

## Damage and Failure of Composite Materials

Understanding damage and failure of composite materials is critical for reliable and cost-effective engineering design. Bringing together materials mechanics and modeling, this book provides a complete guide to damage, fatigue, and failure of composite materials. Early chapters focus on the underlying principles governing composite damage, reviewing basic equations and mechanics theory, before describing mechanisms of damage such as cracking, breakage, and buckling. In subsequent chapters, the physical mechanisms underlying the formation and progression of damage under mechanical loads are described with ample experimental data, and micro- and macro-level damage models are combined. Finally, fatigue of composite materials is discussed using fatigue-life diagrams. While there is a special emphasis on polymer matrix composites, metal and ceramic matrix composites are also described. Outlining methods for more reliable design of composite structures, this is a valuable resource for engineers and materials scientists in industry and academia.

**Ramesh Talreja** is a Professor of Aerospace Engineering at Texas A&M University. He earned his Ph.D. and Doctor of Technical Sciences degrees from the Technical University of Denmark. He has contributed extensively to the fields of damage, fatigue, and failure of composite materials by authoring numerous books and book chapters as well as by editing several encyclopedic works.

**Chandra Veer Singh** is an Assistant Professor of Materials Science and Engineering at the University of Toronto. He earned his Ph.D. in aerospace engineering from Texas A&M University, and worked as a post-doctoral Fellow at Cornell University. His research expertise is in damage mechanics of composite materials, atomistic modeling, and computational materials science. His industry experience includes R&D at GE Aircraft Engines.

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RAMESH TALREJA

Texas A&M University

CHANDRA VEER SINGH

University of Toronto



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## Preface

The field of composite materials has advanced steadily from the early developments during the 1970s when laminate plate theory and anisotropic failure criteria were in focus to today's diversification of composite materials to multifunctional and nanostructured composite morphologies. Throughout the 1970s and 1980s several books appeared along with courses that were developed and taught at advanced levels dealing with mechanics of composite materials and structures. The failure analysis was mostly limited to descriptions of strength that extended previous continuum descriptions of metal yielding and failure. Beginning around the mid-1980s, micromechanics and continuum damage mechanics were applied to multiple cracking observed in composite materials. Under the overall description of "damage mechanics" a flurry of activities took place as evidenced by conferences and symposia. Other than several conference proceedings that recorded such activities, a collection of seminal contributions to the field appeared in a volume (*Damage Mechanics of Composite Materials*, R. Talreja, ed., Composite Materials Series, R.B. Pipes, series ed., Vol. 9, Amsterdam: Elsevier Science Publishers, 1994). The two main avenues of approach to damage in composite materials and its effect on materials response, now referred to as micro-damage mechanics (MIDM) and macro-damage mechanics (MADM), were presented in a balanced form in that volume. In the years since then, many developments have taken place that have brought this field to such level of maturity that a book coherently presenting the material was felt to be timely. It is hoped that this book will help provide impetus for teaching advanced courses in composite damage at universities as well as support short courses for professional development of engineers in industry. The wealth of material covered can also help new researchers in advancing the field further. To this end, the last chapter provides some guidance in identifying gaps and needs for further work.

The structure of the book is as follows. Chapter 1 lays down the overall strategy for durability assessment of composite structures, emphasizing the needs and motivating the content of the book to follow. Chapter 2 provides an easy reference to the basic continuum mechanics topics that are felt to be relevant to the subsequent treatment. Chapter 3 describes the mechanisms of damage that underlie the phenomena aimed for modeling. Many of the physical observations described there are viewed to be vital to developing proper understanding of the complex field of damage in composite materials. Chapters 4 and 5 deal with the

two main approaches stated above, i.e., the MIDM and the MADM. Selected works from the literature, including the authors' own, are given as much treatment as was found justified to generate coherency without overly including details. While these two chapters focus on descriptions of damage and the constitutive property changes caused by it, Chapter 6 is devoted to the progression of damage. The crack multiplication is a distinctive feature of damage in composite materials that distinguishes it from single crack growth in monolithic materials, and therefore justifies treatment in a chapter by itself. Chapter 7 is on fatigue of composite materials. This field suffers from the historical treatments of metal fatigue and is unfortunately the least understood part of damage in composite materials. Multi-axial fatigue illustrates the situation well where the literature displays little understanding of the mechanisms underlying failure. While a separate book on fatigue of composite materials is needed to do full justice to the field, a single chapter here is added to draw attention to the mechanisms-based concepts for proper interpretation and modeling. Finally, Chapter 8 presents a summary of the book and points to the directions in which further advances are seen to be necessary. Particular emphasis is given to the computational incorporation of damage modeling in durability assessment as well as taking account of the manufacturing-induced defects in an integrated manner.

Although the authors have written this book, the credit goes to many researchers who have worked on various aspects of damage in composite materials. Their collective contributions have made it possible for us to present what we have seen as a coherent story at this time. The field is evolving, and future versions of the story will hopefully spur further development and, most importantly, transfer of this knowledge to industry will take place.