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Damage Initiation and Ultimate Tensile Strength of Scaled $[0^\circ_n/90^\circ_n/0^\circ_n]_T$ Graphite-Epoxy Coupons

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Abstract

Previous research on scaling effects in composite materials has demonstrated that the stress levels at first ply failure and ultimate failure of composite laminates are dependent on the size of the laminate. In particular, the thickness dimension has been shown to be the most influential parameter in strength scaling of composite coupons loaded in tension. Geometrically and constitutively scaled laminates exhibit decreasing strength with increasing specimen size, and the magnitude of the strength-size effect is a function of both material properties and laminate stacking sequence. Some of the commonly used failure criteria for composite materials such as maximum stress, maximum strain, and tensor polynomial (e.g., Tsai-Wu) cannot account for the strength-size effect. In this paper, three concepts are developed and evaluated for incorporating size dependency into failure criteria for composite materials. An experimental program of limited scope was performed to determine the first ply failure stress in scaled cross-ply laminates loaded in tension. Test specimens were fabricated of AS-4/3502 graphite-epoxy composite material with laminate stacking sequences of $[0^\circ_n/90^\circ_n/0^\circ_n]_T$ where $n=1-6$. Two experimental techniques were used to determine first ply failure, defined as a transverse matrix crack in the 90° ply: (1) step loading with dye penetrant x-ray of the specimen at each load interval, and (2) acoustic emission. The best correlation between first ply failure analysis and experimental data was obtained using a modified Weibull approach which incorporated the residual thermal stress and the outer ply constraint, as well as the ply thickness effect. Finally, a second set of experiments was performed to determine the tensile response and ultimate failure of the scaled cross-ply laminates. The results of these experiments indicated no influence of specimen size on tensile response or ultimate strength.

Introduction

The strength-size effect (decreasing strength with increasing ply thickness) in composite laminates has been extensively investigated [1-26]. It is well known that the stress level at which the first transverse matrix crack is observed in cross-ply laminates is directly related to the thickness of the 90° core ply. The thicker the 90° ply, the lower the stress level for

first ply failure. For example, experiments were performed by Flaggs and Kural [4] on cross-plyed $[0^\circ_2/90^\circ_n]_T$ laminates to determine the stress level at first ply failure. The chosen lay-up for the experiments contained a constant number of constraint (0°) plies and varying numbers of core (90°) plies. The value of n was increased in increments of 1, 2, 4, and 8. For the most constrained laminate ($n=1$), the stress at first ply failure was reported to be 2.48 times the value of the transverse tensile strength, Y_T . As the thickness of the 90° plies increased, the stress level at first ply failure asymptotically approached the transverse tensile strength. This phenomenon was explained by introducing the concept of 'in situ' strength of the 90° plies to account for the ply thickness effect and the constraint effect provided by the neighboring plies. In general, the effect of increasing ply thickness is believed to reduce the strength of the ply due to an increased probability of the presence of a strength-critical flaw in the greater volume of material present. Conversely, the constraint effect provided by the neighboring 0° ply is believed to strengthen the 90° ply near the interface region and increase the stress level at which a transverse matrix crack will appear in the 90° core ply. As the thickness of the 90° ply increases the constraint effect is limited to the region close to the ply interface, and the interior of the 90° ply is not affected.

In the present paper, a new approach is used to study the strength-size effect in cross-plyed laminates. This study examines scaled laminates having the same relative number of constraint (0°) plies and core (90°) plies, instead of the more typical approach of studying laminates having a constant number of constraint plies and increasing numbers of 90° core plies. The laminate stacking sequence is $[0^\circ_n/90^\circ_n/0^\circ_n]_T$, where $n=1-6$. The composite coupon specimens are designed to be constitutively scaled and geometrically scaled in the thickness dimension. The scaled cross-plyed specimens were examined for pre-existing damage prior to tensile testing. Two specimens of each size and laminate stacking sequence were instrumented and loaded under uniaxial tension to determine the stress-strain response and ultimate strength. The remaining eight specimens were tested to determine the stress level at first ply failure using two different experimental techniques. The experimental program and test results are described in the next section. Finally, three concepts for incorporating size dependency in commonly used failure criteria for composite materials are discussed, and correlation with the test data is presented.

Experimental Program

Six panels with in-plane dimensions of 12 in. x 12 in. were fabricated of AS4/3502 (Grade 190) graphite-epoxy composite material corresponding to the laminate stacking sequences $[0^\circ_n/90^\circ_n/0^\circ_n]_T$ where $n=1-6$. The panels were cured according to manufacturer's recommended specifications. Test specimens were machined from each panel of dimensions 11.0 in. x 1.0 in., for a total of 10 specimens per panel. Specimen thickness varied from three to eighteen plies, corresponding to $n=1$ and 6, respectively. The average thickness per ply was 0.008 in. The average ply thickness is higher than normal due to the special grade of AS4/3502 material which contains approximately 16 fiber diameters through the thickness of a single ply.

One specimen of each laminate stacking sequence was evaluated for damage or defects due to the curing and/or machining processes, prior to testing, using dye penetrant x-ray technique.

Damage was discovered in the n=5 and n=6 laminates in the form of transverse matrix cracks and longitudinal splitting. Two transverse matrix cracks were observed in the n=5 laminate and eight cracks were found in the n=6 laminate. In addition, the n=5 laminate exhibited 2 longitudinal splits, while the n=6 laminate had 3 longitudinal splits. No damage was detected in the n=1-4 laminates. Two specimens of each laminate stacking sequence were instrumented with back-to-back strain gages at the midpoint of the specimen for determination of tensile response and ultimate tensile load. None of the remaining eight specimens per laminate stacking sequence were instrumented.

The material properties of AS4/3502 graphite-epoxy composite material were determined previously [27, 28] to be: $E_1 = 19.94$ Msi, $E_2 = 1.56$ Msi, $\nu_{12} = 0.293$, and $G_{12} = 0.82$ Msi.

Two types of experiments were performed. One set of experiments was conducted to evaluate the tensile response and ultimate strength of the scaled cross-ply laminates. The two strain-gauged specimens of each laminate stacking sequence were loaded in tension until ultimate failure occurred. Ultimate failure is defined as complete loss of load carrying capability. These experiments were performed on a benchtop load test machine under continuous quasi-static loading. The second set of experiments was conducted to determine the stress level at first ply failure, defined as the first transverse matrix crack in the 90° ply. Two different techniques were used to determine first ply failure: (1) step loading with dye penetrant x-ray at each load interval, and (2) acoustic emission. These methods are described in the following sections.

One technique used to determine the stress level at first ply failure of the n=1-4 scaled cross-ply laminates was step loading with dye penetrant x-ray. (Note: the n=5 and 6 specimens were not tested using the step loading technique due to the presence of pre-existing damage in the specimens.) These tests were performed by mounting and aligning the specimen in hydraulic grips of a load test machine and applying tensile load through load control of the machine. At a specified load level, the machine was stopped, with load maintained at a constant level, while the specimen edges were coated with dye penetrant. Then, an x-ray was taken and developed. If a single crack was observed, the specimen was removed and the load level noted. If no crack was observed, the specimen was loaded again, typically in increments of 100 lb., and the procedure was repeated. If several cracks were observed at the first load level, the experiment was repeated with a new specimen and the first x-ray was taken at a lower load level, and the load increments were refined.

Following the first few experiments, the load level at first ply failure for a specific laminate is known, and, for subsequent tests, the load was applied at a much higher level before the first x-ray was taken. Following a test, the load at first ply failure was calculated to be the average of the last load before a crack was found and the load level noted with an observed crack. This procedure worked well for the n = 2-4 specimens except the 3-ply $[0^\circ/90^\circ/0^\circ]_T$ laminate, n=1 case. For this laminate, the first ply failure occurred at or near ultimate failure. In those cases where damage could be assessed prior to ultimate failure, the x-ray film showed many transverse matrix cracks emanating from the edge, but none completely through the width of

the specimen. The load at first ply failure was assumed to be the average of the last load step and the ultimate load.

A refined acoustic emission (AE) technique, developed to detect damage in composite materials, was applied to detect transverse matrix cracks in cross-ply laminates under tensile load. The AE technique not only allowed the detection of the formation of a transverse matrix crack, but was also used to predict the location of the crack. The test method involved applying four broad band sensors to the specimen, two each at either end of the specimen, with the edge of the sensor aligned with the specimen edge. The reason for this sensor arrangement was to determine the site of formation of the transverse cracks. Thus, the technique could determine the location of the crack along the length of the specimen, and provide information on the site of crack initiation. Signals from the sensors were amplified and then input to a digital acoustic emission analysis system. Sensor gains were adjusted depending on specimen thickness. In general, thicker specimens generated signals of larger amplitude. The load at which a given crack signal was detected was obtained from a parametric measurement system in the AE instrument. Predictions of crack location and initiation site were performed after the test using manual, cursor based arrival time determination. More information on this AE procedure is reported in References 29 and 30. Due to difficulty in capturing first ply failure for the $n = 1$ and 2 specimens, no data is reported for these laminates.

Summary of Experimental Results

The stress-strain responses of the scaled cross-ply laminates ($n=1-4$) loaded under uniaxial tension to ultimate failure are shown in Figure 1. In general, the scaled cross-ply laminates

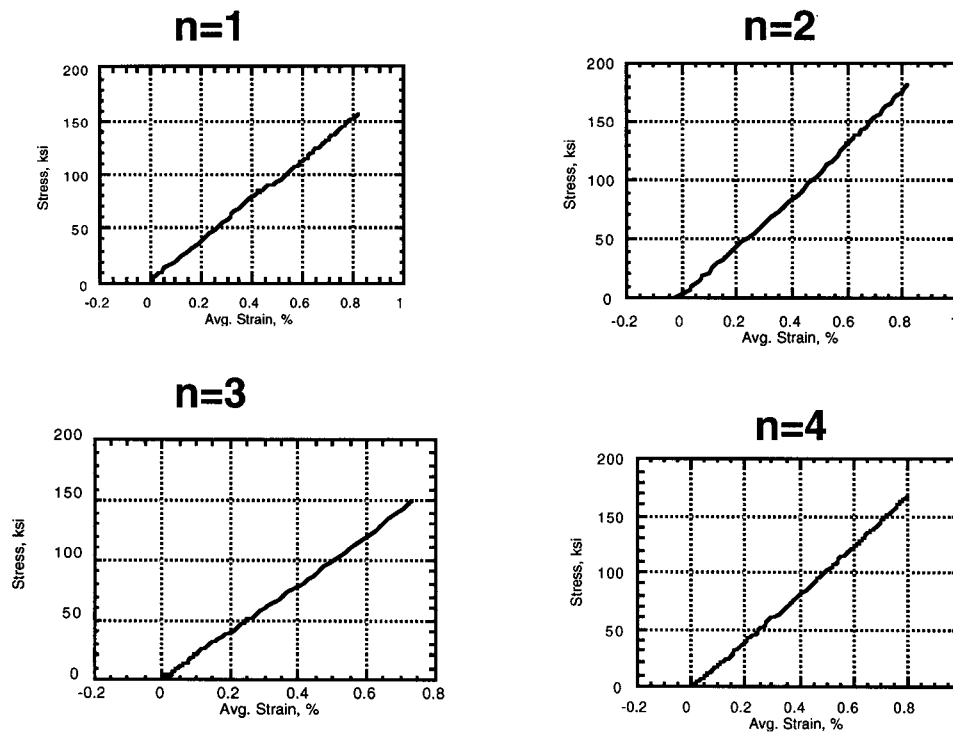


Figure 1. Tensile response of $[0^\circ n/90^\circ n/0^\circ n]_T$, $n=1-4$, scaled cross-ply laminates.

exhibit a linear-elastic response to ultimate failure. The curve for the $n=1$ laminate does show a noticeable shift or “knee” in the tensile response curve, however, the knee is not associated with the first ply failure event. The average values of ultimate strength for the two specimens of each lay-up ($n=1-4$) are plotted versus specimen size in Figure 2. Based on the results shown in Figure 2, it is apparent that the ultimate strength of the cross-ply laminates is not dependent on the size of the laminate. However, previous studies on the effect of specimen size on ultimate strength indicate a much larger influence of the strength-size effect in cross-ply laminates [27]. One explanation for the apparent lack of effect in the present experimental studies is that only the thickness dimension is scaled in the cross-ply laminates, as opposed to truly geometrically scaled laminates in the study reported in Reference 27. The ultimate strength of a cross-ply laminate loaded in tension is controlled by the fiber tensile strength in the unidirectional ply. The statistical strength distribution of unidirectional laminates is highly dependent on the volume of stressed material. For the laminates considered in this study, the in-plane dimensions, length and width, are not scaled, thus reducing the volumetric effect on fiber strength in the unidirectional ply.

The first ply failure load data for each test specimen as determined by step loading with dye-penetrant x-ray and acoustic emission techniques are summarized in Table 1. The load data from Table 1 were used as input to a composite laminated plate analysis to determine the stress, σ_x , in the global x (longitudinal) direction for the 90° core plies. The stress values for both the step loading and AE data are plotted versus laminate size, n , in Figure 3.

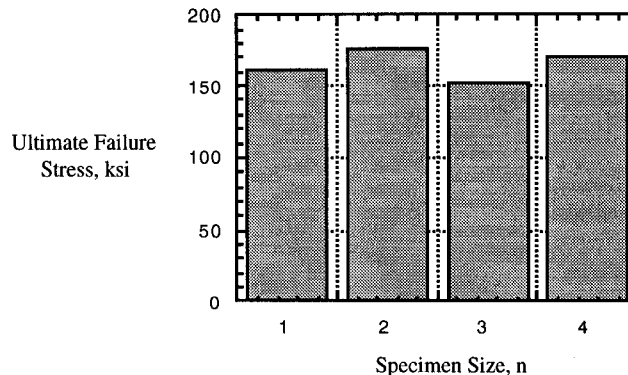


Figure 2. Variation in ultimate tensile failure stress versus increasing specimen size for scaled $[0^\circ n/90^\circ n/0^\circ n]_T$ ($n=1-4$) cross-ply laminates.

These results indicate that the tensile stress in the 90° core ply of the 3-ply $[0^\circ/90^\circ/0^\circ]_T$ laminate is approximately 1.8 times higher than the tensile stress in the 90° core ply of the $[0^\circ_4/90^\circ_4/0^\circ_4]_T$ laminate at first ply failure. In addition, data for the $n=3$ and $n=4$ laminates, obtained through step loading with dye-penetrant x-ray and the AE technique, agree well. The data for the $n=5$ and $n=6$ laminates were determined using the AE technique to capture

the stress level at the next damage formation, as these laminates contained pre-existing damage.

Size Dependent Failure Criteria for Composites

Some of the commonly used failure criteria for composite materials, e.g. maximum stress, maximum strain, and tensor polynomial criteria such as Tsai-Hill and Tsai-Wu, cannot predict the strength-size effect. These criteria assume that the composite strength values (longitudinal tensile and compressive strengths, transverse tensile and compressive strengths, and shear strength) are constant values and are not a function of ply thickness. In the

Table 1. Summary of Experimental Data for First Ply Failure.

n	Step Loading with X-ray (Average of 4 tests)	Acoustic Emission (Average of 4 tests)
1	1936	No Data
2	2520	No Data
3	3351	3220
4	3755	3594
5	No Data	4153
6	No Data	5688

present paper, three concepts for incorporating size dependency in standard failure criteria for composite materials are described in the following sections.

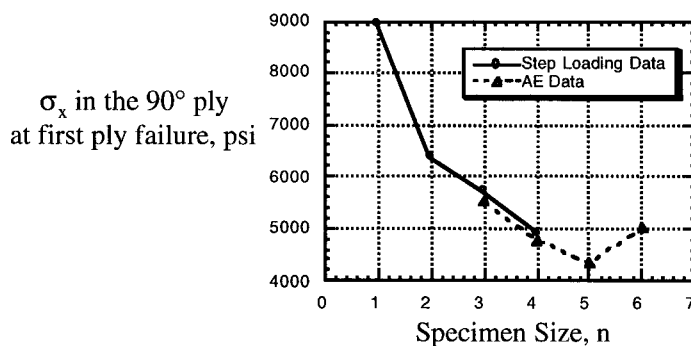


Figure 3. Stress in the global x direction in the 90° core ply at first ply failure versus specimen size, n.

In the first concept, ply-by-ply failure criteria including maximum stress, maximum strain, and Tsai-Hill, are applied in the usual manner except that the composite strength values

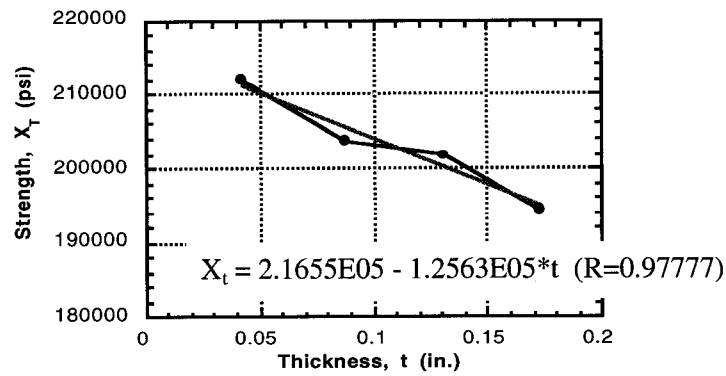
(longitudinal tensile strength, X_t ; transverse tensile strength, Y_t ; and shear strength, S) are determined as a function of ply thickness. The strength versus ply thickness relationships, shown in Figure 4, were generated from experimental data taken from the literature [27,28]. The values of X_t , Y_t , and S are shown to decrease with increasing ply thickness in a linear relationship. An equation of a linear curve fit analysis is shown with each plot, along with the correlation coefficient (R). The implication of these results is that in a ply-by-ply failure analysis, the values of X_t , Y_t , and S cannot be assumed to be constant values, as is usually done. For the first concept, the values of X_t , Y_t , and S for each ply in the laminate are obtained from the equations that represent the best fit of the experimental data of X_t , Y_t , and S versus ply thickness. Thus, the strength values, X_t , Y_t , and S , may be different in each ply of the laminate, based on the thickness of the ply.

Concept two is a variation of the first concept. Since the data for composite strengths versus ply thickness are available for only a limited range of values, as shown in Figure 4, a Weibull approach [31] is used to expand the analysis for a wider range of thickness values. The data shown in Figure 4 were used to determine empirically the shape parameters in the Weibull equations (Eq. 1) which characterize the composite strength values (X_t , Y_t , S) as a function of ply thickness. Then, a ply-by-ply failure analysis is conducted in the usual manner, except that the composite strength values (X_t , Y_t , S) for each ply are determined using the Weibull equations, as opposed to the linear relationships used in the first concept.

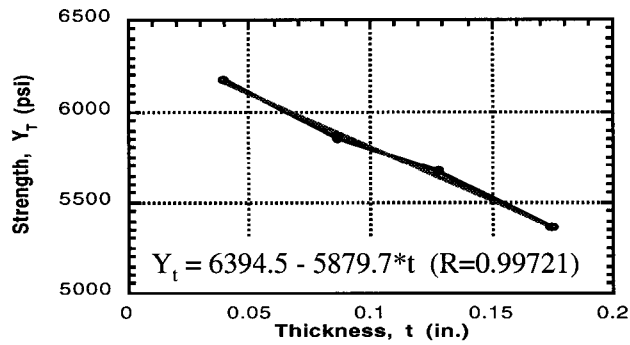
$$\begin{aligned}
 X_t^{\text{ply}} &= 194363.0 \left(\frac{t_{\text{ply}}}{0.173} \right)^{-10.5} \\
 Y_t^{\text{ply}} &= 5359.0 \left(\frac{t_{\text{ply}}}{0.175} \right)^{-9.85} \\
 S^{\text{ply}} &= 12560.0 \left(\frac{t_{\text{ply}}}{0.176} \right)^{-1.55}
 \end{aligned}
 \tag{Equation 1}$$

The third concept is a modified Weibull model which incorporates ply thickness, residual thermal stress, and the constraint provided by the neighboring ply to the 90° core. The concept assumes that the region of the 90° ply which is close to the 0° ply does not behave like a transverse ply, instead it is constrained to behave more like a 0° ply and resists transverse matrix cracking. Consequently, a “layer of influence” near the interface in the 90° ply is defined. The layer of influence, of thickness b , effectively reduces the thickness of the 90° ply which can be considered in the Weibull analysis. The layer of influence is assumed to have a constant thickness for all scaled cross ply laminates, as shown in Figure 5. Also, in the analysis, the total stress (mechanical stress plus residual thermal stress) is considered under the assumption that the residual stress is a constant for all scaled cross-ply laminates, i.e., residual stress is not a function of ply thickness or laminate thickness, as predicted by lamination theory for constitutively scaled laminates. For Concept 3, the unknowns in the modified Weibull equation (Eq. 2) are b , the thickness of the layer of

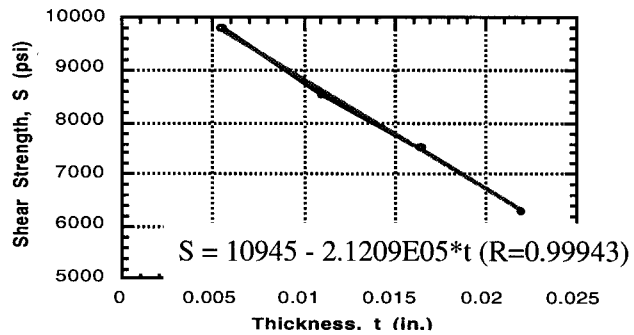
influence; β , the Weibull shape parameter; and σ^R , the residual stress. First ply failure data from tests on the n=1-3 laminates were used in conjunction with Eq. 1 to determine the unknowns, and the resulting equation was then used to predict the value of first ply failure



(a) Longitudinal tensile strength vs. ply thickness



(b) Transverse tensile strength vs. ply thickness



(c) Shear strength vs. ply thickness

Figure 4. Composite strength values versus ply thickness with linear curve fit to the data and R correlation coefficients.

$$\frac{\sigma_{ply}^M + \sigma^R}{\sigma_{ref}^M + \sigma^R} = \left(\frac{t_{ply} - 2b}{t_{ref} - 2b} \right)^{\frac{-1}{\beta}}$$

Equation 2.

- σ_{ply}^M - Mechanical stress in the global x-direction for the 90° ply of the laminate under consideration
- σ_{ref}^M - Mechanical stress in the global x-direction for the 90° ply of the reference laminate
- σ^R - Residual stress in the global x-direction for the 90° ply
- t_{ply} - Total thickness of the 90°ply in the laminate under consideration
- t_{ref} - Total thickness of the 90°ply in the reference laminate
- b - Thickness of the layer of influence
- β - Weibull shape parameter

stress for the n=4-6 laminates. Using this empirical approach, the values of the unknowns were determined to be: the thickness of the layer of influence, $b= 0.00377$ in.; the Weibull shape parameter, $\beta= 21$; and the residual stress in the global x direction for the 90° ply, $\sigma^R= 12,590$ psi.

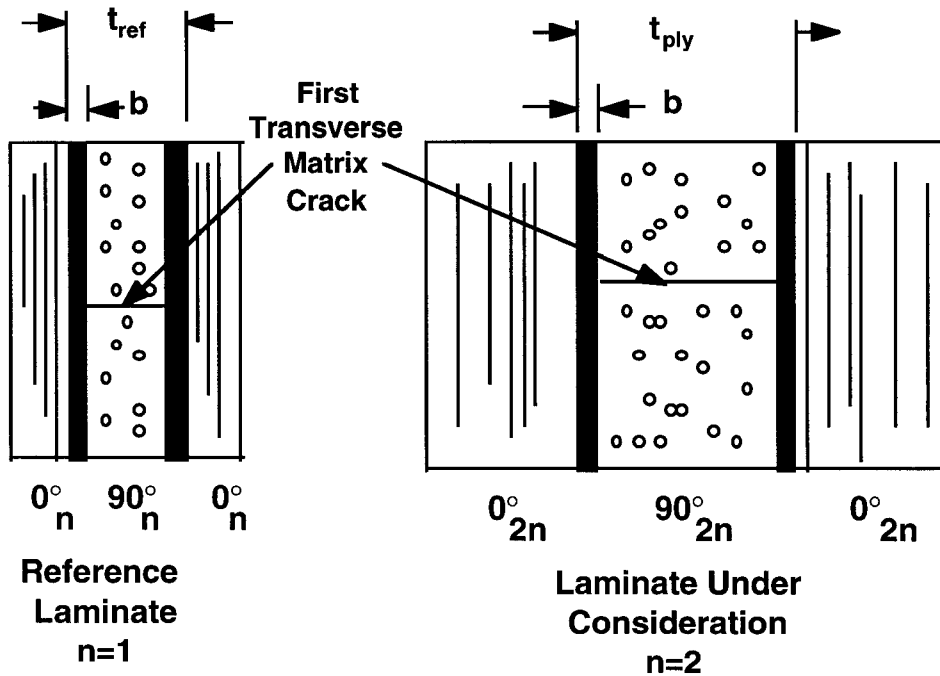


Figure 5. Schematic drawing of Concept 3.

Correlation of Size Dependent Failure Criteria with Experimental Data

The correlation between the first ply failure data and the three concepts presented above that incorporate size dependency in some of the commonly used failure criteria for composite materials is shown in Figure 6. The experimental results represent the average of the first ply failure data for both the step loading and AE techniques. The plot shows the predicted stress in the 90° ply in the global x direction at first ply failure for each concept. The first concept predicts a small change in the stress level at first ply failure in the 90° core due to increasing specimen size. The predicted difference in stress level between the n=1 laminate and the n=6 laminate is approximately 200 psi. The correlation between the predicted stress level and the experimental data for the first concept is poor. For the second concept, a greater change in the stress level at first ply failure is observed, with a difference of approximately 1000 psi between the n=1 and the n=6 laminates. However, the actual correlation between the predicted stress level and the experimental data for the second concept is also poor.

For the third concept, only correlation with the n=4-6 data points is valid due to the fact that first ply failure data from the n=1-3 laminates were used to determine the unknown coefficients in the modified Weibull equation. However, as indicated in Figure 6, the magnitude of the tensile stress at first ply failure in the 90° core plies of the n=4, 5, and 6 laminates was predicted within 11, 20, and 1 percent, respectively, using the third concept.

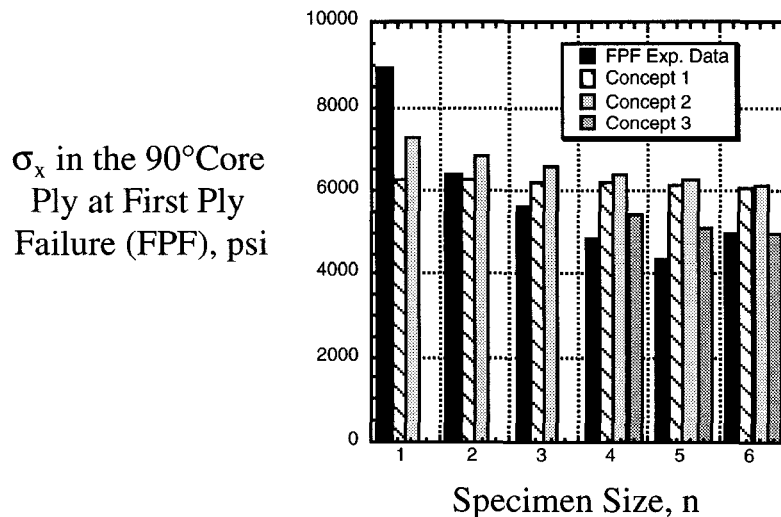


Figure 6. Correlation between the three size dependent failure concepts and the experimental first ply failure data.

Conclusions

An experimental program was conducted to determine the effect of specimen size on the first ply failure stress and ultimate strength of constitutively and geometrically (thickness dimension only) scaled $[0^\circ_n/90^\circ_n/0^\circ_n]_T$ n=1-6 cross-ply laminates. Results of the experimental program demonstrated that the stress level at first ply failure in the 90° core ply

of the $n=1$ laminate was approximately twice that of the $n=4$ laminate. However, no effect of specimen size was seen for the ultimate strength of the scaled cross-ply laminates. Three concepts were developed for incorporating size dependency into standard failure criteria for composite materials to predict the strength-size effect. The first two concepts involved using previous experimental data to determine relationships between composite strength values of longitudinal tensile strength, transverse tensile strength, and shear strength and specimen size, particularly specimen thickness. For the first concept, a linear relationship is determined between the composite strength values and ply thickness. For the second concept, a Weibull model is used to determine the relationships. Thus, the thickness of an individual ply determines the value for composite strengths that can be used in an existing ply-by-ply failure model such as maximum stress, maximum strain, or tensor polynomial approaches such as Tsai-Wu or Tsai-Hill. The third concept is a modified Weibull approach which accounts for the ply thickness effect, the constraint effect provided by the neighboring ply, and the residual thermal stresses. Results from the third concept provided the best correlation with experimental data.

References

1. Peters, P. W. M., "The Strength Distribution of 90° Plies in 0/90/0 Graphite-Epoxy Laminates," *Journal of Composite Materials*, Vol. 18, Nov. 1984, pp. 545-556.
2. Peters, P. W. M., et. al., "On Cross-Ply Cracking in Glass and Carbon Fiber-Reinforced Epoxy Laminates," *Composites*, Vol. 18, No. 1, Jan, 1987, pp. 40-46.
3. Crossman, F. W.; and Wang, A. S. D., "The Dependence of Transverse Cracking and Delamination on Ply Thickness in Graphite/Epoxy Laminates," *Damage in Composite Materials*, ASTM STP 775, K. L. Reifsnider, Ed., American Society for Testing and Materials, 1982, pp. 118-139.
4. Flaggs, D. L.; and Kural, M. H., "Experimental Determination of the In Situ Transverse Lamina Strength in Graphite/Epoxy Laminates," *Journal of Composite Materials*, Vol. 16, March 1982, pp. 103-116.
5. Wang, A. S. D., et. al., "A Theory for Transverse Cracks in Composite Laminates," *Advances in Aerospace Structures and Materials*, Proceedings of the Winter Annual Meeting of ASME, Phoenix, AZ, Nov. 14-19, 1982, pp. 51-60.
6. Han, Y. M.; Hahn, H. T.; and Croman, R. B., "A Simplified Analysis of Transverse Ply Cracking in Cross-Ply Laminates," *Composites Science and Technology*, Vol. 31, 1988, pp. 165-177.
7. Hahn, H. T.; Han, Y. M.; and Kim, R. Y., "Resistance Curves for Ply Cracking in Composite Laminates," 33rd International SAMPE Symposium, March 7-10, 1988, pp. 1101-1108.

8. Han, Y. M.; and Hahn, H. T., "Ply Cracking and Property Degradations of Symmetric Balanced Laminates Under General In-Plane Loading," *Composites Science and Technology*, Vol. 35, 1989, pp. 377-397.
9. Laws, N.; and Dvorak, G. J., "Progressive Transverse Cracking in Composite Laminates," *Journal of Composite Materials*, Vol. 22, October 1988, pp. 900-916.
10. Fukunaga, H.; Chou, T. W.; Schulte, K.; and Peters, P. W. M., "Probabilistic Initial Failure Strength of Hybrid and Non-Hybrid Laminates," *Journal of Materials Science*, Vol. 19, 1984, pp. 3546-3553.
11. Zweben, C., "The Effect of Stress Nonuniformity and Size on the Strength of Composite Materials," *Composites Technology Review*, Vol. 3, No. 1, 1981, pp. 23-26.
12. Zhang, J.; Fan, J.; and Soutis, C., "Analysis of Multiple Matrix Cracking in [$\pm\theta_m/90_n$]s Composite Laminates, Part 2: Development of Transverse Ply Cracks," *Composites*, Vol. 23, No. 5, September 1992, pp. 299-304.
13. Rezaifard, A. H.; Bader, M. G.; and Smith, P. A., "Investigation of the Transverse Properties of a Unidirectional Carbon/Epoxy Laminate: Part 1- Matrix Properties," *Composites Science and Technology*, Vol. 52, 1994, pp. 275-285.
14. Rezaifard, A. H.; Bader, M. G.; and Smith, P. A., "Investigation of the Transverse Properties of a Unidirectional Carbon/Epoxy Laminate: Part 2- Laminate Properties," *Composites Science and Technology*, Vol. 52, 1994, pp. 287-295.
15. Chan, W. S.; and Wang, A. S. D., "A Study on the Effects of the 90° Ply on Matrix Cracks in Composite Laminates," AIAA Paper #86-1018, 27th Structures, Structural Dynamics and Materials Conference, Part 1, San Antonio, TX, May 19-21, 1986.
16. Leaity, M. A.; Smith, P. A.; Bader, M. G.; and Curtis, P. T., "The Behavior of Cross-Ply Hybrid Matrix Composite Laminates, Part 1: Experimental Results," *Composites*, Vol. 23, No. 6, November 1992, pp. 387-395.
17. Leaity, M. A.; Smith, P. A.; and Bader, M. G., "The Behavior of Cross-Ply Hybrid Matrix Composite Laminates, Part 2: Modeling," *Composites*, Vol. 23, No. 6, November 1992, pp. 397-405.
18. Atkins, A. G.; and Caddell, R. M., "The Laws of Similitude and Crack Propagation," *International Journal of Mechanical Science*, Vol. 16, 1974, pp. 541-548.
19. Wang, A. S. D.; Kishore, N. N.; and Li, C. A., "Crack Development in Graphite-Epoxy Cross-Ply Laminates Under Uniaxial Tension," *Composites Science and Technology*, Vol. 24, 1985, pp. 1-31.

20. Crossman, F. W.; Warren, W. J.; and Wang, A. S. D., "Influence of Ply Thickness on Damage Accumulation and Final Fracture," *Advances in Aerospace Structures, Materials, and Dynamics*, ASME AD-06, 1983, pp. 215-226.
21. Wang, A. S. D.; and Crossman, F. W., "Initiation and Growth of Transverse Cracks and Edge Delamination in Composite Laminates Part 1. An Energy Method," *Journal of Composite Materials Supplement*, Vol. 14, 1980, pp. 71-87.
22. Crossman, F. W., et. al., "Initiation and Growth of Transverse Cracks and Edge Delamination in Composite Laminates Part 2. Experimental Correlation," *Journal of Composite Materials Supplement*, Vol. 14, 1980, pp. 88-108
23. O'Brien, T. K.; and Salpekar, S. A., "Scale Effects on the Transverse Tensile Strength of Graphite-Epoxy Composites," *Composite Materials: Testing and Design (Eleventh Volume)*, ASTM STP 1206, 1993, pp. 23-52.
24. Wisnom, M. R., "Relationship Between Strength Variability and Size Effect in Unidirectional Carbon Fibre/Epoxy," *Composites*, Vol. 22, No. 1, January 1991, pp. 47-52.
25. Talreja, R.; Yalvac, S.; Yats, L. D.; and Wetters, D. G., "Transverse Cracking and Stiffness Reduction in Cross Ply Laminates of Different Matrix Toughness," *Journal of Composite Materials*, Vol. 26, No. 2, 1992, pp. 1644-1663.
26. Lagace, P.; Brewer, J.; and Kassapoglou, C., "The Effect of Thickness on Interlaminar Stresses and Delamination in Straight-Edged Laminates," *Journal of Composites Technology and Research*, Vol. 9, Fall 1987, pp. 81-87.
27. Jackson, K. E.; and Kellas, S., "Effect of Specimen Size on the Tensile Strength of Geometrically Scaled ($+\theta_n^\circ/\theta_n^\circ/90^\circ/2n$)s Composite Laminates," *Army Symposium on Solid Mechanics*, Plymouth, Mass, Aug. 17-19, 1993.
28. Kellas, S.; and Morton, J., "Scaling Effects in Angle-Ply Laminates," *NASA CR 4423*, February, 1992.
29. Prosser, W. H., et. al., "Detection and Location of Transverse Matrix Cracks in Cross-Ply Gr/Ep Composites Using Acoustic Emission," 1995 American Society of Nondestructive Testing Spring Conference and Fourth Annual Research Symposium, Las Vegas, Nevada, March 20-24, 1995, pp. 213-215.
30. Prosser, W. H.; Jackson, K. E.; Kellas, S.; et. al., "Advanced, Waveform Based Acoustic Emission Detection of Matrix Cracking in Composites," *Journal of Material Evaluation*, Vol. 53, No.9, September, 1995, pp. 1052-1058.
31. Weibull, W., "A Statistical Distribution Function of Wide Applicability," *Journal of Applied Mechanics*, Vol. 18, 1951, pp. 293-297.