

Research Article

Damage Detection of a Continuous Bridge from Response of a Moving Vehicle

Z. H. Li and F. T. K. Au

Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

Correspondence should be addressed to Z. H. Li; lizhenhu@hku.hk

Received 17 July 2013; Accepted 19 December 2013; Published 2 June 2014

Academic Editor: Nuno Maia

Copyright © 2014 Z. H. Li and F. T. K. Au. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper presents a multistage multipass method to identify the damage location of a continuous bridge from the response of a vehicle moving on the rough road surface of the bridge. The vehicle runs over the bridge several times at different velocities and the corresponding responses of the vehicle can be obtained. The vertical accelerations of the vehicle running on the intact and damaged bridges are used for identification. The multistage damage detection method is implemented by the modal strain energy based method and genetic algorithm. The modal strain energy based method estimates the damage location by calculating a damage indicator from the frequencies extracted from the vehicle responses of both the intact and damaged states of the bridge. At the second stage, the identification problem is transformed into a global optimization problem and is solved by genetic algorithm techniques. For each pass of the vehicle, the method can identify the location of the damage until it is determined with acceptable accuracy. A two-span continuous bridge is used to verify the method. The numerical results show that this method can identify the location of damage reasonably well.

1. Introduction

The safety of bridge structures is very important to economic development of all countries, so it is very important to make sure that the bridges are in good condition. Various damage detection techniques have been developed to meet this need [1, 2]. The aerospace and offshore oil industries conducted early damage detection since the late 1970s and 1980s, respectively, while, in the civil engineering community, structural health monitoring is a relatively vibrant area of current research [3]. Recording the vibration of the structures, extracting modal properties, and then identifying the damage from changes of the structural properties are the most popular methods among them [4]. This is based on the assumption that commonly measured modal parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties, such as reductions in stiffness resulting from the onset of cracks or loosening of a connection, will cause detectable changes in these modal properties. Changes in

modal properties or properties derived from these quantities are being used as indicators of damage [1]. One issue of primary importance is the dependence on prior analytical models and/or prior test data for the detection and location of damage [1]. Damage detection methods in time domain can overcome this problem to certain extent. The time-domain approach has become more popular in recent years to examine nonstationary signals [5]. In the short-time Fourier transform method, the total time interval is divided into shorter time intervals for the fast Fourier transform to be applied to each interval. This time windowing method narrows down the time to that of the interval where the damage is located [6]. However, the constraints of the uncertainty principle limit the obtainable resolutions considerably, prompting the emergence of an alternative approach in multiresolution analysis termed wavelet transform [7]. Wavelet transform allows variable-size windows and this is why it is also called a mathematical microscope. This property makes it a suitable method for detection of damage from a response [8]. There are several other methods used for damage detection. One of them, the genetic algorithm, is modified for use in this

study. Genetic algorithms (GAs), originally developed by Holland, are search algorithms based on the mechanics of natural selection and natural genetics [9]. GAs are different from traditional optimization procedures in four ways: (a) GAs work with a coding of the parameter set, but not the parameters themselves; (b) GAs search from a population of points instead of a single point; (c) GAs use objective function information instead of derivatives or other auxiliary knowledge; and (d) GAs use probabilistic transition rules instead of deterministic rules [10]. Since structural damage detection can be transformed into optimization problems, GAs can be used to do the damage detection.

The above-mentioned methods identify the conditions of a bridge through acquiring the bridge response by putting sensors on the bridge. It is also possible to detect the conditions of the bridge by putting sensors on the passing vehicle. Identifying the damage using the vehicle response has certain advantages over putting sensors on the bridge. Firstly, the vehicle is both a sensor and an exciter. It is much more convenient as it makes the closure of bridge much shorter or even unnecessary. Secondly, it is not much influenced by the locations of damage and distributions of sensors because the vehicle runs over and detects the whole bridge. Yang et al. extracted bridge frequencies from a moving vehicle [11–13]. Inspired by this work, Bu et al. proposed a damaged detection method based on the dynamic response sensitivity analysis and regularization technique [5]. Nguyen and Tran [14] applied wavelet transform to the displacement history of a moving vehicle. Zhang et al. [15] extracted the mode shapes square from the response and conducted damage detection. The above work did not consider the roughness of the bridge, which would be a very important factor affecting the vibration of the vehicle. The authors previously applied a modal strain energy based damaged detection method to analyze the response of the vehicle [16] and came up with two possible locations of the damage. This is due to the limitation of the frequency-based damage detection methods [17]. The authors also conducted damage detection using wavelet transform from the response of the vehicle [18]. This paper will consider the influence of the roughness in the vehicle-bridge interaction system on the damage identification. The strategy is a combination of modal strain energy based method and GA techniques. Modal strain energy based method can narrow down the search space for GA algorithms to save computational time and improve the chance of getting the correct solution.

2. Vehicle-Bridge Interaction System

Figure 1 shows the sketch of a typical vehicle-bridge interaction system. It contains a continuous bridge and a vehicle running over it at a constant speed. The bridge is modeled using the finite element method and the vehicle is modeled as a mass-spring-damper system. The vehicle model contains five parameters. The body is simulated by a concentrated mass m_2 , the spring stiffness k_2 , and the damper c_2 . The wheels are simulated using a concentrated mass m_1 and the stiffness k_1 of the spring connecting the wheel and the road surface.

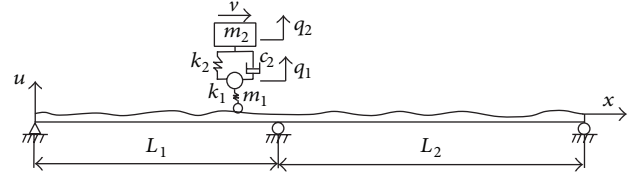


FIGURE 1: A typical vehicle-bridge interaction system.

2.1. Equation of Motion. When the vehicle moves from one end of the bridge to the other end at a constant speed, both the bridge and the vehicle will vibrate vertically. A vector $u_b(t)$ is used to denote the vertical displacements of a series of nodes in the finite element model of the bridge. Its first and second derivatives with respect to time t , that is, $\dot{u}_b(t)$ and $\ddot{u}_b(t)$, are, respectively, the vertical velocity and acceleration of the corresponding nodes. The symbols $q_1(t)$ and $q_2(t)$ denote the vertical displacement of the wheel and the car body, respectively. As they interact with each other by the contact force, the vibration of the vehicle is influenced by the vibration of the bridge and vice versa. So this is a coupled vibration system. It is assumed that the mass of vehicle is insignificant compared to that of the bridge. The governing equation of motion derived using the fully computerized method is expressed as

$$\begin{bmatrix} M_b & 0 \\ 0 & M_v \end{bmatrix} \begin{Bmatrix} \ddot{u}_b \\ \ddot{q}_v \end{Bmatrix} + \begin{bmatrix} C_b & C_{bv} \\ C_{vb} & C_v \end{bmatrix} \begin{Bmatrix} \dot{u}_b \\ \dot{q}_v \end{Bmatrix} + \begin{bmatrix} K_b & K_{bv} \\ K_{vb} & K_v \end{bmatrix} \begin{Bmatrix} u_b \\ q_v \end{Bmatrix} = \begin{Bmatrix} P_b \\ P_v \end{Bmatrix}, \quad (1)$$

where $q_v = [q_1 \ q_2]^T$ is the vertical displacement of the vehicle; \dot{q}_v and \ddot{q}_v are the corresponding velocity and acceleration; M_b and K_b are the mass and stiffness matrices of the bridge obtained by the finite element method, respectively; the damping matrix of the bridge is modeled using Rayleigh damping as $C_b = \alpha_c M_b + \beta_c K_b$, where α_c and β_c are the damping factors;

$$\begin{aligned} M_v &= \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \\ C_v &= \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \\ K_v &= \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \end{aligned} \quad (2)$$

are, respectively, the mass, damping, and stiffness matrices of the vehicle model; C_{bv} , C_{vb} , K_{bv} , and K_{vb} are the coupling damping and stiffness matrices; $K_{vb} = [-k_1 \ 0]^T$ and P_b and P_v are the external loads added to the bridge and vehicle, respectively, due to gravity forces, surface roughness, and so forth. Equation (1) can be solved using Newmark- β method, Wilson- θ method, or similar to calculate the responses of the vehicle and the bridge.

2.2. *Modeling of Roughness.* The random road surface roughness of the bridge can be described by a kind of zero-mean, real-valued, and stationary Gaussian process as [19]

$$r(x) = \sum_{h=1}^{N_T} \alpha_h \cos(2\pi\omega_h x + \phi_h), \quad (3)$$

where

$$\begin{aligned} \alpha_h^2 &= 4S_r(\omega_h) \Delta\omega, \\ \omega_h &= \omega_l + \left(h - \frac{1}{2}\right) \Delta\omega, \quad h = 1, 2, \dots, N_T, \\ \Delta\omega &= \frac{\omega_u - \omega_l}{N_T}, \end{aligned} \quad (4)$$

in which ω_l and ω_u are the lower and upper cut-off spatial frequencies, respectively. The power spectral density function $S_r(\omega_h)$ can be expressed in terms of the spatial frequency ω_h of the road surface roughness as

$$S_r(\omega_h) = \begin{cases} \bar{\alpha}\omega_h^{-\beta} & \text{for } \omega_l < \omega_h < \omega_u \\ 0 & \text{elsewhere,} \end{cases} \quad (5)$$

where $\bar{\alpha}$ is a spectral roughness coefficient and the value of β is determined based on the classification of road surface condition according to ISO specification [20].

The contact force f_c between the vehicle and the bridge can be written as

$$f_c = -(m_1 + m_2)g + k_2(r(x(t)) + u_b|_{x=vt} - q_1). \quad (6)$$

The above equation implies that the roughness and the vertical displacement of the corresponding point influence the contact force in a similar manner. If the height of roughness is obviously larger than the value of the displacement of the bridge, the roughness dominates the contact force. So, to identify the information of the bridge, the response of the bridge should be at least comparable to that of roughness.

2.3. *Measurement Noise.* Measurement noise should also be considered to make the simulation closer to reality. Damage detection is carried out assuming that the signal is contaminated by 5% white noise as shown in

$$\ddot{q}_v^m = \ddot{q}_v(1 + 5\%\eta), \quad (7)$$

where \ddot{q}_v^m is the simulated measured response of vehicle and η is a normally distributed random array with zero mean and unit variance. The measurement noise does influence the response and identification, but its influence is much smaller than that of roughness.

3. Multistage Multipass Damage Detection Method

This method contains two stages which are modal strain energy based method and modified genetic algorithm method. At the first stage, the modal strain energy based

method is simple and fast in roughly estimating the location of damage so as to narrow down the search domain for the second stage. The vehicle can run over the bridge several times and get a series of vehicle responses. Multiple passes are used because different properties of the vehicle and speeds will excite the bridge slightly differently, which will help guarantee the correctness of the identification.

3.1. *Modal Strain Energy Based Method.* Several modal properties based methods are developed for damage detection. Modal strain energy based method is selected because it is very effective and can estimate the location of the damage if only the frequencies of the damaged structure are available [21–23]. For the intact bridge, the first few mode shapes can be simulated by finite element method or obtained by field tests. If changes in mass are neglected, the fractional change in the i th eigenvalue due to damage is given by

$$Z_i = \frac{\omega_i^{*2} - \omega_i^2}{\omega_i^2} = \frac{f_i^{*2} - f_i^2}{f_i^2}, \quad (8)$$

where ω_i is the i th circular frequency, $f_i = \omega_i/(2\pi)$ is the corresponding frequency, and the asterisk denotes those of the damaged state.

For an MDOF structural system of NE elements and N nodes, the damage may be predicted by the sensitivity equation

$$\sum_{j=1}^{NE} F_{ij}\alpha_j = Z_i \quad (9)$$

in which $\alpha_j \in [-1, 0]$ is the fractional reduction in stiffness of j th element and the fraction of modal energy or sensitivity for the i th mode concentrated at the j th element, F_{ij} , is given by

$$F_{ij} = \frac{\int_{x_j}^{x_{j+1}} EI \{\varphi_i''(x)\}^2 dx}{\int_0^L EI \{\varphi_i''(x)\}^2 dx}, \quad (10)$$

where $\varphi_i''(x)$ is the second derivative of the i th mode shape of the bridge; E and I are the elastic modulus and moment of inertia of the bridge, respectively; and x_j and x_{j+1} are the coordinates of the j th and $(j+1)$ th nodes that are the left and right nodes of the j th element, respectively. In practice, only the modal amplitudes at limited nodal points are available. By interpolation using spline functions and the element modal amplitude values from the mode shapes of the finite element model, one can generate the function $\varphi_i(x)$ as necessary.

For any two modes m and n , one may obtain the ratio of sensitivities calculated from (9) as

$$\begin{aligned} \frac{Z_m}{Z_n} &= \frac{\sum_{j=1}^{NE} F_{mj}\alpha_j}{\sum_{j=1}^{NE} F_{nj}\alpha_j} \\ &= \frac{F_{m1}\alpha_1 + F_{m1}\alpha_1 + \dots + F_{mq}\alpha_q + \dots + F_{mNE}\alpha_{NE}}{F_{n1}\alpha_1 + F_{n1}\alpha_1 + \dots + F_{nq}\alpha_q + \dots + F_{nNE}\alpha_{NE}}. \end{aligned} \quad (11)$$

Assuming that damage occurs only at element q , then $\alpha_j = 0$ when $j \neq q$, but $\alpha_j \neq 0$ when $j = q$. The relationship associated with the m th and n th eigenvalues can be established as

$$\frac{Z_m}{Z_n} = \frac{F_{mq}}{F_{nq}}. \quad (12)$$

If NM modes are measured, (12) can be extended to

$$\frac{Z_m}{\sum_{k=1}^{NM} Z_k} = \frac{F_{mq}}{\sum_{k=1}^{NM} F_{kq}}. \quad (13)$$

Based on the above equation, an error index e_{mj} can be developed as

$$e_{mj} = \frac{Z_m}{\sum_{k=1}^{NM} Z_k} - \frac{F_{mj}}{\sum_{k=1}^{NM} F_{kj}}, \quad (14)$$

where $e_{mj} = 0$ indicates in particular that the damage is located at the j th element using the m th modal information. To account for all available modes, one can form a single damage indicator for the j th member as

$$DI_j = \left[\sum_{i=1}^{NM} e_{ij}^2 \right]^{-1/2}. \quad (15)$$

The damage is located at element j if DI_j approaches the local maximum point. It has been validated that the damage can be detected if the surface of the road is assumed to be smooth [16].

3.2. Empirical Mode Decomposition. For this frequency-based method, it is important to extract frequencies from the vehicle response. To help identify the frequencies accurately, several signal processing techniques are used, including common filtering techniques and empirical mode decomposition (EMD) proposed by Huang et al. [24]. EMD is used to decompose a signal into a series of intrinsic mode functions (IMFs). Given a set of measured data $X(t)$, the algorithm of the EMD, characterized by the sifting process, is briefly described below.

- (a) Identify all the local maxima and minima of the data $X(t)$ and then compute the corresponding interpolating signals by cubic spline curves. These signals are the upper and lower envelopes of the signal. All the extrema should be covered in these two envelopes. Let m_1 denote the mean of the upper and lower envelopes. The difference between the data and the mean m_1 is

$$h_1 = X(t) - m_1. \quad (16)$$

- (b) Ideally, h_1 should be the first IMF component. If h_1 does not satisfy the IMF requirements [24], treat it as the original data and repeat the first step until the requirements are satisfied. The first IMF component obtained is designated as c_1 .

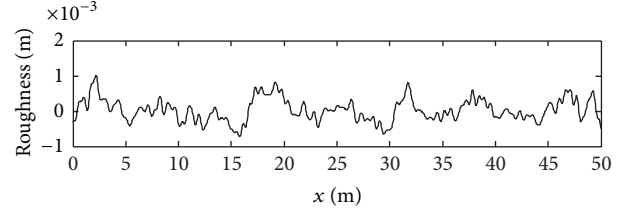


FIGURE 2: A typical profile of roughness.

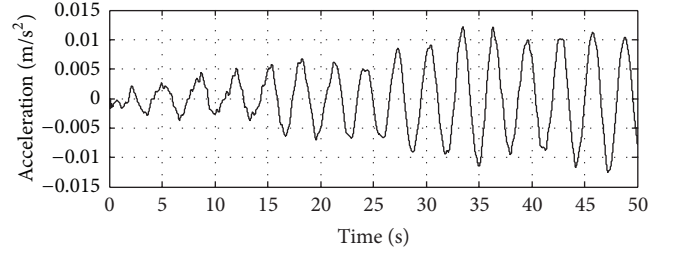


FIGURE 3: Vertical acceleration of the vehicle running on intact and damaged bridges.

- (c) By subtracting c_1 from the original data, one obtains the residue r_1 as

$$r_1 = X(t) - c_1. \quad (17)$$

- (d) Repeat the above sifting processes to obtain the next IMFs. Once an IMF is obtained, remove it from the signal until the predetermined criteria are met: either when the component c_n or the residue r_n becomes too small to be physically meaningful or when the residue r_n becomes a monotonic function from which no more IMF can be extracted. Consequently, the data $X(t)$ is decomposed as

$$X(t) = \sum_{i=1}^n c_i + r_n. \quad (18)$$

Thus, a decomposition of the data into n -empirical modes is achieved. The process is indeed like sifting: to separate the finest local mode from the data first based only on the characteristic time scale. The sifting process, however, has two effects: (a) to eliminate riding waves and (b) to smooth uneven amplitudes. Applying fast Fourier transform to these IMFs, it is easy to extract higher frequencies.

3.3. Modified Genetic Algorithm. Damage detection can be transformed into an optimization problem. The element stiffness and parameters of roughness can be treated as unknowns. It is assumed that the properties of the bridge without damage are known. The objective function can be set as the difference between the responses of the vehicle running on the bridge at the current and the intact state as

$$e = \|\dot{q}_2^{mu} - \dot{q}_2^s\|_2. \quad (19)$$

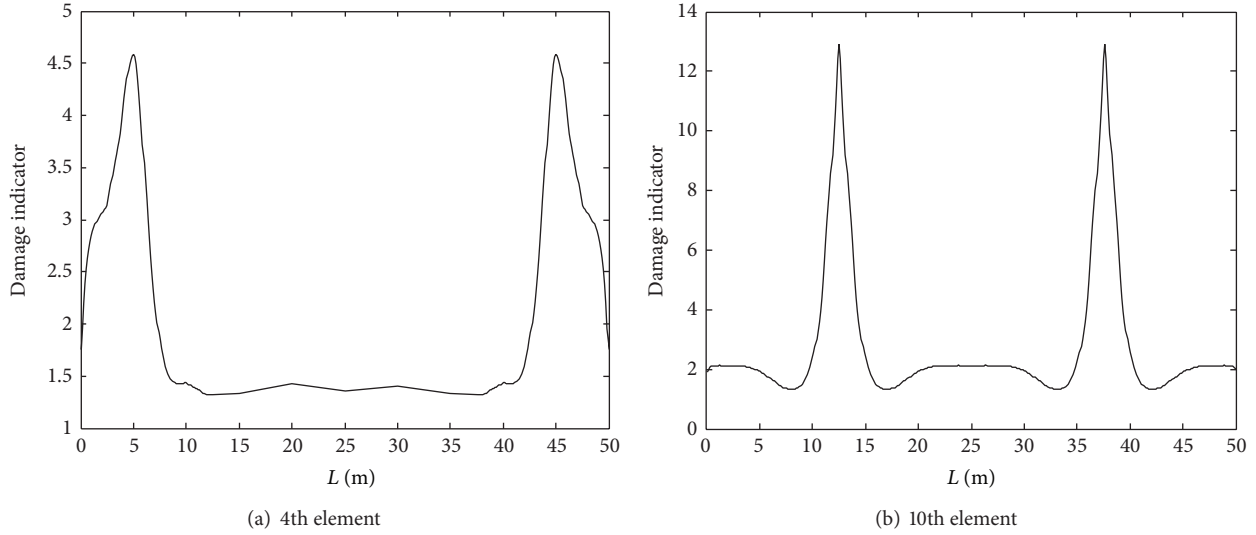


FIGURE 4: Damage indicator when damage is inflicted at (a) fourth element and (b) tenth element.

The nature of randomness of GA makes it possible for a false alarm to occur sometimes, but multiple passes will prevent this. The responses can be divided into two parts. The first stage makes use of the results from the first part and identifies the location of damage. Usually, this part should at least contain responses from three passes. The GA will roughly identify the location of damage based on these responses. If the identified locations from these passes are the same, no further action is needed. If the result shows that the identified locations from the responses are different, these potential locations will be provided to the second stage. Such a way will greatly reduce the possibility of false alarm and may reduce the search domain. The population size and generation used in the GA will thus be reduced, which will save the computational time.

4. Numerical Study

A two-span continuous bridge is used to demonstrate the damage detection strategy. The properties of the bridge are spans $L_1 = L_2 = 25$ m, Young's modulus of the material $E = 2.75 \times 10^{10}$ N/m², density $\rho = 3333$ kg/m³, and the moment of inertia $I = 0.12$ m⁴. The damping is not considered for the moment and the length of elements for finite element analysis is 1.25 m. For the 5-parameter vehicle, the relevant values are chosen as follows: $m_2 = 19840$ kg, $k_2 = 1 \times 10^5$ N/m, $c_2 = 2 \times 10^3$ Ns/m, $m_1 = 160$ kg, and $k_1 = 1 \times 10^5$ N/m. The simulated roughness is shown in Figure 2 as described in the next subsection. The speeds of the vehicle to obtain the vehicle responses are 0.6 m/s, 0.8 m/s, 1 m/s, 1.2 m/s, and 1.4 m/s. The time step for integration is 0.001 second. The damage is modeled as a stiffness reduction at one element of the beam. In this paper, the position of the damage is selected around the first quarter point and middle of the first span of beam which correspond to the fourth and tenth elements of the beam, respectively. The stiffness reduction is set to be 30%. For convenience, the stiffness reduction is reflected in

the equivalent Young modulus instead of the presentation of results of damage detection.

4.1. Profile of Roughness. When the values of $\bar{\alpha}$, β , ω_l , ω_u , and N_T are set to be 1×10^{-8} m²/(m/cycle), 0, 0.05 cycle/m, 2 cycle/m, and 1024, respectively, and two sets of ϕ_k are randomly generated, two profiles of roughness are constructed. One of them is shown in Figure 2.

4.2. Damage Detection at the First Stage. The vertical acceleration of the vehicle can be calculated from (1). When the speed is 1 m/s, the vehicle response is shown in Figure 3. Fast Fourier transform is applied to extract frequencies from these responses. The first five frequencies of both the intact bridge and the damaged bridge can be obtained. Modal strain energy based method is applied to do the damage detection at the first stage. Figure 4 with two peaks each implies that there might be two damaged locations even though the damage is inflicted at one single element. The two nearest elements to each peak are regarded as potential locations of damage. Thus, there are totally four possible solutions for each case.

4.3. Damage Detection at the Second Stage. The first stage can limit the locations of damage to certain elements though it cannot confine the damage to a single element. This will help narrow down the search domain for the subsequent work. For example, Figure 4 shows that the damage may be at the 4th element or the 37th element, which indicates that the second stage only needs to determine which of the two the elements is damaged.

The responses are divided into two parts according to the speeds of the vehicle. The first part contains responses when the vehicle runs at speeds of 0.6 m/s, 1 m/s, and 1.4 m/s, while the remaining responses belong to the second part. Applying GA to the first part, the identified values of Young's modulus of elements are shown in Figure 5. The location

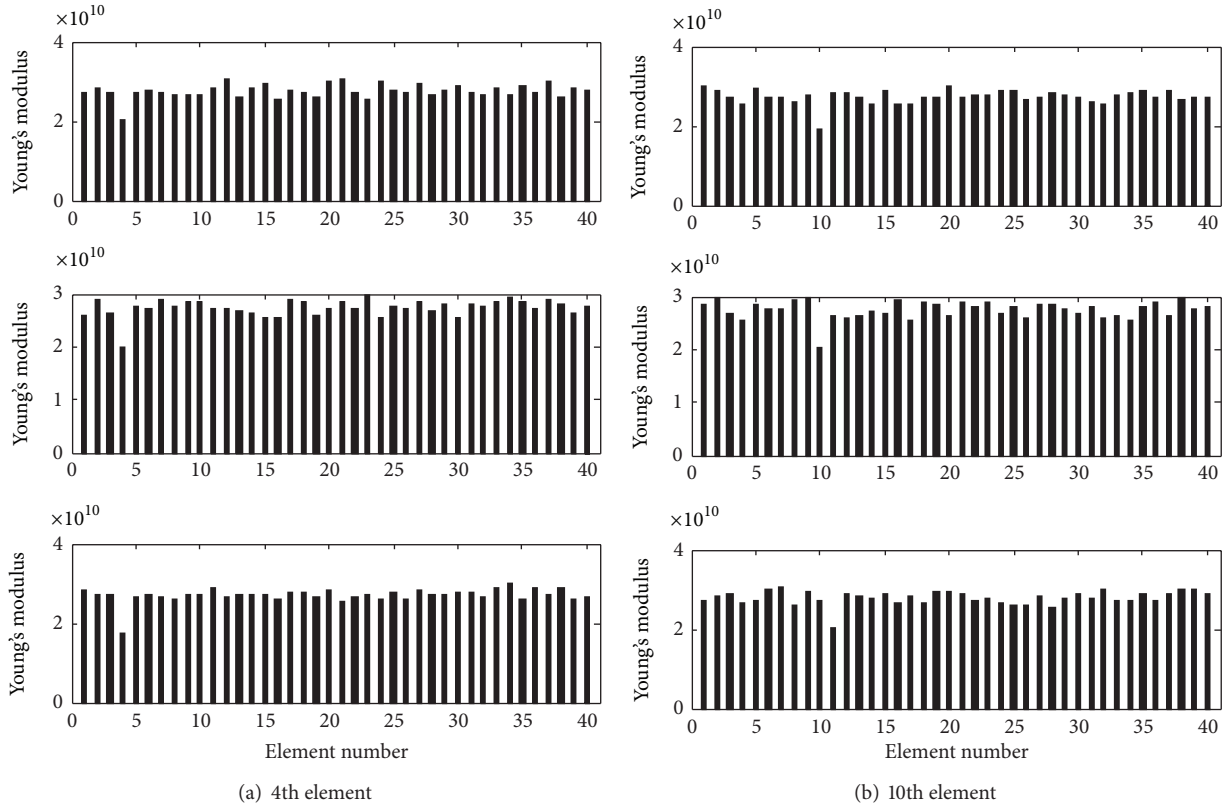


FIGURE 5: Identified equivalent Young's modulus from first part for damage at (a) fourth element and (b) tenth element.

of damage is determined from the three passes when the damage is inflicted at the 4th element. However, the location for the second case of damage is not yet well determined. Even though this stage does not limit the damage to one element, it not only eliminates half of the possibilities but also provides more information on the profile of roughness. Analyzing the second part of the response using GA, the location of the damage can be determined as shown in Figure 6.

5. Conclusions

A multistage multipass strategy is proposed to identify the location of damage from the response of a vehicle moving over a bridge considering the road surface roughness. The frequencies of the bridge are extracted with the help of empirical mode decomposition first. Modal strain energy based damage detection method is then applied to explore the possible damage locations. The potential locations obtained are then given to GAs for further investigation. The algorithm simultaneously identifies the stiffness of each element and the profile of roughness. The numerical study shows that this combined method can successfully determine the location of damage of a two-span continuous bridge when one element is assumed to be damaged. The measurement noise influences the damage detection much less significantly than roughness. For multiple locations of damage, further work is needed.

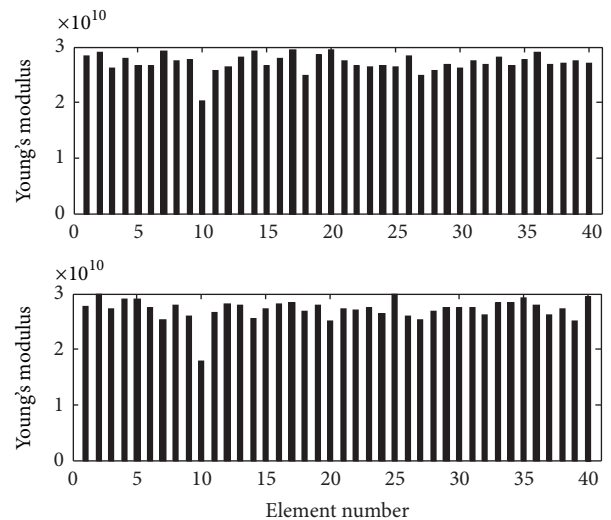


FIGURE 6: Identified equivalent Young's modulus from second part for damage at tenth element.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The work described in this paper has been supported by the Research Grants Council (RGC) of the Hong Kong Special Administrative Region, China (RGC Project no. HKU 7102/08E). The authors would also like to thank Dr. X. T. Si for providing the vehicle-bridge interaction code for computing the responses of vehicle.

References

- [1] S. W. Doebling, C. R. Farrar, and M. B. Prime, "A summary review of vibration-based damage identification methods," *Shock and Vibration Digest*, vol. 30, no. 2, pp. 91–105, 1998.
- [2] C. R. Farrar, S. W. Doebling, and D. A. Nix, "Vibration-based structural damage identification," *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 359, no. 1778, pp. 131–149, 2001.
- [3] E. P. Carden and P. Fanning, "Vibration based condition monitoring: a review," *Structural Health Monitoring*, vol. 3, no. 4, pp. 355–377, 2004.
- [4] K. Worden and J. M. Dulieu-Barton, "An overview of intelligent fault detection in systems and structures," *Structural Health Monitoring*, vol. 3, no. 1, pp. 85–98, 2004.
- [5] J. Q. Bu, S. S. Law, and X. Q. Zhu, "Innovative bridge condition assessment from dynamic response of a passing vehicle," *Journal of Engineering Mechanics*, vol. 132, no. 12, pp. 1372–1379, 2006.
- [6] D. L. Fugal, *Conceptual Wavelets in Digital Signal Processing: An in-Depth, Practical Approach for the Non-mathematician*, Space & Signals Technical Pub, San Diego, Calif, USA, 2009.
- [7] T. Kijewski and A. Kareem, "Wavelet transforms for system identification in civil engineering," *Computer-Aided Civil and Infrastructure Engineering*, vol. 18, no. 5, pp. 339–355, 2003.
- [8] H. Kim and H. Melhem, "Damage detection of structures by wavelet analysis," *Engineering Structures*, vol. 26, no. 3, pp. 347–362, 2004.
- [9] J. H. Holland, *Adaptation in Natural and Artificial Systems*, vol. 2, Ann Arbor, Mich, USA, University of Michigan Press, 1975.
- [10] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, New York, NY, USA, 1989.
- [11] Y. B. Yang, C. W. Lin, and J. D. Yau, "Extracting bridge frequencies from the dynamic response of a passing vehicle," *Journal of Sound and Vibration*, vol. 272, no. 3–5, pp. 471–493, 2004.
- [12] Y. B. Yang and K. C. Chang, "Extraction of bridge frequencies from the dynamic response of a passing vehicle enhanced by the EMD technique," *Journal of Sound and Vibration*, vol. 322, no. 4–5, pp. 718–739, 2009.
- [13] C. W. Lin and Y. B. Yang, "Use of a passing vehicle to scan the fundamental bridge frequencies: an experimental verification," *Engineering Structures*, vol. 27, no. 13, pp. 1865–1878, 2005.
- [14] K. V. Nguyen and H. T. Tran, "Multi-cracks detection of a beam-like structure based on the on-vehicle vibration signal and wavelet analysis," *Journal of Sound and Vibration*, vol. 329, no. 21, pp. 4455–4465, 2010.
- [15] Y. Zhang, L. Wang, and Z. Xiang, "Damage detection by mode shape squares extracted from a passing vehicle," *Journal of Sound and Vibration*, vol. 331, no. 2, pp. 291–307, 2012.
- [16] Z. H. Li and F. T. K. Au, "Damage detection of girder bridges from dynamic response of a moving vehicle," in *Proceedings of the 3rd International Postgraduate Conference on Infrastructure and Environment*, Z. Liu, A. A. Javed, D. Ni, and W. Shen, Eds., pp. 88–95, Hong Kong, 2011.
- [17] O. S. Salawu, "Detection of structural damage through changes in frequency: a review," *Engineering Structures*, vol. 19, no. 9, pp. 718–723, 1997.
- [18] Z. H. Li and F. T. K. Au, "Feasibility of output-only damage detection method using response of a vehicle moving on a bridge," in *Proceedings of the 14th Asia Pacific Vibration Conference*, pp. 1215–1224, Hong Kong, 2011.
- [19] R. Jiang, *Identification of Dynamic Load and Vehicle Parameters Based on Bridge Dynamic Responses*, The University of Hong Kong, Hong Kong.
- [20] ISO8608:1995, Mechanical vibration. Road surface profiles. Reporting of measured data. 1996, International Organization for Standardization (ISO).
- [21] J. T. Kim and N. Stubbs, "Improved damage identification method based on modal information," *Journal of Sound and Vibration*, vol. 252, no. 2, pp. 223–238, 2002.
- [22] J. T. Kim and N. Stubbs, "Crack detection in beam-type structures using frequency data," *Journal of Sound and Vibration*, vol. 259, no. 1, pp. 145–160, 2003.
- [23] J. T. Kim, Y.-S. Ryu, H.-M. Cho, and N. Stubbs, "Damage identification in beam-type structures: frequency-based method vs mode-shape-based method," *Engineering Structures*, vol. 25, no. 1, pp. 57–67, 2003.
- [24] N. E. Huang, Z. Shen, S. R. Long et al., "The empirical mode decomposition and the Hubert spectrum for nonlinear and non-stationary time series analysis," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 454, no. 1971, pp. 903–995, 1998.