Damage Detection on the Z24 Bridge by a Spectral-based Dynamic Identification Technique

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ABSTRACT

The paper tackles the dynamic identification and the damage detection carried out by a spectral-based method on the well-known Z24 bridge, a three-span pre-stressed concrete bridge located in Switzerland. Before being destroyed, the bridge was progressively damaged and tested in the framework of the Brite Euram project SIMCES. Starting from this benchmark, the presented spectral-based identification technique is validated and the usefulness of this method as a non-destructive tool able to catch the dynamic behavior of a structure and locate the damage is widely discussed. Firstly, a FE model of the bridge was built and calibrated in order to analyze its response to different excitation types (free vibration, triangular pulse, swept sine, shaker and random vibrations) and several damage scenarios. Secondly, aiming at identifying both the modal parameters and the damage of the bridge, the spectral-based method is applied making use of the power spectral matrix decomposition. Finally, a proper index is defined and applied to this case-study in order to locate the damage.

KEYWORDS

Concrete bridges, Spectral-based method, Dynamic identification technique, Damage detection, Damage Localization

INTRODUCTION

In the last years the scientific community has been more and more pointing out the relevance of damage detection methods in all the engineering fields in order to catch the occurrence of damage at the earliest possible stage and reduce the seismic vulnerability of a structural system, besides preserving it from the accumulation of physical, chemical and mechanical damage through time. Non-destructive damage detection techniques, such as ultrasonic and acoustic methods, are mostly local approaches and do not allow to catch the global behavior of a structure, as well as conventional visual inspections are often inadequate for identifying damage which is invisible to human eyes. On the contrary, dynamic-based damage identification methods are tools able to provide a 'global' way to assess structural conditions since they are based on changes of dynamic parameters, such as eigenfrequencies, mode shapes and damping coefficients, that are the 'fingerprints' of the system, thus their shift implies a change in the global behaviour of the structure itself. Considering the explicit dependence of outputs power spectral densities on frequency contents, an inverse problem can be solved in order to detect the damage and predict its location, given the dynamic properties of two different structural conditions, namely undamaged and damaged configurations. What mentioned is the key-aspect of the spectral-based damage identification technique hereby introduced [1].

Aimed at studying and validating this method, the well-known case-study of the Z24 Bridge has been taken into account profiting from the benchmark provided by the wide experimental and numerical campaign that was carried out in the framework of the Brite Euram project BE96-3157 system identification to monitor civil engineering structures (SIMCES). A series of damage scenarios was then applied to the pre-stressed concrete bridge located in Switzerland with the purpose to study the dynamic response of the structure and detect, localize and quantify the damage. Detailed information about the progressive damage testing on Z24 can be found in Krämer et al. [2].

After briefly describing the bridge and its geometrical and mechanical features, the present paper deals with the presentation of the FE model of the bridge and the related model updating, firstly tackling preliminary analyses performed to assess the structural conditions before and after the application of different damage scenarios, secondly carrying out a series of transient analyses for evaluating the dynamic characteristics of the system from forced, free and random vibrations data and with respect to two structural configurations (undamaged and damaged). Comparisons among the results obtained by the spectral-based dynamic identification technique and other identification methods are presented. Finally, an appropriate localization index is defined making use of evolutionary complex eigenvectors obtained from the decomposition of the power spectral density matrix. Furthermore, remarks regarding the introduced method and its reliability are widely discussed.

Z24 BRIDGE FE MODEL CALIBRATION

Located in Canton Bern, Switzerland, the Z24 Bridge was an overpass of the national highway A1 between Bern and Zürich connecting the villages of Koppigen and Utzenstorf. Dating back to 1963, it was demolished in 1998 to build a larger side-span bridge, but before being destroyed the bridge was subjected to a long-term continuous monitoring and several progressive damage in order to study the influence of different damage scenarios on its dynamic properties. Z24 was a classical pre-stressed concrete two-cell box girder bridge with a main span of 30 m and two side-spans of 14 m, for an overall length of nearly 60 m and a width of 8.60 m (see Figure 1). Both abutments consisted of three concrete columns connected with concrete hinges to the girder, while both intermediate supports were concrete piers clamped into the girder. A literature review of the benchmark results is provided by Reynders and De Roeck [3] and Teughels and De Roeck [4].

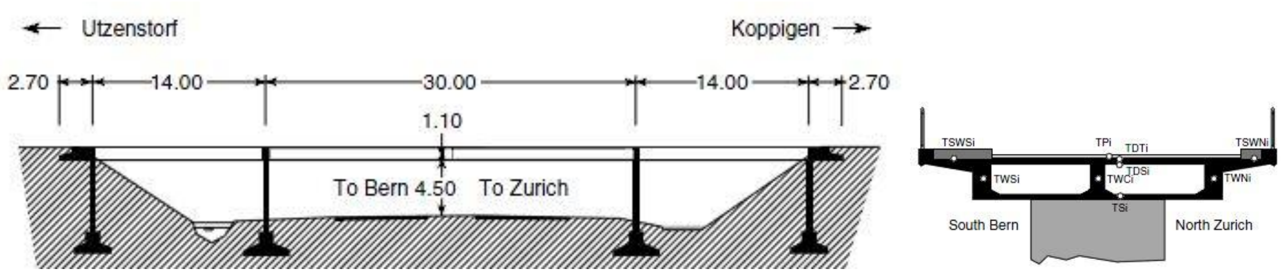


Fig. 1 Side view of the Z24 bridge and related cross section of the girder

Aimed at validating the spectral-based dynamic identification technique, a simplified Finite Element (FE) model of the bridge was built in DIANA (2013) and calibrated using the dynamic features extracted from the experimental data as reference. The numerical model consisted of 126 beam elements (6 degrees of freedom per each node) for the girder, piers and abutment piers; 6 point mass elements with both concentrated translational mass and rotary inertia components for cross girders and foundations and 22 spring elements to simulate the stiffness of the soil (see Figure 2). The concrete is considered to be homogeneous with an initial value of the Young's modulus E_0 = 37.5 GPa and a shear modulus G_0 = 16 GPa. It needs to be stressed that the girder is characterized by higher stiffness values above the supporting piers because of an increased thickness of bottom and top slab. Regarding the soil stiffness parameters, the initial values are: $K_{v,p} = 180 \cdot 10^6$ N/m, $K_{h,p} = 210 \cdot 10^6$ N/m (under the piers, at 14 and 44m), $K_{v,c} = K_{h,c} = 100 \cdot 10^6$ N/m (under the columns, at 0 and 58 m); $K_{v,a} = 180 \cdot 10^6$ N/m, $K_{h,a} = 200 \cdot 10^6$ N/m (at the abutments) and $K_{v,ac} = K_{h,ac} = 100 \cdot 10^6$ N/m (around the columns).

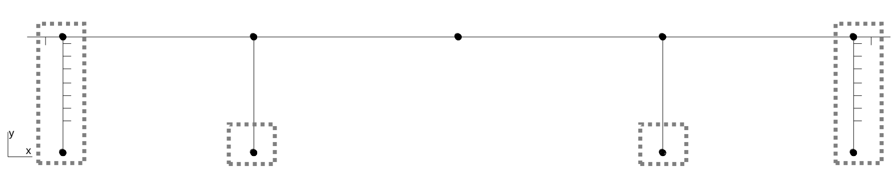


Fig. 2 FE Model of the bridge. Soil springs (dot gey boxes) and point mass elements (black dots) are indicated

In order to represent the dynamic behavior of the structure more accurately, the 806 DOFs FE model was then updated using a direct comparison between experimental and analytical values of the first five identified eigenfrequencies (Modal-based Model Updating). The bending stiffness EI_z and the torsional stiffness GI_t of the beam elements of the girder were detected being the most sensitive parameters affecting the structural response of the system, thus the updating process merely concerned the correction of both the Young's and shear modulus values, E and G, since the geometry of the bridge was well-known and so do the moments of inertia. Even the soil stiffness was not updated since its sensitivity on the dynamic response of the structure was considered negligible. Figure 3 shows the initial and updated bending and torsional stiffness distributions. As the average frequency error was about 3.1%, the global updated results in terms of eigenfrequencies were considered acceptable, see Table I.

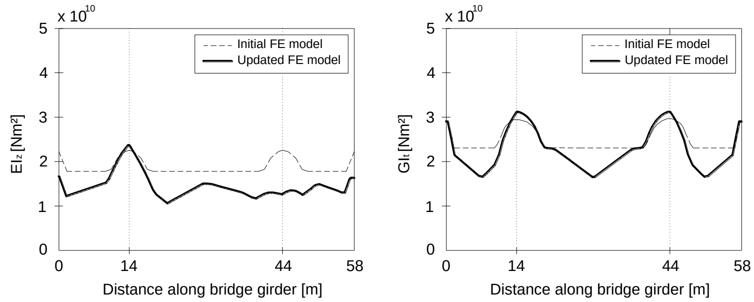


Fig. 3 Bending and torsional stiffness distributions along the bridge girder

TABLE I. UPDATED EIGENFREQUENCIES

Mode	Eigenfrequencies [Hz]			Average ω error [%]
	Experimental	Initial Value (FEM)	Updated Value (FEM)	
1	3.89	3.88	3.89	0.05
2	5.02	5.27	4.93	-1.86
3	9.80	8.56	9.17	-6.39
4	10.30	10.60	10.56	2.48
5	12.67	13.80	13.28	4.85
Average	–	–	–	+3.13

PRELIMINARY ANALYSES

Before proceeding to the spectral-based dynamic identification of the bridge subjected to different types of excitation, a preliminary FE eigenvalue analysis was addressed to the choice of the measurements points (25), the sampling frequency (100 Hz) and the total sampling time to set for the data tabulation. Regarding the eigenmodes of the sound configuration, the first is pure bending mode, the second is a longitudinal mode, the third and fifth are transversal modes and the fourth is a non-symmetric bending mode. Furthermore, stated the necessity to have two structural conditions to detect the possible occurrence of damage and its localization, a series of eigenvalue analyses was also performed taking into account different damage scenarios with the purpose of catching the one better representing the damage scenario induced in the real structure and consisting of a 95 mm lowering of the supporting pier at 44 m with subsequent cracks in the girder above the pier itself. This damage scenario simulated the settlement of the pier foundation (Figure 4).

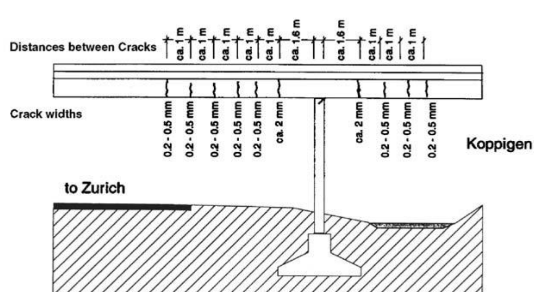


Fig. 4 Cracks in the bridge girder, above the lowered pier (at 44 m).

TABLE II. EXPERIMENTAL AND NUMERICAL EIGENFREQUENCIES FOR DIFFERENT DAMAGE SCENARIOS

Mode	**Eigenfrequencies [Hz]**							
	Undamaged		**Damaged**					
	Experimental	**FEM RS**	**Experimental**	**DS_I fem**	**DS_{II} fem**	**DS_{III} fem**	**DS_{IV} fem**	**DS_V fem**
1	3.89	3.89	3.67	3.77	3.89	3.76	3.69	3.67
2	5.02	4.93	4.95	4.86	4.91	4.84	4.82	4.80
3	9.80	9.17	9.21	8.54	9.17	8.54	8.57	8.36
4	10.30	10.56	9.69	10.24	10.55	10.23	10.33	10.31
5	12.67	13.28	12.03	13.11	13.22	13.05	13.04	13.01
Δ_ω [%]*	–	3.13	–	5.29	5.20	5.20	5.02	5.36

Five different Damage Scenarios (DS_I, DS_{II}, DS_{III}, DS_{IV}, DS_V) were simulated by reducing the Young's modulus of the girder above the pier at 44 m and/or lowering the supporting pier itself. Particularly, the first DS_I consisted of a 30% reduction of the E modulus in the girder above the considered pier; the second DS_{II} consisted of a 95 mm lowering of the Koppigen pier; the third DS_{III} was a combination of both the previous ones; the fourth DS_{IV} consisted of a 60% reduction of the E modulus in the beam elements of the girder just above the Koppigen pier, always 95 mm lowered; finally, the fifth DS_V was a combination of DS_I and DS_{IV}. Table II highlights the first five identified eigenfrequencies for each DS compared with the ones related to the Reference Scenario (RS) or undamaged configuration. As shown in the table, the fourth DS is the one characterized by the lowest average frequency error, thus it was selected as damaged configuration to use for the subsequent analyses. The applied damage scenario caused an overall eigenfrequencies shift up to 3.6%.

NUMERICAL DYNAMIC IDENTIFICATION OF THE BRIDGE

Many researchers have attempted to detect and localize damage using changes in natural frequencies [5]. Kim et al. [6] assert that the most appealing feature associated with using frequencies is that natural frequencies are relatively simple to measure, but significant damage sometimes may cause very small changes in natural frequencies and these changes may go undetected due to measurement or processing errors. Furthermore, temperature variations may introduce additional uncertainties, showing the limited feasibility of using frequency changes for the purpose of damage localization. To overcome these difficulties, research efforts have focused on using changes in mode shapes, as they are much more sensitive to local damage. However, damage is a local phenomenon and may not significantly influence the mode shapes of the lower modes that are usually measured from vibration tests. Another drawback can be also related to the number and position of sensors that may have a crucial effect on the accuracy of the damage detection procedure. In this paper a new methodology based on the knowledge of the second order spectral properties of the nodal response processes is presented. The aim is to detect and localize the damage by a proper combination of eigenvalues and eigenvectors, that are an estimation of eigenfrequencies and mode shapes, respectively.

In order to validate the efficiency of the spectral-based identification method to catch the dynamic characteristics of the bridge, several linear transient analyses were performed to collect the structural response to different types of excitation: triangular pulse, ramp force, shaker and random vibrations. A triangular pulse of 1000 kN was applied on three different points of the bridge deck: in the middle of the Utzenstorf side-span, at one quarter of the main span and at one third of the Koppigen side-span. The data were sampled at 100 Hz, the measurement time for each set-up was 80 s and the number of data points per channel was 7999. Same acquisition parameters were used for the ramp force excitation calibrated to reach a total load of 2000 kN in 20 s and then release the structure. The shaker test was performed by generating a band-limited (3-15 Hz) input signal acting 120 s on the same three points of the bridge deck already used for both the impulse and ramp force. The sampling rate was always 100 Hz, while the measurement time for each set-up was 300 s, thus 29999 data points per channel were measured. Finally, random vibrations with flat spectrum were also generated in order to simulate ambient excitations acting on the bridge and collect its structural response for 10 min, resulting in a number of 59999 data points per channel. All the transient analyses were carried out in DIANA [7] by means of the Newmark's method, time-stepping procedure used to compute the structural response when the system is subjected to a force vector varying in time. It is known that the critical wave length has to be integrated with sufficient accuracy for a correct prediction of the system response, thus the time step for the integration was chosen taking into account either the time variation of the excitation and the natural period of the system ($\Delta t = T_n/20$). Nevertheless, to ensure the stability of the method, the integration parameters were chosen considering the assumption of the average acceleration method, meaning that the variation of the acceleration over a time step is assumed constant implying the stability of the method for any Δt [8]. A damping ratio of 1% is considered in order to compute the Rayleigh damping coefficients for the dynamic analyses and a linear behaviour of the structure is also assumed, thus no equilibrium iteration is required. For each analysis the measurement time was long enough to let the structural response get rid of the transient part due to the excitation.

It must be highlighted that experimentally the bridge was measured in nine set-ups of 33 accelerometers and this lead to identify several mode shapes, whereas in the work hereby presented the whole bridge is measured in three set-ups of 25 measurement points and all the excitations are numerically generated in MATLAB [9]. Both the absence of real excitation sources and the reduction of the number of channels did not allow to achieve an overall dynamic identification of the bridge, but they were enough to catch the meaningful dynamic properties of the structure in order to put into practise the damage localization index by means of a properly combination of the values extracted from the power spectral matrix decomposition. To reach this goal, all the analyses were performed for both the undamaged and damaged configurations since the presence of two different structural conditions is a basic state for detecting and localizing the damage.

A suitable representation of the structural response was achieved by means of the spectral decomposition over the frequency domain, through the evaluation of both the direct and cross power spectral densities of output signals and the construction of the Power Spectrum Matrix, as follows:

$$S_X(\omega)=\begin{bmatrix} S_{X_1X_1}(\omega) & S_{X_1X_2}(\omega) & \cdots & S_{X_1X_m}(\omega) \\ S_{X_2X_1}(\omega) & S_{X_2X_2}(\omega) & \cdots & S_{X_2X_m}(\omega) \\ \vdots & & \ddots & \vdots \\ S_{X_mX_1}(\omega) & \cdots & \cdots & S_{X_mX_m}(\omega) \end{bmatrix} \tag{1}$$

where diagonal elements are the direct power spectral densities (PSD), also called *auto-spectra*, while out of diagonal elements are the cross power spectral densities (CPSD), so-called *cross-spectra*. Generally, the PSD gives the spectral distribution of the average energy of a random process and it is defined as the Fourier transform of the autocorrelation function:

$$S_{XX}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau \tag{2}$$

while the CPSD gives the spectral distribution of the average energy of two random processes, providing information not only about frequencies but also phases, and it is defined as the Fourier transform of the cross-correlation function:

$$S_{XY}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{XY}(\tau)e^{-i\omega\tau}\,d\tau \tag{3}$$

Given a multivariate process **X**(t) and defined the Power Spectrum Matrix (hermitian matrix consisting of a symmetric real part and an anti-symmetric imaginary part), it is always possible to decompose it for each frequency by solving an eigenvalue problem, which has the following expression:

$$\mathbf{S}_X(\omega)\mathbf{\Psi}_X(\omega)=\mathbf{\Psi}_X(\omega)\mathbf{\Lambda}_X(\omega) \quad \Rightarrow \quad \mathbf{S}_X(\omega)=\mathbf{\Psi}_X(\omega)\mathbf{\Lambda}_X(\omega)\mathbf{\Psi}_X^T(\omega) \tag{4}$$

where the elements of the real diagonal matrix $\mathbf{\Lambda}_x$ (ω) are the eigenvalues in decreasing order of the matrix $\mathbf{S}_x$ (ω) and the elements of the complex matrix $\mathbf{\Psi}_x$ (ω) are the orthonormal eigenvectors of $\mathbf{S}_x$ (ω). Each eigenvalue λ_x denotes the energy of the vibration mode at that frequency, whereas each eigenvector $\boldsymbol{\psi}_x$ is a mode shape estimation corresponding to that eigenvalue. Some of the response spectral eigenvalues obtained from the data analysis are shown in Figure 5. It needs to be stressed that two different square matrices [25x25] were computed in MATLAB for X and Y directions with the purpose to speed up the process, thus depending on the involved modes the peaks of the response spectral eigenvalues were caught in one matrix or the other. An overall comparison in terms of identified eigenfrequencies among the three excitation types, namely impulse, ramped force and shaker, is shown in Table III. As mentioned previously, even though both the absence of real excitation sources and the reduction of the number of channels did not enable to achieve a full identification of the bridge if compared with the experimental campaign, the few identified modes showed a good agreement and allowed to detect the frequency decay due to the presence of damage in the structure. Unlike the previous data sets, the data obtained from the ambient excitation simulation were analyzed making use of two methods: the Power Spectral-based Method and the Stochastic Subspace Identification (SSI-PC) [10], implemented in ARTeMIS [11]. In this case more modes were consistently identified and the estimation of damping ratios and coefficients of variation was possible as well (see Tables IV and V).

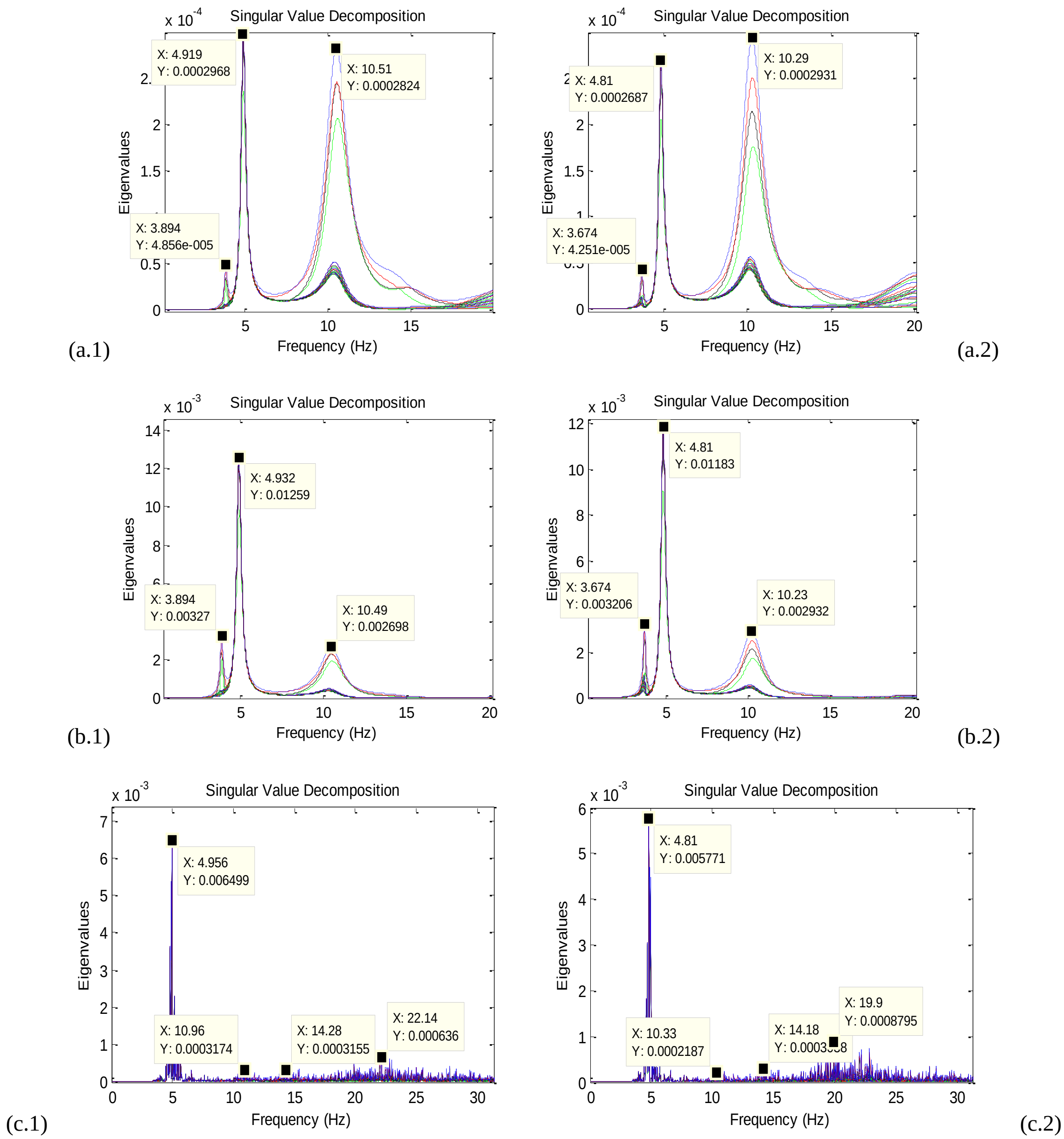


Fig. 5 Response spectral eigenvalues obtained from the decomposition of the Power Spectrum Matrix (X direction) for different excitation types: (a.1) undamaged and (a.2) damaged configurations for triangular pulse; (b.1) undamaged and (b.2) damaged configurations for ramp force; (c.1) undamaged and (c.2) damaged configurations for ambient data

TABLE III. COMPARISON OF IDENTIFIED EIGENFREQUENCIES AMONG DIFFERENT EXCITATION TYPES

	Eigenfrequencies [Hz]								
	Undamaged			Damaged					
Mode	**Impulse**	**Ramp Force**	**Shaker**	**Impulse**	**Ramp Force**	**Shaker**	**$\Delta\omega_{im}$ [%]**	**$\Delta\omega_{rf}$ [%]**	**$\Delta\omega_{sh}$ [%]**
1	3.89	3.89	3.91	3.67	3.67	3.68	-5.66	-5.66	-5.88
2	4.92	4.93	4.95	4.81	4.81	4.82	-2.24	-2.43	-2.62
3	–	–	–	–	–	–	–	–	–
4	10.51	10.49	10.70	10.29	10.23	10.35	-2.09	-2.48	-3.27
5	–	–	–	–	–	–	–	–	–
Average	–	–	–	–	–	–	-3.33	-3.52	-3.92

TABLE IV. IDENTIFIED EIGENFREQUENCIES FOR AMBIENT VIBRATIONS SIMULATION (PSM)

Mode	**Undamaged** ω **[Hz]**	**Damaged** ω **[Hz]**	Δ_ω **[%]**
1	3.95	3.56	-9.87
2	4.96	4.81	-3.02
3	–	–	–
4	10.96	10.33	-5.75
5	–	–	–
6	14.28	14.18	-0.70
7	22.14	19.9	-10.12
Average	–	–	-5.89

TABLE V. IDENTIFIED EIGENFREQUENCIES FOR AMBIENT VIBRATIONS SIMULATION (SSI-PC)

Mode	**Undamaged**		**Damaged**			**Undamaged**		**Damaged**		
	ω **[Hz]**	CV_ω **[%]**	ω **[Hz]**	CV_ω **[%]**	Δ_ω **[%]**	ξ **[%]**	CV_ξ **[%]**	ξ **[%]**	CV_ξ **[%]**	Δ_ξ **[%]**
1	4.09	0.35	3.68	0.19	-10.02	4.22	18.27	4.25	4.88	+0.71
2	4.92	0.05	4.80	0.05	-2.44	3.75	1.16	3.74	0.79	-0.26
3	–	–	–	–	–	–	–	–	–	–
4	10.64	0.50	10.49	0.02	-1.41	4.57	18.64	4.01	0.81	-12.25
5	13.76	0.10	13.82	0.02	+0.44	2.29	5.99	4.89	0.58	+113.5
6	14.63	0.15	14.52	0.04	-0.75	4.07	4.40	1.60	7.17	-60.69
7	22.24	0.03	19.80	0.01	-10.97	3.16	2.98	0.63	2.21	-80.06
Average	–	0.20	–	0.06	-5.12	3.68	8.57	3.18	2.74	+57.12*

*Average value calculated only with positive differences and for comparable modes

As expected, either technique detected a change in terms of dynamic properties of the structure due to the occurrence of damage; particularly, a frequency decrease up to 5.12 % can be noticed between the two structural conditions. Regarding the mode shapes comparison between undamaged and damaged configuration, Figure 6 shows the mode shapes identified from 'ambient data' for either case: the first mode is a pure bending mode, the second is a longitudinal mode, the third is a vertical anti-symmetric bending mode, the fourth and fifth are coupled bending and torsional modes and the sixth one is a highly complex mode. Figure 7 represents the MAC values between mode shape estimates (damaged and undamaged). MAC stands for modal assurance criterion and is nothing more than the squared correlation between two modal vectors [12]. By definition, the MAC is a value ranging from 0 to 1, thus the more the values are far from 1 the more the correlation is poor. In the present case, a very good correlation in terms of MAC values can be stressed.

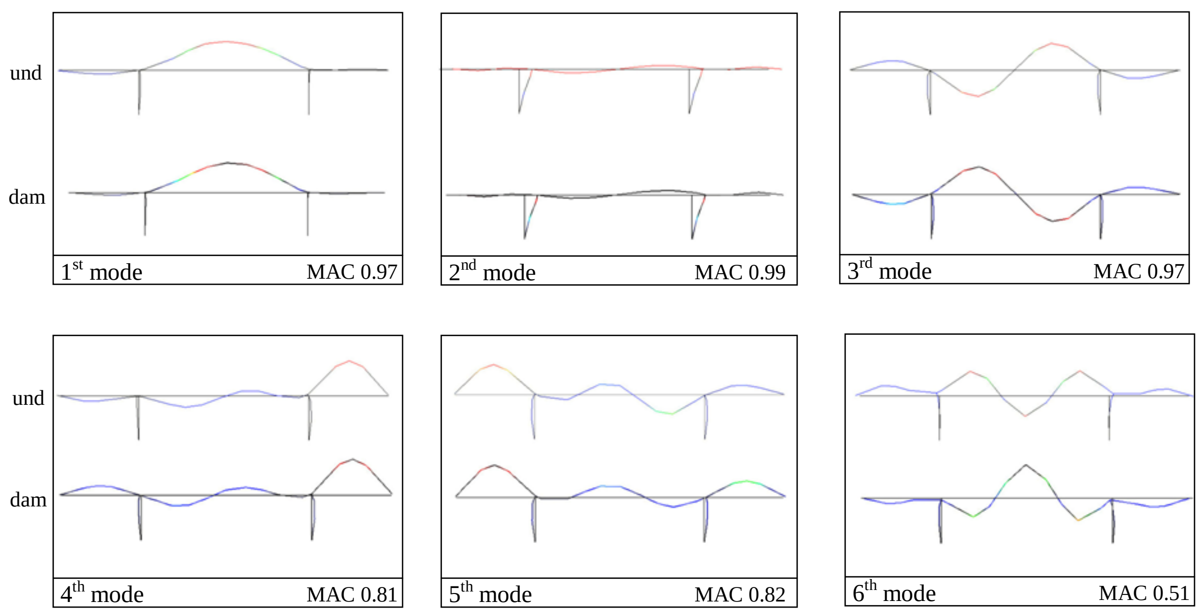


Fig. 6 Identified mode shapes from ambient data simulation

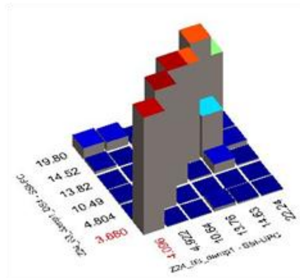


Fig. 7 MAC values between mode shape estimates

DAMAGE IDENTIFICATION OF THE BRIDGE

Making use of evolutionary eigenvectors obtained from the decomposition of the power spectral density matrix and combining them with the related eigenvalues, a damage localization index is properly defined and applied to the present case study:

$$\Delta\psi = \sum_{j=1}^{n}\left\|\left|\sum_{i=1}^{m}\left[\psi_i^d(\omega_i)\cdot\sqrt{\lambda_i^d(\omega_i)}\right]\right| - \left|\sum_{i=1}^{m}\left[\psi_i^u(\omega_i)\cdot\sqrt{\lambda_i^u(\omega_i)}\right]\right|\right\| \qquad (5)$$

where $\boldsymbol{\psi}$ denotes the eigenvector amplified by its related eigenvalue λ over the whole frequency domain, n the mode number, m the frequency range and upper scripts d, u respectively denote damaged and undamaged conditions. The index is based on the differences between spectral modes. Figure 8 presents the point's location were the dynamic response was obtained numerically and where the damage method was applied. Figure 9 shows the obtained results in terms of damage localization for all the four excitation types in X and Y directions, since either direction has to be taken into account to get an exact and exhaustive understanding about damage localization. As one can see, all the indexes allowed to locate the damage where it really occurred, namely at the Koppigen pier and the girder above it, except for the index computed in X direction from ambient data that detects the presence of damage even for the part of the bridge girder that is not affected by it at all.

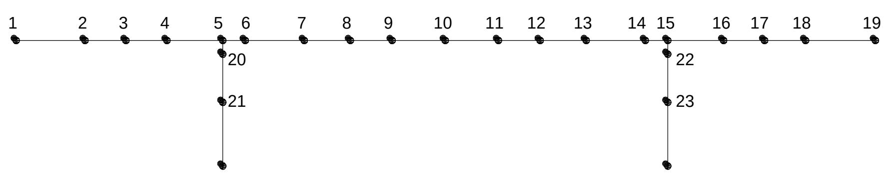
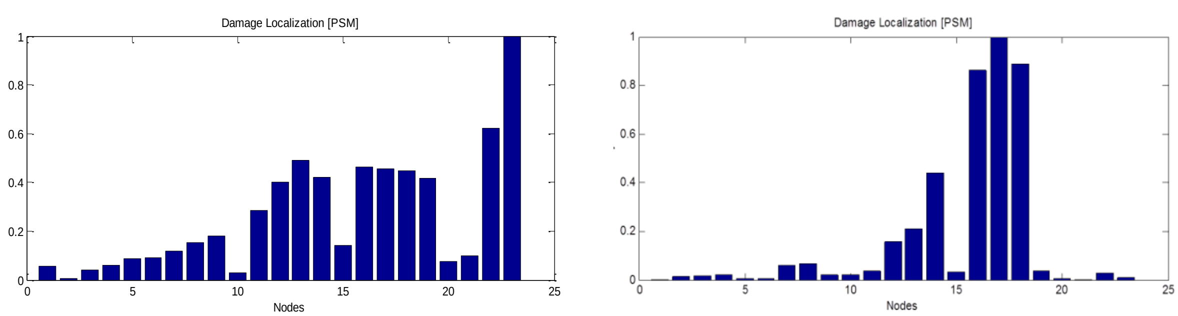


Fig. 8 25 measurement points along the Z24 bridge: 19 monitored points for the deck and 6 for the piers



(a.1) (a.2)

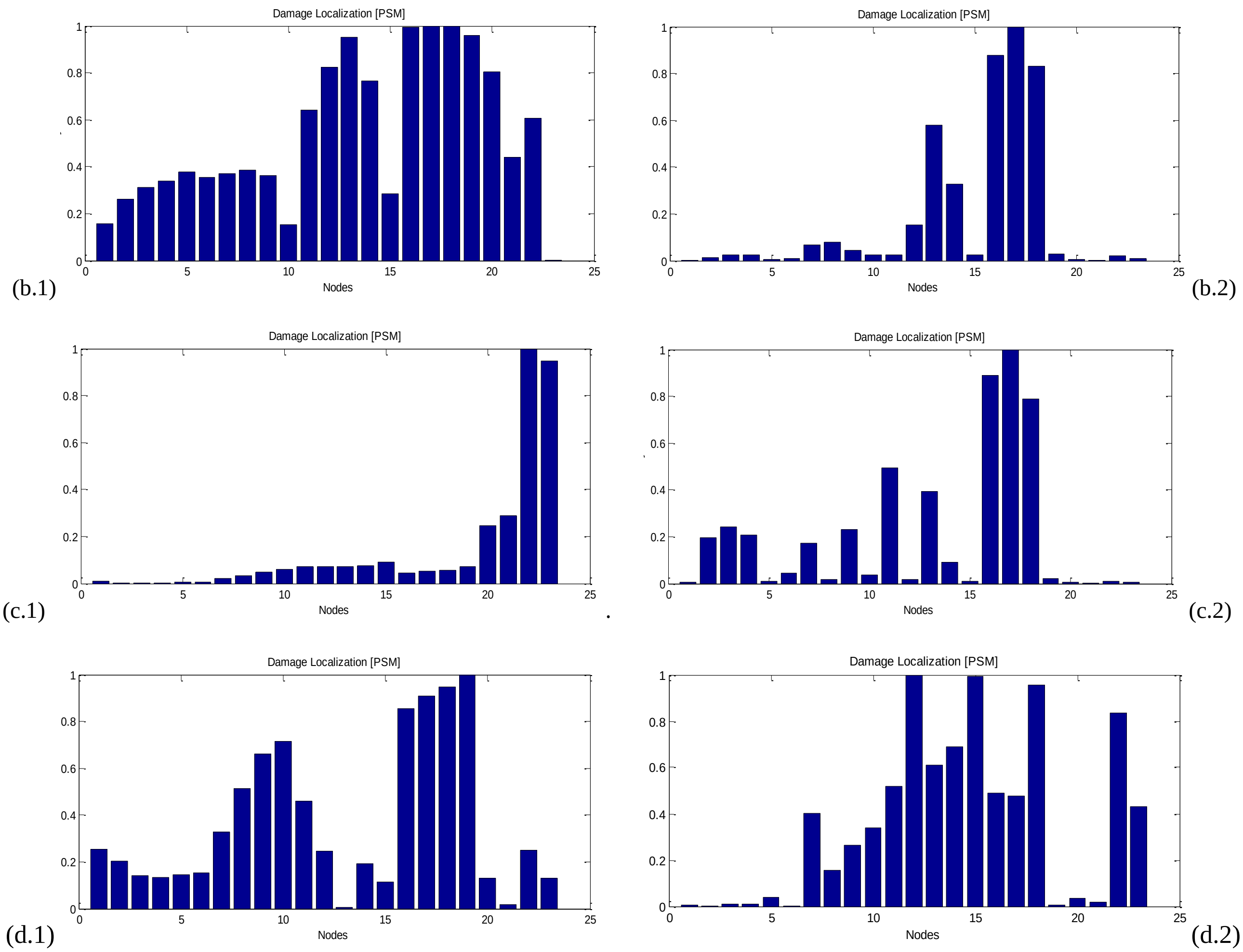


Fig. 9 Impulse: damage localization in (a.1) X and (a.2) Y direction. Ramp force: damage localization in (b.1) X and (b.2) Y direction. Shaker test: damage localization in (c.1) X and (c.2) Y direction. Random vibrations: damage localization in (d.1) X and (d.2) Y direction

CONCLUSIONS

The results of the dynamic identification on the Z24 Bridge by means of a spectral-based method has been presented in this paper, pointing out how the dependence of outputs power spectral densities on frequency contents can allow to detect and locate the damage, given the dynamic properties of two different structural conditions, namely undamaged and damaged configurations. Even though the identification of higher modes has not been possible because of the difficulty in exiting and extracting them from the numerical analyses performed, the main modes characterizing the dynamic behavior of the bridge have been caught by means of the power spectral matrix decomposition. Finally, despite the use of numerically generated excitation sources and a simple beam FE model, satisfactory results have been attained in terms of damage investigation. The efficiency of the introduced spectral-based identification method cannot be generalized since additional studies on this technique and its reliability in terms of damage localization are still ongoing, but regarding the presented case study it can be stressed that the results achieved from the localization index have shown a good agreement with the ones experimentally and numerically obtained.

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