dp-brane wrapping a $(p-3)$ cycle Σ is the sum of Dirac-Born-Infeld (DBI) and Chern-Simons (CS) terms,

$$S_{\mathrm{Dp}} = - \int_{R^4 \times \Sigma} \mathrm{d}^{p+1}\xi\, T_p \sqrt{-g_{\mathrm{ind}}}\ +\ \mu_p \int_{R^4 \times \Sigma} C_{p+1} \,, \qquad (2.4)$$

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[2]To determine the full effective action governing time-dependent solutions, a more general line element is required [31, 32]. In this work we exclusively study the scalar potential as a function of the D3-brane position, for which the much simpler line element (2.2) suffices.

where g_{ind} is the metric induced on the Dp-brane, and

$$T_p = |\mu_p| e^{(p-3)\phi(y)/4} \qquad \text{with} \qquad |\mu_p| = (2\pi)^{-p} (\alpha')^{-(p+1)/2} \,. \tag{2.5}$$

Notice the special status of D3-branes, whose action decouples from fluctuations of the dilaton $\phi(y)$ and of the internal unwarped metric $g_{mn}(y)$. In these variables a D3-brane experiences the potential

$$V_{\text{D3}} = T_3 \left(e^{4A} - \alpha \right) \equiv T_3 \Phi_- \,. \tag{2.6}$$

In the following we are therefore interested in perturbations of the scalar quantity $\Phi_- \equiv e^{4A} - \alpha$. We also define $\Phi_+ \equiv e^{4A} + \alpha$. Furthermore, it will be convenient to use the following parametrization of the three-form fluxes:

$$G_\pm \equiv (\star_6 \pm i) G_3 \,, \tag{2.7}$$

where $\star_6$ is the six-dimensional Hodge star operator on the internal manifold. Then, G_+ is the ISD component of the three-form flux G_3, while G_- is its IASD component. Combining the external Einstein equations with the Bianchi identity for the five-form flux $(\mathrm{d}\tilde{F}_5 = H_3 \wedge F_3 + \mathsf{local})$ we find

$$\nabla^2 \Phi_- = \frac{e^{8A+\phi}}{24} |G_-|^2 + \mathcal{R}_4 + e^{-4A} |\nabla \Phi_-|^2 + \mathcal{S}_{\mathsf{local}} \,, \tag{2.8}$$

where $\mathcal{R}_4$ denotes the four-dimensional Ricci scalar, and ∇^2 is constructed from g_{mn}. This result is a straightforward generalization of eq. (2.30) of [30], with the difference[3] that here we have allowed $g_{\mu\nu}$ to be the metric of a maximally-symmetric four-dimensional spacetime, while with the more restrictive assumptions of [30], $g_{\mu\nu} = \eta_{\mu\nu}$ is required, and $\mathcal{R}_4 = 0$.

The equation of motion for the three-form flux is

$$\mathrm{d}\Lambda + \frac{i}{2} \frac{\mathrm{d}\tau}{\mathrm{Im}(\tau)} \wedge (\Lambda + \bar{\Lambda}) = 0 \,, \tag{2.9}$$

where we have defined

$$\Lambda \equiv \Phi_+ G_- + \Phi_- G_+ \,. \tag{2.10}$$

This must be supplemented by the Bianchi identity, which in the absence of sources reads

$$\mathrm{d}G_3 = \frac{1}{2i} \mathrm{d}(G_+ - G_-) = -\mathrm{d}\tau \wedge H_3 \,. \tag{2.11}$$

2.2 UV perturbations and RG filtering

In an ideal world, one would determine the precise D3-brane potential in a fully-specified compactification, in terms of fluxes, D-brane positions, and closed string moduli vevs. With present methods this is difficult to achieve, except perhaps in a toroidal orientifold setting such as [25, 33]. In this paper, our goal is to determine the general structure of the potential arising from UV deformations of the background,

$$V(\phi) = \sum_i c_i \frac{\phi^{\Delta_i}}{M_{\text{UV}}^{\Delta_i - 4}} \,, \tag{2.12}$$

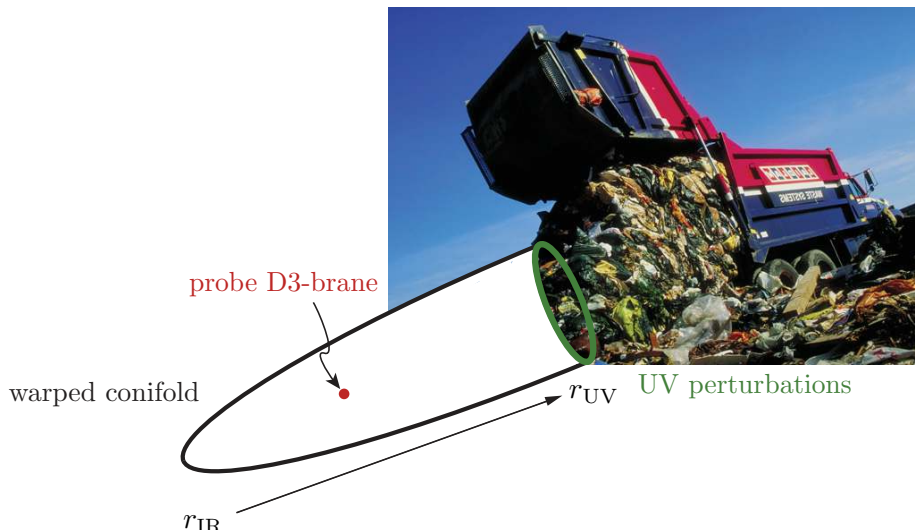[3]We thank David Marsh and Gang Xu for discussions of this point.

Figure 2. Compactification can induce very general UV perturbations of the warped conifold solution, but in the infrared only the lowest-dimension perturbations contribute meaningfully to the D3-brane potential.

where ϕ is the canonically-normalized field related to the D3-brane position and M_{UV} is a UV mass scale (related to r_{UV}, the ultraviolet location at which the throat is glued into the compact bulk; see figure 2). In terms of the parametrization of eq. (2.12), our primary task is to compute the scaling dimensions Δ_i, while leaving the coefficients of individual terms, c_i, undetermined. This undertaking is a necessary precursor to any calculation that does obtain the Wilson coefficients in a concrete model. For comparison, in ref. [16] we argued that in certain special circumstances, the dominant Planck-suppressed contribution to the D3-brane potential comes from interactions with nonperturbative effects on a divisor in the conifold. In the present paper, we are addressing the more general situation in which multiple compactification effects make important contributions to the potential.

Our interest is in the *leading* terms in the potential for a D3-brane that is well-separated from the UV cutoff r_{UV}. The dominant terms come from the Kaluza-Klein modes with the smallest AdS masses, i.e. the modes dual to the most relevant operators in the CFT. Highly irrelevant perturbations are filtered out by the RG flow; in gravity language, these perturbations are described by higher-order terms in a multipole expansion and are subleading at long distances from r_{UV}.

The dominant terms at small radial position r are, of course, those with the smallest Δ_i in eq. (2.12). We choose to work to order r^4 and consistently neglect higher-order terms. Although we are formally expanding in small r, we also assume that the D3-brane is far above the infrared location r_{IR} where the duality cascade [13] terminates and the conifold is deformed. That is, we take $r_{\mathrm{IR}} \ll r \ll r_{\mathrm{UV}}$.

Throughout this paper we restrict our attention to *non-normalizable* perturbations — corresponding to deformations of the gauge theory Lagrangian — sourced by effects in the compact bulk. However, perturbations by normalizable modes of the supergravity fields,

corresponding to perturbations of the state of the dual CFT, can also make important contributions to the D3-brane potential. For example, the Coulomb potential sourced by an anti-D3-brane at the tip of the warped deformed conifold is described in supergravity as a normalizable mode of Φ_- [34] (*cf.* [35]) and plays an important role in D3-brane inflation (see [17] for further details). We focus on non-normalizable perturbations because these encode the effects of Planck-scale physics in the compact bulk, whereas normalizable perturbations are dictated by the better-understood physics in the infrared. Extending the analysis presented here to incorporate any given set of normalizable perturbations is, however, entirely straightforward.

2.3 Perturbative expansion

We now define our perturbative expansion scheme. Our method is general and can be performed in an expansion around any background in which $\Phi_- = G_- = 0$, although we will sometimes specialize to perturbations around the background $AdS_5 \times T^{1,1}$.

We consider perturbations of all fields,

$$X = X_{(0)} + X_{(1)} + X_{(2)} + \dots, \qquad X \equiv \{\Phi_-, \Phi_+, G_-, G_+, \phi, g_{mn}\}, \qquad (2.13)$$

where $X_{(0)}$ is the background, $X_{(1)}$ is the first-order perturbation, $X_{(2)}$ is the second-order perturbation, etc. We assume that all perturbations are small.[4] Importantly, Φ_- and G_- vanish in ISD backgrounds,

$$\Phi_-^{(0)} = G_-^{(0)} = 0. \qquad (2.14)$$

Let us now systematically expand the equations of motion in small perturbations.

Equation of motion for Φ_-. For simplicity we begin by studying the noncompact limit ($M_{\rm pl} \to \infty$) which extracts fluxes as the only source term in the equation of motion for Φ_-,

$$\nabla^2 \Phi_- = \frac{e^{8A+\phi}}{24} |G_-|^2 + e^{-4A} |\nabla \Phi_-|^2. \qquad (2.15)$$

We will incorporate the curvature contribution $\mathcal{R}_4$ for finite $M_{\rm pl}$ in section 4.2. Metric perturbations propagate into perturbations of the Laplacian,

$$\nabla^2 = \nabla_{(0)}^2 + \nabla_{(1)}^2 + \dots \qquad (2.16)$$

At first order in all perturbations we get

$$\nabla_{(0)}^2 \Phi_-^{(1)} = 0, \qquad (2.17)$$

while at second order we find

$$\nabla_{(1)}^2 \Phi_-^{(1)} + \nabla_{(0)}^2 \Phi_-^{(2)} = \frac{g_s}{96} \big(\Phi_+^{(0)}\big)^2 \big|G_-^{(1)}\big|^2 + 2\big|\nabla_{(0)} \Phi_-^{(1)}\big|^2 / \Phi_+^{(0)}. \qquad (2.18)$$

Here, we have defined the string coupling $g_s = e^{\phi_{(0)}}$, where $\phi_{(0)} = const$ is the asymptotic background value of the dilaton. Clearly, the flux source term on the r.h.s. becomes important only at second order. Therefore, we are faced with two very different regimes, both of physical interest:

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[4] For results in the case of large deviations of the metric and dilaton, see appendix B.

- *Case I*

$$\Phi_-^{(1)} \neq 0 \tag{2.19}$$

In this case the linearized equation of motion (2.17) suffices to determine the leading solution. We have analyzed this limit in [17].

- *Case II*

$$\Phi_-^{(1)} = 0 \tag{2.20}$$

In this case flux-induced second-order terms can be important. This limit is the focus of the present paper. In the limit (2.20), the first-order equation (2.17) is identically satisfied, and the second-order equation (2.18) simplifies to

$$\nabla_{(0)}^2 \Phi_-^{(2)} = \frac{g_s}{96} (\Phi_+^{(0)})^2 |G_-^{(1)}|^2 \,. \tag{2.21}$$

In section 5 we will provide a further physical justification for the perturbative expansion scheme we have proposed. We will see that in a spurion analysis of supersymmetry breaking in the compact bulk, perturbations of G_- can arise at linear order in the (small) spurion vacuum expectation value, while perturbations of the homogeneous mode of Φ_- require two spurion insertions. Therefore, it is natural to consider cases in which $\Phi_-^{(1)} = 0$ but $G_-^{(1)} \neq 0$.

In solving (2.21), one must in general include harmonic $\Phi_-^{(2)}$ perturbations, in addition to the $\Phi_-^{(2)}$ solution sourced by the $G_-^{(1)}$ flux. These terms are of comparable size in concrete scenarios, such as that of [16], and indeed our techniques suffice to reproduce the potential of [16] as a special case (see section 6.3).

Note that metric and dilaton perturbations do not appear in either of eq. (2.17) and eq. (2.21): their effects on Φ_- are subleading in both cases. Thus, although the metric and dilaton must obey their own equations of motion, the corresponding solutions for these fields are not required in order to determine the leading contributions to Φ_-. Therefore, we will not pursue explicit solutions of the metric and dilaton equations of motion in this paper. We remark in passing that although we will take $g_{mn}^{(0)}$ to be a Calabi-Yau metric, the perturbed metric g_{mn} will in general not be Calabi-Yau, *cf.* [36].

Flux equation of motion.   Next we consider the flux equation of motion,

$$\mathrm{d}\Lambda + \frac{i}{2} \frac{\mathrm{d}\tau}{\mathrm{Im}(\tau)} \wedge (\Lambda + \bar{\Lambda}) = 0 \,, \tag{2.22}$$

where

$$\Lambda \equiv \Phi_+ G_- + \Phi_- G_+ \,. \tag{2.23}$$

Since the Φ_- equation (2.21) is second order in the fluxes, it suffices to solve (2.22) at first order. We get

$$\mathrm{d}\Lambda_{(1)} = 0 \,, \tag{2.24}$$

where

$$\Lambda_{(1)} = \Phi_+^{(0)} G_-^{(1)} + \Phi_-^{(1)} G_+^{(0)} \,. \tag{2.25}$$

The flux-induced contributions to Φ_- are only important when $\Phi_-^{(1)} = 0$, so that we may take

$$\Lambda_{(1)} \approx \Phi_+^{(0)} G_-^{(1)} . \tag{2.26}$$

This is precisely the source term in eq. (2.21). We can therefore write

$$\nabla_{(0)}^2 \Phi_-^{(2)} = \frac{g_s}{96} \big| \Lambda_{(1)} \big|^2 . \tag{2.27}$$

IASD condition.   In general, metric perturbations induce changes in the definition of IASD fluxes, by perturbing the Hodge star operator. However, since $\Lambda_{(0)} = 0$, the relevant IASD condition at first order is

$$\star_6^{(0)} \Lambda_{(1)} = -i\Lambda_{(1)} , \tag{2.28}$$

i.e. $\Lambda_{(1)}$ is IASD with respect to the background metric. Therefore, one does not need the explicit form of the perturbed metric in order to determine the leading IASD flux solution, a substantial simplification analogous to that occurring in eq. (2.21).

Equations (2.24), (2.27) and (2.28) form the basis for our exploration of flux-induced corrections to the D3-brane potential in warped throats with UV deformations (section 3 and section 4).

2.4   Consistency requirements

Stability of the background.   Let us comment on the stability of the throat solutions in the presence of UV perturbations. We will ultimately allow non-normalizable perturbations dual to *relevant* operators. The corresponding supergravity profiles grow in the infrared, and given enough RG evolution, these modes could become uncontrollably large perturbations of the proposed background solution. We will now argue that this instability is under control whenever the bulk supersymmetry breaking is small enough that decompactification does not ensue (see also appendix A of ref. [17]).

The configuration of interest is a finite throat in a stabilized compactification, with supersymmetry broken controllably in the bulk, and with a moduli potential that provides a finite barrier preventing decompactification. When this system is perturbed so that a positive four-dimensional potential energy is induced, this energy shifts the metastable minimum of the compactification toward larger volume. Sufficiently large perturbations create a decompactification instability. We will insist on studying configurations that remain metastable and hence must impose an upper bound on the four-dimensional potential energy.

As argued in [17], the requirement of metastability implies that the bulk supersymmetry breaking is in fact not large in units of the infrared scale of the throat. A priori these scales were completely unrelated, but demanding an adequate barrier in the moduli potential, and assuming that effects in the infrared region of the throat (e.g., an anti-D3-brane) suffice to uplift to a de Sitter solution, one finds the condition [17],

$$\Phi_-(r) < \Phi_+^{(0)}(r_{\mathrm{IR}}) \le \Phi_+^{(0)}(r) . \tag{2.29}$$

The consequence is that the relevant deformations, evaluated in the ultraviolet, have exponentially small coefficients that are no larger than $(M_{\mathrm{IR}}/M_{\mathrm{UV}})^2$, for modes of flux Λ, and no larger than $(M_{\mathrm{IR}}/M_{\mathrm{UV}})^4$ for harmonic modes of Φ_-. The different scaling for these two classes of modes may be understood to arise from the condition (2.20).

Quite generally, relevant perturbations of the form

$$\delta\mathcal{L} = c\, M_{\mathrm{UV}}^{4-\Delta}\mathcal{O}_\Delta\,, \tag{2.30}$$

with $c \ll 1$, lead to important instabilities after RG evolution to a scale M_{crit} obeying $\frac{M_{\mathrm{crit}}}{M_{\mathrm{UV}}} < c^{\frac{1}{4-\Delta}}$. Above we have argued that in the cases of interest, requiring metastability implies that the coefficients (for operators dual to modes of flux, whose proper treatment is the novelty in this paper) should obey $c \lesssim (M_{\mathrm{IR}}/M_{\mathrm{UV}})^2$. Then, for operators with $\Delta > 2$, the RG evolution does not persist long enough for the relevant perturbations to have unit size.[5]

Constraints from compactness. Finally, let us briefly comment on the consistency of our approach when global constraints are taken into account. It is well-understood that beginning with a compact ISD solution [30], the addition of IASD fluxes alone, with no other new ingredients, is inconsistent with the integrated Einstein equation and Bianchi identity. A consistent compact solution containing both ISD sources (such as ISD fluxes, D7-branes, and O3/O7 orientifold planes) and IASD fluxes requires some additional sources in order to obey, e.g., eq. (2.30) of ref. [30]. These sources could be be additional classical brane sources (for instance, anti-O3-planes), or may arise from quantum effects. In this work we do not explicitly specify any additional sources in the ten-dimensional solution, but anticipate that the contributions of nonperturbative effects will render these solutions consistent, as is strongly suggested by the four-dimensional analyses of [14] *et seq.* The situation is therefore precisely the same as in studies of supersymmetry breaking from an anti-D3-brane, and of soft terms from IASD fluxes.

3 Fluxes in the conifold

One of the main results of this paper is a comprehensive treatment of the contributions of IASD fluxes to the D3-brane potential. To this end, in this section we will present, in closed form, the most general solution to the flux equations of motion,

$$\mathrm{d}\Lambda = 0\,, \quad \text{where} \quad \star_6\Lambda = -i\Lambda\,, \tag{3.1}$$

on general Calabi-Yau cones, including the conifold. The classification of all *closed, IASD three-forms* Λ on Calabi-Yau cones — eq. (3.1) — is a well-defined mathematical problem whose solution is presented in detail in appendix A. Here, we summarize the results, first outlining the problem in terms of a harmonic expansion on the base manifold X_5 (section 3.2), and then giving the flux solutions in closed form (section 3.3).

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[5]There is e.g. an operator with $\Delta = \frac{3}{2}$, but it is dual to a harmonic mode of Φ_- and hence its coefficient should satisfy the stronger constraint $c \leq (M_{\mathrm{IR}}/M_{\mathrm{UV}})^4$ [17]. The leading flux perturbation has dimension $\Delta = \frac{5}{2}$ (see section 3).
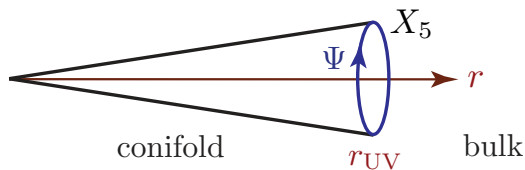
Figure 3. Schematic of the conifold geometry. The five angular coordinates on the base $X_5 = T^{1,1}$ are denoted by $\Psi = \{\theta_i, \phi_i, \psi\}$. The geometry is noncompact, but we imagine that eventually this space smoothly attaches to a compact bulk space at r_{UV}; cf. figure 4.

In section 4 we will integrate the Φ_- equation of motion,

$$\nabla^2 \Phi_- = \frac{g_s}{96}|\Lambda|^2 + \mathcal{R}_4\,, \tag{3.2}$$

incorporating four-dimensional curvature and the flux solutions of section 3 as sources.

3.1 The conifold

Although all of our results in principle apply to arbitrary Calabi-Yau cones, when computing spectral data we will specialize to the conifold. We therefore begin by briefly setting our notation for the conifold (more details may be found in [16, 37]).

The conifold is a singular noncompact Calabi-Yau threefold defined in $\mathbb{C}^4$ by the constraint equation

$$\sum_{a=1}^{4} z_a^2 = 0\,, \qquad z_a \in \mathbb{C}\,. \tag{3.3}$$

Eq. (3.3) describes a cone over the five-dimensional manifold $T^{1,1}$,

$$g_{mn}^{(0)}\,\mathrm{d}y^m \mathrm{d}y^n = \mathrm{d}r^2 + r^2 \mathrm{d}\Omega_{T^{1,1}}^2\,, \tag{3.4}$$

where we have introduced the real coordinates $r^3 \equiv \left(\frac{3}{2}\right)^{3/2} \sum_a |z_a|^2$ and $\Psi = \{\theta_i, \phi_i, \psi\}$, with $i = 1, 2$.

A stack of N D3-branes placed at the singularity $z_a = 0$ backreacts on the geometry, producing the ten-dimensional warped line element

$$\mathrm{d}s^2 = e^{2A_{(0)}(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A_{(0)}(r)}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_{T^{1,1}}^2)\,, \tag{3.5}$$

where

$$e^{-4A_{(0)}(r)} = \frac{L^4}{r^4} \qquad \text{and} \qquad L^4 \equiv \frac{27\pi}{4}g_s N(\alpha')^2\,. \tag{3.6}$$

This ISD solution—$AdS_5 \times T^{1,1}$—is summarized in table 1. In the following we will study small perturbations around this background.

An essential element of our solutions will be harmonic functions on the conifold, i.e. solutions to the Laplace equation, $\nabla^2 f = 0$. Expanding f in angular harmonics on $T^{1,1}$ [23], we have

$$f(r, \Psi) = \sum_{L,M} f_{LM}\left(\frac{r}{r_{\mathrm{UV}}}\right)^{\Delta_f(L)} Y_{LM}(\Psi)\ +\ c.c.\,, \tag{3.7}$$

Fields	$AdS_5 \times T^{1,1}$
e^{4A}	Eq. (3.6)
α	e^{4A}
g_{mn}	Eq. (3.4)
Φ_-	0
$\partial_r \phi$	0
C_0	0
G_3	0
G_-	0
G_+	0

Table 1. Supergravity fields and the $AdS_5 \times T^{1,1}$ solution.

Δ_f	j_1	j_2	R_f
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
2	0	1	0
2	1	0	0
3	1	1	2
$\sqrt{28}-2$	1	1	0

Table 2. Spectrum of lowest-dimension harmonic functions on the conifold.

where f_{LM} are constant coefficients, $L \equiv (j_1, j_2, R_f)$ and $M \equiv (m_1, m_2)$ label the SU(2) $\times$ SU(2) $\times$ U(1)$_R$ quantum numbers under the isometries of $T^{1,1}$, and the radial scaling dimensions $\Delta_f(L)$ are related to the eigenvalues of the angular Laplacian,

$$\Delta_f(L) \equiv -2 + \sqrt{H(j_1, j_2, R_f) + 4}, \tag{3.8}$$

where

$$H(j_1, j_2, R_f) \equiv 6 \left[j_1(j_1 + 1) + j_2(j_2 + 1) - R_f^2/8 \right]. \tag{3.9}$$

Group-theoretic selection rules restrict the allowed quantum numbers [23]. The lowest eigenvalues, i.e. smallest scaling dimensions, are shown in table 2. For chiral modes, $j_1 = j_2 = \frac{1}{2} R_f$, we have $\Delta_f = \frac{3}{2} R_f$.

3.2 Harmonic expansion of flux perturbations

We begin with a harmonic expansion of flux perturbations on Calabi-Yau cones.

We recall that the (linearized) flux equation of motion in an $AdS_5 \times X_5$ background is

$$d\Lambda = d(e^{4A} G_-) = 0, \quad \text{where} \quad G_- = (\star_6 - i)G_3 \quad \text{and} \quad e^{4A} = \frac{r^4}{L^4}. \tag{3.10}$$

This must be supplemented by the Bianchi identity,

$$dG_3 = -d\tau \wedge H_3. \tag{3.11}$$

We solve eq. (3.10) by expanding the three-form G_3 in harmonics on X_5. We introduce the Laplace-Beltrami operator $\star_5 \mathrm{d}$ which acts on two-forms on X_5,

$$\star_5 \, \mathrm{d} \, \Omega_2 = i\delta \, \Omega_2 \,. \tag{3.12}$$

The eigenvalues δ are real and the spectrum is invariant under $\delta \to -\delta$. When the dilaton is constant, G_3 is closed[6] — cf. eq. (3.11) — and can therefore be expressed locally as a sum of exact forms labeled by an index α,

$$G_3 = \sum_\alpha \mathrm{d}(r^\alpha \Omega_2^{(\alpha)}) \,. \tag{3.13}$$

Since eq. (3.10) is linear, we can focus on a single exact form,

$$G_3 = \mathrm{d}(r^\alpha \Omega_2^{(\alpha)}) \,. \tag{3.14}$$

There is no need to consider terms like $\mathrm{d}(r^\# \mathrm{d}r \wedge \Omega_1)$ as these are gauge equivalent to eq. (3.14).

For $\delta \neq 0$, the IASD forms G_- and the ISD forms G_+ are given by

$$G_\mp = -i \frac{\alpha \pm \delta}{\delta} \, r^\alpha \left(\mathrm{d}\Omega_2^{(\alpha)} \pm \delta \, \frac{\mathrm{d}r}{r} \wedge \Omega_2^{(\alpha)} \right) \,. \tag{3.15}$$

The three-form G_- in eq. (3.15) satisfies eq. (3.10) when

$$(\alpha + \delta)(\alpha + 4 - \delta) \, r^{\alpha+4} \frac{\mathrm{d}r}{r} \wedge \mathrm{d}\Omega_2^{(\alpha)} = 0 \,, \tag{3.16}$$

which holds for $\alpha = \delta - 4$ and $\alpha = -\delta$. The form of the resulting linearized perturbation,

$$G_3 = \mathrm{d}(r^{\delta-4}\Omega_2 + r^{-\delta}\Omega_2) \,, \tag{3.17}$$

indicates that the two-form Ω_2 corresponds, via AdS/CFT (see section 5), to an operator of dimension $\Delta = \delta$ for $\delta > 2$, and to an operator of dimension $\Delta = 4 - \delta$ for $\delta < 2$. We are interested only in the *non-normalizable* mode $r^{\Delta-4}\Omega_2$, which corresponds to a perturbation of the field theory Lagrangian. Moreover, we notice that for $\delta < 2$ the IASD three-form in eq. (3.15) vanishes and the corresponding fluctuation ($G_- = 0$, $G_+ \neq 0$) does not affect a probe D3-brane. Therefore, we restrict attention to the modes with $\delta > 2$, for which one has a non-normalizable perturbation containing IASD flux,

$$G_3 = \mathrm{d}(r^{\delta-4}\Omega_2) \,, \tag{3.18}$$

$$G_- = -2i \frac{\delta - 2}{\delta} \, r^{\delta-4} \left(\mathrm{d}\Omega_2 + \delta \, \frac{\mathrm{d}r}{r} \wedge \Omega_2 \right) \,. \tag{3.19}$$

The results above are general, i.e. valid for arbitrary Calabi-Yau cones, but not entirely explicit. In any concrete example, such as the warped conifold, $AdS_5 \times T^{1,1}$, we still need to obtain the eigenfunctions $\Omega_2^{(\alpha)}$ of the Laplace-Beltrami operator, as well as the corresponding spectrum of eigenvalues δ. This is achieved most easily along a slightly

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[6]We consider non-constant dilaton in appendix B.

different route, as we explain in section 3.3. There we derive explicit solutions for three series of IASD flux modes ($k = \mathrm{I}, \mathrm{II}, \mathrm{III}$) with positive δ_k — eqs. (3.27), (3.29) and (3.32) — for which the dimensions of the dual operators are $\Delta_k = \delta_k$. We note that for each series $\mathrm{I}, \mathrm{II}, \mathrm{III}$ with positive $\delta_{\mathrm{I},\mathrm{II},\mathrm{III}}$ there is a complementary series $\bar{\mathrm{I}}, \overline{\mathrm{II}}, \overline{\mathrm{III}}$ with negative $\delta_{\bar{\mathrm{I}},\overline{\mathrm{II}},\overline{\mathrm{III}}} = -\delta_{\mathrm{I},\mathrm{II},\mathrm{III}}$. The dimensions of the dual operators in Series $\bar{\mathrm{I}}, \overline{\mathrm{II}}, \overline{\mathrm{III}}$ are

$$\Delta_{\bar{k}} = 4 + \delta_k \ . \tag{3.20}$$

These operators are dual to G_+ perturbations, and are therefore of limited interest for our considerations, whereas the operators in Series $\mathrm{I}, \mathrm{II}, \mathrm{III}$ are dual to mixtures of G_- and G_+ perturbations and give rise to a D3-brane potential.

For completeness, we remark that if X_5 has a cohomologically nontrivial two-form with $\delta = 0$, the above harmonic analysis yields a special case with vanishing G_3 flux. This mode does not affect the potential of a D3-brane, but is nontrivial in the sense that it corresponds to a perturbation by an operator in the field theory. For example, in the case of $X_5 = T^{1,1}$ there is a single Betti two-form $\Omega_2 = \omega_2$ with

$$G_3 = \mathrm{d}\omega_2 = 0 \qquad \text{and} \qquad G_- = 0 \,. \tag{3.21}$$

This mode changes the difference of the inverse coupling constants $g_1^{-2} - g_2^{-2}$ in the gauge theory.

3.3 Explicit flux solutions

In this section we give the explicit solutions for flux perturbations on arbitrary Calabi-Yau cones (see appendix A for details). Furthermore, for the special case of the conifold background we derive the spectral dimensions of the perturbations.

3.3.1 Building blocks

Our approach is simple. For any Calabi-Yau cone, we directly construct the most general solution to eq. (3.1) using the Kähler form J, the holomorphic $(3,0)$ form Ω, and harmonic functions f on the Sasaki-Einstein base X_5 as building blocks. We will later specialize to $X_5 = T^{1,1}$, in which case the harmonic functions f are known in detail — cf. eq. (3.7).

The components of the Kähler form are

$$J_{\alpha\bar\beta} = i g_{\alpha\bar\beta} \,, \tag{3.22}$$

where $g_{\alpha\bar\beta} \equiv \partial_\alpha \partial_{\bar\beta} k$ is the Kähler metric. The holomorphic $(3,0)$ form has components

$$\Omega_{\alpha\beta\gamma} = q \, \epsilon_{\alpha\beta\gamma} \,, \tag{3.23}$$

where q is a holomorphic function satisfying $q\bar q = \det g$.

Given these basic elements, we are ready to assemble the most general solution for IASD flux on a Calabi-Yau cone.

δ_{I}	j_1	j_2	R	**Type**
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	chiral
4	1	1	0	chiral
$\sqrt{28}-1$	1	1	-2	non-chiral
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	non-chiral
$\frac{9}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	non-chiral
$\sqrt{40}-1$	0	2	-2	non-chiral
$\sqrt{40}-1$	2	0	-2	non-chiral
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	chiral

Table 3. Series I: lowest modes of $(1,2)$ flux.

3.3.2 Classification of fluxes

Three distinct types of closed, IASD three-forms can be constructed using the ingredients of the previous section. We now describe these solutions, leaving detailed derivations to appendix A.

Series I: $(1, 2)$ flux. The first and simplest flux series is of Hodge type $(1,2)$ with components,

$$(\Lambda_{\mathrm{I}})_{\alpha\bar{\beta}\bar{\gamma}} = \nabla_\alpha \nabla_\sigma f_1 \, g^{\sigma\bar{\zeta}} \, \bar{\Omega}_{\bar{\zeta}\bar{\beta}\bar{\gamma}}, \tag{3.24}$$

where f_1 is a harmonic function and ∇_α denotes the covariant derivative with respect to the Kähler metric. More compactly, this flux can be written as

$$\Lambda_{\mathrm{I}} = \nabla\nabla f_1 \cdot \bar{\Omega}. \tag{3.25}$$

In appendix A we prove that Λ_{I} is indeed closed and therefore satisfies the supergravity equations of motion.

We stress that eq. (3.25) is valid for general Calabi-Yau manifolds. However, to quantify the radial scaling of the flux solution for a concrete example we now specialize to the conifold background. We note that the radial scaling of the flux solution descends from the scaling dimension Δ_f of the harmonic function f_1, defined in eq. (3.8) in terms of the quantum numbers j_1, j_2, R_f. The form Λ_{I} has the same $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers j_1, j_2 as f_1, but the R-charge is $R = R_f - 2$. The R-charge is shifted by two because the anti-holomorphic three-form $\bar{\Omega}$ has $R_\Omega = -2$. Given that $g^{\alpha\bar{\beta}}$ scales as r^{-2} and $\Omega_{\alpha\beta\gamma}$ scales as r^3, the three-form Λ_{I} scales as

$$\Lambda_{\mathrm{I}} \sim r^4 G_- \sim r^{\delta_{\mathrm{I}}}, \tag{3.26}$$

with

$$\delta_{\mathrm{I}} = 1 + \Delta_f = -1 + \sqrt{H(j_1, j_2, R+2) + 4}. \tag{3.27}$$

According to the AdS/CFT correspondence, δ_{I} is the dimension of the dual field theory operator (see section 5). The dual operator is chiral if the function f_1 is chiral, i.e. obeys $j_1 = j_2 = \frac{1}{2}R_f$. The lowest-dimension modes of Series I flux are given in table 3.

δ_{II}	j_1	j_2	R	Type
$\frac{7}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	chiral
4	0	1	0	non-chiral
4	1	0	0	non-chiral
5	1	1	2	chiral
$\sqrt{28}$	1	1	0	non-chiral
$\frac{11}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1	non-chiral
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	non-chiral
$\frac{11}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	non-chiral
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	non-chiral

Table 4. Series II: lowest modes of $(2,1)_{\mathrm{NP}} + (1,2)$ flux.

Series II: $(2,1)_{\mathrm{NP}} + (1,2)$ flux. The second flux series is a mixture of fluxes with different Hodge types, non-primitive[7] $(2,1)$, denoted $(2,1)_{\mathrm{NP}}$, and $(1,2)$,

$$\Lambda_{\mathrm{II}} = (\partial + \bar{\partial})\Big(f_2 + \frac{1}{2}k^\alpha \partial_\alpha f_2\Big) \wedge J + \partial(\bar{\partial}f_2 \wedge \bar{\partial}k)\,, \tag{3.28}$$

where $k^\alpha = g^{\alpha\bar{\beta}}\nabla_{\bar{\beta}}k$ is holomorphic (see appendix A).

The R-charge of the three-form Λ_{II} is the same as the R-charge of the harmonic function f_2, i.e. $R = R_f$, while the radial scaling (and hence the dimension of the dual field theory operator) is shifted by two because $J_{\alpha\bar{\beta}} = ig_{\alpha\bar{\beta}} \sim r^2$, so

$$\delta_{\mathrm{II}} = 2 + \Delta_f = \sqrt{H(j_1, j_2, R) + 4}\,. \tag{3.29}$$

The lowest-dimension modes of Series II flux are given in table 4.

Series III: $(3,0) + (2,1)_{\mathrm{NP}} + (1,2)$ flux. The third flux series is a mixture of fluxes with three different Hodge types, $(3,0)$, $(2,1)_{\mathrm{NP}}$, and $(1,2)$,

$$\Lambda_{\mathrm{III}} = (2h + k^\alpha \partial_\alpha h)\Omega + (\bar{\partial}h \cdot \omega) \wedge J + \bar{\partial}(\bar{\partial}f_3 \cdot \omega) \wedge \bar{\partial}k\,, \tag{3.30}$$

where we have defined the following auxiliary forms,

$$\omega^{\bar{\alpha}}_{\ \beta} \equiv \Omega^{\bar{\alpha}}_{\ \beta\gamma}k^\gamma\,, \qquad k^\gamma \equiv g^{\gamma\bar{\zeta}}\partial_{\bar{\zeta}}k\,, \qquad \text{and} \qquad h \equiv 3f_3 + k^\alpha \partial_\alpha f_3\,. \tag{3.31}$$

The R-charge of the three-form Λ_{III} is $R = R_f + 2$ and its radial scaling dimension is

$$\delta_{\mathrm{III}} = 3 + \Delta_f = 1 + \sqrt{H(j_1, j_2, R-2) + 4}\,. \tag{3.32}$$

The lowest-dimension modes of Series III flux are given in table 5. This completes our classification of all closed, IASD fluxes on the singular conifold.

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[7]Let us remark that if our analysis were extended to a compact Calabi-Yau space, there would be no non-primitive $G_{(2,1)}$ that is nontrivial in cohomology.

δ_{III}	j_1	j_2	R	**Type**
3	0	0	2	chiral
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3	chiral
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	non-chiral
5	0	1	2	non-chiral
5	1	0	2	non-chiral

Table 5. Series III: lowest modes of $(3,0) + (2,1)_{\mathrm{NP}} + (1,2)$ flux.

Chiral modes. For chiral perturbations the harmonic functions f_1, f_2, and f_3 are in fact holomorphic functions. For holomorphic f_2 and f_3 the fluxes of Series II and III become a pure $(2,1)$ mode and a pure $(3,0)$ mode, respectively; i.e.

$$\Lambda_{\mathrm{I}}^{(1,2)} = \nabla\nabla\mathsf{f}_1 \cdot \bar{\Omega}, \tag{3.33}$$

$$\Lambda_{\mathrm{II}}^{(2,1)} = \partial\mathsf{f}_2 \wedge J, \tag{3.34}$$

$$\Lambda_{\mathrm{III}}^{(3,0)} = \mathsf{f}_3\,\Omega, \tag{3.35}$$

where $\mathsf{f}_1 \equiv f_1$, $\mathsf{f}_2 \equiv f_2 + \frac{1}{2}k^\alpha\partial_\alpha f_2$ and $\mathsf{f}_3 \equiv 6f_3 + 5k^\alpha\partial_\alpha f_3 + k^\alpha\partial_\alpha(k^\beta\partial_\beta f_3)$ are holomorphic functions related to f_1, f_2 and f_3. The case of a pure $(3,0)$ mode has been considered in [38].

Next, we discuss in detail the effects that these fluxes have on D3-branes.

4 Spectrum of the D3-brane potential

In this section we present the complete spectrum of corrections to the D3-brane potential arising from compactification effects. In section 4.1 we compute the flux-induced corrections sourced by the closed, IASD three-form flux perturbations Λ, constructed in appendix A and reviewed in the previous section. In section 4.2 we include $\mathcal{R}_4$ curvature corrections in the analysis. We summarize the leading correction terms and discuss their physical significance in section 4.3.

4.1 Flux-induced corrections

As we explained above, in the noncompact limit ($M_{\mathrm{pl}} \to \infty$) the equation of motion for Φ_- contains only flux source terms,

$$\nabla^2 \Phi_- = \frac{g_s}{96}|\Lambda|^2\,. \tag{4.1}$$

We will discuss the contribution from four-dimensional curvature $\mathcal{R}_4$ for finite M_{pl} in section 4.2.

4.1.1 Green's function solution

The Green's function solution to eq. (4.1) is

$$\Phi_-(y) = \frac{g_s}{96}\int \mathrm{d}^6 y'\, G(y;y')\,|\Lambda|^2(y') + \Phi_{\mathcal{H}}(y), \tag{4.2}$$

where

$$\nabla_y^2 \, G(y; y') = \delta(y - y') \,, \tag{4.3}$$

and the homogeneous solution $\Phi_{\mathcal{H}}(y)$ is an arbitrary harmonic function, i.e. a solution of

$$\nabla_y^2 \, \Phi_{\mathcal{H}}(y) = 0 \,, \tag{4.4}$$

whose general form is given in eq. (3.7). The spectrum of the homogeneous solution was studied in ref. [17] (see table 2):

$$\Delta_{\mathcal{H}} = \frac{3}{2} \,, \ 2 \,, 3 \,, \ \ldots \tag{4.5}$$

The Green's function on the singular conifold was presented in refs. [15, 39],

$$G(y; y') = \sum_{L,M} Y_{LM}(\Psi) Y_{LM}^*(\Psi') g_L(r; r') \,, \tag{4.6}$$

where

$$g_L(r; r') \equiv -\frac{1}{2\Delta(L) + 4} \begin{cases} \frac{1}{r'^4} \left(\frac{r}{r'}\right)^{\Delta(L)} & r \leq r' \\ \frac{1}{r^4} \left(\frac{r'}{r}\right)^{\Delta(L)} & r \geq r' \end{cases} . \tag{4.7}$$

In eq. (4.7) the scaling $\Delta(L)$ is defined as in eq. (3.8). The D3-brane potential can always be written in the form of eq. (4.2), but in the particularly interesting special case of chiral perturbations, significant simplifications of the final answer can be achieved.

4.1.2 Chiral modes

In eqs. (3.33) – (3.35) we have seen that for chiral perturbations each flux series is of distinct Hodge type. Hence, no mixed terms of the different flux series appear in the flux-squared source term,

$$|\Lambda|^2 = |\Lambda_{\mathrm{I}}|^2 + |\Lambda_{\mathrm{II}}|^2 + |\Lambda_{\mathrm{III}}|^2 \,, \tag{4.8}$$

where

$$|\Lambda_{\mathrm{I}}|^2 = 6 \, g^{\alpha\bar{\alpha}} g^{\beta\bar{\beta}} \nabla_{\alpha\beta}^2 \mathsf{f}_1 \, \overline{\nabla_{\alpha\beta}^2 \mathsf{f}_1} \,, \tag{4.9}$$

$$|\Lambda_{\mathrm{II}}|^2 = 12 \, g^{\alpha\bar{\alpha}} \nabla_{\alpha} \mathsf{f}_2 \, \overline{\nabla_{\alpha} \mathsf{f}_2} \,, \tag{4.10}$$

$$|\Lambda_{\mathrm{III}}|^2 = 6 \, |\mathsf{f}_3|^2 \,. \tag{4.11}$$

The flux-induced potential then becomes

$$\Phi_- = \frac{g_s}{96} \Big[3 g^{\alpha\bar{\alpha}} \nabla_{\alpha} \mathsf{f}_1 \, \overline{\nabla_{\alpha} \mathsf{f}_1} + 12 |\mathrm{Re}(\mathsf{f}_2)|^2 + 6 \nabla^{-2} |\mathsf{f}_3|^2 \Big] + \text{harmonic} \,. \tag{4.12}$$

4.1.3 General solution

In general, we should allow the functions f_i to be harmonic rather than just holomorphic. In this case the fluxes of Series II and III are not of pure Hodge type and the overlap of flux modes from Series I, II, and III is nonzero and can lead to new terms in the D3-brane potential.

We now discuss the resulting spectrum of Φ_- up to r^4,

$$\Phi_- = \sum_{\delta_i, \delta_j} r^{\Delta(\delta_i, \delta_j)} h_{(\delta_i, \delta_j)}(\Psi) , \tag{4.13}$$

where $h_{(\delta_i, \delta_j)}(\Psi)$ are angular wavefunctions describing the overlap of the different flux modes. The explicit forms of the functions $h_{(\delta_i, \delta_j)}(\Psi)$ can be inferred from eq. (4.6), but we will not write them out here. Instead, we focus on the radial scaling dimensions in eq. (4.13), which are

$$\Delta \equiv \delta_i + \delta_j - 4 , \tag{4.14}$$

where δ_i and δ_j are the scaling dimensions of the fluxes Λ_i and Λ_j. Using the scaling dimensions in tables 3, 4, and 5 and recalling that for chiral modes the overlaps between different flux series vanish, we may infer the smallest scaling dimensions of the flux-induced potential:

$$\Delta_\Lambda = 1 , \ 2_s , \ \frac{5}{2} , \ \sqrt{28} - \frac{5}{2} , \ \cdots \tag{4.15}$$

where the subscript on 2_s denotes a singlet.

We notice that the square of a $\delta = \frac{5}{2}$ mode results in a linear term in the potential, $\Phi_- \propto r$. This is the same r-scaling as the leading term in the nonperturbatively-generated D3-D7 potential of ref. [16]. We will expand on this correspondence in the following sections. In fact, in section 6 we will argue that the full nonperturbative potential of ref. [16] can be "geometrized" by turning on appropriate fluxes.

The wavefunctions of the $\delta = \frac{5}{2}$ and $\delta = 3$ modes have zero overlap since the corresponding modes are chiral perturbations in different flux series, i.e.

$$h_{(\frac{5}{2}, 3)}(\Psi) = 0 . \tag{4.16}$$

Hence, there is no $\Delta = \frac{3}{2}$ contribution to the flux-induced potential.

The overlap of the $\delta = \frac{5}{2}$ chiral mode and the non-chiral $\delta = \sqrt{28} - 1$ mode (corresponding to the leading contribution of an operator in a long multiplet) results in a potential term with irrational scaling dimension $\sqrt{28} - 5/2 \approx 2.79$. This term is not protected by supersymmetry or by a global symmetry,[8] and hence would be inaccessible in a field theory analysis at large 't Hooft coupling or in a four-dimensional supergravity analysis such as [15, 16] in which only superpotential interactions are computed. In a general compactification one expects this mode of flux to contribute to the D3-brane potential, and a substantial advantage of the supergravity approach of this paper is the ability to capture such terms.

4.2 Coupling to the Ricci scalar

The potential for a D3-brane in a noncompact warped conifold perturbed by non-normalizable modes of IASD flux is given by the solution (4.2) of (4.1). However, a D3-brane in a warped throat region of a *compact* space receives an additional contribution to

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[8]In [17] we considered non-chiral operators with $\Delta = 2$, but these operators were related by supersymmetry to global symmetry currents and hence were protected.

its potential when the four-dimensional Ricci scalar is nonvanishing: one must then solve

$$\nabla^2 \Phi_- = \frac{g_s}{96} |\Lambda|^2 + \mathcal{R}_4 \,. \tag{4.17}$$

In particular, if we assume that the four-dimensional theory is approximately de Sitter, then the Ricci scalar is given by the Friedmann equation as

$$\mathcal{R}_4 = 12 H^2 \approx \frac{4}{M_{\text{pl}}^2} V = \frac{4}{M_{\text{pl}}^2} \left(V_0 + T_3 \Phi_- \right), \tag{4.18}$$

where V_0, a constant independent of the D3-brane position r, has been extracted from the potential. We have incorporated the fact that the D3-brane potential $T_3 \Phi_-$ contributes to the four-dimensional energy density, but there may be additional contributions from other sectors included in V_0.

4.2.1 The eta problem

For simplicity we first study eq. (4.17) in the absence of IASD flux perturbations,

$$\nabla^2 \Phi_- = \mathcal{R}_4 \,. \tag{4.19}$$

This case is sufficient to understand the appearance of a generic eta problem arising from the curvature coupling of the inflaton in D3-brane models [9] (see also [18, 19]). Combining eqs. (4.18) and (4.19) we find

$$\nabla^2 Q = \lambda \, Q \,, \qquad \lambda \equiv \frac{4 T_3}{M_{\text{pl}}^2} \,, \tag{4.20}$$

where

$$Q(r, \Psi) \equiv \Phi_- + \frac{V_0}{T_3} \,. \tag{4.21}$$

We solve eq. (4.20) by separation of variables,

$$Q(r, \Psi) = \sum_L Q_L(r) h_L(\Psi) \,, \tag{4.22}$$

where the radial functions satisfy

$$\frac{d^2 Q_L}{dr^2} + \frac{5}{r} \frac{dQ_L}{dr} - \frac{H(L)}{r^2} Q_L = \lambda \, Q_L \,. \tag{4.23}$$

Here, $H(L)$ are the eigenvalues of the angular Laplacian defined in eq. (3.9). eq. (4.23) may be solved exactly in terms of (modified) Bessel functions,

$$Q_L = x^{-2} \left[c_1 I_n(x) + c_2 K_n(x) \right], \tag{4.24}$$

where

$$x \equiv \sqrt{\lambda} \, r = 2 \frac{\phi}{M_{\text{pl}}} \qquad \text{and} \qquad n^2(L) \equiv H(L) + 4 \,. \tag{4.25}$$

The functions $I_n(x) = i^{-n} J_n(ix)$ and $K_n(x)$ are modified Bessel functions of the first and second kind, respectively. K_n diverges for small x, so we choose $c_2 = 0$. The second integration constant is fixed by the requirement $\Phi_-(x = 0) \equiv 0$ to be

$$c_1 = \frac{8V_0}{T_3}. \tag{4.26}$$

Hence, we find

$$V(\phi) = V_0 + T_3 \Phi_- = 2V_0 \frac{M_{\text{pl}}^2}{\phi^2} \sum_L I_{n(L)}\left(2\frac{\phi}{M_{\text{pl}}}\right) h_L(\Psi). \tag{4.27}$$

The leading terms in the potential follow from the small x expansion,[9]

$$I_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2k}}{k!\,\Gamma(n+k+1)} \quad \text{for} \quad n \in \mathbb{R}. \tag{4.28}$$

One finds that the leading contribution arises from the term with $L = \{0\}$, and takes the form

$$I_2(x) = \frac{1}{8} x^2 \left(1 + \frac{1}{12} x^2 + \cdots\right). \tag{4.29}$$

This implies

$$V_s \equiv T_3 Q_{L=\{0\}} = V_0 \left(1 + \frac{1}{3}\frac{\phi^2}{M_{\text{pl}}^2} + \cdots\right), \tag{4.30}$$

and

$$\eta = M_{\text{pl}}^2 \frac{V''}{V} = \frac{2}{3} + \cdots \tag{4.31}$$

The leading curvature correction therefore precisely explains the 'eta problem' mass term found in [9]. Higher-order contributions from modes with non-trivial angular dependence may be obtained from eqs. (4.27) and (4.28).

4.2.2 Higher-order corrections

Next, we include IASD flux perturbations in the analysis. The equation of motion for Φ_- now has the form

$$\nabla^2 \Phi_- = \rho(y) + \lambda \Phi_-, \tag{4.32}$$

where

$$\rho(y) \equiv \frac{g_s}{96} |\Lambda|^2(y) + \frac{4}{M_{\text{pl}}^2} V_0. \tag{4.33}$$

One can easily verify that eq. (4.32) is solved by

$$\Phi_- = \sum_{n=0}^{\infty} \Phi_-^{[n]}, \tag{4.34}$$

---

[9]The bound of ref. [40], $x = 2\frac{\phi}{M_{\text{pl}}} < \frac{4}{\sqrt{N}} < 1$, in an $AdS_5 \times X_5$ geometry with D3-brane charge $N \gg 1$, implies that x is a good expansion parameter.

where

$$\Phi_-^{[0]}(y) = \int \mathrm{d}^6 y' \, G(y; y') \, \rho(y') + \Phi_{\mathcal{H}}(y) \,, \tag{4.35}$$

and

$$\Phi_-^{[1]}(y) = \lambda \int \mathrm{d}^6 y' \, G(y; y') \, \Phi_-^{[0]}(y') \quad , \quad \cdots \quad , \quad \Phi_-^{[n]}(y) = \lambda \int \mathrm{d}^6 y' \, G(y; y') \, \Phi_-^{[n-1]}(y') \,. \tag{4.36}$$

Eq. (4.35) describes the solution in the absence of Ricci curvature — *cf.* eq. (4.2) — plus the mass term of the previous section associated with the constant V_0,

$$\Phi_-^{[0]} = \sum_\alpha r^\alpha h_\alpha^{[0]}(\Psi) \,, \quad \text{with} \quad \alpha = \{\Delta_\Lambda, \Delta_{\mathcal{H}}, 2_s\} \,, \tag{4.37}$$

where $h_\alpha^{[0]}(\Psi)$ are angular wavefunctions determined by (4.35). Eq. (4.36) provides the iterative inclusion of higher-order corrections induced by the Ricci curvature. Using eqs. (4.6) and (4.37) to perform the Green's function integrals in eq. (4.36), we find

$$\Phi_-^{[n]} = (\lambda r^2)^n \sum_\alpha r^\alpha h_\alpha^{[n]}(\Psi) \,, \tag{4.38}$$

with angular wavefunctions $h_\alpha^{[n]}$ determined by the above. Evidently, eq. (4.34) is an expansion in the dimensionless parameter $x^2 \equiv \lambda r^2 = 4 \frac{\phi^2}{M_{\mathrm{pl}}^2} < 1$ (see Footnote 9). The iterative inclusion of curvature contributions effectively dresses each term in the potential with factors r^{2n}, $n = 1, 2, \cdots$. We therefore infer from eqs. (4.5) and (4.15) that the corrections induced by the coupling to the Ricci scalar are of the form $\Phi_- \sim r^\Delta$ with

$$\Delta_{\mathcal{R}} = 2_s \,, \, 3 \,, \, \frac{7}{2} \,, \, 4_s \,, \, 4 \,, \, \cdots \tag{4.39}$$

4.3 Summary of supergravity perturbations

Let us briefly summarize the contributions to the D3-brane potential. The potential is given by $T_3 \Phi_-$, where Φ_- is a general solution (4.34) of the equation of motion (4.32). This solution includes a homogeneous solution $\Phi_{\mathcal{H}}$, which is an arbitrary harmonic function on the conifold, and also includes inhomogeneous contributions sourced by $|\Lambda|^2$ and by $\mathcal{R}_4$.

We are interested in the dominant terms in the infrared, i.e. the most relevant contributions for a D3-brane far from the ultraviolet region where the throat is attached to the compact bulk.[10] We write the solution in terms of the canonical coordinate $\phi^2 = T_3 r^2$. The leading terms from the homogeneous solution take the form [17]

$$V_{\mathcal{H}}(\phi) = V_0 + a_{3/2} \phi^{3/2} h_{3/2}(\Psi) + a_2 \phi^2 h_2(\Psi) + \ldots \,, \tag{4.40}$$

where $h_\alpha(\Psi)$ are angular harmonics on $T^{1,1}$ and a_α are constants. We have seen in section 4.1 that the leading terms from the flux-sourced solution take the form

$$V_\Lambda(\phi) = b_1 \phi^1 j_1(\Psi) + b_2 \phi^2 + b_{5/2} \phi^{5/2} j_{5/2}(\Psi) + b_{2.79} \phi^{2.79} j_{2.79}(\Psi) + \ldots \,, \tag{4.41}$$

---

[10]Recall that we also assume that the D3-brane is well above any infrared deformations of the geometry as occur e.g. in the warped deformed conifold solution [13].

where the $j_\alpha(\Psi)$ are angular functions determined by the analysis of section 4.1. The singlet term $b_2\phi^2$ arises from the square of a single mode of flux, so $b_2 \geq 0$. Finally, the leading term from the curvature-sourced solution, as obtained in section 4.2, takes the form

$$V_{\mathcal{R}}(\phi) \;=\; c_2\,\phi^2 \;+\; \dots\,, \tag{4.42}$$

where $c_2 \equiv H_0^2 = \frac{1}{3}\frac{V_0}{M_{\text{pl}}^2}$ is a fixed coefficient. The Wilson coefficients a_Δ, b_Δ and c_Δ are normalized at the UV cutoff, so it is convenient to extract the energy scaling as follows

$$\{\,a_\Delta\,,\,b_\Delta\,,\,c_\Delta\,\} = \{\,\widetilde{a}_\Delta\,,\,\widetilde{b}_\Delta\,,\,\widetilde{c}_\Delta\,\} \cdot M_{\text{UV}}^{4-\Delta}\,, \tag{4.43}$$

so that $\widetilde{a}_\Delta$, $\widetilde{b}_\Delta$ and $\widetilde{c}_\Delta$ are dimensionless coefficients.

Finally, the general form of the potential is

$$\begin{aligned}
V(\phi) = V_0 \;+\; b_1\,j_1(\Psi)\,\phi^1 \;+\; a_{3/2}\,h_{3/2}(\Psi)\,\phi^{3/2} \;+\; \left(c_2 + a_2\,h_2(\Psi) + b_2\,j_2(\Psi)\right)\phi^2 \\
+\; b_{5/2}\,j_{5/2}(\Psi)\,\phi^{5/2} \;+\; b_{2.79}\,j_{2.79}(\Psi)\,\phi^{2.79} \;+\; \dots
\end{aligned} \tag{4.44}$$

This is one of the main results of this paper.

Notice the disparate origin of the various competing terms in eq. (4.44): IASD fluxes, harmonic perturbations of Φ_-, and coupling to the four-dimensional spacetime curvature. The curvature coupling was identified in ref. [9] and further investigated in refs. [18, 19]. The harmonic perturbations of Φ_- were studied in ref. [17]. The flux contribution ϕ^1, which actually dominates at small ϕ, has not been previously identified in ten-dimensional supergravity. Also novel is the contribution $\Delta = \sqrt{28} - \frac{5}{2} \approx 2.79$, which was inaccessible in analyses such as [16] that incorporated only corrections to the superpotential. Clearly, such a mode would be very difficult to guess in field theory, but as we see here it can make an important contribution to the D3-brane potential.

We remark that the leading curvature contribution has a fixed coefficient, leading to the generic eta problem. However, the other contributions have tunable Wilson coefficients that in principle allow a small effective mass term if the different contributions locally cancel against each other, as described in four-dimensional supergravity in [16, 41, 42]. The results obtained above provide a dictionary between physical effects in the compactification and specific terms in the inflaton potential, e.g. the linear term arises only from $G_{(1,2)}$ flux. This characterization of the physical origin of each term is a necessary precursor to any attempt at explicit fine-tuning of the potential.

5 Conformal field theory

Having understood the structure of the D3-brane potential induced by non-normalizable perturbations of the supergravity solution, it is natural to ask whether additional insights, or cross-checks, can be obtained by mapping these results into the dual conformal field theory. As usual when applying the AdS/CFT correspondence at large 't Hooft coupling, the gauge theory is strongly coupled and most computations are far simpler in the gravity picture. However, understanding the D3-brane potential on the gauge theory side will help

us to make contact with four-dimensional reasoning, by clarifying the relation between our analysis and a Wilsonian treatment in the four-dimensional effective theory.

We begin by reviewing the field content of the Klebanov-Witten (KW) theory and listing all protected operators (section 5.1). We then discuss the operators dual to IASD flux perturbations in more detail (section 5.2). We first present all chiral operators (section 5.2.1), explicitly identifying three towers of operators as duals to the three series of holomorphic flux. Next, we discuss the complete AdS/CFT spectroscopy (section 5.2.2) including non-chiral operators. Finally, we present arguments that allow a comparison between the scalar potentials computed in the gauge theory and in supergravity (section 5.3).

5.1 Review of the KW CFT

A canonical class of examples of the AdS/CFT correspondence consists of $\mathcal{N} = 1$ superconformal gauge theories dual to string theory on $AdS_5 \times X_5$, where X_5 is a Sasaki-Einstein space [13, 29, 43]. These theories arise from the near-horizon limit of a stack of N D3-branes placed at the tip of a six-dimensional Calabi-Yau cone M_6 with base manifold X_5. The supergravity results of the previous sections were general enough to capture all $AdS_5 \times X_5$ backgrounds. In principle, the gauge theory analysis could be formulated equally generally, but in practice we will specialize our CFT results to the dual of $AdS_5 \times T^{1,1}$.

Basic symmetries and degrees of freedom. The CFT dual of $AdS_5 \times T^{1,1}$ is an $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with gauge group $\mathcal{G} = \mathrm{SU}(N) \times \mathrm{SU}(N)$ and continuous global symmetries $G = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R$ (inherited from the isometries of $T^{1,1}$) [29]. The matter content consists of two doublets of chiral superfields A_i and B_j $(i, j = 1, 2)$ which are in the $(N, \bar{N})$ and $(\bar{N}, N)$ of $\mathcal{G}$ and in the $(2, 0, \frac{1}{2})$ and $(0, 2, \frac{1}{2})$ of G. Introducing the $\mathrm{SU}(N)$ vector superfields V_i, the chiral gauge field strength superfields for the two gauge symmetries are

$$W_\alpha^{(1)} = \bar{D}\bar{D}(e^{V_1} D_\alpha e^{-V_1}), \tag{5.1}$$

$$W_\alpha^{(2)} = \bar{D}\bar{D}(e^{V_2} D_\alpha e^{-V_2}). \tag{5.2}$$

The $W_\alpha^{(i)}$ fields have dimension $\Delta = \frac{3}{2}$ and R-charge $R = 1$, while the A and B fields have $\Delta = \frac{3}{4}$ and $R = \frac{1}{2}$. The interactions are encoded in the superpotential

$$W \propto \epsilon^{ij} \epsilon^{kl} \mathrm{Tr}(A_i B_k A_j B_l), \tag{5.3}$$

with $\Delta = 3$ and $R = 2$. We now use these results to enumerate the low-dimension operators in the CFT, following Ceresole *et al.* [23, 44].

Operators with protected dimensions. We are interested in determining the most relevant operators in the CFT that contribute to the potential on the Coulomb branch. We first consider operators whose dimensions are protected in the gauge theory; unprotected operators generically acquire large anomalous dimensions in the limit of large 't Hooft coupling and have their dual description only in the full string theory, not in supergravity. Later, in section 5.2.2, we will include the very interesting case of operators that appear unprotected in the gauge theory analysis, but in fact have fixed — and in many cases,

irrational — dimensions at large 't Hooft coupling [45]. For simplicity we refer to such operators as unprotected.

For the CFT in question there are three series of protected operators [23]:

1. *Chiral*

$$S^k = \text{Tr}(AB)^k \ : \qquad \Delta_k = \frac{3}{2}k \qquad R_k = k \,, \qquad (5.4)$$

$$T^k_\alpha = \text{Tr}[W_\alpha (AB)^k] \ : \qquad \Delta_k = \frac{3}{2} + \frac{3}{2}k \qquad R_k = k+1 \,, \qquad (5.5)$$

$$\Phi^k = \text{Tr}[W^\alpha W_\alpha (AB)^k] \ : \qquad \Delta_k = 3 + \frac{3}{2}k \qquad R_k = k+2 \,. \qquad (5.6)$$

2. *Conserved*

$$J^k = \text{Tr}[J(AB)^k)] \ : \qquad \Delta_k = 2 + \frac{3}{2}k \qquad R_k = k \,, \qquad (5.7)$$

$$J^k_{\alpha\dot\alpha} = \text{Tr}[J_{\alpha\dot\alpha}(AB)^k] \ : \qquad \Delta_k = 3 + \frac{3}{2}k \qquad R_k = k \,, \qquad (5.8)$$

$$I^k = \text{Tr}[JW^2(AB)^k] \ : \qquad \Delta_k = 5 + \frac{3}{2}k \qquad R_k = k+2 \,, \qquad (5.9)$$

where $J = \{J_a, J_b\}$, $J_a \equiv Ae^V \bar A e^{-V}$, $J_b \equiv Be^V \bar B e^{-V}$ (with $V \equiv V_1 + V_2$), $J_{\alpha\dot\alpha} \equiv W_\alpha L_{\dot\alpha}$, and $L_{\dot\alpha} \equiv e^V \bar W_{\dot\alpha} e^{-V}$.

3. *Semi-conserved*

$$L^{1,k}_{\dot\alpha} = \text{Tr}[L_{\dot\alpha}(AB)^k] \ : \qquad \Delta_k = \frac{3}{2} + \frac{3}{2}k \qquad R_k = k-1 \,, \qquad (5.10)$$

$$L^{2,k}_\alpha = \text{Tr}[W_\alpha J(AB)^k] \ : \qquad \Delta_k = \frac{7}{2} + \frac{3}{2}k \qquad R_k = k+1 \,, \qquad (5.11)$$

$$L^{3,k}_{\dot\alpha} = \text{Tr}[L_{\dot\alpha} W^2 (AB)^k] \ : \qquad \Delta_k = \frac{9}{2} + \frac{3}{2}k \qquad R_k = k+1 \,. \qquad (5.12)$$

The operators here are written a bit schematically: the trace is over the color degrees of freedom, and we have suppressed the $\text{SU}(2) \times \text{SU}(2)$ flavor indices. Furthermore, the KW theory has two gauge groups, and naively there should be two distinct series for the operators T^k_α and Φ^k, one with $W_\alpha = W^{(1)}_\alpha$ and the other with $W_\alpha = W^{(2)}_\alpha$. However, the commutators of the matter fields and W_α vanish in the chiral ring [46],

$$AW^{(1)}_\alpha - W^{(2)}_\alpha A \ \sim \ W^{(1)}_\alpha B - BW^{(2)}_\alpha \ \sim \ 0 \,. \qquad (5.13)$$

Hence, there is only one chiral mode with protected dimension, $\text{Tr}[(W^{(1)}_\alpha + W^{(2)}_\alpha)(AB)^k]$, while the twisted mode $\text{Tr}[(W^{(1)}_\alpha - W^{(2)}_\alpha)(AB)^k]$ vanishes in the chiral ring and has infinite dimension in the large N limit. Similarly, the chiral primary operator Φ^k should be written as $\text{Tr}[(W^2_{(1)} + W^2_{(2)})(AB)^k]$. Finally, there is an exceptional chiral operator with protected dimension,

$$\text{Tr}[W^2_{(1)} - W^2_{(2)}] \,, \qquad (5.14)$$

that does not belong to any tower.

We define the conformal dimension and R-charge of a supermultiplet as the conformal dimension and R-charge of the bottom ($\theta = 0$) component of the corresponding superfield. The conformal dimensions and R-charges of descendants are then shifted in half-integer and integer steps, respectively. In the following we will refer to the individual components of a superfield by the notation

$$F = [F]_{\rm b} + \cdots + [F]_{\theta^n \bar{\theta}^m} \, \theta^n \bar{\theta}^m + \cdots \, . \tag{5.15}$$

This completes the bookkeeping required for this section.

5.2 CFT duals of IASD fluxes

Having enumerated the lowest-dimension protected supermultiplets of operators, we now focus on the components of these supermultiplets that generate a potential on the Coulomb branch.

5.2.1 Chiral operators

We consider first the three sets of chiral operators in eqs. (5.4) – (5.6):

$$\mathcal{O}_{\rm I} = {\rm Tr}(AB)^k \, , \tag{5.16}$$

$$\mathcal{O}_{\rm II}^\alpha = {\rm Tr}[W_+^\alpha (AB)^k] \, , \tag{5.17}$$

$$\mathcal{O}_{\rm III} = {\rm Tr}[W_+^2 (AB)^k] \, , \tag{5.18}$$

where we have introduced the notation $W_\pm^\alpha \equiv W_{(1)}^\alpha \pm W_{(2)}^\alpha$ and $W_\pm^2 \equiv W_{(1)}^2 \pm W_{(2)}^2$. The integer k obeys $k \geq 1$ for Series I and II and $k \geq 0$ for Series III.

Our interest is in the components of the above supermultiplets that induce a D3-brane potential, i.e. the components dual to Φ_- and G_- perturbations. The bottom component of $\mathcal{O}_{\rm I}$ corresponds to a perturbation of Φ_- [17]:

Φ_-	Operator	Δ	R	j_1	j_2
f	$[{\rm Tr}(AB)^k]_{\rm b}$	$\frac{3}{2}k$	k	$\frac{1}{2}k$	$\frac{1}{2}k$

while the top, middle and bottom components, respectively, of $\mathcal{O}_{\rm I}$, $\mathcal{O}_{\rm II}^\alpha$, and $\mathcal{O}_{\rm III}$ are dual to G_- perturbations:

Flux	Operator	Δ	R	j_1	j_2
$\nabla \nabla f_1 \cdot \bar{\Omega}$	$[{\rm Tr}(AB)^k]_{\theta^2}$	$\frac{3}{2}k + 1$	$k - 2$	$\frac{1}{2}k$	$\frac{1}{2}k$
$\partial f_2 \wedge J$	$[{\rm Tr}[W_+^\alpha (AB)^k]]_\theta$	$\frac{3}{2}k + 2$	k	$\frac{1}{2}k$	$\frac{1}{2}k$
$f_3 \Omega$	$[{\rm Tr}[(W_+^2 (AB)^k)]]_{\rm b}$	$\frac{3}{2}k + 3$	$k + 2$	$\frac{1}{2}k$	$\frac{1}{2}k$

Evidently, these three towers of chiral operators are dual to the three series of 'holomorphic' flux perturbations found in section 3. A few comments are in order:

- *Series I operators.* The perturbations $\int d^2\theta \, \mathcal{O}_{\rm I} = [\mathcal{O}_{\rm I}]_{\theta^2}$ are *superpotential perturbations*, consistent with the supersymmetry of the unperturbed CFT. These operators

Δ	j_1	j_2	R	**Operator**		**Multiplet**	**Type**
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[S^1]_{\mathrm{b}}$	$[\mathrm{Tr}(AB)]_{\mathrm{b}}$	V.I	chiral
2	0	1	0	$[{}_aJ^0]_{\mathrm{b}}$	$[\mathrm{Tr}J_a]_{\mathrm{b}}$	V.I	semi-long
2	1	0	0	$[{}_bJ^0]_{\mathrm{b}}$	$[\mathrm{Tr}J_b]_{\mathrm{b}}$	V.I	semi-long
3	1	1	2	$[S^2]_{\mathrm{b}}$	$[\mathrm{Tr}(AB)^2]_{\mathrm{b}}$	V.I	chiral

Table 6. Matching between supergravity Φ_- modes and CFT operators [17].

are dual to perturbations by $G_{(1,2)}$ fluxes in Series I, eq. (3.25), with holomorphic functions $\mathsf{f}_1 \sim (AB)^k$. This is consistent with the results of [26], where it was found that $G_{(3,0)}$ and non-primitive $G_{(2,1)}$ fluxes are incompatible with the original supersymmetry of the ISD background, while $G_{(1,2)}$ flux generates superpotential interactions for a D3-brane.

- *Series II operators.* The operators $\int d\theta_\alpha \, \mathcal{O}_{\mathrm{II}}^\alpha = [\mathcal{O}_{\mathrm{II}}]_\theta$ are chiral but not supersymmetric. They correspond to the non-primitive $G_{(2,1)}$ flux of Series II, eq. (3.28), with holomorphic $\mathsf{f}_2 \sim (AB)^k$. This is in agreement with the result of [26, 47] where it was shown that non-primitive $(2,1)$ flux couples the gaugino to the fermions from the chiral superfields.

- *Series III operators.* Finally, the operators $[\mathcal{O}_{\mathrm{III}}]_{\mathrm{b}}$ are mapped to fluxes in Series III, eq. (3.30), with holomorphic $\mathsf{f}_3 \sim (AB)^k$. Again this is supported by the fact that $(3,0)$ flux creates a mass term for the gaugino [26, 47].

5.2.2 Non-chiral operators

Next, we study generalizations of the chiral operators of the previous section:

$$\mathcal{O}_{\mathrm{I}} = \mathrm{Tr}f_1\,, \tag{5.19}$$

$$\mathcal{O}_{\mathrm{II}}^\alpha = \mathrm{Tr}[W_+^\alpha f_2]\,, \tag{5.20}$$

$$\mathcal{O}_{\mathrm{III}} = \mathrm{Tr}[W_+^2 f_3]\,, \tag{5.21}$$

where $f_i \equiv f_i(A, B, \bar{A}, \bar{B})$ are harmonic, but not holomorphic, functions of the matter fields A and B. The low-lying operators corresponding to Φ_- and G_- perturbations are collected in tables 6 and 7.[11]

Let us make a few comments about each of these operators and their supermultiplet structure:

- $\mathcal{O}_{\mathrm{I}} = \mathrm{Tr}f_1$:

  These superfields correspond to Vector Multiplet I of [23] (see table 7 of [23]). The bottom components are operators made out of scalar fields only; they are dual to perturbations that are certain combinations of the four-form potential C_4 and the

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[11]The subscripts 'a' and 'b' on $J_{a,b}$ and $f_{a,b}$ indicate that these are functions of A only or of B only. The dimensions of operators dual to fluxes are denoted by δ (rather than Δ) to agree with the notation used in section 3.

δ	j_1	j_2	R	Operator		Multiplet	Type	Flux Series
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$[S^1]_{\theta^2}$	$[\mathrm{Tr}(AB)]_{\theta^2}$	V.I	chiral	I
3	0	0	2	$[\Phi^0_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)}+W^2_{(2)})]_{\mathrm{b}}$	V.IV	chiral	III
$\frac{7}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[T^1_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB))]_\theta$	G.I	chiral	II
4	0	0	0	$[\Phi^0_-]_{\theta^2}$	$[\mathrm{Tr}(W^2_{(1)}-W^2_{(2)})]_{\theta^2}$	V.III	chiral	$\star$
4	0	1	0	$[_aL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_a)]_\theta$	G.I+G.III	semi-long	II
4	1	0	0	$[_bL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_b)]_\theta$	G.I+G.III	semi-long	II
4	1	1	0	$[S^2]_{\theta^2}$	$[\mathrm{Tr}(AB)^2]_{\theta^2}$	V.I	chiral	I
$\sqrt{28}-1$	1	1	-2	—	$[\mathrm{Tr}(f)]_{\theta^2}$	V.I	long	I
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3	$[\Phi^1_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)}+W^2_{(2)})(AB)]_{\mathrm{b}}$	V.IV	chiral	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[\bar{\Phi}^1_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)}+W^2_{(2)})(\overline{AB})]_{\mathrm{b}}$	V.IV	—	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$[_aJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_a(AB))]_{\theta^2}$	V.I	semi-long	I
$\frac{9}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$[_bJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_b(AB))]_{\theta^2}$	V.I	semi-long	I
5	1	1	2	$[T^2_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB)^2)]_\theta$	G.I	chiral	II
5	0	1	2	$[_aI^0]_{\mathrm{b}}$	$[\mathrm{Tr}((W^2_{(1)}+W^2_{(2)})J_a)]_{\mathrm{b}}$	V.IV	semi-long	III
5	1	0	2	$[_bI^0]_{\mathrm{b}}$	$[\mathrm{Tr}((W^2_{(1)}+W^2_{(2)})J_b)]_{\mathrm{b}}$	V.IV	semi-long	III
$\sqrt{28}$	1	1	0	—	$[\mathrm{Tr}(W_\alpha f)]_\theta$	G.I+G.III	long	II
$\sqrt{40}-1$	0	2	-2	—	$[\mathrm{Tr}(f_a)]_{\theta^2}$	V.I	long	I
$\sqrt{40}-1$	2	0	-2	—	$[\mathrm{Tr}(f_b)]_{\theta^2}$	V.I	long	I
$\frac{11}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1	$[_aL^{2,1}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_a(AB))]_\theta$	G.I+G.III	semi-long	II
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	1	$[_bL^{2,1}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_b(AB))]_\theta$	G.I+G.III	semi-long	II
$\frac{11}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$[_a\bar{L}^{2,1}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_a(\overline{AB}))]_\theta$	G.I+G.III	—	II
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$[_b\bar{L}^{2,1}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_b(\overline{AB}))]_\theta$	G.I+G.III	—	II
$\frac{11}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	$[S^3]_{\theta^2}$	$[\mathrm{Tr}(AB)^3]_{\theta^2}$	V.I	chiral	I

Table 7. Matching between supergravity G_- flux modes and CFT operators.

trace of the metric (denoted by b in [23]). Such fluctuations induce Φ_- perturbations that contribute to the D3-brane potential at linear order, as discussed in our previous paper [17] (see table 6 above). The θ^2 components of $\mathcal{O}_\mathrm{I}$ are operators bilinear in fermions with R-charge $R-2$, and correspond to the G_- perturbations from Series I. Here we disagree with table 7 of [23], which states that these operators are dual to metric perturbations.

We may further consider the superfields

$$\widetilde{\mathcal{O}}_\mathrm{I} = \mathrm{Tr}[\overline{W}^2_+ f_1]\,, \tag{5.22}$$

where $\overline{W}^2_+ \equiv \overline{W}^2_{(1)} + \overline{W}^2_{(2)}$. The $\bar{\theta}^2\theta^2$ components of these superfields are operators with dimensions $\Delta+4$. As shown in section 3.2, these additional operators arise from Series $\bar{\mathrm{I}}$ G_+ perturbations.

- $\mathcal{O}^\alpha_\mathrm{II} = \mathrm{Tr}[W^\alpha_+ f_2]$:

These superfields correspond to Gravitino Multiplet I (table 3 of [23]). In particular, the θ components correspond to G_- perturbations from Series II, as can be confirmed by comparing their dimensions Δ. This agrees with the field assignment in [23].

We may further consider the superfields

$$\widetilde{\mathcal{O}}_{\mathrm{II}}^{\alpha} = \mathrm{Tr}[W_\alpha \overline{W}_+^2 f_2]. \tag{5.23}$$

The $\bar{\theta}^2\theta$ components of these superfields are operators with dimensions $\Delta + 4$, corresponding to Series $\overline{\mathrm{II}}$ G_+ perturbations. More generally, we note that these superfields correspond to Gravitino Multiplet II (table 4 of [23]).

- $\mathcal{O}_{\mathrm{III}} = \mathrm{Tr}[W_+^2 f_3]$:

  These superfields correspond to Vector Multiplet IV (table 10 of [23]). The bottom components, bilinear in the gauginos, correspond to G_- perturbations from Series III. The θ^2 components instead correspond to the dilaton and RR scalar of type IIB theory.

  We may further consider the superfields

$$\widetilde{\mathcal{O}}_{\mathrm{III}} = \mathrm{Tr}[W_+^2 \overline{W}_+^2 f_3] . \tag{5.24}$$

The $\bar{\theta}^2$ components of these superfields are operators whose dimensions are higher by 4 than those of the operators in the previous paragraph. As shown in section 3.2, these additional operators arise from Series $\overline{\mathrm{III}}$ G_+ perturbations. We find that the superfields (5.24) correspond to Vector Multiplet II (table 8 of [23]). The bottom components correspond to metric deformations squashing the S^1 fiber of $T^{1,1}$ relative to the base [48, 49], while the $\theta^2\bar{\theta}^2$ components correspond to well-known operators of the form $F^4 f_3$ that appear as a combination of fluctuations of C_4 and of the trace of the metric. These fluctuations induce Φ_+ perturbations. These assignments agree with table 8 of [23]. However, in disagreement with that table we find that the θ^2 and $\bar{\theta}^2$ components of these superfields are dual to G_+ perturbations.

In summary, we have matched the three series of flux modes with holomorphic f (see section 3) to the three series of chiral operators. Similarly, the three corresponding series of non-chiral operators obtained by taking $f_i \equiv f_i(A, B, \bar{A}, \bar{B})$ to be harmonic but not holomorphic match the three series of flux modes with harmonic, but non-holomorphic f. The apparent complexity of table 7 is therefore reduced to three distinct series of flux or three different types of operator perturbations.

There is also one special case, denoted $\star$ in the table, which corresponds to the vanishing three-form flux (3.21). This mode changes the difference between the complex coupling constants $g_1^{-2} - g_2^{-2}$ in the KW theory and hence corresponds to the top component of the chiral operator (5.14). The corresponding bottom component $\mathrm{Tr}(\lambda_1^2 - \lambda_2^2)$, with R-charge 2 and dimension 3, corresponds to a traceless perturbation of the metric on $T^{1,1}$. As this mode is not dual to a G_- or Φ_- perturbation, the operator $\mathrm{Tr}(\lambda_1^2 - \lambda_2^2)$ does not create a D3-brane potential at quadratic order.

Notice that some of the non-chiral operators described above reside in long multiplets and have irrational dimensions. In an analysis conducted exclusively in the gauge theory, it would be rather difficult to determine the dimensions of these operators, but on the gravity

side of the correspondence it is straightforward to identify these contributions. Moreover, these long operators can have non-negligible effects on the Coulomb branch potential: e.g. an interference term between fluxes dual to the long operator with $\delta = \sqrt{28} - 1$ and the chiral operator with $\delta = \frac{5}{2}$ gives rise to a term in the scalar potential of the form $r^{\sqrt{28} - \frac{5}{2}}$, which can substantially affect the structure of the scalar potential.

5.3 Potential on the Coulomb branch

Having identified the leading operators dual to perturbations of Φ_- and G_-, we can now write the leading perturbations to the CFT Lagrangian and compute the resulting potential on the Coulomb branch.

5.3.1 Perturbed lagrangian and SUSY breaking

We begin by introducing an efficient representation for the perturbed Lagrangian in terms of spurion fields. Some of the perturbations of interest are consistent with the supersymmetry of the gauge theory, while others break this supersymmetry explicitly.[12] To characterize the bulk supersymmetry breaking, and more generally the bulk sourcing of perturbations of the gauge theory, we introduce spurion fields X and Y_α. We will take X to be a chiral superfield external to the gauge theory, each of whose components will in general have a nonvanishing expectation value,

$$X = [X]_{\rm b} + [X]_\theta \, \theta + [X]_{\theta^2} \, \theta^2 \equiv x + x^\alpha \theta_\alpha + F_X \theta^2 \,. \tag{5.25}$$

Similarly, for Y_α we have

$$Y_\alpha = [Y_\alpha]_{\rm b} + [Y_\alpha]_\theta \, \theta + [Y_\alpha]_{\theta^2} \, \theta^2 \equiv y_\alpha + y \, \theta_\alpha + F_{Y_\alpha} \theta^2 \,. \tag{5.26}$$

The perturbations to the CFT Lagrangian involving *chiral* supermultiplets $\mathcal{O}_{\rm I}, \mathcal{O}_{\rm II}^\alpha, \mathcal{O}_{\rm III}$ may then be written as

$$\Delta\mathcal{L} = \int d^2\theta \left[\mathcal{O}_{\rm I} X + \mathcal{O}_{\rm II}^\alpha Y_\alpha + \mathcal{O}_{\rm III} X \right] \ + \ c.c. \,. \tag{5.27}$$

Among the resulting terms, some correspond to (the chiral subset of) G_- perturbations,

$$\Delta\mathcal{L}_{\rm flux} = [\mathcal{O}_{\rm I}]_{\theta^2}[X]_{\rm b} + [\mathcal{O}_{\rm II}^\alpha]_\theta[Y_\alpha]_\theta + [\mathcal{O}_{\rm III}]_{\rm b}[X]_{\theta^2} \ + \ c.c. \,, \tag{5.28}$$

while the remaining terms are not dual to G_- perturbations and can be neglected for the present purposes.

Similarly, the perturbations involving non-chiral supermultiplets $\mathcal{O}_{\rm I}, \mathcal{O}_{\rm II}^\alpha, \mathcal{O}_{\rm III}$ may be written as

$$\Delta\mathcal{L} = \int d^4\theta \left[\mathcal{O}_{\rm I} X X^\dagger + \mathcal{O}_{\rm II}^\alpha Y_\alpha X^\dagger + \mathcal{O}_{\rm III} X X^\dagger \right] \ + \ c.c. \,, \tag{5.29}$$

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[12]We must distinguish spontaneous breaking of supersymmetry in the conifold gauge theory from breaking that is spontaneous in a larger supersymmetric theory, but appears as explicit breaking in the conifold gauge theory. If dynamics in a distant region of the compactification (e.g. gauge dynamics on D-branes at some local singularity distant from the conifold) leads to mass splittings in bulk supermultiplets, this spontaneous breaking of supersymmetry will manifest itself as an explicit breaking of the supersymmetry of the conifold gauge theory after we integrate out the distant physics.

with the operators dual to G_- taking the form

$$\Delta\mathcal{L}_{\text{flux}} = [\mathcal{O}_{\text{I}}]_{\theta^2}[X]_{\text{b}}[X^\dagger]_{\bar\theta^2} + [\mathcal{O}_{\text{II}}^\alpha]_\theta[Y_\alpha]_\theta[X^\dagger]_{\bar\theta^2} + [\mathcal{O}_{\text{III}}]_{\text{b}}[X]_{\theta^2}[X^\dagger]_{\bar\theta^2} \; + \; c.c. \,, \qquad (5.30)$$

and the operators dual to harmonic Φ_- modes taking the form

$$\Delta\mathcal{L}_{\Phi_-} = [\mathcal{O}_{\text{I}}]_{\text{b}}[X]_{\theta^2}[X^\dagger]_{\bar\theta^2} \; + \; c.c. \,. \qquad (5.31)$$

We remark that the spurion analysis presented here gives further justification for the expansion scheme proposed in *Case II* in section 2.3. Chiral modes, including the leading modes of G_-, are proportional to a single power of the small spurion expectation values $[X]_{\text{b}}, [X]_\theta, [X]_{\theta^2}$, and $[Y_\alpha]_{\text{b}}, [Y_\alpha]_\theta, [Y_\alpha]_{\theta^2}$, whereas the harmonic modes of Φ_- [17] require two spurion insertions. This motivates considering the case $\Phi_-^{(1)} = 0$ in which harmonic perturbations of Φ_- are neglected at linear order, while perturbations of G_- are retained at linear order. The leading potential, $\Phi_-^{(2)}$, then receives important contributions both from harmonic $\Phi_-^{(2)}$ perturbations and from the particular $\Phi_-^{(2)}$ solution sourced by $G_-^{(1)}$, as in the analysis of section 4.

5.3.2 Scalar potential in the gauge theory

To give further evidence for the correspondence between the operators of this section and the flux modes of the previous sections, let us discuss[13] how one can arrive at the scalar potential (4.12) in the gauge theory:

- The first term in (4.12), $g^{\alpha\bar\beta}\partial_\alpha\mathsf{f}_1\partial_{\bar\beta}\bar{\mathsf{f}}_1$, is simply the F-term potential due to the superpotential perturbation $\int d^2\theta\,\mathsf{f}_1$, for holomorphic $\mathsf{f}_1 \sim (AB)^k$.[14]

- To derive the second term in (4.12), $(\text{Re}\,\mathsf{f}_2)^2$, we consider the chiral operator dual to the flux in Series II as a superpotential correction,

$$\int d^2\theta\,\mathcal{O}_{\text{II}}^\alpha Y_\alpha \sim y\,[\mathcal{O}_{\text{II}}^\alpha]_{\theta_\alpha} \,. \qquad (5.32)$$

Utilizing the component expansion of W_α, we obtain a D-term of the form

$$y\,\text{Tr}[(D_{(1)} + D_{(2)})\mathsf{f}_2] \; + \; c.c. \,, \qquad (5.33)$$

where $\mathsf{f}_2 \sim (AB)^k$. After the D fields are integrated out, the resulting potential is proportional to $(\text{Re}\,\mathsf{f}_2)^2$. The exact proportionality coefficient depends on the coupling constants at the IR fixed point, but at leading order in perturbations this can be taken to be the KW value. Although the coupling (5.32) changes the renormalization of the Kähler potential, there is no corresponding Kähler potential contribution to the scalar potential at first or second order in y: because y is accompanied by only one power of θ in Y_α, terms from the Kähler potential $\int d^4\theta\,K(Y_\alpha, \bar{Y}_\alpha, A, B, \bar{A}, \bar{B})$ that are linear or quadratic in y could contribute e.g. to fermion masses, but not to the scalar potential.

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[13]We thank Zohar Komargodski, Juan Maldacena and Nathan Seiberg for extensive discussions of these issues.

[14]As explained in section 2.3, we consistently omit corrections to the inverse metric $g^{\alpha\bar\beta}$, taking it to be the unperturbed conifold metric $g_{(0)}^{\alpha\bar\beta}$.

- The third term in (4.12), $\nabla^{-2}|f_3|^2$, is more involved. This term corresponds to the contribution of the operator $\mathcal{O}_{\mathrm{III}}$ dual to flux in Series III, and is very difficult to calculate in field theory, even for chiral $\mathcal{O}_{\mathrm{III}}$. The problem is that if we represent this operator as a perturbation of the superpotential, $\int d^2\theta\, \mathcal{O}_{\mathrm{III}} X$, the scalar potential will receive a new contribution proportional to Kähler potential corrections quadratic in X.

Although we will not calculate the resulting potential in field theory, we can provide an interesting consistency check. The perturbation

$$\int d^2\theta\, \mathcal{O}_{\mathrm{III}} X \tag{5.34}$$

gives rise both to

$$[\mathcal{O}_{\mathrm{III}}]_{\theta^2}[X]_{\mathrm{b}} \tag{5.35}$$

and to

$$[\mathcal{O}_{\mathrm{III}}]_{\mathrm{b}}[X]_{\theta^2} = [\mathcal{O}_{\mathrm{III}}]_{\mathrm{b}}\, F_X\,. \tag{5.36}$$

The former is a perturbation of the gauge coupling function, and is well-known to be dual to a perturbation of the axio-dilaton in supergravity, $\delta\tau = [X]_{\mathrm{b}}f_3$. However, (5.36) is a gaugino mass term and is dual to Series III flux. Supersymmetry manifestly relates (5.35) and (5.36).

Next, we note that adding the interaction (5.34) changes the renormalization of the Kähler potential, so that

$$K_0(A, \bar{A}, B, \bar{B}) \quad \to \quad K_0(A, \bar{A}, B, \bar{B}) + \delta K(X, \bar{X}, A, \bar{A}, B, \bar{B})\,. \tag{5.37}$$

We treat the chiral spurion X as a dynamical field with a large mass and a nonzero vev acquired through the superpotential $\int d^2\theta\, w(X)$. Before the coupling $\int d^2\theta\, \mathcal{O}_{\mathrm{III}} X$ is introduced, X does not couple to the KW theory, and has diagonal metric $g^{(0)}_{X\bar{X}}$. After the coupling $\int d^2\theta\, \mathcal{O}_{\mathrm{III}} X$ is introduced, the F-term potential for X,

$$V_X = g^{X\bar{X}} \left| \frac{\partial w}{\partial X} \right|^2\,, \tag{5.38}$$

acquires a correction because eq. (5.37) induces a change in the metric for X,

$$\delta V = -\left| \frac{\partial w}{\partial X} \right|^2 (g^{(0)}_{X\bar{X}})^2 \frac{\partial^2}{\partial X \partial \bar{X}}\, \delta K\,. \tag{5.39}$$

At the same time, there is a correction to the metric of the KW fields,

$$\delta g_{\alpha\bar{\beta}} = \frac{\partial^2}{\partial z^\alpha \partial z^{\bar{\beta}}}\, \delta K\,, \tag{5.40}$$

where we have changed from A, B to the geometric notation z^α for the coordinates on the conifold. Therefore, supersymmetry provides a relationship between the perturbations to the metric and the potential,

$$\partial_\alpha \partial_{\bar{\beta}}\, \delta V \propto -\delta g_{\alpha\bar{\beta}}\,, \tag{5.41}$$

where the unspecified constant of proportionality is positive. Moreover, from the definition of the Ricci tensor, $R_{\alpha\bar{\beta}} = -\partial_\alpha \partial_{\bar{\beta}} \log \det g$, we have

$$\delta R_{\alpha\bar{\beta}} = -\partial_\alpha \partial_{\bar{\beta}} \nabla^2 \delta K \,. \tag{5.42}$$

One then easily shows that

$$\partial_\alpha \partial_{\bar{\beta}} \nabla^2 \delta V \propto \delta R_{\alpha\bar{\beta}} \,. \tag{5.43}$$

We will now verify that the supergravity solution implied by our analysis satisfies the relationship (5.43), providing a strong consistency check. To do this, we compute the perturbation of the Ricci tensor induced by the dilaton perturbation (5.35) and show that this is related by (5.43) to the perturbed potential (4.12) computed in supergravity in the presence of the flux perturbation (5.36). We recall that the dilaton perturbation sources a perturbation of the metric via the Einstein equation (*cf.* appendix B),

$$\delta R_{\alpha\bar{\beta}} = \frac{1}{4(\operatorname{Im} \tau)^2} \partial_\alpha \delta\tau \, \partial_{\bar{\beta}} \delta\bar{\tau} \,. \tag{5.44}$$

Substituting $\delta\tau \propto \mathsf{f}_3$ into eq. (5.44) and inserting $\delta V = \nabla^{-2} |\mathsf{f}_3|^2$ into eq. (5.43), we confirm that our result (4.12) obeys the relationship (5.43) required by supersymmetry.

5.4 Summary of CFT perturbations

Let us recapitulate our results from the CFT viewpoint. The simplest and most important perturbations,

$$\int d^2\theta \operatorname{Tr}(AB)^k \,, \tag{5.45}$$

are perturbations of the CFT superpotential. This tower of perturbations is dual to the *chiral* subseries, in Series I, of $G_{(1,2)}$ flux perturbations. The $k=1$ mode is relevant in the RG sense, and is dual to the lowest-dimension ($\delta = \frac{5}{2}$) mode of flux; it is responsible for the *leading* r^1 contribution to the D3-brane potential. The bottom component of the same supermultiplet is a linear Φ_- perturbation, and corresponds to the $r^{3/2}$ term of [17].

Non-chiral generalizations of Series I operators are of the form

$$\int d^2\theta \, f_1 \Big|_{\bar{\theta}^2=0} \,, \tag{5.46}$$

for f_1 a harmonic function of the matter fields A and B. The lowest-dimension non-chiral operator of this form has irrational dimension $\sqrt{28} - 1$. Adding it to the Lagrangian, together with $\int d^2\theta \operatorname{Tr}(AB)$, leads to a term $r^{\sqrt{28} - \frac{5}{2}} \approx r^{2.79}$ in the scalar potential.

Similarly, the Series II and III chiral operators are

$$\int d\theta_\alpha \, [W_+^\alpha \operatorname{Tr}(AB)^k] \qquad \text{and} \qquad [W_+^2 \operatorname{Tr}(AB)^k]_{\mathrm{b}} \,, \tag{5.47}$$

with the non-chiral extensions

$$\int d\theta_\alpha \, [W_+^\alpha f_2] \Big|_{\bar{\theta}^2=0} \qquad \text{and} \qquad [W_+^2 f_3]_{\mathrm{b}} \,. \tag{5.48}$$

The leading contributions to the D3-brane potential from chiral Series II and III modes scale as r^3 and r^2, respectively. Furthermore, the leading non-chiral Series II mode ($\delta = 4$) can interfere with the leading chiral Series I mode ($\delta = \frac{5}{2}$) to give a term scaling as $r^{5/2}$.

6 D3-brane superpotentials from fluxes

We have seen that perturbations of the supergravity solution by chiral modes of flux in Series I,

$$(\Lambda_{\mathrm{I}})_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha\nabla_\sigma f \, g^{\sigma\bar\zeta} \, \bar\Omega_{\bar\zeta\bar\beta\bar\gamma}\,, \tag{6.1}$$

with f a holomorphic function on the conifold, correspond to superpotential perturbations in the CFT,

$$\Delta W \sim f(A,B)\,. \tag{6.2}$$

In this section we will apply this correspondence to derive a useful representation of non-perturbative superpotentials in terms of fluxes.

6.1 Superpotentials in global supersymmetry

First, we observe that the D3-brane potential[15] in global supersymmetry,

$$V = g^{\alpha\bar\beta}\nabla_\alpha W\overline{\nabla_\beta W}\,, \tag{6.3}$$

is reproduced by the supergravity calculation in the corresponding flux background. Specifically, given any holomorphic function W on the conifold, we turn on the flux

$$\Lambda_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha\nabla_\sigma W \, g^{\sigma\bar\rho} \, \bar\Omega_{\bar\rho\bar\beta\bar\gamma} \equiv \nabla\nabla W \cdot \bar\Omega\,. \tag{6.4}$$

Solving (4.1) in the background (6.4), we obtain the desired potential,

$$T_3\Phi_- = g^{\alpha\bar\beta}\nabla_\alpha W\overline{\nabla_\beta W}\,. \tag{6.5}$$

Therefore, for any superpotential interaction added to the conifold gauge theory, there is a $G_{(1,2)}$ flux that geometrizes this superpotential. This is implicit in the analysis of [50], in which it was found that $G_{(1,2)}$ fluxes induce superpotential interactions for D3-branes. In appendix B, we show that this correspondence can be extended to backgrounds with large dilaton gradients and Ricci curvature sourced by D7-branes.

We now turn to an interesting application of this representation of superpotentials by fluxes.

6.2 Superpotentials from D7-branes

In the presence of a nonperturbative superpotential on wrapped D7-branes, a D3-brane feels a nontrivial potential [15]. In this section, we briefly review this result, then observe that the corresponding potential can be represented in ten dimensions by a background of three-form fluxes.

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[15]For simplicity, in this subsection we take all derivatives to be with respect to the canonically-normalized field $\phi_\alpha \equiv \sqrt{T_3}z_\alpha$.
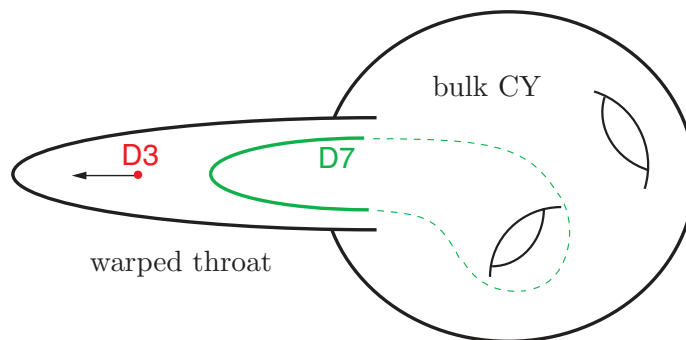
Figure 4. Schematic of a finite throat with an embedded stack of D7-branes wrapping a four-cycle. The four-cycle is compact and resides partially in the bulk and partially in the throat. Gaugino condensation on the D7-branes induces a potential for the D3-brane.

The scalar potential for a D3-brane in a compactification with nonperturbative moduli stabilization has been analyzed in ref. [16]. The scenario of interest is a finite throat containing a holomorphically-embedded stack of D7-branes (or Euclidean D3-branes), and this configuration is approximated by a noncompact conifold containing D7-branes wrapping a noncompact divisor (see figure 4).

The four-cycle Σ wrapped by the D7-branes is defined by the holomorphic embedding condition

$$h(z_\alpha) = 0, \tag{6.6}$$

the simplest examples being the Kuperstein embedding [51, 52],

$$z_1 = \mu, \tag{6.7}$$

and the Ouyang embedding [51],

$$z_3 + iz_4 = \mu. \tag{6.8}$$

The warped volume of the four-cycle, $V_\Sigma = \int \mathrm{d}^4 y \sqrt{-g} e^{-4A}$, determines the gauge coupling of the D7-brane theory. D3-branes are local sources for Φ_+ and hence contribute corrections to V_Σ. If the D7-brane gauge theory generates a gaugino condensate superpotential, then the dependence of the four-cycle volume on the D3-brane position introduces a dependence of the superpotential on the D3-brane position [15],

$$W_{\rm np}(z_\alpha) = \mathcal{A}(z_\alpha) e^{-a\rho}, \tag{6.9}$$

where ρ is the Kähler modulus associated with the overall volume of the four-cycle and $\mathcal{A}(z_\alpha)$ is given by the embedding function $h(z_\alpha) = 0$,

$$\mathcal{A}(z_\alpha) = \mathcal{A}_0 \, h(z_\alpha)^{1/N_c}, \tag{6.10}$$

with N_c the number of D7-branes and $\mathcal{A}_0$ a constant proportional to N_c^2. The full superpotential is then

$$W = W_0 + W_{\rm np}(z_\alpha, \rho), \tag{6.11}$$

with W_0 the Gukov-Vafa-Witten flux superpotential [53], which is constant after stabilization of the complex structure moduli. Finally, the Kähler potential is [32, 54] $K = -3\log[\rho + \bar{\rho} - \gamma k(z_\alpha, \bar{z}_\alpha)]$ (with γ a constant, *cf.* [16]). In terms of the above $\mathcal{N} = 1$ data, one can derive the supergravity F-term potential $V_F[\rho, \bar{\rho}, z_\alpha, \bar{z}_\alpha]$ in the usual way.

6.3 Flux representation of nonperturbative superpotentials

We will now demonstrate that for any specified superpotential $W(z_\alpha)$ for a D3-brane in the conifold, *cf.* eq. (6.11), there exists a noncompact supergravity solution in which the Born-Infeld plus Chern-Simons potential of a probe D3-brane precisely equals the F-term potential $V_F[W(z_\alpha), K(z_\alpha, \bar{z}_\alpha)]$ computed in four-dimensional supergravity with $W(z_\alpha)$ and $K(z_\alpha, \bar{z}_\alpha)$ as the input data.[16] That is, given a superpotential $W(z_\alpha)$, we will provide a noncompact supergravity solution containing suitable IASD fluxes that *geometrizes* this superpotential.

At leading order in an expansion in $\frac{\gamma k}{\sigma} = \frac{\phi^2}{3M_{\rm pl}^2}$, where $\sigma \equiv \frac{1}{2}(\rho + \bar{\rho})$, the F-term potential is [16]

$$V = \frac{\kappa^2}{12\sigma^2}\frac{e^{-2a\sigma}}{\gamma}\left(g^{\alpha\bar{\beta}}\mathcal{A}_\alpha\bar{\mathcal{A}}_{\bar{\beta}} + 2a\gamma(a\sigma + 3)\mathcal{A}\bar{\mathcal{A}} - a\gamma(\bar{\mathcal{A}}g^{\alpha\bar{\beta}}k_{\bar{\beta}}\mathcal{A}_\alpha + c.c.)\right) \; + \; \text{harmonic}. \tag{6.12}$$

We now exhibit the ten-dimensional supergravity solution in which a probe D3-brane experiences the four-dimensional supergravity F-term potential (6.12) [16]. To accomplish this we turn on fluxes in the first two series of section 3. We turn on $(1, 2)$ flux of the form

$$\Lambda_1 = \nabla\nabla f_1 \cdot \bar{\Omega}, \tag{6.13}$$

with f_1 a holomorphic function. In addition, we turn on a non-primitive $(2, 1)$ flux of the form

$$\Lambda_2 = \partial f_2 \wedge J, \tag{6.14}$$

with f_2 a holomorphic function. As we have shown in section 4, this leads to the potential[17]

$$\Phi_- = \frac{V}{T_3} = \frac{g_s}{32}\left[g^{\alpha\bar{\beta}}\nabla_\alpha f_1 \overline{\nabla_\beta f_1} + 2|f_2|^2\right] \; + \; \text{harmonic}. \tag{6.15}$$

Comparing eq. (6.15) to eq. (6.12) suggests the matching conditions

$$f_1 = c_1 \mathcal{A}, \tag{6.16}$$

$$f_2 = c_2 \mathcal{A} + c_3 k^\beta \mathcal{A}_\beta, \tag{6.17}$$

which yields

$$\Phi_- = \frac{g_s}{32}\left[|c_1|^2 g^{\alpha\bar{\beta}}\mathcal{A}_\alpha\bar{\mathcal{A}}_{\bar{\beta}} + 2|c_2|^2\mathcal{A}\bar{\mathcal{A}} + 2(c_2\bar{c}_3\,\mathcal{A}\,\overline{k^\beta}\mathcal{A}_\beta + c.c.)\right], \tag{6.18}$$

---

[16]In this discussion we will suppress the dependence on ρ for brevity, but one should keep in mind that ρ is taken to be a dynamical field in the computation of V_F.

[17]Notice that $(\mathrm{Re}f_1)^2$ and $\frac{1}{2}|f_1|^2$ are equal up to harmonic terms.

once we drop the term proportional to $|c_3|^2$, which is subleading in the $\gamma k/\sigma$ and $(a\sigma)^{-1}$ expansions employed in eq. (6.12).[18]

Choosing

$$c_1 = c \,, \tag{6.19}$$

$$c_2 = c \sqrt{a\gamma(a\sigma + 3)} \,, \tag{6.20}$$

$$c_3 = -c \frac{a\gamma}{2c_2} \,, \tag{6.21}$$

with

$$c^2 \equiv \frac{\kappa^2}{\sigma^2} \frac{e^{-2a\sigma}}{\gamma T_3} \frac{8}{3g_s} \,, \tag{6.22}$$

we recover eq. (6.12). The omitted harmonic terms can, of course, be adjusted by adding a harmonic piece to Φ_-, no matter the flux background. Notice also that

$$\left| \frac{c_3}{c_2} \right| = \frac{1}{2(a\sigma + 3)} \ll 1 \,. \tag{6.23}$$

Finally, we remark that in the decompactification limit ($M_{\mathrm{pl}} \to \infty$ and $\gamma \to 0$), Series I flux dominates over Series II flux and the supergravity F-term potential reduces to the rigid supersymmetry potential.

7   Towards a D7-brane geometric transition

As remarked in the introduction, an important open question in compactifications with nonperturbatively-stabilized Kähler moduli is the computation of nonperturbative contributions to the open string effective action. One can argue very generally that every divisor supporting nonperturbative effects will contribute to the potential for a mobile D3-brane [55]. When a coordinate chart containing the divisor is available, it is straightforward to determine the superpotential, and this has been done from a variety of perspectives in toroidal orientifolds [25] and in noncompact cones [15].[19] In a more generic F-theory compactification, the result of [55] still applies [56], but it is difficult to translate this fact into a meaningful contribution to the potential for a D3-brane far from a given divisor.

It is therefore natural to pursue an understanding of nonperturbative effects on D7-branes that more efficiently describes the potential for a D3-brane interacting with multiple distant divisors, each bearing nonperturbative effects. We have suggested in the preceding section that, for the purpose of determining the D3-brane potential, nonperturbative effects on D7-branes can be represented by appropriately-chosen IASD fluxes. This result, however, is not completely satisfying, as the fluxes were engineered to achieve the desired result, rather than emerging from the inclusion of local source terms in the equation of motion. We will now establish a much more interesting fact: *nonperturbative effects*

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[18]The $\gamma k/\sigma$ expansion can be seen to be an expansion in powers of $\frac{\phi}{M_{\mathrm{pl}}}$, with ϕ the canonically-normalized scalar describing D3-brane motion. As shown in [40], this quantity is bounded from above by $\frac{2}{\sqrt{N}}$ in an $AdS_5 \times X_5$ geometry with D3-brane charge N, and hence is a suitable expansion parameter.

[19]It has also been argued that this result can be encoded in a generalized complex geometry [27].

on D7-branes source IASD fluxes.[20] In particular, in a background containing D7-branes wrapping a rigid four-cycle, gaugino condensation in the D7-brane theory corrects the ten-dimensional equation of motion for the fluxes, by introducing a source term localized to the four-cycle. The ten-dimensional solution in the presence of this source contains IASD fluxes that geometrize, in the spirit of section 6, the nonperturbative superpotential of the D7-brane theory.

For noncompact configurations, Series I flux suffices to geometrize the entire scalar potential, while for compact spaces, Series II flux is also necessary, as shown in section 6. In this section we restrict attention to the sourcing of Series I flux by gaugino condensation, but we emphasize that the results obtained below are valid for compact spaces, with the proviso that in such cases the D3-brane potential is not fully determined by Series I flux.

We make the further simplifying assumption that any dilaton gradients are small, in the sense of section 2.3. This can be consistent with the classical backreaction of D7-branes, *e.g.* four D7-branes coincident with an O7-plane have vanishing total charge and tension, and hence source neither dilaton monodromies nor a deficit angle. We expect that the inclusion of dilaton gradients constitutes a purely technical complication, and we anticipate that our conclusions can be extended to general F-theory backgrounds. Although a calculation with non-trivial dilaton is beyond the scope of the present work, in appendix B we have paved the way for such a computation.

We begin in section 7.1 by outlining our general strategy, explaining how worldvolume couplings of the gaugino can serve as source terms, and indicating the anisotropic regime in which our methods are most reliably applied. Then, in section 7.2, we identify the particular coupling of the gaugino that is responsible for sourcing $G_{(1,2)}$ flux. We provide an indirect argument, via AdS/CFT, for the existence of such a coupling (section 7.2.1), and then confirm this by direct computation in ten dimensions (section 7.2.2). We show that the resulting $G_{(1,2)}$ flux is precisely what is required to geometrize the nonperturbative superpotential in field theory.

7.1 4D fermion bilinears as 10D sources

First, we observe that the ten-dimensional equations of motion for the fluxes in principle include source terms[21] proportional to open string fermion bilinears. These terms are dropped when considering classical solutions, but nonperturbative effects can induce an expectation value for e.g. the D7-brane gaugino bilinear, $\langle \lambda\lambda \rangle \neq 0$. Then, ten-dimensional closed string fields that couple to $\lambda\lambda$ will obtain D7-brane-localized source terms when gaugino condensation occurs. For example, a coupling of the schematic form[22]

$$\delta S \ \sim \ \int \mathrm{d}^8\xi \, \sqrt{-g} \, G_3 \, \lambda\lambda \tag{7.1}$$

could plausibly source bulk fluxes.

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[20]We thank Juan Maldacena and Gonzalo Torroba for discussions on this topic.

[21]We thank James Gray for helpful discussions of this point.

[22]Here and in the following, $\lambda\lambda$ is shorthand for $\mathrm{Tr}(\lambda_\alpha \lambda^\alpha)$.
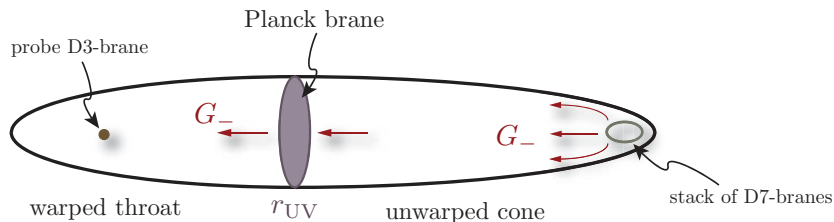
Figure 5. Schematic of a configuration in which nonperturbative effects on D7-branes wrapping a small four-cycle are represented, at long distances, by IASD fluxes G_-.

Let us emphasize that we are proposing to solve the *ten-dimensional* equations of motion for the fluxes, incorporating fermion expectation values that are nonvanishing as a result of *four-dimensional* nonperturbative physics.[23] This proposal, albeit unconventional, is plausibly consistent, as we will explain in an example. Consider a stack of D7-branes wrapping the base of a complex cone over a del Pezzo surface (see figure 5). Gaugino condensation in the D7-brane gauge theory leads to a nonperturbative superpotential that generates a potential for a D3-brane probing the cone. At radial distances that are large compared to the size of the base, there should be a local supergravity solution describing this system. Any non-locality caused by the four-dimensional nonperturbative effects should be confined to the region near the collapsed surface, and at larger distances the net effect of gaugino condensation should be to source corrections to the ten-dimensional supergravity solution. Equivalently, when the four-cycle is small compared to the remainder of the internal space, then the nonperturbative effects will arise at an energy scale that is large compared to the lowest Kaluza-Klein mass of the internal space. In such a case, it is reasonable to study the ten-dimensional background fields incorporating four-dimensional nonperturbative effects. We will restrict our attention to anisotropic configurations of this sort.

7.2 D7-brane couplings to flux

We will now identify a specific D7-brane worldvolume coupling that causes gaugino condensation to source fluxes. In section 7.2.1 we will begin by using AdS/CFT to identify the corresponding coupling in field theory. Then, in section 7.2.2, we will demonstrate that the well-known tree-level coupling of D7-brane gauginos to flux [47] serves to source $G_{(1,2)}$ fluxes that geometrize the gaugino condensate superpotential. The latter result is more general and applies to any Calabi-Yau geometry; specifically, it is not restricted to asymptotically AdS spaces.

7.2.1 Gauginos as a source for flux: field theory perspective

Four-dimensional supersymmetry requires that the gauge coupling function f of the D7-brane gauge theory is a holomorphic function of the chiral superfields Φ,[24]

$$\mathcal{L} \supset - \int d^2\theta \, f(\Phi) W^\alpha W_\alpha + c.c. \tag{7.2}$$

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[23]For a related investigation in the heterotic string, see [57]. The backreaction of nonperturbative effects on fluxes has also been considered in type IIB string theory in [58].

[24]To facilitate comparison with the results of [59] we absorb a factor of $1/32\pi^2$ in $W^\alpha W_\alpha$.
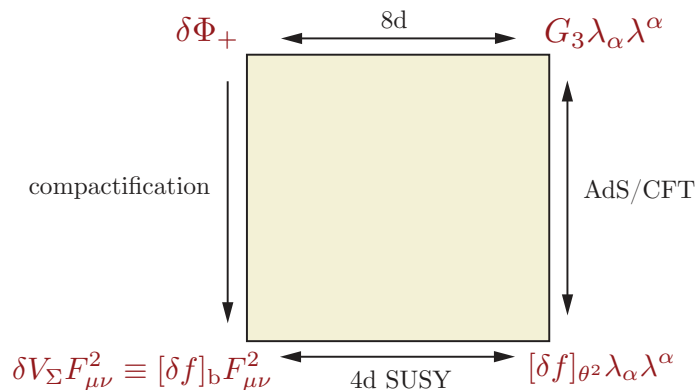
Figure 6. Indirect argument that a nonvanishing D7-brane gaugino bilinear $\lambda_\alpha \lambda^\alpha$ sources $G_{(1,2)}$ flux. The D7-brane gauge coupling depends on the D3-brane position via the perturbation $\delta\Phi_+$ [15] (left edge). Four-dimensional supersymmetry relates this to a gaugino mass (bottom edge), which in turn is related by AdS/CFT to a flux-gaugino coupling (right edge).

Expanding the superspace integral, one finds the usual gauge kinetic term, as well as the gaugino mass term

$$\lambda\lambda \times \int d^2\theta \, f(\Phi) = F_\Phi \frac{\partial f}{\partial \Phi} \, \lambda\lambda \,. \tag{7.3}$$

Notice that the D7-brane gaugino mass depends on the F-components of the fields appearing in f; this is just the familiar statement that in four-dimensional supersymmetry, the gaugino mass term is the F-component of the gauge kinetic function, $F_f \equiv [f]_{\theta^2}$ (bottom edge of figure 6).

eq. (7.3) may also be viewed as a source for the F-term F_Φ, and can be represented as a superpotential contribution $\int d^2\theta \, W(\Phi)$ with $W(\Phi)$ satisfying

$$\frac{\partial W}{\partial \Phi} = -\frac{\partial f}{\partial \Phi}\lambda\lambda \,. \tag{7.4}$$

This is in agreement with the well-known expression for the gaugino condensate and its relation to the superpotential [59],

$$W = N_c\lambda\lambda \quad \text{and} \quad \lambda\lambda \sim e^{-f/N_c} \,. \tag{7.5}$$

Let us now specialize to the warped conifold in order to make use of the AdS/CFT dictionary of section 5. The general chiral superfields Φ are therefore replaced by the local coordinates z_α (related to gauge-invariant combinations of A, B). We argued in section 5 that the F-term of the chiral fields is dual to $G_{(1,2)}$ flux of Series I. From eq. (4.12) we find that the superpotential W and the defining function f_1 of the Series I flux are related by

$$\frac{\partial W}{\partial z_\alpha} = \zeta \, \frac{\partial f_1}{\partial z_\alpha} \,, \tag{7.6}$$

where we have defined the dimensionful parameter

$$\zeta \equiv T_3\sqrt{\frac{g_s}{32}} \,, \tag{7.7}$$

and in (7.6) we have suppressed an undetermined phase. Comparing this with eq. (7.4), we see that the gaugino condensate $\langle \lambda\lambda \rangle$ in field theory must source $G_{(1,2)}$ flux in the bulk.

In fact, we can be more precise in predicting the form of the sourced flux. From [15, 55] we know how f depends on the chiral superfields. For a D7-brane embedded along a divisor that is defined by the holomorphic equation $h(z_\alpha) = 0$, the coupling constant is

$$f = 2\pi\rho - \log(h(z_\alpha)) \,. \tag{7.8}$$

Combining the above results, we obtain a nontrivial prediction for the harmonic function f_1 (which defines the Series I flux),

$$\frac{\partial f_1}{\partial z^\alpha} = \zeta^{-1} \lambda\lambda \, \frac{\partial \log h}{\partial z^\alpha} \,, \tag{7.9}$$

up to an undetermined phase.

Our argument uses four-dimensional supersymmetry and the AdS/CFT dictionary to deduce a coupling in four dimensions between the gauginos of D7-branes in a warped throat, and CFT fields dual to $G_{(1,2)}$ fluxes in that throat. We expect that this coupling in the four-dimensional theory arises from the dimensional reduction of a corresponding eight-dimensional coupling between D7-brane fermions and G_3. Moreover, it is reasonable to expect, and we shall demonstrate, that such a coupling is present even in spaces that are not asymptotically AdS. In the next section we will demonstrate that the coupling in question is nothing but the tree-level gaugino mass term of [47].

7.2.2 Gauginos as a source for flux: bulk perspective

The tree-level couplings between D7-brane worldvolume fermions and the bulk fields have been studied in [26, 47] by expanding the Born-Infeld plus Chern-Simons actions around the D7-brane location. Ignoring the trivial Minkowski part and assuming constant dilaton, the unique tree level coupling involving $G^*_{(0,3)}\lambda\lambda$ is [47],

$$\mathcal{L} = 16 \, c_0 \, \zeta \int\limits_\Sigma \sqrt{g} \, G_3 \cdot \Omega \, \bar\lambda\bar\lambda \; + \; c.c. \tag{7.10}$$

Here $c_0 \sim \mathcal{O}(1)$ is a dimensionless coefficient that could be obtained from a careful uplifting to eight dimensions of the four-dimensional action of [47]. In the following we set $c_0 \to 1$, but we emphasize that the matching of the overall coefficient in the analysis below would require the computation of c_0. Adding this local coupling (7.10) to the bulk action modifies the flux equation of motion, and, as we shall show, links the gaugino expectation value $\langle \lambda\lambda \rangle$ to the bulk flux $G_{(1,2)}$.

Before we proceed, let us recall some facts about divisor delta-functions.[25] The divisor Σ wrapped by the D7-branes is defined via the holomorphic embedding equation $h = 0$. The Poincare-Lelong Equation then defines a delta-function two-form [60],

$$\delta^{(2)}_{\alpha\bar\beta} = \frac{1}{\pi} \partial_\alpha \partial_{\bar\beta} \operatorname{Re}(\log h) \,, \tag{7.11}$$

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[25]We thank Luca Martucci for discussions.

which localizes the integral of any four-form $F^{(4)}$ onto Σ,

$$\int\limits_M F^{(4)} \wedge \delta^{(2)} = \int\limits_\Sigma F^{(4)} \,. \tag{7.12}$$

Eq. (7.11) is a generalization of the scalar delta-function, which may be obtained from (7.11) by contraction with $g^{\alpha\bar\beta}$,

$$\delta^{(0)} = g^{\alpha\bar\beta} \delta^{(2)}_{\alpha\bar\beta} = \frac{1}{2\pi} \nabla^2 \mathrm{Re}(\log h) \,. \tag{7.13}$$

Eq. (7.13) was derived for conifold geometries in [15].

We use eq. (7.11) to transform the local action on Σ, eq. (7.10), into an integral over the whole compactification manifold M,

$$\int\limits_\Sigma \sqrt{g}\, G_3 \cdot \Omega\, \bar\lambda\bar\lambda = \frac{1}{2!} \int\limits_\Sigma J \wedge J\, (G_3 \cdot \Omega\, \bar\lambda\bar\lambda) = \frac{1}{2!} \int\limits_M J \wedge J \wedge \delta^{(2)}\, (G_3 \cdot \Omega\, \bar\lambda\bar\lambda) \,. \tag{7.14}$$

Using that $\frac{1}{2!} J \wedge J \wedge \delta^{(2)} = \frac{1}{3!} J \wedge J \wedge J\, \delta^{(0)}$, we can then write

$$\frac{1}{3!} \int\limits_M J \wedge J \wedge J\, (G_3 \cdot \Omega\, \bar\lambda\bar\lambda\, \delta^{(0)}) = -i \int\limits_M G_3 \wedge \Omega\, (\bar\lambda\bar\lambda\, \delta^{(0)}) \,. \tag{7.15}$$

Next, using (7.15), we compare the variation of (7.10) with respect to C_2 and B_2,

$$-16i\,\zeta \int\limits_M \delta G_3 \wedge \Omega\, (\bar\lambda\bar\lambda\, \delta^{(0)}) \; + \; c.c. \,, \tag{7.16}$$

to the variation of the bulk supergravity action,

$$-\frac{g_s}{4\kappa_{10}^2} \int\limits_M \delta G_3 \wedge \bar\Lambda \; + \; c.c. \,, \tag{7.17}$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 \alpha'^4 = \pi/T_3^2$. We obtain the modified equation of motion

$$\mathrm{d}\Lambda = \mathrm{d}\left(\frac{2\pi}{i\zeta}\, \bar\Omega\, \lambda\lambda\, \delta^{(0)} \right), \qquad \text{with} \quad \star_6 \Lambda = -i\Lambda \,. \tag{7.18}$$

To solve this equation we consider non-trivial IASD fluxes from Series I, II, and III. The source term on the r.h.s. of (7.18) is a singular $(1,3)$ form, and therefore the defining functions f_1, f_2, and f_3 must be harmonic everywhere except on the divisor Σ. Consulting eqs. (A.36) and (A.18), we see that Series III flux does not lead to a non-trivial $(1,3)$ form, and that Series II flux leads to a combination of $(1,3)$ and $(2,2)$ forms. The only remaining possibility, therefore, is Series I flux (3.24), which obeys (cf. eq. (A.14))

$$\mathrm{d}\Lambda = \mathrm{d}\left(\frac{1}{2} \nabla^2 f_1 \bar\Omega \right). \tag{7.19}$$

$$- 45 -$$

Comparing eqs. (7.18) and (7.19), we obtain

$$\nabla^2 f_1 = \frac{4\pi}{i\zeta} \lambda\lambda\, \delta^{(0)}\,. \tag{7.20}$$

The gaugino condensate $\lambda\lambda$ depends on the location of any D3-branes $\widehat{z}_\alpha$, but is a constant with respect to the bulk coordinates z_α. Solving eq. (7.20) then yields

$$f_1 = \frac{2}{i\zeta}\,\lambda\lambda\,\operatorname{Re}(\log h(z_\alpha))\,, \tag{7.21}$$

in agreement with eq. (7.9). This, in fact, is the bulk dual of the field-theory mechanism of section 7.2.1.

To calculate the potential for a D3-brane we first calculate the flux $G_{(1,2)}$ using eq. (7.21) and the form of the embedding function $h(z_\alpha)$. Then we calculate Φ_- using eq. (4.12). At this step Φ_- depends on the coordinates z_α in the bulk, and also on the location of the D3-branes through $\lambda\lambda(\widehat{z}_\alpha) \propto \frac{1}{N_c} h(\widehat{z}_\alpha)^{1/N_c}$, *cf.* (6.10) [15]. Finally, we compute the potential for the D3-brane by evaluating the D3-brane probe action $T_3\Phi_-$ at the location of the D3-brane. The resulting potential is given by eq. (6.3), with W the gaugino condensate superpotential (6.9). Therefore, the scalar potential takes precisely the same form when calculated in four-dimensional supersymmetry or from the ten-dimensional probe action in the flux background (7.21) sourced by gaugino condensation.

We remark in passing that our result is valid for any number of D3-branes. To illustrate this point, let us consider a conifold region that contains N_c D7-branes wrapping the four-cycle $h(z_\alpha) = \mu - z_1$, as well as N_f well-separated D3-branes. We further assume that any adjoint chiral multiplets living on the D7-branes have obtained a high-scale mass from appropriate ISD flux, so the low-energy effective theory on the D7-branes in the absence of probe D3-branes is pure $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N_c)$ QCD. The D3-branes at finite separation from the D7-branes correspond to massive flavors in the $\mathcal{N} = 1$ super-Yang-Mills theory on the D7-branes, and in this theory with mass matrix m for the flavors, the superpotential is [59]

$$W = N_c\lambda\lambda \;\propto\; \det(m)^{1/N_c} = \det\left(\mu\,\mathbb{I}_{N_f\times N_f} - \widehat{\mathbb{Z}}_1\right)^{1/N_c}, \tag{7.22}$$

in agreement with [15, 61]. The above superpotential describes an interaction among the D3-branes sitting at locations specified by the eigenvalues of the matrix $\widehat{\mathbb{Z}}_1$.

We note that our proposal for the long-distance supergravity description of the D7-brane nonperturbative effects has certain similarities to the well-known geometric transition [13, 62, 63], for which it is now firmly established that nonperturbative dynamics on a stack of D5-branes wrapping a small curve can be captured by a modified background geometry and suitable G_3 flux. However, at present our only method of probing the supergravity solution sourced by the D7-brane nonperturbative effects is by examining the potential of a probe D3-brane. Therefore, our speculations are necessarily limited for the time being to the modes of Φ_- (and hence, IASD fluxes) that nonperturbative effects on D7-branes may produce. It would be interesting to identify additional probes of the geometry around the D7-branes [28].

Another ten-dimensional description of nonperturbative effects was given in [27], where a generalized complex geometry was argued to encode the nonperturbative superpotential in the decompactification limit. In our language this would correspond to the presence of Series I flux. It would be interesting to understand the relation between the flux solution described in this paper and the configuration studied in [27]; it is conceivable that the ten-dimensional equations of motion relate these results.

7.3 Comments on noncommutativity

Our proposal provides a direct connection between the works of Cecotti *et al.* [64] and Marchesano and Martucci [61]. In [64] it was observed that the noncommutative superpotential induced on coincident D3-branes by three-form fluxes [47, 65] can solve a problem in F-theory model-building by increasing the rank of the Yukawa matrix. However, in [61] it was argued that a noncommutative superpotential should not appear at tree level in a no-scale compactification, but should instead arise from nonperturbative effects on D7-branes. We have argued here that nonperturbative effects on D7-branes can be represented in ten-dimensional supergravity by suitable IASD fluxes. Therefore, our analysis provides a concrete link between these approaches. To illustrate this point we briefly sketch how one can arrive at the same answer for the noncommutativity parameter from either perspective. Marchesano and Martucci derived the noncommutativity parameter on a stack of N_f D3-branes in the presence of nonperturbative effects on N_c D7-branes [61][26]

$$\theta_{\alpha\beta} \propto \epsilon_{\alpha\beta\gamma} h^{N_f-1} \partial_\gamma h \,, \tag{7.23}$$

where as before $h = 0$ defines the four-cycle wrapped by the D7-branes (or by a Euclidean D3-brane), and special complex coordinates have been chosen such that the metric is canonical, $g_{\alpha\bar\beta} = \delta_{\alpha\bar\beta}$. On the other hand, Cecotti *et al.* [64] (see also [47]) showed that the noncommutativity parameter arises from $(1, 2)$ flux,

$$\nabla_\alpha \theta_{\bar\beta\bar\gamma} \propto \Lambda^{(1,2)}_{\alpha\bar\beta\bar\gamma} \,. \tag{7.24}$$

Our conjecture for relating nonperturbative effects on D7-branes to bulk IASD fluxes directly relates the results in (7.23) and (7.24). In our approach, the $(1, 2)$ flux of (7.24) is sourced by gaugino condensation on D7-branes in the presence of the N_f D3-branes, and hence depends on their locations $\widehat{z}_\alpha$. Namely, for coincident D3-branes $\lambda\lambda \propto h^{N_f}(\widehat{z}_\alpha)$, and hence $\partial_\alpha f_1 \propto h^{N_f}\partial_\alpha \log h(z_\beta)$. Comparing the expression (3.24) for the $(1, 2)$ flux, $\Lambda^{(1,2)}_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha \nabla_\sigma f_1 g^{\sigma\bar\zeta}\epsilon_{\bar\zeta\bar\beta\bar\gamma}$, with (7.24) we conclude that

$$\theta_{\alpha\beta} \propto \epsilon_{\alpha\beta\gamma}\partial_\gamma f_1 \,, \tag{7.25}$$

in agreement with (7.23). It would be interesting to extend this analysis to include the effects of a running dilaton by using the expression for the Series I flux from appendix B, thereby generalizing the results of [64] and [47].

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[26]For simplicity, [61] considered the case $N_c = 1$, corresponding to replacing the effect of gaugino condensation on D7-branes by that of Euclidean D3-branes. To compare our result to [61] we also take $N_c = 1$.

8 Conclusions

We have provided a comprehensive analysis of the structure of the scalar potential for a D3-brane in a noncompact conifold background subject to arbitrary ultraviolet deformations, in an expansion around the ISD solution with $G_- = \Phi_- = 0$. This strongly constrains the form of the effective action of a D3-brane in a finite warped throat attached to a compact space. Our general analysis of ultraviolet deformations provides the gravity-side version of a Wilsonian treatment of the D3-brane action, in that the perturbations of the supergravity solution are dual to Planck-suppressed contributions to the D3-brane action. As is clear from our description of these results in the dual CFT, strong coupling makes a complete treatment on the field theory side extremely difficult.

Our investigation yielded a general solution for IASD fluxes in the singular conifold, and can be extended to arbitrary Calabi-Yau cones given the scalar harmonics on the corresponding Sasaki-Einstein bases. For a given flux background, it is straightforward to find the scalar potential, and for $G_{(1,2)}$ and $G_{(2,1)}$ fluxes parametrized by a holomorphic function, we gave the result in closed form. In the remaining cases we provided an integral formula involving the flux density and the (known) Green's function, which is easily evaluated in any case of interest. Assembling these results, we presented the spectrum of radial scaling dimensions for the leading correction terms, eq. (4.44).

We then systematically matched our non-normalizable flux solutions to sources for dual operators in the corresponding CFT. This provided a nontrivial check of the completeness of our results, as well as a useful perspective for comparing to four-dimensional field theory analyses. We showed that for two out of three towers of chiral operators, the scalar potential computed in the field theory matches that found in supergravity. For the third tower of chiral operators, we argued that knowledge of the perturbed Kähler potential would be necessary to compare to the supergravity result, but we did provide a consistency check relating the scalar potential to the perturbations of the metric.

Next, we observed that the D3-brane potential specified by any given superpotential $W(z_\alpha)$ on the conifold can be *geometrized*, in the sense that there exists a ten-dimensional supergravity solution in which a probe D3-brane experiences the supergravity F-term potential dictated by $W(z_\alpha)$. We have verified that for the rigid supersymmetry terms, this correspondence persists even in the presence of large perturbations of the metric and dilaton sourced by D7-branes.

Finally, we showed that gaugino condensation on D7-branes wrapping a four-cycle *sources* IASD flux in ten dimensions. For the purpose of computing the potential of a space-filling D3-brane, these fluxes serve as a useful dual representation of the nonperturbative effects. Using AdS/CFT, we argued that a suitable gaugino coupling to flux must exist, then demonstrated that in fact the well-known tree-level gaugino mass term [47] serves to source IASD flux that is dual, in the above sense, to the nonperturbative superpotential. This observation is similar to the D5-brane geometric transition [13, 62, 63], but we leave a thorough investigation of this intriguing possibility as a subject for future work [28].

Our results have a range of applications. An important problem of general interest in string theory is to determine Planck-suppressed contributions to the effective action in

sectors that are sensitive to Planck-scale physics. In inflation, one needs to compute these contributions in order to ensure that they do not spoil the inflationary dynamics, while in particle physics scenarios with high-scale supersymmetry breaking, one needs to determine the soft terms from gravity mediation in order to assess the flavor structure. Direct computation of these effects in general compact models is prohibitively complicated. In this work, by first decoupling gravity in a noncompact configuration, then systematically reincorporating compactification effects, we have provided an approach in which the structure of the Planck-suppressed contributions can be computed.

A   IASD fluxes in Calabi-Yau cones

In this appendix we derive the solutions for imaginary anti-self dual (IASD) flux perturbations on a general Calabi-Yau cone in terms of the scalar harmonics on the Sasaki-Einstein base. These results provide the primary input for computing the flux-induced potential felt by probe D3-branes.

Preliminaries.   We are searching for closed, IASD three-forms Λ,

$$\mathrm{d}\Lambda = 0, \qquad \star_6 \Lambda = -i\Lambda. \tag{A.1}$$

We construct these forms using the Kähler form J and the holomorphic $(3,0)$ form Ω, as well as harmonic functions f. The Kähler form is a $(1,1)$ form with components

$$J_{\alpha\bar\beta} = i g_{\alpha\bar\beta}\,, \tag{A.2}$$

where $g_{\alpha\bar\beta}$ is the Kähler metric

$$g_{\alpha\bar\beta} = \partial_\alpha \partial_{\bar\beta} k\,. \tag{A.3}$$

The holomorphic $(3,0)$ form has components

$$\Omega_{\alpha\beta\gamma} = q\,\epsilon_{\alpha\beta\gamma}\,, \tag{A.4}$$

where q is a holomorphic function satisfying $q\bar q = \det g$. We will make use of the identity

$$\frac{1}{2!}\Omega_{\alpha\beta\gamma}\bar\Omega_{\bar\alpha\bar\beta\bar\gamma}g^{\beta\bar\beta}g^{\gamma\bar\gamma} = g_{\alpha\bar\alpha}\,. \tag{A.5}$$

Using these ingredients we construct IASD three-forms:

- We write the non-primitive $(2,1)$ component of the three-form Λ as the exterior product of the Kähler form J and a holomorphic one-form P,

$$\Lambda^{(2,1)} = P^{(1,0)} \wedge J^{(1,1)}\,, \qquad \text{or} \qquad \Lambda_{\alpha\beta\bar\gamma} = iP_{[\alpha}g_{\beta]\bar\gamma}\,. \tag{A.6}$$

- We write the $(1,2)$ component of the three-form Λ as the contraction of the anti-holomorphic $(0,3)$ form $\bar\Omega$ and a holomorphic $(2,0)$ form P,

$$\Lambda^{(1,2)} = P^{(2,0)} \cdot \bar\Omega^{(0,3)}\,, \qquad \text{or} \qquad \Lambda_{\alpha\bar\beta\bar\gamma} = P_{(\alpha\sigma)}g^{\sigma\bar\zeta}\bar\Omega_{\bar\zeta\bar\beta\bar\gamma}\,. \tag{A.7}$$

Only the symmetric part of P gives an IASD form.

- Finally, the holomorphic $(3,0)$ form Ω is IASD.

The forms P will be constructed out of derivatives of harmonic functions f.

Series I: Three-Form of Type $(1,2)$. We construct the two-form P in eq. (A.7) out of mixed covariant derivatives of a harmonic function f,

$$P_{(\alpha\sigma)} = \nabla_\alpha \nabla_\sigma f\,, \tag{A.8}$$

where $\nabla^2 f = g^{\rho\bar\zeta}\nabla_\rho \nabla_{\bar\zeta} f = 0$. Below we prove that the resulting three-form is closed. Hence, we obtain the first three-form, $\Lambda_{\rm I} = \Lambda^{(1,2)}$,

$$\Lambda_{\rm I} = \nabla\nabla f \cdot \bar\Omega\,, \tag{A.9}$$

or

$$\Lambda_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha \nabla_\sigma f\, g^{\sigma\bar\zeta}\bar\Omega_{\bar\zeta\bar\beta\bar\gamma}\,. \tag{A.10}$$

1. *The three-form $\Lambda_{\rm I}$ defined in eq. (A.9) is closed.*

Proof. We aim to show that $\mathrm{d}\Lambda_{\mathrm{I}} = (\partial + \bar{\partial})\Lambda_{\mathrm{I}} = 0$. First, we consider the anti-holomorphic derivative

$$(\bar{\partial}\Lambda_{\mathrm{I}})_{\alpha\bar{\beta}\bar{\gamma}\bar{\delta}} = \nabla_{[\bar{\delta}}\nabla_{\alpha}\nabla_{\sigma}f\,g^{\sigma\bar{\zeta}}\bar{\Omega}_{\bar{\zeta}|\bar{\beta}\bar{\gamma}]} \equiv T_{\alpha}\,\bar{\Omega}_{\bar{\beta}\bar{\gamma}\bar{\delta}}\,, \tag{A.11}$$

where

$$T_{\alpha} \propto g^{\sigma\bar{\zeta}}\nabla_{\bar{\zeta}}\nabla_{\alpha}\nabla_{\sigma}f\,. \tag{A.12}$$

We can exchange the order of $\nabla_{\bar{\zeta}}$ and ∇_{α} because the difference is proportional to

$$g^{\sigma\bar{\zeta}}R^{\rho}{}_{\sigma\bar{\zeta}\alpha}\nabla_{\rho}f = R_{\alpha\bar{\beta}}\,g^{\rho\bar{\beta}}\,\nabla_{\rho}f\,, \tag{A.13}$$

which vanishes because the cones we consider are Calabi-Yau manifolds with vanishing Ricci tensor, $R_{\alpha\bar{\beta}} = 0$. Hence, we find

$$(\bar{\partial}\Lambda_{\mathrm{I}})_{\alpha\bar{\beta}\bar{\gamma}\bar{\delta}} \propto \nabla_{\alpha}\nabla^2 f\,\bar{\Omega}_{\bar{\beta}\bar{\gamma}\bar{\delta}} = 0\,, \tag{A.14}$$

where we used that f is harmonic, $\nabla^2 f = 0$.

Similarly, we can show that the holomorphic derivative of Λ_{I} vanishes,

$$(\partial\Lambda_{\mathrm{I}})_{\alpha\beta\bar{\gamma}\bar{\delta}} = \nabla_{[\alpha}P_{\beta]\zeta}\bar{\Omega}^{\zeta}{}_{\bar{\gamma}\bar{\delta}} = 0\,. \tag{A.15}$$

To see this we use $\nabla_{[\alpha}P_{\beta]\zeta} = \nabla_{[\alpha}\nabla_{\beta]}\nabla_{\zeta}f$, which vanishes because the holomorphic covariant derivatives ∇_{α} and ∇_{β} commute; their commutator is the Riemann tensor, which has no nontrivial components with only holomorphic indices if the metric is Kähler. We have therefore shown that $\mathrm{d}\Lambda_{\mathrm{I}} = (\partial + \bar{\partial})\Lambda_{\mathrm{I}} = 0$. $\qquad\square$

Series II: three-form of type $(1,2) + (2,1)_{\mathrm{NP}}$. Our next ansatz for the two-form in eq. (A.7) is

$$P_{(\alpha\sigma)} = \nabla_{\bar{\zeta}}\nabla_{(\alpha}f\omega^{\bar{\zeta}}{}_{\sigma)}\,, \tag{A.16}$$

where

$$\omega^{\bar{\zeta}}{}_{\sigma} \equiv \Omega^{\bar{\zeta}}{}_{\sigma\rho}k^{\rho}\,, \qquad k^{\rho} \equiv g^{\rho\bar{\xi}}\partial_{\bar{\xi}}k\,. \tag{A.17}$$

The resulting three-form,

$$\Lambda_{\mathrm{II}}^{(1,2)} = \partial\bar{\partial}f \wedge \bar{\partial}k + \frac{1}{2}J \wedge \bar{\partial}(\partial_{\rho}f k^{\rho}) - \frac{1}{2}\nabla^2 f\,J \wedge \bar{\partial}k\,, \tag{A.18}$$

is *not* closed. However, it becomes closed when an appropriate $(2,1)$ piece is added,

$$\Lambda_{\mathrm{II}}^{(2,1)} = \partial\left(f + \frac{1}{2}\partial_{\rho}f k^{\rho}\right) \wedge J\,, \tag{A.19}$$

so that we have $\Lambda_{\mathrm{II}} \equiv \Lambda_{\mathrm{II}}^{(1,2)} + \Lambda_{\mathrm{II}}^{(2,1)}$,

$$\Lambda_{\mathrm{II}} = (\partial + \bar{\partial})\left(f + \frac{1}{2}\partial_{\rho}f k^{\rho}\right) \wedge J + \partial(\bar{\partial}f \wedge \bar{\partial}k)\,. \tag{A.20}$$

2. *The three-form Λ_{II} defined in eq. (A.20) is closed.*

Proof. We aim to show that $d\Lambda_{\mathrm{II}} = (\partial + \bar{\partial})\Lambda_{\mathrm{II}} = 0$. We will begin by showing that k^σ is holomorphic, i.e.

$$\partial_{\bar{\zeta}} k^\sigma = \nabla_{\bar{\zeta}} k^\sigma = g^{\sigma\bar{\rho}} \nabla_{\bar{\zeta}} \nabla_{\bar{\rho}} k = 0 \,. \tag{A.21}$$

To prove this, it suffices to prove that

$$\nabla_m \nabla_n k = g_{mn} \,, \tag{A.22}$$

where m, n are *real* indices. This result is easily checked for any cone with metric $ds^2 = dr^2 + r^2 d\Omega_{X_5}^2$ and Kähler potential $k = \frac{r^2}{2}$.

We now show that $\bar{\partial}\Lambda_{\mathrm{II}}^{(1,2)}$ vanishes. Since k^σ is holomorphic and Ω is covariantly constant, we can extract $\omega^{\bar{\zeta}}{}_\beta$ from the covariant derivative of eq. (A.16). We find

$$\bar{\partial}_{[\bar{\delta}} \Lambda_{\alpha|\bar{\beta}\bar{\gamma}]} = T_\alpha \, \bar{\Omega}_{\bar{\beta}\bar{\gamma}\bar{\delta}} \,, \tag{A.23}$$

where

$$T_\alpha \equiv \frac{1}{2} \nabla_{\bar{\zeta}} \nabla_{\bar{\sigma}} \nabla_\alpha f \, \Omega^{\bar{\zeta}\bar{\sigma}}{}_\rho k^\rho + \frac{1}{2} g^{\rho\bar{\sigma}} \nabla_{\bar{\sigma}} \nabla_{\bar{\zeta}} \nabla_\rho f \, \omega^{\bar{\zeta}}{}_\alpha \,. \tag{A.24}$$

Commuting $\nabla_{\bar{\zeta}}$ and $\nabla_{\bar{\sigma}}$ with ∇_α in the first term we find

$$\nabla_{\bar{\zeta}} \nabla_{\bar{\sigma}} \nabla_\alpha f = \nabla_\alpha \nabla_{\bar{\zeta}} \nabla_{\bar{\sigma}} f - R^{\bar{\rho}}{}_{\bar{\zeta}\bar{\sigma}\alpha} \nabla_{\bar{\rho}} f \,. \tag{A.25}$$

The result is symmetric in the $\bar{\zeta}$ and $\bar{\sigma}$ indices because

$$R^{\bar{\rho}}{}_{\bar{\zeta}\bar{\sigma}\alpha} = \partial_\alpha \Gamma^{\bar{\rho}}{}_{\bar{\zeta}\bar{\sigma}}. \tag{A.26}$$

Since $\Omega^{\bar{\zeta}\bar{\sigma}}{}_\rho$ is antisymmetric in $\bar{\zeta}$ and $\bar{\sigma}$, the first term in eq. (A.24) vanishes. The second term vanishes because f is harmonic.

Before we calculate $\partial\Lambda_{\mathrm{II}}^{(1,2)}$, let us rewrite eq. (A.16) in a slightly different form

$$P_{(\alpha\sigma)} = \nabla_{\bar{\zeta}} \nabla_{(\alpha} f \omega^{\bar{\zeta}}{}_{\sigma)} = \nabla_{\bar{\zeta}} \nabla_\alpha f \omega^{\bar{\zeta}}{}_\sigma + \nabla_{\bar{\zeta}} \nabla_{[\alpha} f \omega^{\bar{\zeta}}{}_{\sigma]} \,. \tag{A.27}$$

After contracting eq. (A.27) with $\bar{\Omega}$ as in eq. (A.7) the first term gives

$$\nabla_{\bar{\beta}} \nabla_\alpha f k_{\bar{\gamma}} - \nabla_{\bar{\gamma}} \nabla_\alpha f k_{\bar{\beta}} \,, \tag{A.28}$$

while the second term should be proportional to $P^{(0,1)} \wedge J$ for some appropriate one-form $P^{(0,1)}$,

$$\nabla_{\bar{\zeta}} \nabla_{[\alpha} f \omega^{\bar{\zeta}}{}_{\sigma]} \bar{\Omega}^\sigma{}_{\bar{\beta}\bar{\gamma}} = P_{\bar{\beta}} g_{\alpha\bar{\gamma}} - P_{\bar{\gamma}} g_{\alpha\bar{\beta}} \,. \tag{A.29}$$

To find $P^{(0,1)}$ we contract eq. (A.29) with $g^{\alpha\bar{\beta}}$. Because f is harmonic we find

$$P_{\bar{\gamma}} = -\frac{1}{2} \nabla_{\bar{\gamma}} (\partial_\rho f k^\rho) \,. \tag{A.30}$$

If we combine this result with eq. (A.28) the resulting $\Lambda_{\mathrm{II}}^{(1,2)}$ acquires the simple form

$$\Lambda_{\mathrm{II}}^{(1,2)} = \partial\bar{\partial} f \wedge \bar{\partial} k + \frac{1}{2} J \wedge \bar{\partial}(\partial_\rho f k^\rho) \,. \tag{A.31}$$

We are now ready to calculate the holomorphic derivative $\partial\Lambda_{\text{II}}^{(1,2)}$,

$$\partial\Lambda_{\text{II}}^{(1,2)} = \partial\bar{\partial}\left(f + \frac{1}{2}\partial_\rho f k^\rho\right) \wedge J\,. \tag{A.32}$$

This does *not* vanish, so we need to introduce a $(2,1)$ piece

$$\Lambda_{\text{II}}^{(2,1)} = \partial\left(f + \frac{1}{2}\partial_\rho f k^\rho\right) \wedge J\,, \tag{A.33}$$

for which obviously $\partial\Lambda_{\text{II}}^{(2,1)} = 0$. We see that

$$\partial\Lambda_{\text{II}}^{(1,2)} + \bar{\partial}\Lambda_{\text{II}}^{(2,1)} = 0\,. \tag{A.34}$$

We have therefore shown that $\mathrm{d}\Lambda_{\text{II}} = (\partial + \bar{\partial})\Lambda_{\text{II}} = 0$, where $\Lambda_{\text{II}} \equiv \Lambda_{\text{II}}^{(1,2)} + \Lambda_{\text{II}}^{(2,1)}$. $\qquad\square$

Series III: three-form of Type $(1,2) + (2,1)_{\text{NP}} + (3,0)$. Our next ansatz for the two-form in eq. (A.7) uses two derivatives acting on a harmonic function f,

$$P_{(\alpha\sigma)} = \nabla_{\bar{\zeta}}\nabla_{\bar{\rho}}f\,\omega^{\bar{\zeta}}{}_\alpha\omega^{\bar{\rho}}{}_\sigma\,. \tag{A.35}$$

The resulting $(1,2)$ form,

$$\Lambda_{\text{III}}^{(1,2)} = \bar{\partial}(\bar{\partial}f \cdot \omega) \wedge \bar{\partial}k\,, \tag{A.36}$$

is *not* closed, but the three-form becomes closed when appropriate $(2,1)$ and $(3,0)$ pieces are added,

$$\Lambda_{\text{III}}^{(2,1)} = (\bar{\partial}h \cdot \omega) \wedge J\,, \tag{A.37}$$

$$\Lambda_{\text{III}}^{(3,0)} = (2h + k^\xi\partial_\xi h)\Omega\,, \tag{A.38}$$

where $h \equiv 3f + k^\rho\partial_\rho f$.

Our final closed, IASD three-form therefore is $\Lambda_{\text{III}} \equiv \Lambda_{\text{III}}^{(1,2)} + \Lambda_{\text{III}}^{(2,1)} + \Lambda_{\text{III}}^{(3,0)}$,

$$\Lambda_{\text{III}} = (2h + k^\xi\partial_\xi h)\Omega + (\bar{\partial}h \cdot \omega) \wedge J + \bar{\partial}(\bar{\partial}f \cdot \omega) \wedge \bar{\partial}k\,. \tag{A.39}$$

3. *The three-form Λ_{III} defined in eq. (A.39) is closed.*

Proof. We aim to show that $\mathrm{d}\Lambda_{\text{III}} = (\partial + \bar{\partial})\Lambda_{\text{III}} = 0$.

Contracting eq. (A.35) with $\bar{\Omega}$ we get

$$\Lambda_{\alpha\bar{\beta}\bar{\gamma}} = \omega^{\bar{\zeta}}{}_\alpha\left(\nabla_{\bar{\zeta}}\nabla_{\bar{\beta}}f k_{\bar{\gamma}} - \nabla_{\bar{\zeta}}\nabla_{\bar{\gamma}}f k_{\bar{\beta}}\right)\,. \tag{A.40}$$

The derivative $\bar{\partial}\Lambda_{\text{III}}^{(1,2)}$ vanishes because $\omega^{\bar{\zeta}}{}_\alpha$ and $k_{\bar{\gamma}}$ can be extracted from the differentiation and $\nabla_{\bar{\delta}}\nabla_{\bar{\zeta}}\nabla_{\bar{\beta}}f$ will vanish after antisymmetrization with respect to $\bar{\delta}$ and $\bar{\beta}$ (see the discussion following eq. (A.15)).

Next we consider $\partial\Lambda_{\text{III}}^{(1,2)}$. To simplify the expression we contract $(\partial\Lambda)_{\alpha\beta\bar{\gamma}\bar{\delta}}$ with $\bar{\Omega}^{\alpha\beta}{}_{\bar{\rho}}$

$$\frac{1}{2}\bar{\Omega}^{\alpha\beta}{}_{\bar{\rho}}(\partial\Lambda)_{\alpha\beta\bar{\gamma}\bar{\delta}} = g^{\alpha\bar{\alpha}}\nabla_\alpha(\nabla_{\bar{\alpha}}\nabla_{\bar{\gamma}}f k_{\bar{\delta}}k_{\bar{\rho}}) - \nabla_\alpha(\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}f k^\alpha k_{\bar{\delta}}) - [\bar{\gamma} \leftrightarrow \bar{\delta}]\,. \tag{A.41}$$

We expand the covariant derivatives and notice that many terms cancel. The first term in eq. (A.41) is

$$g^{\alpha\bar{\alpha}}\nabla_\alpha(\nabla_{\bar{\alpha}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}}k_{\bar{\rho}}) = g^{\alpha\bar{\alpha}}\nabla_\alpha\nabla_{\bar{\alpha}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}}k_{\bar{\rho}} + \nabla_{\bar{\delta}}\nabla_{\bar{\gamma}}fk_{\bar{\rho}} + \nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}}. \tag{A.42}$$

The first term in eq. (A.42) vanishes because the metric is Ricci-flat and f is harmonic. The second term vanishes after antisymmetrization with respect to $\bar{\gamma}$ and $\bar{\delta}$, and only the third term survives. Similarly, the second term in eq. (A.41) gives

$$\nabla_\alpha(\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}fk^\alpha k_{\bar{\delta}}) = -k^\alpha\nabla_\alpha\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}} - 4\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}}. \tag{A.43}$$

The first term in eq. (A.43) can be simplified as follows: we will use the fact that Kähler manifolds, for which

$$\partial_\alpha(\nabla_{\bar{\beta}}\nabla_{\bar{\gamma}}k) = 0, \tag{A.44}$$

have the property

$$k^\alpha R_{\alpha\bar{\beta}\gamma\bar{\delta}} = 0. \tag{A.45}$$

Therefore, we see that $k^\alpha\nabla_\alpha$ and $\nabla_{\bar{\beta}}$ commute. eq. (A.43) then becomes

$$-\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}(\partial_\alpha fk^\alpha)k_{\bar{\delta}} - 4\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}fk_{\bar{\delta}}. \tag{A.46}$$

Together with eq. (A.41) this gives

$$\frac{1}{2}\bar{\Omega}^{\alpha\beta}{}_{\bar{\rho}}(\partial\Lambda_{\text{III}}^{(1,2)})_{\alpha\beta\bar{\gamma}\bar{\delta}} = -\nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}(3f + \partial_\alpha fk^\alpha)k_{\bar{\delta}} + [\bar{\gamma} \leftrightarrow \bar{\delta}]. \tag{A.47}$$

To cancel this term we introduce the $(2,1)$ form

$$\Lambda_{\alpha\beta\bar{\gamma}}^{(2,1)} = \partial_{\bar{\zeta}}h\left(\omega^{\bar{\zeta}}{}_\alpha g_{\beta\bar{\gamma}} - \omega^{\bar{\zeta}}{}_\beta g_{\alpha\bar{\gamma}}\right), \tag{A.48}$$

for some harmonic function h. We get

$$\frac{1}{2}\bar{\Omega}^{\alpha\beta}{}_{\bar{\rho}}(\bar{\partial}\Lambda_{\text{III}}^{(2,1)})_{\alpha\beta\bar{\gamma}\bar{\delta}} = \nabla_{\bar{\gamma}}(\nabla_{\bar{\rho}}hk_{\bar{\delta}}) - \nabla_{\bar{\gamma}}(\nabla_{\bar{\delta}}hk_{\bar{\rho}}) - [\bar{\gamma} \leftrightarrow \bar{\delta}]. \tag{A.49}$$

The second term vanishes after antisymmetrization so that we find

$$\frac{1}{2}\bar{\Omega}^{\alpha\beta}{}_{\bar{\rho}}(\bar{\partial}\Lambda_{\text{III}}^{(2,1)})_{\alpha\beta\bar{\gamma}\bar{\delta}} = \nabla_{\bar{\rho}}\nabla_{\bar{\gamma}}hk_{\bar{\delta}} - [\bar{\gamma} \leftrightarrow \bar{\delta}]. \tag{A.50}$$

If we choose $h \equiv 3f + \partial_\alpha fk^\alpha$ this cancels eq. (A.47). However, the (2,1) form (A.48) produces a non-trivial $(3,1)$ form after holomorphic differentiation,

$$T_{\bar{\delta}} \equiv \frac{1}{6}\bar{\Omega}^{\alpha\beta\gamma}(\partial\Lambda_{\text{III}}^{(2,1)})_{\alpha\beta\gamma\bar{\delta}} = g^{\alpha\bar{\alpha}}\nabla_\alpha(\nabla_{\bar{\alpha}}hk_{\bar{\delta}}) - \nabla_\alpha(k^\alpha\nabla_{\bar{\delta}}h). \tag{A.51}$$

Because h is harmonic the result is simply

$$T_{\bar{\delta}} = -\nabla_{\bar{\delta}}(2h + k^\alpha\partial_\alpha h). \tag{A.52}$$

To cancel this term we introduce the $(3,0)$ form

$$\Lambda_{\text{III}}^{(3,0)} = (2h + k^\alpha\partial_\alpha h)\Omega_{\alpha\beta\gamma}. \tag{A.53}$$

We have therefore shown that $d\Lambda_{\text{III}} = (\partial + \bar{\partial})\Lambda_{\text{III}} = 0$, where $\Lambda_{\text{III}} \equiv \Lambda_{\text{III}}^{(1,2)} + \Lambda_{\text{III}}^{(2,1)} + \Lambda_{\text{III}}^{(3,0)}$. $\qquad\square$

B Including dilaton gradients

We established in section 2 that small perturbations of the dilaton and metric do not contribute to the leading-order flux-induced potential for a D3-brane. However, many interesting compactifications arising from F-theory contain substantial dilaton gradients that cannot be treated perturbatively. If D7-branes are the only source for these dilaton gradients[27] then the axio-dilaton will be holomorphic, so that the equation of motion (2.9) for the three-form flux reduces to

$$d\Lambda + \partial\phi \wedge (\Lambda + \bar{\Lambda}) = 0 \,, \tag{B.1}$$

where the exterior derivative has been split into holomorphic and anti-holomorphic parts, $d = \partial + \bar{\partial}$. Moreover, the internal space will not be Ricci-flat, but will obey[28]

$$R_{\alpha\bar{\beta}} = \partial_\alpha\phi \, \partial_{\bar{\beta}}\phi \,, \tag{B.2}$$

where $R_{\alpha\bar{\beta}}$ is the Ricci tensor of the internal space. Finally, the Φ_- equation of motion takes the form

$$\nabla^2\Phi_- \;=\; \frac{e^\phi}{96}\,|\Lambda|^2 \,, \tag{B.3}$$

where we call attention to the non-constant prefactor e^ϕ.

In a general compactification, it is challenging to determine the metric and dilaton in the presence of D7-brane sources (*cf.* e.g. ref. [66]). However, we will now show that some of our considerations can be extended to D7-brane backgrounds *without* determining the metric and dilaton explicitly. First, we will generalize the fluxes in Series I to D7-brane backgrounds, solving eq. (B.1). Then, generalizing section 6.1, we will prove that the global supersymmetry interactions encoded in an arbitrary holomorphic superpotential $W(z_a)$ can be geometrized by these fluxes. That is, in the globally-supersymmetric theory arising on a D3-brane probing a noncompact cone containing D7-branes, for any specified superpotential $W(z_a)$ there is a solution of the ten-dimensional equations of motion in which fluxes in Series I give rise to this superpotential. Let us remark that for warped cones with gauge theory duals, the AdS/CFT correspondence guarantees that such a supergravity solution exists for any specified superpotential in the gauge theory; we will specify the fluxes in this solution, in terms of the metric and dilaton.

We begin by presenting a solution of the flux equation of motion (B.1) in the presence of non-constant dilaton perturbations, which we do not assume to be small. Given any holomorphic function f, we turn on the $(1,2)$ flux

$$\Lambda_{\alpha\bar{\beta}\bar{\gamma}} = g_s\, e^{-\phi}\, \nabla_\alpha\nabla_\sigma f\, g^{\sigma\bar{\rho}}\, \bar{\Omega}_{\bar{\rho}\bar{\beta}\bar{\gamma}} \,, \tag{B.4}$$

and the $(3,0)$ flux

$$\Lambda_{\alpha\beta\gamma} = g_s\, \overline{\nabla_\sigma f\, g^{\sigma\bar{\rho}}\, \nabla_{\bar{\rho}} e^{-\phi}}\, \Omega_{\alpha\beta\gamma} \,. \tag{B.5}$$

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[27] The IASD flux G_- discussed in this section sources a running dilaton if the combination $G_+ \cdot G_-$ is non-zero, but this effect is subleading in our expansion scheme (see section 2.3).

[28] In this appendix we set $\kappa_{10} \equiv 1$.

For constant dilaton, $\phi = const = -\ln g_s$, this reduces to the flux in Series I, eq. (A.10). For notational simplicity we now set $g_s \equiv 1$ (or absorb it into the function f). One easily verifies that this combination of fluxes solves eq. (B.1).

4. *The fluxes in eqs. (B.4) and (B.5) satisfy eq. (B.1).*

Proof. We now explain how we constructed the fluxes in (B.4) and (B.5) that solve (B.1). First we establish a few useful identities. We assume that the background manifold is complex, the metric $g_{\alpha\bar\beta}$ is Kähler, the axio-dilaton $\tau = C + ie^{-\phi}$ is holomorphic and the metric is related to the dilaton through (*cf.* eq. (2.11) in ref. [66]),

$$\det g_{\alpha\bar\beta} = q\bar q \, e^{-\phi} \,, \tag{B.6}$$

where q is the same holomorphic function appearing in $\Omega_{\alpha\beta\gamma} = q\epsilon_{\alpha\beta\gamma}$. Because the metric is Kähler the following identity is satisfied

$$\partial_\alpha \left(g^{\alpha\bar\beta} \det g \right) = \partial_\alpha \left(g^{\alpha\bar\beta} q e^{-\phi} \right) = 0 \,. \tag{B.7}$$

(This identity ensures that $\nabla^2 = 2g^{\alpha\bar\beta}\partial_\alpha\partial_{\bar\beta}$.) Holomorphicity of τ implies the following identities

$$\partial_\alpha \tau = -2i\partial_\alpha\phi e^{-\phi} \,, \tag{B.8}$$

$$\partial_\alpha\partial_{\bar\beta} e^{-\phi} = 0 \,, \tag{B.9}$$

$$\partial_\alpha\partial_{\bar\beta}\phi = \partial_\alpha\phi\partial_{\bar\beta}\phi \,. \tag{B.10}$$

The flux equation of motion is then eq. (B.1), and the expression (B.2) for the Ricci tensor follows from $R_{\alpha\bar\beta} = -\partial_\alpha\partial_{\bar\beta}\log\det g$ and (B.6). We are now ready to prove that eq. (B.1) is solved by eqs. (B.4) and (B.5).

Our goal is to construct an IASD three-form Λ that satisfies (B.1) and reduces to the IASD (1,2) form of appendix A for constant dilaton,

$$\Lambda^{\phi=const}_{\alpha\bar\beta\bar\gamma} = \partial_\alpha \left(\partial_\sigma f g^{\sigma\bar\zeta} \bar q \epsilon_{\bar\zeta\bar\beta\bar\gamma} \right) \,. \tag{B.11}$$

Clearly, this form is closed if the dilaton is constant (see appendix A). Now we will try to modify (B.11) to account for a running dilaton. Let us first notice that a (1,2) form will produce (3,1), (2,2) and (1,3) terms in eq. (B.1). If we leave eq. (B.11) unchanged, then $\partial\Lambda^{\phi=const} = 0$ and the (2,2) term $\partial\phi \wedge \Lambda^{\phi=const}$ is not canceled. With this in mind we change eq. (B.11) by an overall factor of $e^{-\phi}$,

$$\Lambda^{(1,2)}_{\alpha\bar\beta\bar\gamma} = \partial_\alpha \left(\partial_\sigma f g^{\sigma\bar\zeta} \bar q \epsilon_{\bar\zeta\bar\beta\bar\gamma} \right) e^{-\phi} \,. \tag{B.12}$$

Now the (2,2) piece $\partial\phi \wedge \Lambda^{(1,2)}$ is canceled by $\partial\Lambda^{(1,2)}$. However, the form in eq. (B.12) is not closed with respect to $\bar\partial$, leading to (1,3) and (3,1) terms in eq. (B.1). To cancel these terms we introduce a (3,0) form

$$\Lambda^{(3,0)}_{\alpha\beta\gamma} = \psi \, q\epsilon_{\alpha\beta\gamma} \,, \tag{B.13}$$

with function ψ to be determined. Letting $\Lambda \equiv \Lambda^{(1,2)} + \Lambda^{(3,0)}$, eq. (B.1) implies two constraints

$$\partial_{\bar{\alpha}} \psi \, q + \partial_{\beta} \phi \partial_{\bar{\alpha}} \left[\overline{\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}} \right] e^{-\phi} = 0 \,, \tag{B.14}$$

$$-\partial_{\bar{\beta}} \left[\partial_{\alpha} (\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] + \partial_{\alpha} \phi \, \bar{\psi} \, \bar{q} = 0 \,. \tag{B.15}$$

Eq. (B.14) is easily solved if we notice that

$$\partial_{\beta} \phi \partial_{\bar{\alpha}} \left[\overline{\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}} \right] e^{-\phi} = -\partial_{\beta} e^{-\phi} \partial_{\bar{\alpha}} \left[\overline{\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}} \right] = \partial_{\bar{\alpha}} \left[\overline{\partial_{\sigma} f g^{\sigma \bar{\zeta}} \bar{q}} \partial_{\zeta} e^{-\phi} \right] \,, \tag{B.16}$$

where in the last step we used eq. (B.9). Thus eq. (B.14) is solved by

$$\psi = \overline{\partial_{\sigma} f g^{\sigma \bar{\beta}} \partial_{\bar{\beta}} e^{-\phi}} \,. \tag{B.17}$$

It turns out that eq. (B.15) is also solved by (B.17). To show this we rewrite the first term in (B.15),

$$\partial_{\bar{\beta}} \left[\partial_{\alpha} (\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] = \partial_{\bar{\beta}} \partial_{\alpha} \left[(\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] - \partial_{\bar{\beta}} \left[(\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) \partial_{\alpha} e^{-\phi} \right] \,, \tag{B.18}$$

$$= \partial_{\alpha} \partial_{\bar{\beta}} \left[(\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] + \partial_{\bar{\beta}} \left[(\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \partial_{\alpha} \phi \right] \,. \tag{B.19}$$

Using eq. (B.7) we then notice that the first term in (B.19) vanishes,

$$\partial_{\bar{\beta}} \left[(\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] = 0 \,, \tag{B.20}$$

if f is harmonic. The second term in (B.19) can be simplified with the help of (B.10),

$$\partial_{\bar{\beta}} \left[\partial_{\alpha} (\partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q}) e^{-\phi} \right] = \partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q} e^{-\phi} \partial_{\bar{\beta}} \partial_{\alpha} \phi = \partial_{\sigma} f g^{\sigma \bar{\beta}} \bar{q} e^{-\phi} \partial_{\bar{\beta}} \phi \partial_{\alpha} \phi \,. \tag{B.21}$$

This proves that (B.17) solves (B.15) and hence, the fluxes in eqs. (B.4) and (B.5) satisfy the equation of motion eq. (B.1) in the case of a running dilaton. $\qquad\square$

Next, we consider the potential of a probe D3-brane in the presence of the above fluxes, specified by a holomorphic function f. This flux is dual to a perturbation of the superpotential $W \propto f$, and the corresponding potential should be

$$V = g^{\alpha \bar{\beta}} \nabla_{\alpha} W \overline{\nabla_{\beta} W} \,. \tag{B.22}$$

Now we will show how eq. (B.22) will arise from eq. (B.3). To verify the solution, we must show that

$$\nabla^2 \left[g^{\alpha \bar{\beta}} \nabla_{\alpha} f \overline{\nabla_{\beta} f} \right] = \frac{1}{2 \cdot 3!} e^{\phi} |\Lambda|^2 \,, \tag{B.23}$$

with Λ given by eqs. (B.4) and (B.5) and $W^2 = \frac{T_3}{8} f^2$. First, we have

$$\nabla^2 \left[g^{\alpha \bar{\beta}} \nabla_{\alpha} f \overline{\nabla_{\beta} f} \right] = g^{\rho \bar{\sigma}} g^{\alpha \bar{\beta}} \left(2 \nabla_{\rho} \nabla_{\alpha} f \overline{\nabla_{\sigma} \nabla_{\beta} f} + 2 \nabla_{\bar{\sigma}} \nabla_{\alpha} f \nabla_{\rho} \nabla_{\bar{\beta}} \bar{f} \right) + \tag{B.24}$$

$$+ \ g^{\alpha \bar{\beta}} \nabla^2 \nabla_{\alpha} f \nabla_{\bar{\beta}} \bar{f} + g^{\alpha \bar{\beta}} \nabla_{\alpha} f \nabla^2 \nabla_{\bar{\beta}} \bar{f} \,.$$

The second term in (B.24) — $g^{\rho\bar\sigma}g^{\alpha\bar\beta}\nabla_{\bar\sigma}\nabla_\alpha f\nabla_\rho\nabla_{\bar\beta}\bar f$ — vanishes because f is holomorphic. The first term — $g^{\rho\bar\sigma}g^{\alpha\bar\beta}\nabla_\rho\nabla_\alpha f\overline{\nabla_\sigma\nabla_\beta f}$ — is equal to $\frac{1}{3!}e^\phi|\Lambda^{(1,2)}|^2$. We therefore just need to show that the last two terms combine into

$$\frac{2}{3!}e^\phi|\Lambda^{(3,0)}|^2 = 2|\partial_\alpha f g^{\alpha\bar\beta}\partial_{\bar\beta}\phi|^2 \ . \tag{B.25}$$

To calculate $\nabla^2\nabla_\alpha f$ we use real notation,

$$\nabla^2\nabla_k f = g^{ij}\nabla_i\nabla_j\nabla_k f = g^{ij}(\nabla_k\nabla_i\nabla_j f - R^l_{jik}\nabla_l f) \ . \tag{B.26}$$

The first term vanishes because f is harmonic and the second is equal to $R_{mk}g^{ml}\nabla_l f$. Hence, using (B.2),we obtain

$$g^{\alpha\bar\beta}\nabla^2\nabla_\alpha f\overline{\nabla_\beta f} + \text{c.c.} = g^{\alpha\bar\beta}R_{\bar\sigma\alpha}g^{\bar\sigma\rho}\nabla_\rho f\overline{\nabla_\beta f} + \text{c.c.} = 2|\partial_\alpha f g^{\alpha\bar\beta}\partial_{\bar\beta}\phi|^2 \ . \tag{B.27}$$

Comparing to (B.25), we conclude that (B.23) is satisfied.

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