

# Damage tolerance and residual strength of composite sandwich structures

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*The reasonable man adapts himself to the world,  
whereas the unreasonable man persists in trying to adapt the world to himself.  
Therefore, all progress depends on the unreasonable man.*

George Bernard Shaw



## **Preface**

The work in this thesis has been carried out at the Department of Aeronautics at the Royal Institute of Technology, Stockholm, Sweden. The department merged with the department of Vehicle Engineering in 2002 and thus changed name to Aeronautical and Vehicle engineering.

The first two papers have been financed by the Office of Naval Research, grant No. N0014-95-0671, through Program Officer Dr. Yapa D.S. Rajapakse. The other papers, excluding paper E, consist of work which was part of the SaNDI (THALES JP3.23) project with participants from Norway, Denmark, Sweden, Finland and the United Kingdom. Additional funding was provided by ONR's Research Program on "Composites For Marine Structures" grant No. N00014-99-1-0316 through Program Officer Dr. Yapa D.S. Rajapakse.

I wish to thank Professor Emeritus Karl-Axel Olsson and Dr. Stefan Hallström for persuading me to start my graduate studies, and for pointing me in the right direction whenever I wandered off into the mist. Considering my fascination for large toys, the possibility to shoot with a 40 mm anti aircraft gun was to tempting to resist. Of course, the possibility to break large things with even larger tools did add to the temptation. I also wish to thank my supervisor Professor Dan Zenkert for guidance and many interesting discussions.

From January 2001 I started to work half time as a research engineer for the Defense Materiel Administration of Sweden. This gave me the unique opportunity to bring what I had learned, and was learning, into practice. Parts of this work led to paper E. It all started one day when Anders Lönnö, of said governmental institution, asked the seemingly very simple question: "Which is the better fiber reinforcement?". Little did I know that it would take me the better part of three years working half time to figure out parts of the answer to that question. But as in the case of the answer to the great question of life, the universe and everything, the answer and the question cannot co-exist. Therefore I only found parts of the answer however hard I tried.

The guys in the lab, Anders, Bosse, Gilles and Peter, deserve warm thanks for all the help they have provided.

Without the possibility to discuss problems with my fellow graduate students, either problems of work related nature or simple world problems to be solved over a whisky or a beer, this work would not have been half as fruitful as it has.

Had it not been for my mother telling me to stop selling other peoples soul for money, i.e. working in automotive insurance, and for the financial support of my father through my masters studies, I would not have been where I am today.

Solna, April 2004

Peter H. Bull



## **Abstract**

The exploitation of sandwich structures as a means to achieve high specific strength and stiffness is relatively new. Therefore, the knowledge of its damage tolerance is limited compared to other structural concepts such as truss bars and monocoque plate solutions.

Several aspects of the damage tolerance of sandwich structures are investigated. The influence of impact velocity on residual strength is investigated. Sandwich panels with faces of glass fiber reinforced vinylester are impacted both with very high velocity and quasi static. The residual strength after impact is found to be similar for both cases of impact velocity.

Curved sandwich beams subjected to opening bending moment are studied. Face–core debonds of varying size are introduced between the compressively loaded face sheet and the core. Finite element analysis in combination with a point stress criterion is utilized to predict the residual strength of the beams. It is shown that it is possible to predict the failure load of the beams with face–core debond.

Using fractography the governing mode of failure of compressively NCF-carbon is characterized. Sandwich panels subjected to compression after impact are shown to fail by plastic micro buckling.

The residual compressive strength after impact of sandwich panels is investigated. Sandwich panels with face sheets of non-crimp fabric (NCF) carbon are subjected to different types of impact damages. Predictions of residual strength are made using the Budiansky, Soutis, Fleck (BSF) model. The residual strength is tested, and the results are compared to predictions. Predictions and tests correlate well, and indicate that the residual strength is dependent on damage size and not the size of the damaged panel.

A study of the properties of a selection of fiber reinforcements commonly used in sandwich panels is conducted. The reinforcements are combined with two types of core material and three types of matrix. Also the influence of laminate thickness is tested. Each combination materials is tested in uni-axial compression, compressive strength after impact and energy absorption during quasi static indentation. The specimens which are tested for residual strength are either subjected to quasi-static or dynamic impact of comparable energy level. Prediction of the residual strength is made and correlates reasonably with the test results. The tests show that if weight is taken into account the preferred choice of fiber reinforcement is carbon.





## Dissertation

This dissertation consists of a short introduction to damage tolerance of composite sandwich structures and the following appended papers:

### Paper A

P.H. Bull and S. Hallström: “*High-velocity and quasi-static impact of large sandwich panels*”, published in Journal of Sandwich Structures and Materials, Vol. 6, No. 2, 2004

### Paper B

P.H. Bull and S. Hallström: “*Curved sandwich beams with face–core debond subjected to bending moment*”, published in Journal of Sandwich Structures and Materials, Vol. 6, No. 2, 2004.

### Paper C

F. Edgren, P.H. Bull, and L.E. Asp: “*Compressive failure of impacted NCF composite sandwich panels – Characterisation of the failure process*”, published in Journal of Composite Materials, Vol. 38, No. 6, 2004.

### Paper D

P.H. Bull and F. Edgren: “*Compressive strength after impact of CFRP-foam core sandwich panels in marine applications*”, accepted for publication in Composites Part B: Engineering.

### Paper E

P.H. Bull and P. Jolma: “*Residual strength of impact damaged scaled down CFRP-foam core sandwich panels*”, to be submitted for publication.

### Paper F

P.H. Bull: “*Damage tolerance and compressive strength after impact of a selection of fiber reinforcements for composite sandwich structures*”, to be submitted for publication.



## Division of work between authors

### Paper A

P.H. Bull and S. Hallström: “*High-velocity and quasi-static impact of large sandwich panels*”.

Bull and Hallström outlined the work, Bull conducted the experimental and analytical studies and wrote the paper under the supervision of Hallström.

### Paper B

P.H. Bull and S. Hallström: “*Curved sandwich beams with face–core debond subjected to bending moment*”.

Bull and Hallström outlined the work, Bull conducted the experimental and analytical studies and wrote the paper under the supervision of Hallström.

### Paper C

F. Edgren, P.H. Bull, and L.E. Asp: “*Compressive failure of impacted NCF composite sandwich panels – Characterisation of the failure process*”

Edgren conducted the fractography modeling and wrote the paper under supervision of Asp. Bull conducted the materials tests.

### Paper D

P.H. Bull and F. Edgren: “*Compressive strength after impact of CFRP-foam core sandwich panels in marine applications*”

Bull conducted the analysis, the testing and wrote the paper. Edgren conducted the fractography.

### Paper E

P.H. Bull and P. Jolma: “*Residual strength of impact damaged scaled down CFRP-foam core sandwich panels*”

Bull conducted the analysis and the testing of the sandwich panels loaded in in-plane compression and wrote the paper. Jolma has conducted the FE-analysis and the testing of the sandwich panels loaded laterally.



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## Introduction

### Background

This dissertation investigates the damage tolerance of composite sandwich structures. Here composite refers to a material which consists of two or more materials. An example of a commonly used composite is glass fiber reinforced plastics, which is used to build pleasure boats. Even though it is commonly referred to as glass fiber, it consists of glass fiber and a plastic resin, or matrix.

When two layers of material are separated by another material they make a sandwich. Composite sandwich structures usually consist of two stiff thin layers of composite separated by a relatively soft core material. The sandwich concept is not new as such. For instance the human bone structure is composed as a sandwich. The human development and exploitation of the sandwich principle as a means to achieve high specific stiffness and strength is, however, relatively new.

The probably most famous of early sandwich constructions produced is the De Havilland Mosquito fighter/bomber, figure 1, made in the mid 1940's as a monocoque structure of plywood skins and balsa core [1].



Figure 1: De Havilland Mosquito fighter/bomber. Photo courtesy of NASA.

Offshore powerboats and pleasure boats made from glass fiber reinforced composites first appeared on the market in the early 1960's. In the beginning they were manufactured as single skin structures, but the desire for higher speeds and lower weight has brought forth sandwich boats such as the 42' Cigarette Tiger offshore powerboat, figure 2, which is made from unidirectional glass fiber reinforced vinyl ester and poly vinyl chloride (PVC) foam core. For more extreme applications, such as offshore class 1 race catamarans, figure 3, carbon fiber is often the preferred reinforcement material.

The power boat industry early recognized the potential of composites, maybe more because they proved simple to shape than due to their high performance. The development was, to a great extent, based on trial and error, sometimes resulting in very durable boats, many of which are still in service.

The naval forces have seemingly taken a more cautious approach towards composite and sandwich vessels. They have mainly chosen to purchase existing powerboat concepts which have been converted to armed vessels, or utilized composite and sandwich structures to shield sensitive equipment such as antennas and radars.

In the 1960's the Swedish navy identified the need for new mine counter measure vessels. Glass fiber with non-magnetic properties, seemed like a good alternative to wood,



Figure 2: Cigarette Tiger 42' offshore powerboat. Photo courtesy of Cigarette Racing Team LLC.



Figure 3: 43' Tencara catamaran "Spirit of Norway". Photo courtesy of Intl. Offshore Team Assoc.

of which mine counter measure vessels traditionally had been built. The existing wooden mine hunters were also plagued with extreme maintenance costs, which the navy wanted to reduce. Nobody knew then how such a glass fiber sandwich vessel would perform. Research led to the building of the HMS "Viksten" mine hunting vessel in the early 1970's, figure 4. It was a hand laminated glass fiber PVC foam core sandwich construction. The concept proved to work well, even when subjected to mine explosions. HMS "Viksten" is still in service, after having been fitted with a new engine [2]. Other sandwich vessels like the Swedish Landsort and the Danish Standardflex series have followed suit, culminating in the YS 2000 or KV "Visby", figure 5, which was, when launched, the worlds largest carbon fiber sandwich structure.



Figure 4: HMS "Viksten". Photo courtesy of Kockums AB



Figure 5: KV "Visby". Photo courtesy of Kockums AB

As sandwich structures have been more optimized, the understanding of governing mechanisms concerning damage tolerance has grown in importance. Early glass fiber sandwich vessels had very thick face sheets and relatively heavy core material, very large safety margins were used. Large safety margins usually results in unnecessary heavy vessels. For a planing craft or a surface effect ship (SES), weight is an important issue. In the case of a SES, low weight is necessary in order to properly exploit the advantage of the surface effect principle. Reduced fuel consumption, increased payload and increased operating speed, either by themselves or in combination are other advantages which follow from reduced structural weight. One possible drawback of weight reduction is an increased sensitivity to damages. In order to reduce the structural weight, the face thickness and core density are reduced, fiber lay up directions are optimized and new production



methods leading to increased fiber content are introduced. An optimized sandwich structure has the potential to be both light and very durable. Without a solid knowledge on how the composite structure performs and what service conditions it is supposed to endure the result can however be both fragile and heavy.

When building a composite sandwich structure it is possible to tailor the material properties to fit most requirements as part of the design procedure. Unfortunately, some properties can be contradictory. It can be difficult to make a construction both stiff and durable, but by combining the right materials it is possible to get very close. The list of constituent materials to choose from is very long. A thorough knowledge of the constituent materials is therefore essential.

### Impact damages

Damage tolerance can be referred to the capability of a structure to carry load after different types of impacts, e.g. things dropped, projectile hits, rough seas and foundering. It can also refer to the capability of the same structure to absorb the energy from, and stop, impacting objects such as projectiles or shrapnel from explosions. This work is focused on damage tolerance according to the former definition.

Because of its composition a composite sandwich can show a magnitude of failure modes. Hildebrand [3] has identified a number of different failure modes which are listed below.

- *Face crushing*, where the face fails in through-thickness compression, figure 6 I.
- *Face shear*, where the face fails in interlaminar shear at the edges of the impacting body, figure 6 II.
- *In-plane face failure*, where the face fails in in-plane tension or compression near the edges of the impacting body, figure 6 III.
- *Flexural face failure*, where the face fails in bending near the edges of the impacting body, figure 6 IV.
- *Core crushing*, where the core is damaged in out-of-plane compression, figure 6 V.
- *Core shear failure*, where the core fails in shear near the impactor, or is sheared out as a plug, figure 6 VI.
- *Face/core debonding*, where the impacting body tears the face from the core, figure 6 VII.

The modes of failure in sandwich structures depicted in figure 6 can be divided into two main categories, the point load related damages and the structural response related damages. The point load damages occurs as a direct result of something hitting the sandwich structure. Damage type I, II, IV, V and VII are typical examples of point load damages. Structural response related damages occurs because the hitting object forces the structure to deflect too much, resulting in damages such as type III and VI. An example of a structural response damage can be seen in figures. 7 and 8. The example is the result of a laboratory experiment, but it illustrates the complexity of the modes of failure in

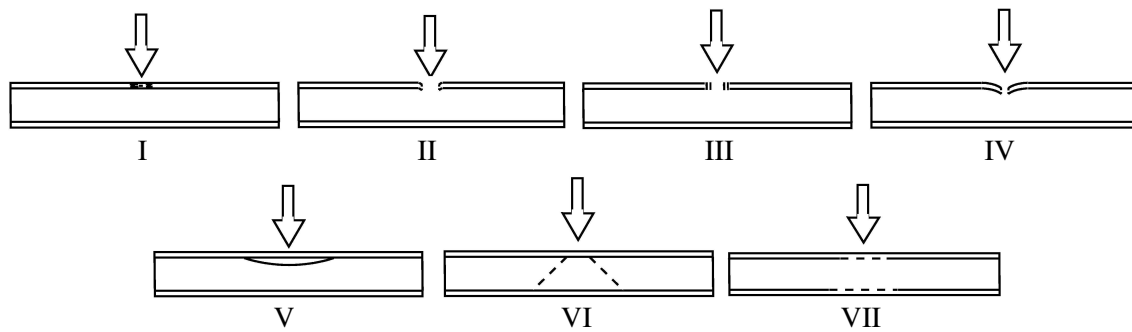


Figure 6: Different types of impact damages on sandwich panels

sandwich structures. A shot put ball was dropped from a height of about one meter onto a square sandwich panel which was simply supported along all edges. The panel had thin faces of woven glass fiber roving and a low density PVC foam core. On the impacted side, figure 7, there were no visible damages. On the reverse side, figure 8, there was a circular mark which could be easily hidden behind gel coat or some internal structure if the panel was part of a larger structure. Although the panel looks undamaged, its load bearing capacity is severely reduced. The circular mark on the reverse side is from a core shear crack, centered at the point where the shot put ball hit. The crack goes through the core material and thus reduces the load carrying capacity of the panel to fractions of its undamaged capacity. The square marks inside the circular marks which can be seen in figure 8 are related to a post failure test conducted on the panel and not to the dropped shot put ball.

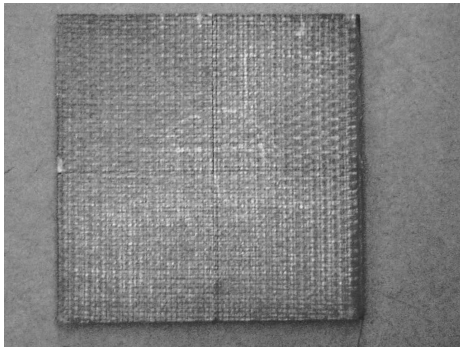


Figure 7: Impacted side of a sandwich panel hit by a shot put ball.

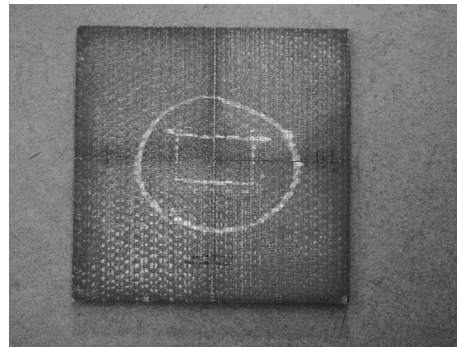


Figure 8: Reverse side of same sandwich panel.

Although not the case in the above example, a damage to a sandwich structure often consists of a combination of the types listed above. If, for instance, a sandwich plate with a brittle core is hit by a sharp object, a crushed face sheet and a face/core debond together with a crushed core will probably be the resulting damage.

### Impact testing

Composite sandwich structures are made of very different materials, i.e. one or more stiff fiber reinforcements, a polymer matrix and a relatively soft core material, where the constituent materials are required to bond well to each other. The structure as such can therefore be sensitive to impact damages, which obviously can degrade the bond line between the constituent materials and damage the constitutive materials. Compared to

other building methods, such as riveting and welding, it is quite new and research into damage tolerance has not reached as far as for more conventional methods and materials. Even though impact damages may have a very critical effect on a sandwich structure, few standardized methods for impact testing of sandwich structures, e.g. from ASTM<sup>1</sup>, ISO<sup>2</sup>, DIN<sup>3</sup>, exist. Some methods for impact testing of single skin composites, such as ISO 6603 [4] or ASTM D6264 [5], could be slightly changed to be used for sandwich materials. One test method which is developed for composite sandwich structures is NordTest Mech42 [6]. It is however common that the organizations conducting impact tests choose to use their own test methods. Tests conducted by Hildebrand [3] shows that it is very difficult, even on a qualitatively level, to compare the results from different test methods.

The only limitation to impact testing seems to be the imagination of the persons conducting the tests. Experimental set-ups, procedures and indentors are custom made for the impact event one wants to simulate. This is reflected by the vast selection of indentors and procedures that can be found in the literature.

Some examples of the most common indentors used is shown in figure (9). Indentors usually consist of two parts, excluding equipment for mounting them into a test machine. The head or tup, which is often made of hardened steel, and the rod. The rod makes it possible to push the head through thick plates.

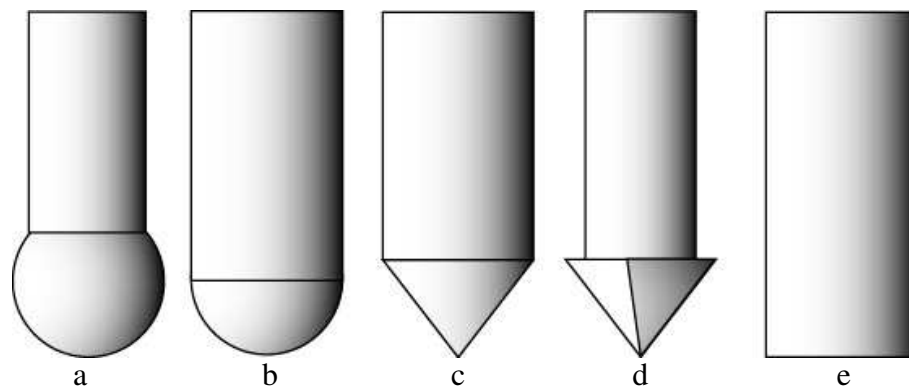


Figure 9: Common types of indentors.

Indentors with spherical heads [7]– [12] can be mounted on a rod which has the same diameter, as shown in figure 9b. For thick panels this adds an extra factor of friction between the rod and the panel. In order to avoid, or reduce, this friction a rod with a smaller diameter than the head can be used, see figure 9a. Projectiles from guns or other sharp objects may be better modeled with a conical head [13], figure 9c, where it is possible to adjust the angle of the cone to best represent the modeled object.

Horizontal surfaces, such as ship decks or aircraft floors can be subjected to impacts from almost any dropped object. Which object creates the most damage is a matter of circumstance. A small object dropped from a great height may create more damage than a large object dropped from only a few centimeters. For example, the corner of a heavy box can make a hole in most sandwich panels. Such impacts can be simulated using a pyramid shaped indenter head, figure 9d. Usually a four sided pyramid [14], [15] is used, but it is

<sup>1</sup>American Society for Testing and Materials, [www.astm.org](http://www.astm.org)

<sup>2</sup>International Organization for Standardization, [www.iso.ch](http://www.iso.ch)

<sup>3</sup>Deutsches Institut für Normung, [www.din.de](http://www.din.de)

argued that a three sided one is better suited [16], because it actually looks like a corner of a right angled block. The last indenter showed, figure 9e, is flat-ended [17], [18] and also quite common. Wen et al. [13] conducted a comparison of three different indentors, flat-faced cylinder, conical and hemispherical, used in quasi-static impact. The flat-faced indenter was found to shear off the entry face sheet, crush and shear out a core plug and tear off the exit face sheet. Conical indentors crushed the entry face and the core, and the exit face was debonded from the core. With a hemispherical indenter core crushing started before the entry face failed from bending, the core was crushed and the exit face was bent until failure.

For slow impact the global response of the panel will affect the impact damage, figure 10. Depending on the size of the panel, the boundary conditions and the point of impact, the mode of failure will change.

If the impact velocity exceeds some critical value, which depends on the sandwich panel and the impacting object, the global response of the sandwich will have a reduced effect on the impact damage. The damage is instead dependent on the local properties of the sandwich, figure 11

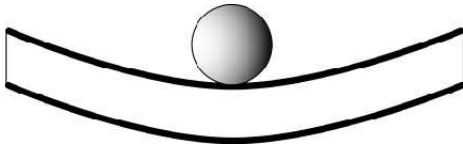


Figure 10: Typical response of a low velocity, or quasi-static, impact.

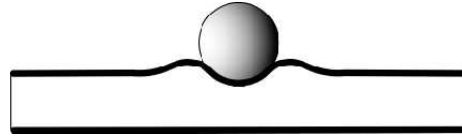


Figure 11: Typical response of a high velocity impact.

Caprino and Teti [7] found that for increasing core stiffness the contact forces during impact would increase, thus reducing the impact tolerance of a sandwich panel. Nilsen et al. [8] on the other hand, showed that increasing core stiffness was an important factor for increasing the impact tolerance of a sandwich panel. These papers serves to illustrate the complexity of the impact damage tolerance of a sandwich structures.

## Analytical methods

For a flat sandwich structure, be it a beam or a plate, it is usually assumed that bending moments are carried by tensile and compressive stresses in the face sheets, and transverse or shear forces are carried by shear stresses in the core [19]. A closer study of the stress field of a sandwich shows that this is a reasonable assumption if the face sheets are thin compared to the core, and if the stiffness of the face sheets is much larger than the stiffness of the core.

Failure of an undamaged straight sandwich beam subjected to an in-plane load is governed by one of the following expressions, with dimensions as defined in figure 12. The width  $b$  is usually set to unity, which is the case for the following expressions. Tensile or compressive face failure is governed by

$$\hat{P}_c = 2t_f \hat{\sigma}_f, \quad (1)$$

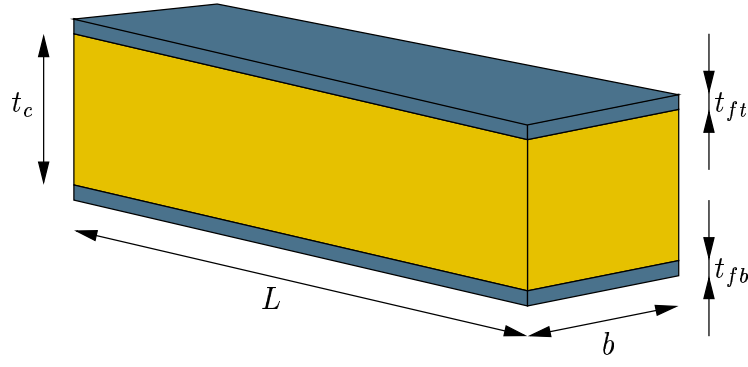


Figure 12: Typical sandwich beam.

where  $\hat{\sigma}_f$  is the tensile or compressive strength of the face sheet. Fiber reinforced composites usually exhibit a higher failure load in tension than in compression. Local waviness of the reinforcement, voids in the matrix and wrinkles as a result of manufacturing errors are more critical to the compressive load carrying capacity.

Rosen [20] proposed that the compressive strength of a uni-axially loaded composite consisting of uniaxial fibers is governed by an internal instability mode. This mode of failure is known as micro buckling or kink band formation. He assumed perfectly aligned fibers in a linearly elastic matrix, which resulted in the equation.

$$\hat{\sigma}_c = \frac{G_m}{1 - v_f} + \frac{\pi^2}{12} \left( \frac{d}{\lambda} \right)^2 E, \quad (2)$$

where  $E$  is the elastic modulus of the composite,  $G_m$  is the shear modulus of the matrix,  $v_f$  is the volumetric fraction of fibers,  $d$  is fiber diameter and  $\lambda$  is the buckling wave length. Assuming long waves compared to fiber diameter, and that  $G = G_m/(1 - v_f)$  leads to

$$\hat{\sigma}_c \approx G, \quad (3)$$

where  $G$  is the shear modulus of the composite. This predicted value is usually much larger than the compressive strength observed in uni-axial fiber reinforced composites. Polymer matrix and long fiber reinforced composites usually display a non-linear behaviour. They are typically sensitive to fiber misalignment. Argon [21] considered a rigid perfectly plastic composite with a shear yield stress  $k$ . This resulted in the following expression for the compressive strength.

$$\hat{\sigma}_c = \frac{k}{\bar{\phi}}, \quad (4)$$

where  $\bar{\phi}$  is the initial fiber misalignment. Budiansky [22] extended the expression to an elastic perfectly plastic composite, with yield strain  $\gamma_Y$ ,

$$\hat{\sigma}_c = \frac{k}{\gamma_Y + \bar{\phi}} = \frac{k}{1 + \gamma_Y/\bar{\phi}}. \quad (5)$$

Budiansky and Fleck [23] developed the theory to take into account a more general loading condition on the compressive strength of a uni-directional composite. Assuming far field compressive and shear stresses  $\sigma^\infty$  and  $\tau^\infty$  respectively, and an angle  $\beta$  between the fiber direction and the kink band leads to the following equation

$$\sigma^\infty - 2\tau^\infty \tan(\beta) = \frac{\tau_Y - \tau^\infty + \sigma_T \tan(\beta)}{\phi + \bar{\phi}}, \quad (6)$$

where  $\phi$  is the additional rotation due to the far field stress,  $\sigma_T$  and  $\tau$  are the stresses that develop inside the micro buckle. Figure 13 shows a schematic of a micro buckle. Considering the case where  $\beta = 0$  (6) reduces to

$$\sigma^\infty = \frac{\tau_Y - \tau^\infty}{\phi + \bar{\phi}}. \quad (7)$$

Jelf and Fleck [24] conducted a series of experiments using a composite tube which was loaded both in compression and torsion, showing that the compressive strength of the composite is knocked down by the added shear stress.

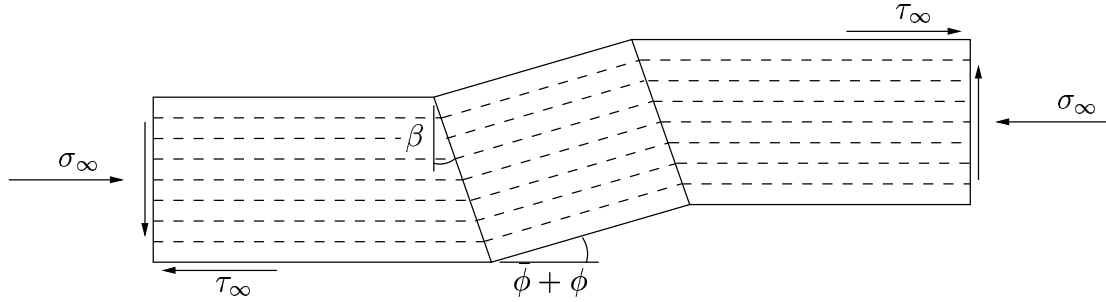


Figure 13: Schematic of micro buckle, redrawn from [23].

A sandwich beam subjected to a compressive load can also fail in face wrinkling according to Hoff and Mautner [25]

$$\hat{P}_w = 2t_f C \sqrt[3]{E_f E_c G_c}, \quad (8)$$

where  $E_f$  and  $E_c$  are the Young's moduli of the face sheet and core material respectively,  $G_c$  is the shear stiffness of the core and  $C$  is a constant usually in the order of 0.5. For isotropic face sheets this equation holds, but if the face sheets are orthotropic the bending stiffness of the face may have to be taken into account. The following expression by Plantema [26] can then be used instead.

$$\hat{P}_w = 2t_f \frac{3}{2} \sqrt[3]{2D_f E_c G_c}, \quad (9)$$

where  $D_f$  is the bending stiffness of the face.

Global buckling is governed by both shear and bending properties of the beam [19]. With the assumption of face sheets of equal thickness which are thin compared to the

core,  $t_f \ll t_c$ , and a core which is weak compared to the face sheets,  $E_c \ll E_f$ , the shear stiffness,  $S$ , and the bending stiffness,  $D$ , of a sandwich beam are defined by

$$S = \frac{G_c d^2}{t_c} \quad \text{and} \quad D = \frac{E_f t_f d^2}{2}, \quad \text{where} \quad d = \frac{t_{ft} + t_{fb}}{2} + t_c \quad (10)$$

Based on these assumptions, the critical buckling load is

$$\hat{P}_c = \frac{n^2 \pi^2 \frac{D}{L^2}}{1 + n^2 \pi^2 \frac{D}{L^2 S}} \quad \text{and} \quad \hat{P}_c = \frac{4n^2 \pi^2 \frac{D}{L^2}}{1 + 4n^2 \pi^2 \frac{D}{L^2 S}}, \quad (11)$$

for a simply supported and a clamped sandwich beam respectively, where  $n$  is the number of buckling waves. If the beam is either long or stiff in shear the above expressions simplify to

$$\hat{P}_b = \frac{n^2 \pi^2 D}{L^2} \quad \text{and} \quad \hat{P}_b = \frac{4n^2 \pi^2 D}{L^2}. \quad (12)$$

If, however, the beam is weak in shear, or short, both the above expressions simplify to

$$\hat{P}_s = S \quad (13)$$

which indicates that the shear buckling, or shear crimping, of a sandwich beam is independent of the boundary conditions.

If the beam is curved, tensile or compressive stresses appear in the core even if the beam is subjected to a pure bending moment. Whether the stresses are tensile or compressive depends on whether the bending moment acts in an opening or closing way. It is often simpler to study a curved sandwich beam in a cylindrical coordinate system, instead of a Cartesian. Here in-plane-direction refers to the  $\varphi$ -direction and out-of-plane, or radial, direction refers to the  $r$ -direction as defined in figure 14.

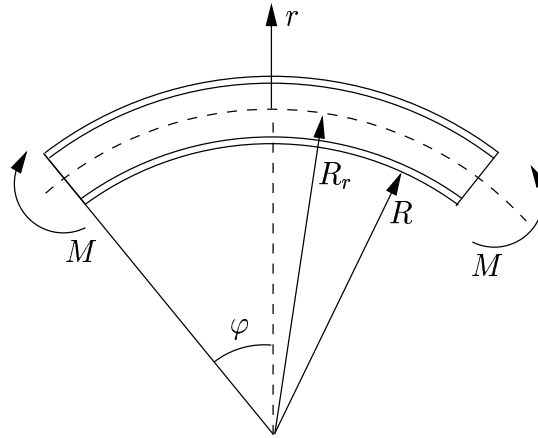


Figure 14: Definition of a curved sandwich beam

Under the assumption of thin faces and a weak core, the radial stresses in the core of a sandwich beam subjected to a pure bending moment can be expressed as [27]

$$\sigma_r = \frac{M}{(R+r)t_c} \text{ for } -t_c/2 \leq r \leq t_c/2. \quad (14)$$

This expression has been found to correspond well with experiments by Smidt [28].

A more elaborate expression for the radial stresses can be found utilizing an expression by Seely and Smith [29]. The expression is originally for curved I-beams, but can be used for curved sandwich beams with the equilibrium requirements that  $\sigma_r$  and  $\varepsilon_\varphi$  are continuous across material boundaries and that  $\sigma_r = 0$  at both surfaces. Because of the complexity of the complete solution for a curved sandwich beam, the details of the procedure are left out.

### Strength reduction due to damaged core

For all the above expressions to be true in reality, however, a good bond between the constituent materials is required. If the bond between face sheet and core is not good, the interaction between face sheet and core, which makes the sandwich such an efficient concept, is lost. The consequence of a poor bond between the fiber and matrix is similar.

The simplest approach to predicting the damage tolerance of a sandwich structure is to utilize elementary cases from solid mechanics [30]. With careful handling they can deliver useful results.

In polymer foam core sandwich beams a very useful phenomenon has been shown by Shipsha et al. [31]. A crack in the interface between the face sheet and the core material actually propagates the distance of a few cells into the core. It is therefore reasonable to assume that delamination properties of a sandwich is governed by the properties of the core material, and not by the interaction of different materials. Based on this assumption, the stress field in the vicinity of a crack can be described by

$$\sigma = \frac{K_I}{\sqrt{2\pi r}}, \quad (15)$$

where  $K_I$  is a stress intensity factor and  $r$  is the distance from the crack tip. Some manipulation of eqn. (15) leads to the expression

$$l_0 = \left( \frac{K_{Ic}}{\sqrt{2\pi}\sigma_0} \right)^2, \quad (16)$$

where  $K_{Ic}$  is the fracture toughness and  $\sigma_0$  is found from testing. When the stresses in the core  $\sigma$  exceeds  $\sigma_0$  at the distance  $l_0$  from the crack tip the sandwich beam is predicted to fail. This approach has been shown to work well with PVC foam core materials by Grenestedt et al [32].

Finite element (FE) analysis can of course be used in damage tolerance prediction. The great difficulty is the application of failure criteria to the model. Expressions such as eqns. (15) and (16) has been used successfully together with FE analysis [32]. FE codes such as Stripe [33] are designed to accurately calculate stress intensity factors. Utilizing this, failure can be predicted to occur when the ratio  $K_I/K_{Ic}$ , for a given problem, reaches unity.



### Compressive strength after impact

When it comes to compressively loaded composite laminates with holes and cracks, there are no simple analytical models available known to the author. Schemes like the point stress criterion presented by Whitney and Nuismer [34] and the damage zone criterion proposed by Eriksson and Aronsson [35] working well for laminates loaded in tension, do not provide the necessary accuracy for compression loaded laminates.

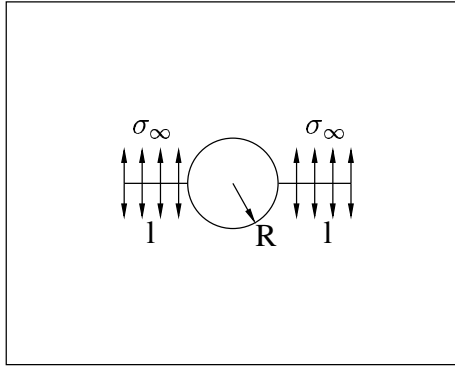


Figure 15: Schematic of the Dugdale cohesive zone model

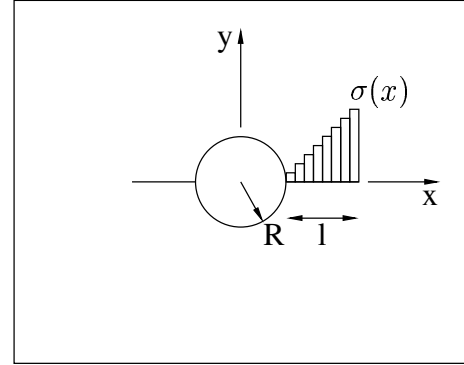


Figure 16: Local stress varies with crack displacement

Soutis and Fleck [36] proposed that stable growth of micro buckles would occur over a certain distance when the average stress over that distance reached the unnotched strength of the laminate. Unstable growth, leading to failure, would then occur when the stress intensity factor at the tip of the buckled region reached the fracture toughness of the laminate.

Soutis et al. [37] assumed that the stress is finite everywhere in a specimen containing micro buckles. The assumption was similar to Dugdale's plastic zone model [38], see figure 15. Utilizing the model, they were able to predict the compressive strength of a carbon fiber laminate with a hole.

However, the predicted growth of micro buckles was underestimated. As an alternative to the previous approach the micro buckle was replaced by an equivalent crack [37], figure 16. The relationship between the normal traction  $\sigma$  and the crack displacement  $2v$  was assumed to be linear where the area under the  $\sigma - v$  curve corresponds to the fracture energy  $G$ . Good agreement was found with experiments both for prediction of failure load and for growth of micro buckles see Soutis et al. [39].

It has been argued by Soutis and Curtis [40] that the Budianski, Fleck, Soutis (BFS) model [37] could be utilized to predict compressive strength of an impact damaged carbon fiber composite laminate. This is performed by the use of an equivalent hole. From c-scans of impact damaged laminates it was evident that the damaged area had the appearance of a hole with cracks emanating from it. A reasonable agreement between predictions and tests was achieved by Soutis et al. [39], [41]. Edgren et al. [42], found that growth of micro buckles was the governing mode of failure for compressively loaded sandwich structures with faces from NCF carbon fiber. By interrupting compressive tests of impact damaged sandwich panels prior to final failure, kink bands were found in the laminates. Comparison of predictions using the BSF model with compression tests of sandwich panels showed good agreement [43]. Compression tests of a selection of fiber reinforcements conducted

by Bull [44] showed that the BSF model works reasonable also for other reinforcements that carbon.

## **Future work**

This work has investigated damage tolerance of sandwich panels of varying size. From small pieces of 150 mm  $\times$  150 mm to scaled down models of deck panels measuring 800 mm  $\times$  1000 mm in size. It has been shown that for relatively small impact damages the residual strength in compression is dependent on the absolute size of the damage. The dependancy of the size of the panel on the residual strength was found to be minor.

The next step would be to conduct tests on actual vessels. On real life sandwich structures supporting and surrounding structure is always present. Their effect on the residual strength is difficult to evaluate in a laboratory environment. The Budiansky, Soutis, Fleck (BSF) model is capable of predicting the residual strength of a sandwich sandwich panel. The remaining question is, how does it work on a real structure such as a planing vessel. Implementing the BSF model in a finite element model of a whole vessel might provide an answer to that question. Full scale tests of either sections of a vessel or a whole vessel might give a more accurate answer.

There are other modes of failure in sandwich structures than failure due to compressive over loading of an impact damaged one. To include all different modes of failure into a design tool would be the ultimate goal.

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