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Daily Scheduling of Nurses in Operating Suites

Abstract

This paper provides a new multi-objective integer programming model for the daily scheduling of nurses in operating suites. The model is designed to assign nurses to different surgery cases based on their specialties and competency levels, subject to a series of hard and soft constraints related to nurse satisfaction, idle time, overtime, and job changes during a shift. To find solutions, two methodologies were developed. The first is based on the idea of a solution pool and the second is a variant of modified goal programming. The model and the solution procedures were validated using real data provided by the University of Texas MD Anderson Cancer Center in Houston, Texas. The results show that the two methodologies can produce schedules that satisfy all demand with 50% less overtime and 50% less idle time when benchmarked against current practice.

1 Introduction

The growing shortage of registered nurses continues to hamper the effective delivery of healthcare throughout the United States and much of Europe (Ulrich et al., 2002). In a clinical setting, not being able to assign a sufficient number of qualified nurses to a shift places undue stress on those who are called upon to take up the slack. This, in turn, leads to reduced patient safety and substandard treatment (Bester et al., 2007, Blythe et al., 2005, Cheng et al., 1997, Chiaramonte and Chiaramonte, 2008). In the face of personnel shortages, hospital managers are under constant pressure to balance the need to cover demand with the available staff, but without jeopardizing job performance and satisfaction.

Assigning each available nurse to the right place at the right time to do the right job is a major concern for healthcare organizations. As the types of services and treatments offered by such organizations increase so do the skill requirements of their staff. In particular, nurses are specifically trained and certified to work in a subset of units such as orthopedics or gerontology and cannot be floated outside their areas of expertise. To make the most efficient use of the available workforce then, it is necessary to adjust their schedules daily as the patient population changes. When constructing individual schedules, one must consider the relevant skill categories, time-off requests, shift preferences, demand variability by unit, break times and seniority, as well as system-wide factors such as regulatory and union requirements, contractual agreements, and overtime. In addition, each unit of a hospital may have a host of specific rules that further restrict the staffing decisions. Although most of these rules are common throughout a hospital, there are a number of exceptions, such as the way breaks are handled, when it comes to the operating suites, the main focus of this paper. These exceptions further frustrate our ability to generate efficient daily schedules.

An operating suite is an area in a hospital consisting of several operating rooms (ORs) where surgical procedures are performed. Each OR is designed to accommodate specific types of surgeries using specialized equipment. Because the operating suites are the largest revenue source in a hospital (Belien and Demeulemeester, 2008) it is critical that they be scheduled to achieve maximum utilization. Although we do not address this issue directly, it does have an impact on our research since underutilized facilities may result in unnecessary nurse idle time, while overbooked ORs may produce excessive overtime and high levels of staff dissatisfaction. When the prevailing nursing shortage (Cardoen et al., 2010) is coupled with an increasing demand for surgeries linked to an aging population, it is essential for hospital managers to maximize the use of their ORs and to make the most efficient use of their nursing staff.

With this background in mind, the primary contribution of this research is the development of a nurse scheduling tool designed to minimize overtime and idle time, delays due to staff shortages, and assignment changes during the day, as well as to maximize case demand satisfaction. With respect to the latter, it is critical to take into account nurse specialties and competency levels along with various types of constraints such as shift limitations, contractual agreements and break requirements when assigning them to cases. Considering the conflicting nature of these objectives and insufficient resources to satisfy all constraints, it is not possible to identify and solve a single optimization model. Instead, we attempt to create a pool of Pareto-optimal solutions such that each optimizes at least one objective.

The rest of the paper is organized as follows. In Section 2 we provide a summary of the recent nurse scheduling literature most relevant to our work. In Section 3 the planning environment is outlined and a formal problem statement is given. In Section 4 we introduce our optimization model for assigning nurses to surgery cases and, in Section 5, propose two competing solution algorithms whose operations are described in detail. The first is based on the idea of a solution pool and the second takes a modified goal programming approach. Numerical results are presented in Section 6 where it is shown that reductions in idle time and overtime of 50% or more are achievable with respect to current practice. We close in Section 7 with several observations and suggestions for future research. This work was done in conjunction with the University of Texas MD Anderson Cancer Center (MDACC) who defined the problem and provided the test data.

2 Literature Review

Given the ever increasing labor costs in the healthcare industry and the fact that the nursing staff accounts for approximately 50% of a hospital's budget, a great deal of effort has been spent trying to improve the nurse scheduling process (Ernst et al., 2004). Recent surveys include Cheang et al. (2003) and Burke et al. (2004). With respect to the mid-term planning problem (up to 6 weeks), integer programming (IP) methods have been widely used to either minimize cost or maximize nurse preferences (Aickelin and Li, 2007, Bard and Purnomo, 2007, Ogulata et al., 2008). Others have taken a heuristic approach including (Bai et al., 2010, Bard and Purnomo, 2007, Goodman et al., 2009, He and Qu, 2009, Maenhout and Vanhoucke, 2010, Oddoye et al., 2006, Parr and Thompson, 2007, Sundaramoorthi et al., 2009, Tsai and Li, 2009).

In contrast, there has been only a limited amount of research on the daily scheduling problem which begins with the mid-term schedule and makes daily adjustments to account for changes in demand, absenteeism, and skill mismatches (Burke et al., 2010, Hansen et al., 2010). Bard and Purnomo (2005) formulated the problem as a mixed integer program and developed a branch-and-price algorithm to find solutions. The objective was to minimize a combination staff dissatisfaction costs due to preference violations and the cost of using agency nurses to cover shortages. The model took a hospital-wide view and considered a float pool, overtime, and call-in nurses on their days off. Optimal solutions for instances with up to 200 nurses were obtained using a rolling horizon framework for each shift.

As mentioned in Section 1, due the importance of surgery and the unique teaming requirements for each case, scheduling nurses in operating suites has been considered separately from scheduling them in other areas of the hospital. Cardoen et al. (2010) provide a comprehensive review of recent applications of operations research methods to operating room planning and scheduling. They evaluated multiple domains related to problem settings, performance measures, solution methods and uncertainty considerations. Much of the ongoing work is aimed at improving existing methods to find the the best surgery-room assignments, surgery-physician assignments, and surgery-block time assignments (Cardoen, 2010, Dexter et al., 1999, Ozkarahan, 2000).

Feia et al. (2008) developed an integer programming model for assigning surgical cases to several multi-functional ORs with the objective of minimizing total operating cost. A branch-and-price procedure was used to find solutions. The results were promising and showed that the approach is capable of solving large instances In related work, Fei et al. (2009) addressed a tactical OR planning problem and used a set-partitioning model to assign surgical cases to ORs one week at a time. The assignments were based on an open scheduling strategy. Two conflicting objectives were considered: maximize OR utilization and minimize overtime. To find solutions quickly, they relied on a column generation based heuristic. Lamiri et al. (2008) proposed a stochastic model for OR scheduling for both elective and emergency patients. The elective patients were assigned to the different OR blocks depending on the type of surgery needed while emergency patients were assigned to available time slots when they arrived at the facility. The objective was to minimize related costs and overtime. The problem was formulated as a mixed integer program and solved with a combination of optimization techniques and Monte Carlo simulation to deal with the uncertainty of surgical durations and emergency arrivals.

Belien and Demeulemeester (2008) developed an integrated nurse and surgery scheduling system for a weekly planning problem. The underlying model took the form of an integer program with the objective of meeting nurse preferences while assuring that demand requirements were satisfied in general, but not necessarily with respect to skill type. It was assumed that adequate resources were available to cover all demand. The basic model was extended by enumerating all possible ways of assigning operating blocks to the different surgeons subject to surgery demand and capacity restrictions. A branch-and-price algorithm was proposed to find solutions. The pricing subproblems used to generate columns for the master problem were solved with a standard dynamic program and the results compared with those obtained with a commercial optimizer.

Although there exists a vast amount of literature on OR scheduling and nurse scheduling, there has been little if any research on the problem of scheduling nurses in operating suites, even after the surgeons and physicians have themselves been scheduled. Procedures for subsequently assigning nurses are ad hoc, inefficient, and often give poor results. To the best of our knowledge, we are the first to address this problem using an optimization approach.

3 Planning Environment and Problem Statement

Services in an OR are provided by surgeons, nurses and anesthesia professionals who have been trained for different specialties. Figure 1 depicts the layout of MD Anderson Cancer Center's main operating suite, which consists of 32 ORs. Most are multi-functional and can accommodate different types of cases depending on the equipment and instruments that are required. For example, rooms 31 and 32 can be used for both robotic and brain surgery and have some permanently installed equipment. Most equipment, though, is stored in a nearby block of rooms which also contains break rooms for the nurses where they can rest between assignments and do their paperwork. All ORs are monitored from the control room where nurse managers continually check the status of each and coordinate the resource needs of the upcoming cases. After surgery, patients are transferred to the post-anesthesia care unit (PACU) for recovery and perhaps additional treatment.

Although operating suites are generally run the same from one hospital to the next, our description below is limited to the processes and terminology used at MD Anderson Cancer Center.

Surgery case. A *surgery case* is defined as a series of operations performed by a medical team on a single patient in a single OR over the course of a day. Each case generally falls under one of two dozen specialties typically related to a specific organ or section of the body, and can vary greatly in procedural complexity. We define the *surgery duration* as the time required to finish a case starting from transferring the patient to the OR (patient-in time), performing the procedure (surgery time), and moving the patient to the PACU (patient-out time). For modeling purposes, all surgery durations are given in 1/2-hour increments. Also *surgery demand* is defined as the number of nurses by skill type required for each case in each time period. All of these factors are assumed to be known at the time the schedule is constructed.

Nurse categories in an operating suite. Nurses can have different roles during a surgery based on their skill level. The most recognized roles are *circulation* and *scrub*. A circulator is a registered nurse who coordinates all the needs of the surgical team in the OR during a case. A scrub is really a technician who prepares and passes supplies, equipment, and instruments to the surgeon during the procedure. Registered nurses can do both jobs while scrub techs can only do their job. Each surgery needs at least one circulator and one scrub – both being essential. Nurses are also distinguished by their area of expertise, skill level and certifications. In particular, they are categorized by specialty (e.g., Head and Neck, OPTH, ORTHO, Plastic, Surgical Oncology) and different competency levels based on their experience and qualifications. Not all nurses can work all surgical procedures, though, even when they are adequately trained. Given the complexity of

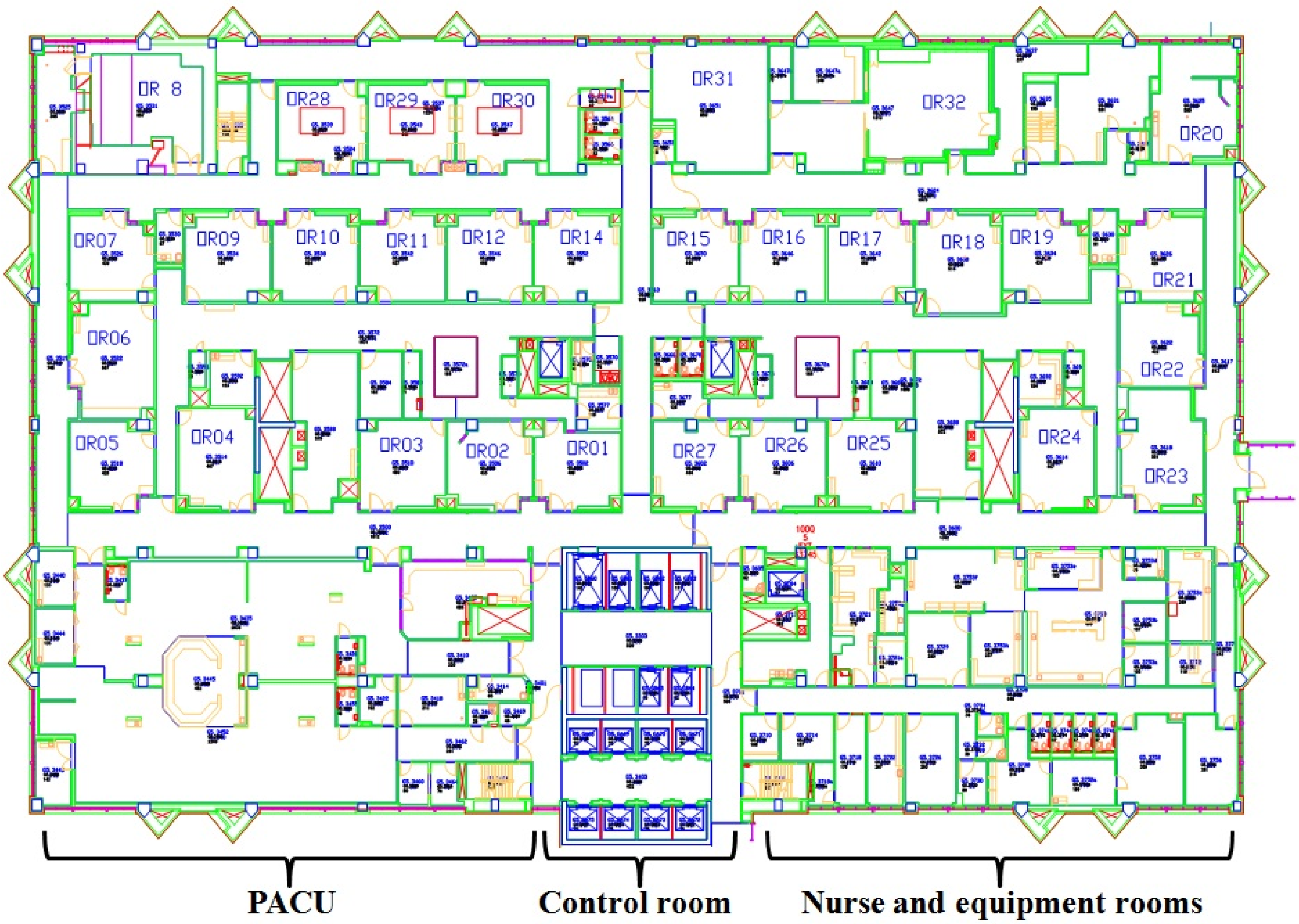

Figure 1: MD Anderson Cancer Center main operating suite

a procedure assignments are made based on the skill level and experience. Of course, nurses who have enough experience to work on harder procedures can also work on easier ones.

Each day the scheduling department provides the nurse managers with the schedule sheets for the next day. These sheets list the planned surgeries, the physician scheduled for each, his or her instrument preferences, the room number, the estimated duration and procedural complexity, as well as the nursing demand. From this information, the nurse managers construct and post the case assignments for the nurses under their jurisdiction.

Shift limitations. Operating suites have shift restrictions along with regulatory and union requirements similar to those imposed on the nurses in other areas of the hospital. We work with five shift types defined by their length and starting time. All nurses upon being hired are assigned to one of these shifts based on their contract. Each shift has its own lunch break and overtime rules. During the scheduling process, available nurses are assigned to the various cases in accordance with their qualifications and surgical needs. However, depending on the criticality of the surgery and the condition of the patient, these rules can be overridden on a case-by-case basis. For instance, nurses cannot leave a surgery to take a break unless the surgery is finished or someone else is available to relieve them.

Given this environment, our aim is to develop a decision support tool that can be used by nurse managers to assign nurses to surgery cases on a daily basis considering nurse availability, shift restrictions, and demand requirements. It is assumed that complete information is available on the number of nurses to be scheduled during each shift, their specialties and competencies, their qualifications to assume certain roles, the surgery schedules and durations, case specialties and procedural complexities, break requirements, and working contract options. The corresponding parameter values are treated as inputs.

In next section, we present an integer programming model for making the daily nurse assignments. As explained in Section 1, conflicting goals must be taken into account when deriving the rosters so we chose not to optimize a single measure but instead to provide a set of high quality solutions to the nurse managers. Then, by further taking into account fairness, seniority and related issues, they can decide which assignments are "best" for their units. For comparative purposes, the candidate schedules will be judged by staff utilization, the need for overtime, the degree to which demand is met, and nurse satisfaction.

4 Nurse Assignment Model (NAM) - Problem Formulation

In formulating our integer programming model, it is necessary to take into account the specialties required for each case and the corresponding procedural complexities. As mentioned, nurses have different competency levels and can only work on surgery cases that match their skills and qualifications. In all, we wish to provide daily rosters that, to some extent balance five objectives to be described presently. We start with the modeling assumptions and a series of hard and soft constraints.

4.1 Assumptions and Notation

The operating suites run five days a week up to 17 hours per day. We assume that each working day can be divided into a fixed number of time intervals of 30 minutes each, giving 38 planning periods. Shifts are defined over 8- or 10-hour periods (not including a 1-hour lunch break) and have different starting times. All shifts include regular hours and authorized overtime hours that bring them up to 13 hours when lunch is included. Authorized overtime hours are additional hours that a nurse can be assigned if a surgery is not finished by the end of her regular shift and either there is no qualified nurse available to provide relief, or some in-progress cases have unfulfilled demand. Note that all case durations are expressed in terms of the basic time interval.

Surgery complexity levels are classified as simple, moderate or complex. Nurses with a higher competency can perform surgery procedures with lower complexity but not vice versa. If a nurse is idle during a shift, it is assumed that she will be assigned to a surgery case as a learning fellow (i.e., a nurse who is assigned to an OR room for the purpose of learning) in order to increase her experience and competency. A final assumption is that each day is independent of another so a daily decomposition is possible; that is, a separate model needs to be solved for each day rather than for multiple days.

We start by introducing some basic notation in this and the next subsection.

$\mathcal{C}$ set of cases scheduled for surgery on the current day

$\mathcal{H}$ time intervals in a working day

$\mathcal{I}$ set of available nurses

$\mathcal{J}$ set of available ORs

$\mathcal{K}$ set of roles that are required for each surgery case

$\mathcal{P}$ set of competency/complexity levels (1: Simple, 2: Moderate, 3: Complex)

$\mathcal{Q}$ set of specialties

$\mathcal{S}$ set of available shifts

$\mathcal{T}$ nurse job category(1 = RN, 2 = scrub tech)

4.2 Input Parameters

The constraints in our model are a function of ten parameters that are defined below. Parameters P_1 and P_2 respectively specify the shift that each nurse works, the role, specialty and competency level of that nurse, and her job category. The next five parameters P_3, P_4, P_5, P_6 and P_7 characterize the surgery cases and their attributes. They provide information about each case duration, specialty and procedural complexity, number of required nurses in each role, and the time intervals in which each is performed. Finally, parameters P_8 and P_9 indicate the starting time of each shift, the regular hours of the shift and authorized overtime hours.

d^2_{ick}	1 if nurse i is assigned to case c to do job k, and 0 otherwise
P^1_{is}	1 if nurse $i \in \mathcal{I}$ is working in shift $s \in \mathcal{S}$, 0 otherwise
P^2_{ikqp}	1 if nurse $i \in \mathcal{I}$ can do role $k \in \mathcal{K}$ in specialty $q \in \mathcal{Q}$ with competency level $p \in \mathcal{P}$, 0 otherwise
P^3_{cj}	1 if case $c \in \mathcal{C}$ is scheduled to happen in OR $j \in \mathcal{J}$, 0 otherwise
P^4_{cqph}	1 if case $c \in \mathcal{C}$ needs specialty $q \in \mathcal{Q}$ and has procedural complexity $p \in \mathcal{P}$ in time interval $h \in \mathcal{H}$, 0 otherwise
P^5_{ckh}	required number of nurses for case $c \in \mathcal{C}$ who can perform role $k \in \mathcal{K}$ in time interval $h \in \mathcal{H}$
P^6_{ch}	1 if case $c \in \mathcal{C}$ is in progress during time interval $h \in \mathcal{H}$, 0 otherwise
P^7_{c}	case $c \in \mathcal{C}$ duration (length of surgery)
P^8_{sh}	1 if shift $s \in \mathcal{S}$ contains time interval $h \in \mathcal{H}$ as regular working hours, 0 otherwise
P^9_{sh}	1 if shift $s \in \mathcal{S}$ contains time interval $h \in \mathcal{H}$ as authorized overtime hours, 0 otherwise
M	big number estimated from other parameter values

4.3 Decision Variables

Our aim is to determine which nurse should be assigned to which surgery case, during which time intervals, and what their role should be. The corresponding decision variable is:

$$y_{ickh} = \begin{cases} 1, & \text{if nurse } i \in \mathcal{I} \text{ works on case } c \in \mathcal{C} \text{ in time interval } h \in \mathcal{H} \text{ doing role } k \in \mathcal{K} \\ 0, & \text{otherwise} \end{cases}$$

4.4 Constraints

The constraints listed in this section are applicable for all nurses without considering break hours. They are divided into two sets. The first set corresponds to those constraints that must be satisfied (hard) and the second to those that may be violated but at a cost (soft). The latter are related to our various objective functions and are referred to as *objective constraints*.

Hard constraints. These constraints cannot be violated under any circumstances, and concern shift restrictions, nurse skill levels, and case requirements, to name a few. Although some nurses are qualified to work in different roles, each nurse must be assigned to one case in each time interval to perform one specific job. During a shift, each nurse can be given at most 8 regular hours of work and 4 overtime hours, or 10 regular hours and 2 overtime hours. Nurses must be assigned to surgery cases based on their specialties and competency levels.

$$\sum_{c\in\mathcal{C}}\sum_{k\in\mathcal{K}} y_{ickh} \leq 1, \qquad i\in\mathcal{I}, h\in\mathcal{H} \tag{1}$$

$$y_{ickh} \leq \sum_{s\in S}\big(P^1_{is}\cdot(P^8_{sh}+P^9_{sh})\big), \qquad i\in\mathcal{I}, c\in\mathcal{C}, k\in\mathcal{K}, h\in\mathcal{H} \tag{2}$$

$$\sum_{c\in\mathcal{C}}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}} y_{ickh} \leq \sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}}\big(P^1_{is}\cdot(P^8_{sh}+P^9_{sh})\big), \qquad i\in\mathcal{I} \tag{3}$$

$$y_{ickh} \leq P^6_{ch}\cdot\sum_{q\in\mathcal{Q},}\sum_{p\in\mathcal{P}}\big(P^4_{cqph}\cdot P^2_{ikqp}\big), \qquad i\in\mathcal{I}, c\in\mathcal{C}, k\in\mathcal{K}, h\in\mathcal{H} \tag{4}$$

$$\sum_{i\in\mathcal{I}} y_{ickh} \geq P^6_{ch}, \qquad c\in\mathcal{C}, k\in\mathcal{K}, h\in\mathcal{H} \tag{5}$$

$$\sum_{h\in\mathcal{H}} y_{ickh} \leq M\cdot d^2_{ick}, \qquad i\in\mathcal{I}, c\in\mathcal{C}, k\in\mathcal{K} \tag{6}$$

$$\sum_{k\in\mathcal{K}} d^2_{ick} \leq 1, \qquad i\in\mathcal{I}, c\in\mathcal{C} \tag{7}$$

Constraints (1) limit the number of cases and roles that nurse i can be assigned in time interval h to at most 1. Constraints (2) and (3) ensure that nurses will only be assigned to cases that are in progress during their shift s, and that the total working hours scheduled will be less than or equal to the total number of regular and overtime hours associated with s. Constraints (4) and (5) ensure that, if a case is in progress, a nurse can only be assigned to it if she has the requisite skills and competency to deal with its procedural complexities.

It is preferred that nurses be scheduled to work continuously during their shift and perform the same job for the duration of each surgery to which they are assigned, rather than alternate between jobs. This is enforced with constraints (6) and (7.

The latter constraints restricts the maximum number of times that a nurse can be assigned to serve in different roles during a specific case and it is at most 1.

Soft (objective) constraints. These constraints need not be satisfied exactly but penalties will be imposed to minimize the magnitude of their violations. Demand satisfaction, room assignments and job changes are considered as objective constraints. Each surgery must be assigned at least one circulator and one scrub, but it may be desirable to include more in each category. Under staffing represents our first permissible deviation that is penalized. In a similar vein, mismatches between supply and demand can lead to a need for overtime, job changes, room changes or excessive breaks during a shift. The corresponding deviations will also be penalized. With respect to the individual nurses, we wish to minimize (i) the number of breaks in a shift, (ii) the amount of overtime assigned, (iii) the number of different cases assigned, (iv) the number of job changes, (v) and the number of breaks over the day. Nurses prefer to work continuously during their shifts and perform the same role, at least for a given surgery, rather than move from one case to another.

To formulate these goals as soft constraints, we define a pair of deviation variables for each; that is,

de_{ckh} 1 if there is a deviation in demand in case $c \in \mathcal{C}$ for job $k \in \mathcal{K}$ in time interval $h \in \mathcal{H}$, 0 otherwise

$\mathcal{DE}$ maximum demand deviation for any case $c \in \mathcal{C}$ and job $k \in \mathcal{K}$

d^7_{ih} 1 if nurse $i \in \mathcal{I}$ is idle in time interval $h \in \mathcal{H}$, 0 otherwise

$\mathcal{DS}$ maximum number of nonconsecutive idle intervals for any nurse $i \in \mathcal{I}$

d^4_{ih} 1 if nurse $i \in \mathcal{I}$ is assigned overtime in time interval $h \in \mathcal{H}$, 0 otherwise

$\mathcal{DF}$ maximum amount of overtime assigned to any nurse $i \in \mathcal{I}$

x_{ij} 1 if nurse $i \in \mathcal{I}$ is assigned to room $j \in \mathcal{J}$, 0 otherwise

$\mathcal{X}$ maximum number of room assignments for any nurse $i \in \mathcal{I}$

nc_{ic} 1 if nurse $i \in \mathcal{I}$ is assigned to case $c \in \mathcal{C}$, 0 otherwise

cd_{ic} 1 if the assignment of nurse $i \in \mathcal{I}$ to case $c \in \mathcal{C}$ is broken, 0 otherwise

$\mathcal{NCT}$ maximum number of case assignments given to any nurse $i \in \mathcal{I}$

$\mathcal{CDT}$ maximum number of times an individual assignment to a case is broken due to shortages for any nurse $i \in \mathcal{I}$

Based on these definitions, the soft constraints can be stated as follows.

$$\sum_{i \in \mathcal{I}} y_{ickh} + de_{ckh} \geq P^5_{ckh} \cdot P^6_{ch}, \qquad c \in \mathcal{C}, k \in \mathcal{K}, h \in \mathcal{H} \tag{8}$$

$$\mathcal{DE} \geq \sum_{h \in \mathcal{H}} de_{ckh}, \qquad c \in \mathcal{C}, k \in \mathcal{K} \tag{9}$$

$$\left| \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}} y_{ick(h+1)} - \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}} y_{ickh} \right| \leq d^7_{ih}, \qquad i \in \mathcal{I}, h \in \mathcal{H} \tag{10}$$

$$\mathcal{DS} \geq \sum_{h \in \mathcal{H}} d^7_{ih}, \qquad i \in \mathcal{I}, \tag{11}$$

$$\sum_{c\in\mathcal{C}}\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}}\left(P^3_{cj}\cdot y_{ickh}\right)\leq M\cdot x_{ij}, \qquad i\in\mathcal{I}, j\in\mathcal{J} \tag{12}$$

$$\mathcal{X}\geq\sum_{j\in\mathcal{J}}x_{ij}, \qquad i\in\mathcal{I} \tag{13}$$

$$d^4_{ih}-\sum_{s\in\mathcal{S}}P^1_{is}\cdot P^9_{sh}\cdot\sum_{c\in\mathcal{C}}\sum_{k\in\mathcal{K}}y_{ickh}\geq 0, \qquad i\in\mathcal{I}, h\in\mathcal{H} \tag{14}$$

$$\mathcal{DF}\geq\sum_{h\in\mathcal{H}}d^4_{ih}, \qquad i\in\mathcal{I} \tag{15}$$

$$\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}}y_{ickh}-\sum_{h\in\mathcal{H}}P^6_{ch}+M\cdot cd_{ic}+M\cdot(1-nc_{ic})\geq 0, \qquad i\in\mathcal{I}, c\in\mathcal{C} \tag{16}$$

$$\sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}}y_{ickh}\leq M\cdot nc_{ic}, \qquad i\in\mathcal{I}, c\in\mathcal{C} \tag{17}$$

$$\mathcal{NCT}\geq\sum_{c\in\mathcal{C}}nc_{ic}, \qquad i\in\mathcal{I} \tag{18}$$

$$\mathcal{CDT}\geq\sum_{c\in\mathcal{C}}cd_{ic}. \qquad i\in\mathcal{I} \tag{19}$$

Constraints (8) and (9) calculate the maximum amount of under coverage for each case and job. If a case is in progress during the time interval h, then the required number of nurses will be assigned to it as long as a sufficient number are available. The two integer deviation variables de_{ckh} and $\mathcal{DE}$ are used to account for shortages for case c and role k in time interval h and to determine the maximum shortage, respectively. As discussed in the next section, by minimizing $\mathcal{DE}$, we assure that the demand for circulators and scrubs for each case will be satisfied as closely as possible. The remaining variables and constraints are used for similar purposes.

To control nonconsecutive breaks in a shift, we introduce constraints (10) and (11) which are a function of the binary deviation variables d^7_{ih} and the integer deviation variable $\mathcal{DS}$. By minimizing $\mathcal{DS}$, we force the idle intervals in a nurse's schedule to be reduced whenever it is possible to do so without violating any of the hard constraints. As an example, given the following two feasible assignments for five periods: on-on-off-off-on and on-off-on-off-on, the first would be preferred since it contains only one idle interval while the second contains two.

Constraints (12) and (13) address the preference for assigning nurses to work continuously in one OR during their shift rather than jumping from one room to another. To minimize these movements, we make use of the two binary deviation variables x_{ij} and $\mathcal{X}$. By minimizing $\mathcal{X}$, we ensure that the maximum number of ORs to which a nurse is assigned is reduced as much as possible. To account for overtime, we introduce the binary variable d^4_{ih}, which takes a value 1 in constraints (14) if nurse i is assigned overtime in period h, 0 otherwise, and an integer deviation variable $\mathcal{DF}$. Constraints (14) count the number of overtime periods assigned to each nurse. By minimizing $\mathcal{DF}$, we assure that overtime will only be assigned when regular hours are not available.

Finally, constraints (16), (17), (18) and (19) with deviation variables nc_{ic}, cd_{ic}, $\mathcal{NCT}$ and $\mathcal{CDT}$ assure that if a nurse is assigned to a case, it will be for its entire duration unless staffing shortages dictate that she be reassigned elsewhere to avoid violating a hard constraint. Also, by minimizing the maximum number of cases that a nurse can work, we limit her movement from period to period.

4.5 Objective Function

As implied in the above discussion, we wish to minimize six conflicting objectives. Each corresponds to a maximum deviation from a stated goal. In particular, the objective of NAM is to minimize:

$\mathcal{DE}$ maximum demand deviation for any case $c \in \mathcal{C}$ and job $k \in \mathcal{K}$

$\mathcal{DS}$ maximum number of nonconsecutive breaks for any nurse $i \in \mathcal{I}$

$\mathcal{DF}$ maximum amount of overtime assigned to any nurse $i \in \mathcal{I}$

$\mathcal{X}$ maximum number of room assignments to any nurse $i \in \mathcal{I}$

$\mathcal{NCT}$ maximum number of cases assigned to any nurse $i \in \mathcal{I}$

$\mathcal{CDT}$ maximum number of times an individual assignment to a case is broken due to shortages for any nurse $i \in \mathcal{I}$

The corresponding objective function can be written as

$$\text{Minimize } [\mathcal{DE}, \mathcal{DF}, \mathcal{DS}, \mathcal{X}, \mathcal{NCT}, \mathcal{CDT}] \tag{20}$$

and is designed to ensure that the maximum deviation for each goal (worst case results) will be minimized for all nurses and cases.

5 Solution Methods

A variety of solution techniques are available to deal with the multi-objective nature of the nurse assignment model. One of the most common is to construct a single objective function by weighting each term (deviation) and summing them. This was the first scheme we tried but were unable to get optimal solutions with CPLEX after 10 hours of computations. Nevertheless, we make use of this idea indirectly in one of our two proposed methods. The first is based on the idea of constructing a pool of high quality solutions; the second makes use of goal programming techniques (e.g., see Schniederjans, 1995). Each is described in the remainder of this section.

5.1 Solution Pool Approach

Rather than terminate with an "optimal" solution to an optimization problem, several of the more powerful commercial codes offer a feature that accumulates feasible solutions in a pool. CPLEX, for example, now has the option to save a specified number of solutions that are within a given percentage of the optimum (IBM, 2010). In our first approach to the NAM, called SPM, we use this feature in CPLEX to generate a set of candidate rosters which are then evaluated with respect to the six objectives. The methodology is outlined in Algorithm 1 and consists of three steps. In Step 1, all input data sets, parameters and goals are initialized, and an index j is assigned to each variable contained in (20). In Step 2, we solve a single objective problem for each deviation. All hard constraints as well as those soft constraints associated with the current deviation are included. For example, for the the first deviation $\mathcal{DE}$, the problem is:

$$\begin{aligned} \text{Minimize} \quad & \mathcal{DE} \\ \text{subject to} \quad & \text{hard constraints: } (1) - (7) \\ & \text{soft constraints: } (8) - (9) \end{aligned} \tag{21}$$

The solution pool feature is applied to this problem to generate S candidates, and the process repeated for the remaining deviations. Since soft constraints do not affect feasibility, regardless of the objective function in model (21), all the feasible regions contain the same hard constraints so a feasible solution with objective j will also be feasible for objective $j' \in \{1, \ldots, J\} \setminus \{j\}$. Thus, given a solution s for objective j, we can calculate the objective values associated with the other deviation j' at s. To be able to compare different candidates, we also calculate a cumulative weighted index CWI_{js} for each solution $s \in \{1, \ldots, S\}$ and objective $j \in \{1, \ldots, J\}$. The weights used in forming the index are based on the decision maker's perceived importance of each deviation. The comparisons are performed in Step 3 and the candidate that yields the smallest CWI is reported as the NAM solution.

Upon termination, the value of the decision variables $Y_{ickh}^{*j^*s^*}$ associated with CWI^* are reported as the solution, where CWI^* is the smallest cumulative weighted index found at Step 3. The values of the six deviations are also reported.

Example 1. To illustrate the approach, suppose that we have a problem with three deviations defined in terms of the goals g_1, g_2 and g_3, a set of hard constraints $Ax \leq B$, and a pair of soft constraints for each deviation j given by $C_j(x) \leq D_j$, where, goal $g_j(x)$ is associated with constraint $C_j(x) \leq D_j$ for $j = 1, 2, 3$. The corresponding multi-objective optimization model can be stated as follows.

$$\begin{aligned} \text{Minimize} \quad & z_1 = g_1(x) \\ \text{Minimize} \quad & z_2 = g_2(x) \\ \text{Minimize} \quad & z_3 = g_3(x) \\ \text{subject to} \quad & Ax \leq B \\ & C_j(x) \leq D_j \qquad \forall\ j = 1, 2, 3 \\ & x \in Z_+ \end{aligned} \tag{23}$$

For each j, the single objective model is

Algorithm 1 Solution Pool Method (SPM)

Step 1: Initialization

• Input: data sets $\mathcal{I}$, $\mathcal{C}$, $\mathcal{K}$, $\mathcal{H}$, $\mathcal{S}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{J}$; parameters P^1, P^2, P^3, P^4, P^5, P^6, P^7, P^8, P^9, P^{10}, M.

• Number of objectives $J = 6$, $j \in \{1, 2, \ldots, J\}$, where $j = 1$ for deviation $\mathcal{DE}$, $j = 2$ for deviation $\mathcal{DF}$, $j = 3$ for deviation $\mathcal{DS}$, $j = 4$ for deviation $\mathcal{X}$, $j = 5$ for deviation $\mathcal{NCT}$, $j = 6$ for deviation $\mathcal{CDT}$.

• Number of solutions S to be generated by the solution pool feature, $s \in \{1, 2, \ldots, S\}$.

• Deviation weights $w^j, j = 1, ..., J$; CPLEX optimality gap α.

Step 2: Solution Generation

for each deviation j **do**

  • Develop a single objective optimization model equivalent to (21).

  • Apply the solution pool feature to generate S candidates using an optimality gap α in CPLEX.

  for each candidate solution s **do**

    • Record components of optimal solution vector as Y^{*js}_{ickh}

    • For all $j' \neq j$, calculate the value of deviation j in (20) using the optimal values Y^{*js}_{ickh}, i.e., $\widehat{\mathcal{DE}}$, $\widehat{\mathcal{DF}}$, $\widehat{\mathcal{DS}}$, $\widehat{\mathcal{X}}$, $\widehat{\mathcal{NCT}}$, and $\widehat{\mathcal{CDT}}$.

    • Calculate the cumulative weighted index as follows.

$$CWI_{js} = w^1\widehat{\mathcal{DE}}_{js} + w^2\widehat{\mathcal{DF}}_{js} + w^3\widehat{\mathcal{DS}}_{js} + w^4\widehat{\mathcal{X}}_{js} + w^5\widehat{\mathcal{NCT}}_{js} + w^6\widehat{\mathcal{CDT}}_{js} \tag{22}$$

    • When $s = S$ or when a predefined time limit is reached, terminate and go to Step 3.

  end for

end for

Step 3: Solution Comparison

• Initialize $CWI^* = M$, $j^* = 0$ and $s^* = 0$.

for $j = \{1, \ldots, J\}$ **do**

  for $s = \{1, \ldots, S\}$ **do**

    IF ($CWI^{\geq}CWI_{js}$); THEN ($CWI^* = CWI_{js}, j^* = j, s^* = s$).

  end for

end for

• Output CWI^*, $\mathcal{DE}^*$, $\mathcal{DF}^*$, $\mathcal{DS}^*$, $\mathcal{X}^*$, $\mathcal{NCT}^*$, $\mathcal{CDT}^*$, Y^*_{ickh}.

$$\begin{aligned} \text{Minimize} \quad & z_j = g_j(x) \\ \text{subject to} \quad & Ax \leq B \\ & C_j(x) \leq D_j \\ & x \in Z_+ \end{aligned} \tag{24}$$

After solving each model, assume that the optimal objective values are $g^*_1 = 1$, $g^*_2 = 0$, and $g^*_3 = 1.5$. Next, we apply the solution pool feature for each goal j to get, say, $S = 10$ candidates. For each candidate s and goal j the value of the other goals can be evaluated. Table 1 provides the results for $j = 2$ (the corresponding results for $j = 1$ and 3 are reported in the Appendix). As expected, objective values for $j = 2$ are noticeably smaller than those associated with the other two objectives. Now, using the weights 0.5, 0.3 and 0.2 for goals $g_1(x)$, $g_2(x)$ and $g_3(x)$, respectively, we obtain the value of CWI_{js} from Eq. (25). The calculations for $j = 2$ are given in the bottom row of Table 1.

$$CWI_{js} = 0.5 \cdot \hat{g}_{1s} + 0.3 \cdot \hat{g}_{2s} + 0.2 \cdot \hat{g}_{3s} \tag{25}$$

Table 1: Results obtained from SPM for single objective model for goal $j = 2$

Goal	Solution number, s									
j	1	2	3	4	5	6	7	8	9	10
1	1.5	1.2	1	1.2	1.1	1	1.3	1	1.1	1.05
2	0	0	0	0.01	0.02	0	0.015	0.03	0	0
3	1.5	2	3	1.7	1.9	2.5	1.8	1.6	2.1	3.1
CWI_{2s}	1.05	1	1.1	0.943	0.936	1	1.0145	**0.829**	0.97	1.145

Comparing the CWI_{js} values for all goals j and solutions s in the corresponding tables, we see that the eighth solution ($s = 8$) obtained from solving the single objective model for $j = 2$ provided the smallest value of CWI. Therefore $CWI^* = 0.829$ as highlighted in Table 1. For the given weights and solution pool this was the best solution obtained using SPM.

5.2 Modified Goal Programming Approach

The second algorithm we developed for solving the daily nurse scheduling problem is based on preemptive goal programming (GP) which similarly requires a prioritization of goals (Taha, 2006). The alternative known as the weights method is closer to our first algorithm where the values of $w^j, j = 1, ..., J$ need to be specified. In preemptive GP, a sequence of mixed-integer programs (MIPs) are solved where each successive MIP has an additional constraint derived from the previous solution.

To illustrate, assume that we have a bi-objective optimization problem with integer variables x and two goals given by the functions $g_1(x)$ and $g_2(x)$. Assuming that $g_1(x)$ is the more important goal, we begin by minimizing this function to get g_1^*. When the second MIP is constructed with objective function $g_2(x)$, the constraint $g_1(x) \leq g_1^*$ is added to the formulation. The main drawback of this approach is that the new constraint may drastically curtail the size of the feasible region to the point where no other solutions exist. This would be the case if the optimal solution to the first problem were unique. To avoid this situation and to allow for a broader exploration of the solution space, we propose a modified approach based on an "elastic" constraint of the form

$$\begin{aligned} &\text{if } g_1^* \neq 0, \text{ then, } g_1(x) \leq g_1^* \cdot (1 + \beta) \\ &\text{else, } g_1(x) \leq \beta \end{aligned} \tag{26}$$

where β is a new variable that measures the fractional deviation from the optimal objective value g_1^*.

Unlike SPM, our modified goal programming method (MGPM) considers all deviations simultaneously by minimizing their sum in a single objective problem. The first step (see Algorithm 2) is to initialize all input data sets, parameters and goals, and an index j is assigned to each objective. In Step 2, a single objective optimization model equivalent to (21) is solved for each deviation j; call the optimal objective function value OPT_j^* for $j = 1, ...J$. In Step 3, we formulate the new constraints equivalent to (26) by introducing J deviation variables $\beta_1, \ldots, \beta_J$. For example, if $\mathcal{DE}^* > 0$ is the optimal value of the first deviation $\mathcal{DE}$, then (26) becomes $\mathcal{DE} \leq \mathcal{DE}^* \cdot (1 + \beta_1)$. if $\mathcal{DE}^* = 0$, then the constraint is $\mathcal{DE} \leq \beta_1$.

The final model is formulated in Step 4 with the objective of minimizing the sum of the β variables. The hard constraints (1) – (7), the soft constraints (8) – (19), and the elastic constraints are included in the model. By minimizing $\sum_{j\in\{1,\ldots,J\}} \beta_j$ we ensure that the total deviations are as small as possible, and that the final schedules are Pareto-optimal. Although there is no guarantee that the deviations will be evenly spread over all goals, our empirical results show a near-uniform distribution.

Algorithm 2 Modified Goal Programming Method (MGPM)

Step 1: Initialization

- Input: data sets $\mathcal{I}$, $\mathcal{C}$, $\mathcal{K}$, $\mathcal{H}$, $\mathcal{S}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{J}$; parameters P^1, P^2, P^3, P^4, P^5, P^6, P^7, P^8, P^9, P^{10}, M.
- Number of objectives $J = 6$, $j \in \{1, 2, \ldots, J\}$, where $j = 1$ for deviation $\mathcal{DE}$, $j = 2$ for deviation $\mathcal{DF}$, $j = 3$ for deviation $\mathcal{DS}$, $j = 4$ for deviation $\mathcal{X}$, $j = 5$ for deviation $\mathcal{NCT}$, $j = 6$ for deviation $\mathcal{CDT}$.

Step 2: Minimum Deviation Computations

for each deviation j **do**

- Set up and solve a single objective optimization problem equivalent to (24) for each deviation z_j.
- For the j^{th} problem, let the optimal objective function value be OPT_j^*.

end for

Step 3: Optimality Constraint Generation

for each deviation j **do**

- Introduce deviation variable β_j.
- Construct the elastic constraints as follows:

$$\begin{aligned} &\text{If } OPT_j^* \neq 0, \text{ then } z_j \leq OPT_j^* \cdot (1 + \beta_j) \\ &\text{else } z_j \leq OPT_j^* + \beta_j \end{aligned} \tag{27}$$

end for

Step 4: Final Model Formulation

- Add the constraints developed in Step 3 to all hard and soft constraints.
- Solve the following problem:

$$\hat{Z} = \text{Minimize} \left\{ \sum_{j\in\{1,\ldots,J\}} \beta_j : \text{subject to } (1) - (7), (8) - (19), (27) \right\} \tag{28}$$

- Output CWI^*, $\mathcal{DE}^*$, $\mathcal{DF}^*$, $\mathcal{DS}^*$, $\mathcal{X}^*$, $\mathcal{NCT}^*$, $\mathcal{CDT}^*$, Y_{ickh}^*, β^*.

Example 2. We illustrate MGPM using problem (23) introduced in Section 5.1. First, the single objective model given by (24) is solved for each goal j. Let $g_1^* = 1$, $g_2^* = 0$, and $g_3^* = 1.5$ be the optimal objective function values, and let $\beta_j, j = 1, 2, 3$ be the corresponding deviation variables. The elastic constraint for goal $j = 1$ is

$$g_j(x) \leq g_j^* \cdot (1 + \beta_j) \tag{29}$$

Substituting the values g_j^* into Eq. (29) gives

$$g_1(x) \leq 1 \cdot (1 + \beta_1)$$

$$g_2(x) \leq \beta_2$$

$$g_3(x) \leq 1.5 \cdot (1 + \beta_3)$$

Therefore, the final model is

$$\begin{array}{lll} \text{Minimize} & \beta_1 + \beta_2 + \beta_3 & \\ \text{subject to} & Ax \leq B & \\ & C_j(x) \leq D_j & \forall\, j = 1, 2, 3 \\ & g_1(x) \leq 1 + \beta_1 & \\ & g_2(x) \leq \beta_2 & \\ & g_3(x) \leq 1.5 \cdot (1 + \beta_3) & \\ & x \in Z_+ & \end{array} \tag{30}$$

If the partial solution to (30) is $\beta_1 = 0.001$, $\beta_2 = 0$ and $\beta_3 = 0.027$, then the final goal values in model (23) are $g_1(x^*) = 1.001$, $g_2(x^*) = 0$ and $g_3(x^*) = 1.54$, and from Eq. (13) we get $CWI^{MGPM} = 0.809$. The latter value compares favorably with $CWI^{SPM} = 0.829$ which was obtained with Algorithm 1. Thus, for this example at least, MGPM provides a better solution than SPM.

6 Computational Experience

The two algorithms developed to solve the NAM were tested using data provided by MD Anderson Cancer Center, one of the world's most respected cancer treatment facilities. A typical daily roster for a subset of nurses is depicted in Table 10 in the Appendix. In the discussion accompanying the table, we highlight several issues that continue to thwart those responsible for constructing the schedules.

By way of background, MD Anderson Cancer Center handles an estimated 1500 surgeries per month with 148 nurses and scrub techs, and 140 surgeons. Most surgeries are scheduled in advance and patients either arrive at the hospital on the day of their procedure or are transferred from another department beforehand. The task of scheduling nurses is done manually over a 4-hour period from 3 PM to 7 PM the day before with the actual assignments being finalized on the current day. Call-ins, cancellations, overtime and case-duration variability make the problem even more complex and frustrating.

On average, 60 nurses with RN and scrub tech titles are available a various times during the day, depending on their contractual agreements. The following five different shifts are used in the operating suites, each spanning either 8 or 10 regular hours plus overtime and lunch.

Shift 1: 6:30 AM - 2:30 PM (8 regular hours, 4 overtime hours, 1 hour for lunch)

Shift 2: 6:30 AM - 5:30 PM (10 regular hours, 4 overtime hours, 1 hour for lunch)

Shift 3: 10:30 AM - 5:30 PM (8 regular hours, 4 overtime hours, 1 hour for lunch)

Shift 4: 10:30 AM - 7:30 PM (10 regular hours, 2 overtime hours, 1 hour for lunch)

Shift 5: 2:30 PM - 11:30 PM (8 regular hours, 4 overtime hours, 0 hours for lunch)

To construct a problem instance, we collected surgery case data and nurse attribute data for one day in May and one day in November 2010 for the main operating suite. These data, in collaboration with several nurse managers, allowed us to define all model parameters and preference settings. Input information included nurse specialties and competency levels, their shifts and job titles, case specialties, procedural complexities, and demand per case. The basic data set was built around 28 cases to be staffed by 23 RNs and 14 scrubs each working one of the five shifts defined above. The demand for each case was 1 circulator and 1 scrub. For modeling purposes, the day was broken into 38 half-hour time periods starting at 6:30 AM and ending at 11:30 PM. Both algorithms were implemented in a C++ environment and run under Windows Server 2008 R2 on a 2.83 GHz Dell workstation with 16 GB of memory. All integer programs were solved with CPLEX 12.2.

6.1 Results for Solution Pool Method

To evaluate the performance of SPM we conducted four experiments using the different weight values shown in Table 2. Experiment 1 assigns equal weight to all six deviations. The remainder reflect the preferences of the various nurse managers at MD Anderson Cancer Center. In all cases, surgery demand satisfaction ($\mathcal{DE}$) was deemed to be of highest importance. In Experiment 2, less preference is given to overtime ($\mathcal{DF}$) and nonconsecutive assignments ($\mathcal{DS}$) than to moving between cases ($\mathcal{NCT}$) and unfinished surgery assignments ($\mathcal{CDT}$). The third and fourth experiments can be similarly interpreted.

Table 2: Experimental design

Scenario	Weights for deviations					
	$\mathcal{DF}$ (overtime)	$\mathcal{DS}$ (nonconsecutive)	$\mathcal{DE}$ (demand)	$\mathcal{X}$ (rooms)	$\mathcal{NCT}$ (number of cases)	$\mathcal{CDT}$ (breaks)
Experiment 1	100	100	100	100	100	100
Experiment 2	40	40	100	70	80	80
Experiment 3	60	70	100	80	60	80
Experiment 4	90	80	100	70	60	80

Table 3 gives the SPM results for all experiments as well as the implied results for the actual May roster. The first output row reports the values of the deviations found by separately minimizing each of the six deviations using a single objective model equivalent to (24). These values represent lower bounds. The last column identifies the deviation that provided the minimum CWI^* value for the row obtained in Step 3 of Algorithm 1.

The first observation is that when compared to the actual roster, all four experiments provided superior results as measured by CWI^*. For Experiment 2, $CWI^* = 780$, which is only 11.4% above the 700 minimum. Moreover, all experiments yielded schedules with less overtime ($\mathcal{DF}$) and fewer nonconsecutive beaks ($\mathcal{DS}$) than the actual schedules without sacrificing coverage, which was always satisfied. A third observation is that the values associated with the six deviations did not vary much among experiments, with $\mathcal{DF}$, $\mathcal{DS}$ and $\mathcal{X}$ being the same in all cases regardless of their individual weights. For the first three experiments, the best solutions were obtained from model (24) when $\mathcal{CDT}$ was minimized; for the fourth experiment, the best solution was found when $\mathcal{X}$ was minimized. Finally, it should be mentioned that in the actual schedules, some nurses were assigned to cases that were scheduled to start prior to their contracted shift start times. Such disparities are not permitted in our algorithms.

Table 3: SPM results

Scenario	$\mathcal{DF}$	$\mathcal{DS}$	$\mathcal{DE}$	$\mathcal{X}$	$\mathcal{NCT}$	$\mathcal{CDT}$	CWI^*	Optimal deviation (j^*)
Single objective model (24)	2	2	0	1	2	0	700	–
Actual roster	8	10	0	2	2	0	2200	–
Experiment 1	6	4	0	2	3	0	1500	$\mathcal{CDT}$
Experiment 2	6	4	0	2	3	0	780	$\mathcal{CDT}$
Experiment 3	6	4	0	3	3	0	1060	$\mathcal{CDT}$
Experiment 4	6	4	0	1	3	1	1190	$\mathcal{X}$

Table 4 gives the individual assignments for the circulator and scrub roles by case for each experiment. From the results, we see, for example, that nurse 27 is assigned to work as a circulator in case 28, which starts at period and runs through period 27. This suggests that nurse 27 is the best qualified person for the circulation job on case 28. In contrast, looking at case 21, which starts at period 2 and finishes at the end of period 9, we see that nurse 4 is assigned as either a circulator or scrub, depending on the experiment, or is assigned to a different case altogether in Experiment 4 (case 23). In the first three experiments, we can see that some nurses are assigned to cases as learning fellows, while in Experiment 4, there are no such assignments. Given that the demand is one circulator and 1 scrub per case, this observation comes from the fact that more than one nurse is associated with some cases in these roles. For example, case 17 is assigned two scrubs in Experiments 1 and 3.

In sum, the results indicate that good solutions can be obtained with SPM that are relatively insensitive to the weights selected for each goal. In a hospital setting, a nurse manager might run several scenarios, compare schedules, and choose the one that comes closest to her needs or to her nurses' preferences.

Table 4: Individual nurse assignments obtained with SPM

	Duration	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
Case number	(start time - end time)	Nurse # circulator	Nurse # scrub	Nurse # circulator	Nurse # scrub	Nurse # circulator	Nurse # scrub	Nurse # circulator	Nurse # scrub
1	2-5	6	5	1	13	27	33	25	24
2	6-9	27	5	1	24	30	24	25	24
3	10-13	27	24/33	1/4	24	4	24	25	23
4	2-5	16	35	16	37	16	21	36	21
5	8-13	16/34	31	16	37	16	17	36	21
6	14-17	1	24	10	24/37	1	33	36	21
7	2-15	30	3/28	30	28	29	22	3	22
8	16-22	6	3	14	8	23	17	3	22
9	9-14	29	12	6	13	19	31	11	8
10	15-20	34	15	2	35	19	10	16	15
11	2-17	2	13	19	12/22	14	8	19	12
12	2-9	14	10	34	15	34	20	34	2
13	10-18	10	35	34	15	34	21	34	2
14	2-15	11	8	11	14	18	37	30	37
15	2-9	25	33	27	7	3	25	23	31
16	10-17	25	32	25	7	25	7	23	31
17	18-24	30	22/33	30	22	25	22/31	23	31
18	2-9	1	20	3	25	32	7	6	20
19	10-20	14	20	29	31	30	20	6	20
20	2-17	23	22	23	33	26	6	14	33
21	2-9	32	4	4	31	4	5	1	5
22	10-18	4	5	32	5	32	5	1	5
23	2-5	29	31	6/29	5	30	24	28	4
24	2-13	18	15/17	10	35	11	10	10	35
25	2-18	19	37	18	21	2	12/3	18	13
26	2-19	36	21	36	17	36	15	29	17
27	2-14	26	7	26	20	23	28	32	26
28	15-27	27	7	27	3	27	3	27	7

6.2 Results for Modified Goal Programming Method

Table 5 reports the results obtained with MGPM, again using the May 2010 data set. Rows 2 and 3 give the same statistics as Table 3 for model (24) and the actual roster, respectively. The last column gives the optimal objective function value for model (28).

From the third row, we see that the MGPM schedules are as good or better than the actual schedules in all deviations except movement between rooms ($\mathcal{NCT}$), and as good or better than all results obtained by SPM. That is, MGPM dominates SPM. Also, the deviation values for demand satisfaction ($\mathcal{DE}$), room changes ($\mathcal{X}$) and complete case assignments ($\mathcal{CDT}$) are optimal or near-optimal with respect to the lower bounds obtained with the single objective model (row 2). In other words, by assigning each nurse to at most 3 surgery cases, optimal values for deviations $\mathcal{DE}$, $\mathcal{CDT}$ and $\mathcal{X}$ can be attained.

Table 5: MGPM results for each deviation

Scenario	$\mathcal{DF}$	$\mathcal{DS}$	$\mathcal{DE}$	$\mathcal{X}$	$\mathcal{NCT}$	$\mathcal{CDT}$	$\hat{Z} = \sum_j \beta_j$
Single objective model (24)	2	2	0	1	2	0	0
Actual roster	8	10	0	2	2	0	7.5
MGPM	6	4	0	1	3	0	3.5

The roster produced by MGPM is shown in Table 6. Compared with the SPM results, for example, nurse 27 now works as a circulator on cases 27 and 28 rather than on case 28 only, and nurse 4 now works on cases 21 and 22 in the single role of a scrub rather than as a circular and scrub on different cases. Another interesting result is that the MGPM schedules have no nurses assigned as learning fellows; that is, there are no surplus nurses, implying that the schedules are generally more efficient.

Table 6: Individual nurse assignments obtained with MGPM

Case Number	Duration (Start Time - End Time)	Schedule	
		Nurse # circulator	Nurse # scrub
1	2-5	3	24
2	6-9	3	24
3	10-13	3	24
4	2-5	26	8
5	8-13	26	8
6	14-17	26	8
7	2-15	25	22
8	16-22	25	22
9	9-14	30	2
10	15-20	16	13
11	2-17	18	37
12	2-9	34	35
13	10-18	34	35
14	2-15	11	21
15	2-9	32	20
16	10-17	32	20
17	18-24	32	20
18	2-9	29	6
19	10-20	29	6
20	2-17	23	31
21	2-9	1	4
22	10-18	1	4
23	2-5	36	5
24	2-13	14	17
25	2-18	19	12
26	2-19	10	15
27	2-14	28	33
28	15-27	27	7

To further compare the rosters produced by SPM and MGPM with each other as well as with the actual roster, we calculated the β values for each deviation introduced in Algorithm 2 for each experiment presented in Table 2. Table 7 reports the results. The first observation is that the $\hat{Z}$ values associated with the SPM and MGPM solutions are smaller than the $\hat{Z}$ value of 7.5 calculated for the actual roster. This indicates that both our methods generate more efficient schedules than the current method, at least for the sample data set.

A second observation is that the $\hat{Z}$ value of 3.5 obtained from MGPM is smallest among all experiments. This is not surprising since we saw that MGPM uniformly produced deviations no larger than did SPM and often smaller. With respect to the computational effort, MGPM ran in less than half the time required by SPM, a significant advantage in the real environment given the magnitude of the runtimes. Although both methods have similar complexity, the elastic constraints in (27) have the effect of tightening the feasible region of the MIP which is solved by

Table 7: Comparison between SPM and MGPM results

Scenario	β_1	β_2	β_3	β_4	β_5	β_6	$\hat{Z} = \sum_j \beta_j$	Runtime (hours)
Actual roster	3	4	0	1	0	0	7.5	–
SPM, Experiment 1	2	1	0	1	0.5	0	4.5	7.77
SPM, Experiment 2	2	1	0	1	0.5	0	4.5	7.63
SPM, Experiment 3	2	1	0	2	0.5	0	5.5	7.59
SPM, Experiment 4	2	1	0	0	0.5	1	4.5	7.73
MGPM	2	1	0	0	0.5	0	3.5	3.18

MGPM. Empirically, this was seen to reduce the size of the branch-and-bound tree generated by CPLEX and hence speed convergence. However, SPM has the advantage of producing a pool of solutions allowing the decision maker to choose an alternative if the scheduler does not like the one associated with CWI^*. In contrast, if the single solution produced by MGPM has undesirable properties, such as too many interrupted assignments or room changes, the decision maker may not be able to modify it to her liking as witnessed by the shortcomings of the current scheduling process.

7 Summary and Ongoing Work

In this paper, we developed and compared two approaches for solving a daily nurse scheduling problem that arises in large operating suites. The problem is complicated by the need to account for individual specialties, competency levels, preferences and shift limitations, as well as the procedural complexities of each case. In addition, it was not possible to identify a single objective such as cost or quality to optimize. Instead we worked with a variety of objectives and aimed at Pareto-optimal solutions.

The first approach (called SPM) used the idea of generating a pool of representative solutions by solving a series of optimization problems, each aimed a minimizing a single objective. A cumulative weighted index was then constructed for each solution and the one with the smallest index was reported as the "optimum." The second approach (called MGPM) was based on preemptive goal programming, where we first found the optimal solution for each goal separately, and then solved a derivative optimization problem whose objective was to minimize the sum of the deviations from those goals. Both methods were computationally challenging since they required the solution of large-scale mixed-integer programs at intermediate steps.

The scheduling model and accompanying algorithms were developed and validated over a two-year period. During this time, we had extensive meetings with nurse managers and continually gathered data by shadowing nurses in the main operating suite. Based on the information derived from this experience, we constructed several databases, organized the model parameters, and obtained the real schedules used in the analysis.

Our computational experience showed that MGPM outperformed SPM by generating schedules with smaller deviations in significantly less time. However, SPM offers the advantage of providing many good solutions from which the decision maker can choose, based on her individual preferences or perceptions of fairness that might not have been captured in the models. It is an easy matter to rank the solutions in the pool or filter out those that do not meet certain implicit goals. It is also simple to re-rank them by adjusting the weights used to compute CWI. In any case, the results from either model proved superior to those obtained with the current techniques in use at MD Anderson Cancer Center so it is anticipated that the implementation of either of approach will lead to measurable savings in the coming years.

Nevertheless, the numerical results showed that neither algorithm could provide solutions quickly, so developing more efficient computational methods would be valuable. We are currently looking into the use of column generation to get improved relaxed solutions and heuristics for converting them to feasible solutions. We believe that this approach will lead to at least a twofold reduction in runtimes.

One of the main characteristics of any OR is case continuity. A nurse cannot leave the OR until the case is finished or she is relieved by someone with comparable qualifications. In practice, this means that there must be a pool of nurses on call, so to speak, during a portion of the day to take over when a lunch break is requested. We are now developing a complementary optimization model to generate lunch schedules for those shifts that require a break. Upon completion, we expect to wrap all the models with a graphical user interface that will facilitate daily planning, OR monitoring, and real-time control. When nurses call out or surgeries extend beyond their planned durations, a user friendly system is needed to get the schedules and cases back on track.

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8 Appendix

Tables 8 and 9 provide the remaining results for Example 1.

Table 8: Results obtained from SPM for single objective model for goal $j = 1$

Goal	Solution number, s									
j	1	2	3	4	5	6	7	8	9	10
1	1	1	1.01	1	1	1	1.01	1.05	1	1.05
2	0	0	1	1	1.1	1.2	2	2.5	0	0.05
3	3	2.5	1.8	1.9	2.1	2.2	2	1.5	2.3	3.2
CWI_{1s}	1.1	1	1.165	1.18	1.25	1.3	1.505	1.575	0.96	1.18

Table 9: Results obtained from SPM for single objective model for goal $j = 3$

Goal	Solution number, s									
j	1	2	3	4	5	6	7	8	9	10
1	2	2.1	2.5	1.9	1.9	1.8	1.4	3	1.3	1.8
2	0.05	0.01	0	0	0.02	0.02	0.5	0	1	0.1
3	1.5	1.5	1.5	1.6	1.55	1.52	1.62	1.5	1.5	1.52
CWI_{3s}	1.315	1.353	1.55	1.27	1.266	1.21	1.174	1.8	1.25	1.234

Table 10 provides a portion of a real roster obtained from MDACC. The case number is listed in the first column, the second column gives the case duration, and the last two columns indicate the circulators and scrubs assigned to the case. From this information we see, for example, that nurse 9 is assigned to work on cases 21 and 22 both scheduled in OR 11. The surgery information indicates that she is to start her work on case 21 at the beginning of period 2 (7 AM) and finish at the end of period 9 (10:30 AM). At that point, she transitions to case 22 that extends from period 10 (11 AM) through period 18 (3 PM) in the same room. However, based on her contract, her shift runs from 10:30 AM to 7 PM.

This type of mismatch is typical in the daily schedules produced by the manual techniques used at MDACC. A second example relates to the quality of schedules produced, and concerns their policy of trying to assign each nurse to only one OR per day but no more than two. In the roster in Table 10, nurse 25 is assigned to work on case 27 in OR 16 from period 2 through period 14 and then move to OR 17 to work on case 28 from 15 to 27. While these assignments do not violate the stated policy, our results show that it is possible to limit nurse 25 to one OR during the day without affecting the quality of the schedules of the other nurses.

Table 10: Example of a real schedule obtained from MD Anderson Cancer Center

Case number	Duration (starting time - end time)	Actual schedule: Nurse # circulator	Actual schedule: Nurse # scrub
1	2-5	14	13
2	6-9	14	13
3	10-13	14	13
4	2-5	10	21
5	8-13	10	21
6	14-17	10	21
7	2-15	6	7
8	16-22	6	7
9	9-14	11	31
10	15-20	16	2/8
11	2-17	2/19	8
12	2-9	16/18	5
13	10-18	18	5
14	2-15	2	37
15	2-9	3	33
16	10-17	27/3	33
17	18-24	27	3
18	2-9	29	4
19	10-20	29	4
20	2-17	1	25
21	2-9	9	24
22	10-18	9/26	35
23	2-5	26	20
24	2-13	36	15
25	2-18	30	12
26	2-19	28	17
27	2-14	23/25	22
28	15-27	25	20

In a similar vein, many nurses have significant idle time in their schedules while others work continuously without a break. For example, nurse 35 starts her shift at period 1 (6:30 AM) and finishes at the end of period 18 (3 PM) but is only assigned to case 22 from period 10 (11 AM) through period 18 (3 PM). She is idle for 5 hours between periods 1 and 10. The same is true for nurses 11, 20 and 31, implying that the value of the deviation variable $\mathcal{DS}$ is 10. As a final example, it is informative to examine the schedule for nurse 20, whose shift runs from period 1 through period 18 (6:30 AM to 3 PM). Her first assignment is case 23 between periods 2 and 5 and her second is case 28 between periods 15 and 27 (1:30 PM to 7:30 PM). Thus, she has 5 hours of idle time and 4 hours off overtime – a poor schedule – which translates into a value of 8 for the overtime deviation variable $\mathcal{DF}$.