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BY

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Damping in Dynamic Structure–Foundation Interaction

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Two methods of calculating the damping ratio for structures on compliant foundations are presented. One method employs the calculation of the system damping ratio from the dynamic amplification factor, the other the modal damping ratio from energy considerations. The numerical results for both methods are compared and interpreted. Three sources of damping are considered: inter-storey damping, radiation damping, and foundation material damping. The numerical results demonstrate that with the introduction of compliant foundations the damping ratio of the system can be larger or smaller than that of the corresponding fixed-base structure. Material damping in the foundation soil has been shown to contribute significantly to the over-all damping ratio.

Deux méthodes de calcul du facteur d'amortissement des structures sur fondations déformables sont présentées. Une méthode emploi le calcul du facteur d'amortissement du système à partir du facteur d'amplification dynamique, l'autre le facteur d'amortissement modal basé sur des considérations énergétiques. Les résultats numériques obtenus par les deux méthodes sont comparés et interprétés. On considère trois sources d'amortissement: l'amortissement interétage, l'amortissement par radiation et l'amortissement par le matériau de fondation. Les résultats numériques démontrent que, par suite de l'introduction d'une fondation déformable, le facteur d'amortissement du système peut être plus grand ou plus petit que celui d'une structure correspondante sur une base fixe. On montre que l'amortissement dans le sol de fondation contribue de façon importante au facteur d'amortissement global.

[Traduit par la Revue]

Introduction

Dynamic structure–foundation interaction is of importance in the prediction and interpretation of earthquake and wind effects in structures with compliant foundations. It is particularly significant when structures such as nuclear power plants, high-rise buildings, and towers are founded on moderately or highly compressible soils such as till, sand or clay.

Dynamic structure–foundation interaction depends on the properties of the structure (including the foundation elements) and those of the underlying soil. These properties can be expressed by the two most important parameters affecting dynamic response: natural frequency and the damping ratio of the system. The natural frequency is a measure of the degree of 'tuning' of the structure to the characteristics of a dynamic disturbance, whereas damping of the system is a measure of the energy dissipated and thus is the main parameter that limits the maximum response. Natural frequencies can be determined simply from the mass and stiffness properties of the structure and the underlying soil; this applies to fixed-base structures as well as to those on compliant foundations. The literature on dynamic structure–foundation interaction includes work by Parmelee (1967), Rainer (1971), Sarrazin *et al.* (1972), Meek and Veletsos (1972), Jennings and Bielak (1973); and of Roesset *et al.* (1973) who compared various methods of determining modal damping for soil–structure systems. The study now reported tends to support their findings. Although the finite element method can be employed with great refinement to the solution of problems of structure–ground interaction (for example, Isenberg and Adham 1972), the application is complex and does not lend itself readily to generalizations. It is hoped that the simpler approach and the numerical results now presented may advance understanding of the phenomenon.

The present study is limited to incorporating the various sources of damping in a single system damping ratio. Having such a system damping ratio greatly simplifies the response calculations for structures with foundation flexibilities. The complex mathematical computations for interaction structures can then be replaced by the relatively simple methods available for single-degree-of-freedom systems. Structural damping, radiation damping, and material damping of the foundation soil are considered. The numerical results obtained from the two methods are compared and interpreted. The methods presented are judged to be suitable for design applications for structures having shallow foundations.

Damping in Single-degree-of-freedom Systems (Fixed Base Structure)

The damping ratio λ in the single-degree-of-freedom structure may be characterized by the ratio of the damping coefficient C to the critical damping coefficient of the structure C_{cr}:

$$[1] \qquad \lambda = C/C_{cr}$$

Critical damping is defined by the relation

$$[2] \qquad C_{cr} = 2\lambda\sqrt{km}$$

where k = spring stiffness and m = mass of structure. From the differential equation of motion of a single-degree-of-freedom system subjected to a harmonic base motion with frequency ω, the dynamic response factor for relative displacement of the mass is (for example, Jacobsen and Ayre 1958):

$$[3] \quad T_s = \frac{(\omega/\omega_0)^2}{[(1-(\omega/\omega_0)^2)^2 + (2\lambda\omega/\omega_0)^2]^{1/2}}$$

At the undamped resonance frequency ω_0, the magnitude M_s of the dynamic response factor T_s is related to the ratio of critical damping λ by

$$[4] \qquad \lambda = \frac{1}{2M_s}$$

With the resonance frequency and the ratio of critical damping known, the response of this type of oscillator to an arbitrary input such as an earthquake can be found from response calculations or a response spectrum (for example, Wiegel 1970).

Damping in Structure–Ground Interaction Systems

General Characteristics of Single-storey Interaction Systems

When a single-degree-of-freedom system is placed on a compliant foundation, the following changes occur:

(1) the fundamental frequency of the system decreases from that of the fixed-base structure;

(2) energy is removed from the compliant system by the foundation medium during a dynamic disturbance owing to the propagation of waves into this support medium and is commonly called geometric or radiation damping;

(3) energy is dissipated in the foundation soil medium by intergranular friction and is commonly called material damping.

When the interaction system shown in Fig. 1a is subjected to a base disturbance, the mass m_1 will undergo a total relative displacement u composed of the following components: (1) relative horizontal foundation displacement u_h, (2) rocking displacement $h\theta$, and (3) inter-

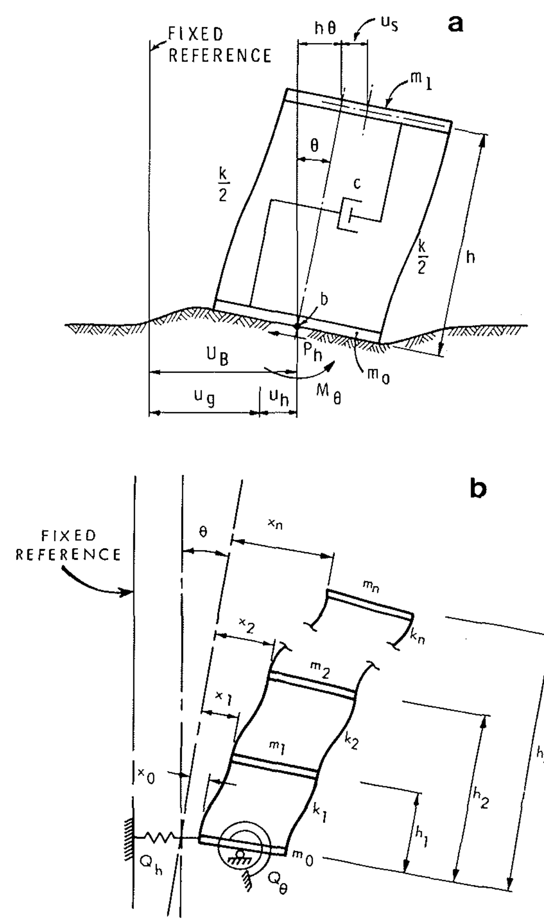

FIG. 1. (a) Single-storey structure–ground interaction system, (b) multi-storey structure–ground interaction system.

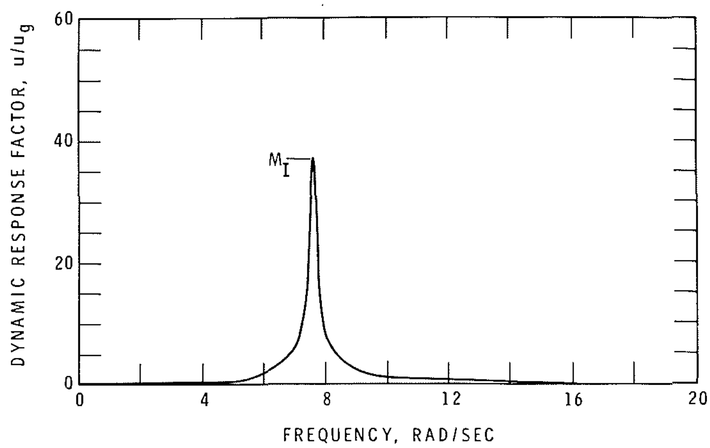

FIG. 2. Dynamic response factor for total relative displacement of top mass.

storey structural displacement u_s, so that

$$[5] \qquad u = u_h + h\theta + u_s$$

From the differential equations of motion the dynamic amplification factor for this system can be derived for a harmonic forcing function as presented by Parmelee (1967) and Rainer (1971).

An example of the variation of this factor is plotted in Fig. 2.

Damping Determined from Dynamic Response Factor

As for the single-degree-of-freedom fixed-base structure, damping for the system with a compliant base can be characterized by the system damping ratio λ_I defined as

$$[6] \qquad \lambda_I = \frac{1}{2M_I}$$

where M_I is the magnitude of the dynamic response factor T at the fundamental frequency ω of the interaction system (illustrated in Fig. 2). A relation similar to Eq. [6] was previously employed in the derivation of an equivalent single-degree-of-freedom system for relative displacement in flexible-base systems (Rainer 1971). If X, Y, and Z are the dynamic amplification factors for absolute base displacement $u_h + u_g$, base rocking θ, and inter-storey displacement u_s, respectively, the dynamic amplification for the total relative displacement of the top mass is

$$[7] \qquad T = (X - 1) + hY + Z$$

Values of M_I, the dynamic magnification factor at the fundamental resonance frequency of the flexible system, were calculated for the set of structural parameters given in Table 1. From these values the system damping ratios λ_I were computed and are presented in Tables 3 to 7. In these calculations the dynamic foundation properties for a circular footing given by Bycroft (1956) were employed.

Modal Damping Determined from Energy Considerations

A second method of determining the damping ratio was presented by Novak (1974) and by Roesset *et al.* (1973). It consists of computing the ratio of the total energy ΔE dissipated in the system to the maximum potential energy E for a particular mode of vibration:

$$\lambda_E = \frac{\Delta E}{4\pi E}$$

For the interaction structure the total energy dissipated, ΔE, during one cycle vibration of a given natural mode is 2π times the summation of the work done by the modal displacements u_h, $h\theta$, and u_s against the corresponding damping forces. As the maximum potential energy in the system is equal to the maximum kinetic energy, the damping ratio λ_E of mode j for the entire structure is given by (Novak 1974):

$$[8] \qquad \lambda_E = \frac{\Delta E}{4\pi E} = \frac{\pi\omega_j(W_h + W_0 + W_s)}{4\pi\omega_j(\frac{1}{2}\Sigma m_i u_i^2)} = \frac{(C_h u_h^2 + C_0 \phi^2 + C_s u_s^2)}{2\omega_j[m_0 u_h^2 + m_i(u_h + h\theta + u_s)^2 + I_0\theta^2]}$$

TABLE 1. Structure and foundation properties

Variable	Structure group 1	Structure group 2	Units
m_o	1000	100 000	lb s²/in.
m_1	4000	100 000	lb s²/in.
r	20	60	ft
h	80	80, 120, 160	ft
ω_o	10	20	rad/s
λ	1, 2, 5	2, 5	% of critical
G	15 200	66 280	p.s.i.
ρ	110	120	lb/ft³
V_s	800	1600	ft/s

where C is the damping coefficient for various modal coordinates.

Determination of Damping Coefficients

The determination of damping coefficients required in the numerical evaluation of the system damping ratio λ_I from Eq. [5] and the modal damping ratio λ_E from Eq. [8] is now presented. Inter-storey damping, radiation damping, and material damping in the foundation are considered.

Inter-storey Damping

Single-storey Structures. For single-storey structures the damping coefficients C_s can be found from the relation

$$[9] \qquad C_s = 2\lambda\sqrt{km}$$

Multi-storey Structures. As the damping ratio due to relative displacement of a multi-degree-of-freedom structure is usually expressed as a modal damping ratio, the energy dissipated by the structure is determined here from a definition of the modal damping ratio of Eq. [9]; C_{cr} is determined from Eq. [2]. The modal mass M is given by

$$[10] \qquad M = \sum_{i=1}^{n} m_i x_i^2$$

where x_i is the amplitude of the fixed-base mode shape of the structure and m_i is storey mass. The corresponding modal stiffness K can be determined from the resonance frequency of the fixed-base structure f_o,

$$[11] \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

The fixed-base frequency is chosen because the damping ratio for inter-storey displacement is generally known or assumed for the fixed-base structure. As the fundamental mode shape for relative displacement has not changed significantly in going from the fixed-base structure to the interaction structure, the modal damping coefficient C_s will have remained substantially unchanged with the introduction of base flexibility. A multi-storey ground–structure interaction system is illustrated in Fig. 1b.

Radiation Damping

In general, the damping factor c_j, due to radiation damping, is part of the complex stiffness Q_j for the foundation (Veletsos and Verbic 1973):

$$[12] \qquad Q_j = K_j(k_j + ia_o c_j)$$

where: the subscript $j = h, \theta$ for horizontal motion and rocking of the foundation, respectively;

K_j = static stiffness of footing on an elastic = half-space;
k_j = dynamic spring factor;
c_j = dynamic damping factor;
a_0 = non-dimensional frequency = $\omega r/V_s$;
ω = frequency, rad/s;
r = radius or equivalent radius of footing;
V_s = shear wave velocity of ground;
i = $\sqrt{-1}$

The factors k_j and c_j are functions of coefficients that have been calculated and presented for various foundation shapes by, among others, Bycroft (1956), Kobori *et al.* (1966), Veletsos and Wei (1971), and Veletsos and Verbic (1974), for footings resting on an elastic half-space.

The damping factor c_j for partially buried structures can also be determined from coefficients presented by Beredugo and Novak (1972) and Novak (1973, 1974).

For circular foundations

$$[13] \quad K_h = \frac{32(1-\nu)Gr}{7-8\nu} \quad \text{(Bycroft 1956)}$$

$$K_0 = \frac{8Gr^3}{3(1-\nu)}$$

where G = shear modulus, r = radius, and ν = Poisson's ratio for the foundation material. The damping coefficient that represents the energy radiated or dissipated for any particular degree of freedom j is then

$$[14] \quad C_j = \frac{a_0}{\omega} K_j c_j = K_j c_j r / V_s$$

Material Damping in the Foundation

Damping in the foundation soil material arises from the energy dissipated through intergranular friction and reveals itself in a hysteretic load deformation curve for the soil (Fig. 3). Such a load deformation curve is characteristic of viscoelastic materials. The energy dissipated per cycle, ΔW, may be expressed as a fraction of the total strain energy W by means of the damping ratio D, as employed by Hardin and Drnevich (1972*a*, *b*).

$$[15] \quad D = \frac{\Delta W}{4\pi W}$$

By making use of the correspondence principle in the theory of visco-elasticity, Veletsos and Verbic (1973) arrived at a most useful approximation: for values of a_0 up to about two the damping coefficient ξ due to material behavior may be added linearly to the coefficient

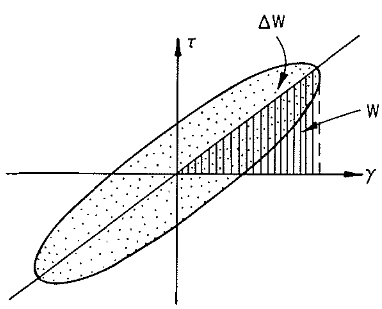

FIG. 3. Stress–strain ellipse for viscoelastic material.

c_j representing energy radiation from the vibrating foundation:

$$[16] \quad c_j{}^t = c_j + \xi$$

Veletsos and Verbic (1973) defined ξ as

$$\xi = \frac{1}{2\pi a_0} \frac{\Delta W}{W};$$

it follows then that the total foundation damping coefficient $c_j{}^t$ is

$$[17] \quad c_j{}^t = c_j + 2D/a_0.$$

This relation enables one to incorporate the contribution of material damping in the same manner as was done for radiation damping in the previous sections.

As the material energy dissipation depends to a major degree on the strain levels to which the soil is subjected, it is necessary to obtain the damping data corresponding to the required loading conditions. For example, for an evaluation of the foundation damping coefficient under wind-induced vibrations the energy dissipated under low levels of strain would usually be desired. For earthquake conditions a strain level approaching that of failure may be needed. These damping parameters can be obtained from appropriate laboratory or field tests or from semi-empirical methods, as for example those outlined by Hardin and Drnevich (1972*b*).

Numerical Examples

Single-storey Structures on Flexible Foundations

For structure group 1 having parameters given in Tables 1 and 2 the system damping ratio λ_E was calculated using Eq. [8] for structural damping ratios of 1, 2, and 5% of critical. The results are presented in Table 3. The magnitudes of energy dissipated for horizontal base displacement, rocking, and inter-storey structural displacement are shown in columns W_h, W_θ, and W_s, respectively. The associated damping coefficients c_h are also given. Finally, the modal damping ratio λ_E obtained from Eq. [8] and the corresponding interaction system damping ratio λ_I obtained from the dynamic amplification factor, Eqs. [6] and [7], are presented.

The same method of calculation was used to find the modal and system damping ratio for

TABLE 2. Mode shapes from eigenvalue calculation

Modal coordinate	Structure group 1	Structure group 2		
		$h = 80$ ft	$h = 120$ ft	$h = 160$ ft
x_h	0.0174	0.1128	0.0775	0.0544
x_s	0.781	0.5309	0.3797	0.2741
$h\theta$	0.480	0.3563	0.5428	0.6715
θ	6×10^{-3}	0.271×10^{-3}	0.377×10^{-3}	0.350×10^{-3}

TABLE 3. System damping for structure group 1

λ (%)	c_h	W_h	W_θ *	W_s	$W_h + W_\theta + W_s$	$\Sigma m_i u_i^2$	λ_E (%) from Eq. [8]	λ_I (%)
1	0.61	84	0	490	574	6460	0.57	—
2	0.61	84	0	980	1064	6460	1.06	1.04
5	0.61	84	0	2440	2524	6460	2.49	2.50

*Rocking damping is finite, but is assumed to be negligible.

structure group 2 whose properties are also given in Tables 1 and 2. These represent massive, stubby structures such as nuclear reactors. Again, the modal damping ratio λ_E computed from Eq. [8] and the corresponding system damping ratio, λ_I, obtained from the dynamic amplification factor are presented in Table 3 for purposes of comparison.

Multi-storey Structures on Flexible Foundations

To determine the damping ratio of the multi-storey interaction system the transfer function approach may be employed, as for the single-storey structure above. Only the energy method, Eq. [8], is used, however, to calculate the damping ratio of multi-storey structures.

The modal damping ratio for the fundamental mode is calculated for a structure whose characteristics were determined from ambient vibration measurements (Rainer 1973); structural and foundation parameters have been given by Eden *et al.* (1973). The energy dissipated at the base in the form of rocking and radiation damping in the horizontal direction has been determined from theoretical results for rectangular footings (Kobori *et al.* 1966). For the experimentally determined mode shapes of $u_b = 0.24$, $\theta = 0.00184$, $u_s = 1$, and modal mass $\sum m_i x_i^2 = 246\,000$ lb s²/in., the amounts of energy dissipated in the horizontal and rocking base motion and inter-storey displacement are shown in Table 5, which also presents the modal damping ratios λ_E calculated from Eq. [8] for an inter-storey damping ratio λ of 1, 2, and 5%.

It may be observed from Table 5 that the computed modal damping ratio λ_E is substantially less than the fixed-base modal damping ratio of $\lambda = 5\%$ and slightly less for $\lambda = 2\%$. For $\lambda = 1\%$ an increase in system damping may be seen to be due to the influence of the flexible foundation.

Material Damping in the Foundation

To illustrate the influence of foundation material damping on the system damping ratio, two assumed levels of material damping are chosen: $D = 3\%$ and $D = 15\%$, and the same value of D is used for horizontal motion as well as for rocking. The structure group 2 and foundation properties of Tables 1 and 2 are used; results without foundation material damping are presented in Tables 3 and 4.

The results of the calculations for λ_E with foundation material damping are presented in Tables 6 and 7. Energy dissipated due to material damping is designated W_θ^v and W_h^v for rocking and horizontal motion, respectively.

Discussion

A comparison of the system damping ratio λ_I from the dynamic response factor and modal damping ratio λ_E from the energy method, Eq. [8], is presented in Tables 3 and 4 for the case without foundation material damping, and in

TABLE 4. System damping for structure group 2

Case No.	h (ft)	ω_o rad/s	c_h	W_h $\times 10^4$	c_θ	W_θ $\times 10^4$	λ (%)	W_s $\times 10^4$	$(W_h + W_\theta + W_s)$ $\times 10^4$	$\Sigma m_i u_i^2$ $\times 10^5$	λ_E(%) from Eq. [8]	λ_I (%) from dynamic response function
1	80	14.63	0.61	6.35	0.062	2.13	2	2.25	10.73	1.031	3.56	3.16
2			0.61	6.35	0.062	2.13	5	5.63	14.11	1.031	4.69	4.23
3	120	12.35	0.62	3.05	0.040	1.43	2	1.15	5.63	1.024	2.23	2.06
4			0.62	3.05	0.040	1.43	5	2.88	7.36	1.024	2.90	2.71
5	160	10.49	0.62	1.52	0.023	0.75	2	0.602	2.87	1.019	1.35	1.24
6			0.62	1.52	0.023	0.75	5	1.50	3.77	1.019	1.77	1.76

TABLE 5. Modal damping ratios for multi-storey structure

λ (%)	$W_h \times 10^4$	$W_\theta \times 10^4$	$W_s \times 10^4$	λ_E (%) from Eq. [8]
1	4.55	3.31	0.80	1.77
2	4.55	3.31	1.61	1.93
5	4.55	3.31	4.03	2.43

TABLE 6. System damping for structure group 2 with foundation material damping of $D = 0.15$

Case No.	λ (%)	$W_h^c \times 10^4$	$W_\theta^c \times 10^4$	$(W_h + W_\theta + W_s + W_h^c + W_\theta^c) \times 10^4$	λ_E (%) from Eq. [8]	λ_I (%) from dynamic response function
1	2	4.95	18.8	34.5	11.4	9.7
2	5	4.95	18.8	37.9	12.5	10.8
3	2	2.82	23.1	31.5	12.5	11.2
4	5	2.82	23.1	33.3	13.1	11.8
5	2	1.64	25.0	29.5	13.8	12.0
6	5	1.64	25.0	30.4	14.2	12.4

TABLE 7. System damping for structure group 2 with foundation material damping of $D = 0.03$

Case No.	λ (%)	$W_h^c \times 10^4$	$W_\theta^c \times 10^4$	$(W_h + W_\theta + W_s + W_h^c + W_\theta^c) \times 10^4$	λ_E (%) from Eq. [8]	λ_I (%) from dynamic response function
1	2	0.99	3.75	15.5	5.1	4.6
2	5	0.99	3.75	18.9	6.3	5.7
3	2	0.56	4.61	10.8	4.3	4.0
4	5	0.56	4.61	12.5	5.0	4.6
5	2	0.33	5.00	8.20	3.8	3.5
6	5	0.33	5.00	9.10	4.3	3.9

Tables 5 and 6 for the case with material damping. Throughout the range of parameters considered the comparison between the values of λ_I and λ_E is quite favorable, with a maximum deviation of about 15 to 20% for the low structures considered. This difference decreases as the structures become taller and foundation damping becomes smaller.

The discrepancy in the computed damping ratios may be explained as follows. First, the energy method for computing λ_E uncouples the modal amplitudes from the damping effects, thereby slightly over-estimating the modal amplitudes that are associated with high damping coefficients. This is particularly pronounced for low structures where the energy dissipated by the horizontal base motion dominates the other sources of damping, as is evident from Tables 3 and 4, and 5 and 6. The values of λ_E from the energy method will therefore be smaller than λ_I from the dynamic response factor. Second, the transfer function at the fundamental resonance frequency also contains small components of the other modes (Novak 1974). This tends to overestimate the damping ratio computed by Eq. [6].

Examination of the values for transfer function amplitude peaks M_I/M_s and the resulting damping ratios λ_I in Table 3 shows that the system damping ratio can be substantially reduced in comparison with fixed-base structural damping λ. The reduction in system damping ratio is larger for higher values of damping, as is evident from a comparison of the results for $\lambda = 2\%$ and $\lambda = 5\%$ in Table 4. Furthermore, the system damping ratio becomes smaller the taller the structure. This can be explained by means of the energy method of computing the modal damping ratio. As the modal damping ratio computed from Eq. [8] depends on the sum of the contributions from the various sources of damping, a change in any one of the damping coefficients will affect this damping ratio. Consequently, when the contributions of energy dissipation from material damping and radiation damping are low, structures that have large fixed-base structural damping ratios may experience a substantial reduction in damping ratio when they are founded on compliant bases. On the other hand, for structures having small structural damping ratios an increase in system damping ratio can be expected with the introduction of a compliant base. This effect is illustrated quantitatively by the numerical results for the single-storey structure as well as for the multi-storey building (Tables 3–7).

The results obtained from the energy method illustrate the following principle in a quantitative manner: With a compliant foundation, the major contribution to the over-all system damping shifts from the structure to the foundation. In order to achieve or maintain satisfactory levels of system damping (as is desirable for limiting the response of structures to dynamic loads) adequate sources of damping in foundations have to be provided. The damping ratio present for any particular problem can be determined to a reasonable degree of accuracy by the energy method outlined herein.

Another important result that emerges from the numerical calculations is that, for the structures considered, the large damping coefficient C_H associated with horizontal base motion yields relatively modest contributions to the total energy dissipated and hence does not result in large over-all system damping ratios. The reason is that for moderately tall structures the modal amplitude of horizontal base displacement is small, compared with the other degrees of freedom. As the energy dissipated per cycle is the product of damping coefficient and the square of the modal amplitude, this product is greatly affected by a small modal base displacement and will, in general, be smaller than the large damping coefficient would lead one to expect. For a structural configuration, however, where the ratio of base modal amplitude to rocking and structural displacement amplitudes becomes relatively large, significant contributions to system damping can be expected from the horizontal base component.

For the multi-storey example an examination of the relative magnitudes of the damping energies W_h, W_θ, and W_s shows that structural damping is a relatively small proportion of the total damping energy. Consequently, system damping is essentially governed by foundation damping. This is reflected in the results shown in Table 5, where the damping ratios λ_E change from 1.77 to 2.43% while the structural damping ratio λ varies between 1 and 5%.

The inclusion of foundation material damping has substantial influence on calculated system damping, as is evident in comparing the results in Table 4 with those of Tables 6 and 7. For the relatively high material damping ratio assumed, $D = 0.15$, Table 6 shows that the system damping ratios λ_I are substantially larger than their counterparts in Table 4, in which foundation material damping was not included. Similar results, but less pronounced, can be observed in Table 7 for the smaller assumed value of foundation material damping ratio, $D = 0.03$. An examination in Tables 4, 5 and 6 of the magnitudes of each of the contributions to the total energy dissipated indicates that energy dissipated as a result of material damping in base rocking is the major contributor when material damping is increased. This is a desirable and welcome trend, since the rocking component is responsible for a large portion of the total kinetic energy of the system in the denominator of Eq. [8]. Without the corresponding damping mechanism in the rocking displacement, the over-all system damping ratio may be considerably smaller than the structural damping ratio, as is illustrated by the results of Table 4.

Conclusion

Two methods of calculation have been presented for determining the over-all damping ratio of structures on compliant foundations. The first makes use of the amplitude of the resonance peak of the dynamic response function for the system; the other is an approximate procedure using energy considerations. Comparison of numerical results for a series of single-storey structures demonstrates that the results of the energy method compare favorably with those obtained by means of the dynamic response function approach. The energy method of determining damping ratio has also been applied to multi-storey structures; and a procedure for incorporating the contributions of foundation material damping has been described and illustrated by numerical examples.

The numerical results show that the introduction of foundation flexibilities changes the system damping ratio. Depending on the structural configuration and the degree of foundation material damping present, increased or decreased system damping ratios can be realized. Quantitative evaluation of the system damping ratio is of importance in determining the response of structures and foundations to dynamic disturbances such as earthquakes and wind. The energy method described enables one to perform this calculation in a relatively simple manner.

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