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Dark Energy Density in SUGRA models and degenerate vacua

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In $N = 1$ supergravity the tree-level scalar potential of the hidden sector may have a minimum with broken local supersymmetry (SUSY) as well as a supersymmetric Minkowski vacuum. These vacua can be degenerate, allowing for a consistent implementation of the multiple point principle. The first minimum where SUSY is broken can be identified with the physical phase in which we live. In the second supersymmetric phase, in flat Minkowski space, SUSY may be broken dynamically either in the observable or in the hidden sectors inducing a tiny vacuum energy density. We argue that the exact degeneracy of these phases may shed light on the smallness of the cosmological constant. Other possible phenomenological implications are also discussed. In particular, we point out that the presence of such degenerate vacua may lead to small values of the quartic Higgs coupling and its beta function at the Planck scale in the physical phase.

Keywords: Supergravity; Cosmological constant; Higgs boson.

1. Introduction

It is expected that at ultra-high energies the Standard Model (SM) is embedded in an underlying theory that provides a framework for the unification of all interactions such as Grand Unified Theories (GUTs), supergravity (SUGRA), String Theory, etc. At low energies this underlying theory could lead to new physics phenomena beyond the SM. Moreover the energy scale associated with the physics beyond the SM is supposed to be somewhat close to the mass of the Higgs boson to avoid a fine-tuning problem related to the need to stabilize the scale where electroweak (EW) symmetry is broken. Despite the successful discovery of the 125 GeV Higgs boson in 2012, no indication of any physics beyond the SM has been detected at the LHC so far. On the other hand, there are compelling reasons to believe that the SM is extremely fine-tuned. Indeed, astrophysical and cosmological observations indicate that there is a tiny energy density spread all over the Universe (the cosmo-

logical constant), i.e. $\rho_\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^{4,1,2}$, which is responsible for its acceleration. At the same time much bigger contributions must come from the EW symmetry breaking ($\sim 10^{-67} M_{Pl}^4$) and QCD condensates ($\sim 10^{-79} M_{Pl}^4$). Because of the enormous cancellation between the contributions of different condensates to ρ_Λ , which is required to keep ρ_Λ around its measured value, the smallness of the cosmological constant can be considered as a fine-tuning problem.

Here, instead of trying to alleviate fine-tuning of the SM we impose the exact degeneracy of at least two (or even more) vacua. Their presence was predicted by the so-called Multiple Point Principle (MPP)³. According to the MPP, Nature chooses values of coupling constants such that many phases of the underlying theory should coexist. This corresponds to a special (multiple) point on the phase diagram where these phases meet. At the multiple point these different phases have the same vacuum energy density.

The MPP applied to the SM implies that the Higgs effective potential, which can be written as

$$V_{eff}(H) = m^2(\phi)H^\dagger H + \lambda(\phi)(H^\dagger H)^2, \quad (1)$$

where H is a Higgs doublet and ϕ is a norm of the Higgs field, i.e. $\phi^2 = H^\dagger H$, has two degenerate minima. These minima are taken to be at the EW and Planck scales⁴. The corresponding vacua can have the same energy density only if

$$\lambda(M_{Pl}) \simeq 0, \quad \beta_\lambda(M_{Pl}) \simeq 0. \quad (2)$$

where $\beta_\lambda = \frac{d\lambda(\phi)}{d \log \phi}$ is the beta-function of $\lambda(\phi)$. It was shown that the MPP conditions (2) can be satisfied when $M_t = 173 \pm 5 \text{ GeV}$ and $M_H = 135 \pm 9 \text{ GeV}$ ⁴. The application of the MPP to the two Higgs doublet extension of the SM was also considered⁵⁻⁷.

The measurement of the Higgs boson mass allows us to determine quite precisely the parameters of the Higgs potential (1). Furthermore using the extrapolation of the SM parameters up to M_{Pl} with full 3-loop precision it was found⁸ that

$$\begin{aligned} \lambda(M_{Pl}) = & -0.0143 - 0.0066 \left(\frac{M_t}{\text{GeV}} - 173.34 \right) \\ & + 0.0018 \left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \right) + 0.0029 \left(\frac{M_H}{\text{GeV}} - 125.15 \right). \end{aligned} \quad (3)$$

The computed value of $\beta_\lambda(M_{Pl})$ is also rather small, so that the MPP conditions (2) are basically fulfilled.

The successful MPP predictions for the Higgs and top quarks masses⁴ suggest that we may use this idea to explain the tiny value of the cosmological constant as well. In principle, the smallness of the cosmological constant could be related to an almost exact symmetry. Nevertheless, none of the generalizations of the SM provides any satisfactory explanation for the smallness of this dark energy density. An exact global supersymmetry (SUSY) guarantees that the vacuum energy density

vanishes in all global minima of the scalar potential. However the non-observation of superpartners of quarks and leptons implies that SUSY is broken. The breakdown of SUSY induces a huge and positive contribution to the dark energy density which is many orders of magnitude larger than M_Z^4 . Here the MPP assumption is adapted to $(N = 1)$ SUGRA models, in order to provide an explanation for the tiny deviation of the measured dark energy density from zero.

2. SUGRA models inspired by degenerate vacua

The full $(N = 1)$ SUGRA Lagrangian can be specified in terms of an analytic gauge kinetic function $f_a(\phi_M)$ and a real gauge-invariant Kähler function $G(\phi_M, \phi_M^*)$. These functions depend on the chiral superfields ϕ_M . The function $f_a(\phi_M)$ determine, in particular, the gauge couplings $Ref_a(\phi_M) = 1/g_a^2$, where the index a represents different gauge groups. The Kähler function can be presented as

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2, \quad (4)$$

where $K(\phi_M, \phi_M^*)$ is the Kähler potential while $W(\phi_M)$ is the superpotential of the SUGRA model under consideration. Here we shall use standard supergravity mass units: $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$.

The SUGRA scalar potential can be written as a sum of F - and D -terms, i.e.

$$\begin{aligned} V(\phi_M, \phi_M^*) &= \sum_{M, \bar{N}} e^G \left(G_M G^{M\bar{N}} G_{\bar{N}} - 3 \right) + \frac{1}{2} \sum_a (D^a)^2, \\ D^a &= g_a \sum_{i, j} (G_i T_{ij}^a \phi_j), \quad G_M \equiv \frac{\partial G}{\partial \phi_M}, \quad G_{\bar{M}} \equiv \frac{\partial G}{\partial \phi_M^*}, \\ G_{\bar{N}M} &\equiv \frac{\partial^2 G}{\partial \phi_M^* \partial \phi_M}, \quad G^{M\bar{N}} = G_{\bar{N}M}^{-1}. \end{aligned} \quad (5)$$

In Eq. (5) g_a is the gauge coupling associated with the generator T^a . In order to achieve the breakdown of local SUSY in $(N = 1)$ supergravity, a hidden sector is introduced. The superfields of the hidden sector (z_i) interact with the observable ones only by means of gravity. It is expected that at the minimum of the scalar potential (5) hidden sector fields acquire vacuum expectation values (VEVs) so that at least one of their auxiliary fields

$$F^M = e^{G/2} G^{M\bar{P}} G_{\bar{P}} \quad (6)$$

is non-vanishing, leading to the spontaneous breakdown of local SUSY, giving rise to a non-zero gravitino mass $m_{3/2} \simeq \langle e^{G/2} \rangle$. The absolute value of the vacuum energy density at the minimum of the SUGRA scalar potential (5) tends to be of order of $m_{3/2}^2 M_{Pl}^2$. Therefore an enormous degree of fine-tuning is required to keep the cosmological constant in SUGRA models around its observed value.

The successful implementation of the MPP in $(N = 1)$ SUGRA models requires us to assume the existence of a supersymmetric Minkowski vacuum^{9,10}. According to the MPP this second vacuum and the physical one must be degenerate. Since the vacuum energy density of supersymmetric states in flat Minkowski space vanishes,

the cosmological constant problem is solved to first approximation. Such a second vacuum exists if the SUGRA scalar potential (5) has a minimum where

$$W(z_m^{(2)}) = 0, \quad \left. \frac{\partial W(z_i)}{\partial z_m} \right|_{z_m=z_m^{(2)}} = 0 \quad (7)$$

where $z_m^{(2)}$ are VEVs of the hidden sector fields in the second vacuum. Eqs. (7) indicate that an extra fine-tuning is needed to ensure the presence of the supersymmetric Minkowski vacuum in SUGRA models.

The simplest Kähler potential and superpotential that satisfies conditions (7) are given by

$$K(z, z^*) = |z|^2, \quad W(z) = m_0(z + \beta)^2. \quad (8)$$

The hidden sector of the corresponding SUGRA model involves only one singlet superfield z . If $\beta = \beta_0 = -\sqrt{3} + 2\sqrt{2}$, the corresponding SUGRA scalar potential possesses two degenerate vacua with zero energy density at the classical level. The first minimum associated with $z^{(2)} = -\beta$ is a supersymmetric Minkowski vacuum. In the other minimum, $z^{(1)} = \sqrt{3} - \sqrt{2}$, local SUSY is broken so that it can be identified with the physical vacuum. Varying β around β_0 one can get a positive or a negative contribution from the hidden sector to the total vacuum energy density of the physical phase. Thus parameter β can be always fine-tuned so that the physical and supersymmetric Minkowski vacua are degenerate.

The fine-tuning associated with the realisation of the MPP in ($N = 1$) SUGRA models can be to some extent alleviated within the no-scale inspired SUGRA model with broken dilatation invariance¹⁰. Let us consider the no-scale inspired SUGRA model that involves two hidden sector superfields (T and z) and a set of chiral supermultiplets φ_σ in the observable sector. These superfields transform differently under the dilatations ($T \rightarrow \alpha^2 T, z \rightarrow \alpha z, \varphi_\sigma \rightarrow \alpha \varphi_\sigma$) and imaginary translations ($T \rightarrow T + i\beta, z \rightarrow z, \varphi_\sigma \rightarrow \varphi_\sigma$), which are subgroups of the $SU(1, 1)$ group^{10,11}. The full superpotential of the model can be written as a sum¹⁰:

$$W(z, \varphi_\alpha) = W_{hid} + W_{obs}, \quad (9)$$

$$W_{hid} = \varkappa \left(z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n \right), \quad W_{obs} = \sum_{\sigma, \beta, \gamma} \frac{1}{6} Y_{\sigma\beta\gamma} \varphi_\sigma \varphi_\beta \varphi_\gamma.$$

The superpotential (9) includes a bilinear mass term for the superfield z and higher order terms $c_n z^n$ which explicitly break dilatation invariance. A term proportional to z is not included since it can be forbidden by a gauge symmetry of the hidden sector. Here we do not allow the breakdown of dilatation invariance in the superpotential of the observable sector to avoid potentially dangerous terms that may lead to the so-called μ -problem, etc.

The full Kähler potential of the SUGRA model under consideration is given

by¹⁰:

$$\begin{aligned}
 K(\phi_M, \phi_M^*) = & -3 \ln \left[T + \bar{T} - |z|^2 - \sum_{\alpha} \zeta_{\alpha} |\varphi_{\alpha}|^2 \right] + \\
 & + \sum_{\alpha, \beta} \left(\frac{\eta_{\alpha\beta}}{2} \varphi_{\alpha} \varphi_{\beta} + h.c. \right) + \sum_{\beta} \xi_{\beta} |\varphi_{\beta}|^2,
 \end{aligned} \tag{10}$$

where ζ_{α} , $\eta_{\alpha\beta}$, ξ_{β} are some constants. If $\eta_{\alpha\beta}$ and ξ_{β} have non-zero values the dilatation invariance is explicitly broken in the Kähler potential of the observable sector. Here we restrict our consideration to the simplest set of terms that break dilatation invariance. Moreover we only allow the breakdown of the dilatation invariance in the Kähler potential of the observable sector, because any variations in the part of the Kähler potential associated with the hidden sector may spoil the vanishing of the vacuum energy density in global minima. When the parameters $\eta_{\alpha\beta}$, ξ_{β} and \varkappa go to zero, the dilatation invariance is restored, protecting supersymmetry and a zero value of the cosmological constant.

In the SUGRA model under consideration the tree-level scalar potential of the hidden sector is positive definite

$$V_{hid} = \frac{1}{3(T + \bar{T} - |z|^2)^2} \left| \frac{\partial W_{hid}(z)}{\partial z} \right|^2, \tag{11}$$

so that the vacuum energy density vanishes near its global minima. In the simplest case when $c_n = 0$, the SUGRA scalar potential of the hidden sector (11) has two minima, at $z = 0$ and $z = -\frac{2\mu_0}{3}$. At these points V_{hid} attains its absolute minimal value i.e. zero. In the first vacuum, where $z = -\frac{2\mu_0}{3}$, local SUSY is broken and the gravitino gains mass

$$m_{3/2} = \left\langle \frac{W_{hid}(z)}{(T + \bar{T} - |z|^2)^{3/2}} \right\rangle = \frac{4\varkappa\mu_0^3}{27 \left\langle \left(T + \bar{T} - \frac{4\mu_0^2}{9} \right)^{3/2} \right\rangle}. \tag{12}$$

All scalar particles get non-zero masses $m_{\sigma} \sim \frac{m_{3/2}\xi_{\sigma}}{\zeta_{\sigma}}$ as well. This minimum can be identified with the physical vacuum. Assuming that ξ_{α} , ζ_{α} , μ_0 and $\langle T \rangle$ are all of order unity, a SUSY breaking scale $M_S \sim 1 \text{ TeV}$ can only be obtained when \varkappa is extremely small, i.e. $\varkappa \simeq 10^{-15}$. In the second vacuum, where $z = 0$, the superpotential of the hidden sector vanishes and local SUSY remains intact giving rise to the supersymmetric Minkowski vacuum. If the high order terms $c_n z^n$ are present in Eqs. (9), V_{hid} can have many degenerate vacua, with broken and unbroken SUSY, in which the vacuum energy density vanishes. As a result the MPP conditions are fulfilled without any extra fine-tuning at the tree-level.

It is worth noting that the inclusion of perturbative and non-perturbative corrections to the Lagrangian of the no-scale inspired SUGRA model should spoil the degeneracy of vacua, giving rise to a huge energy density in the minimum of the scalar potential where local SUSY is broken. Furthermore, in the SUGRA model

under consideration the mechanism for the stabilization of the VEV of the hidden sector field T remains unclear. Therefore this model should be considered as a toy example only. It demonstrates that in $(N = 1)$ supergravity there might be a mechanism which ensures the vanishing of the vacuum energy density in the physical vacuum. This mechanism can also result in a set of degenerate vacua with broken and unbroken SUSY, leading to the realization of the MPP.

3. Implications for cosmology and collider phenomenology

3.1. Model with intermediate SUSY breaking scale

Now let us assume that the MPP inspired SUGRA model of the type just discussed is realised in Nature. In other words there exist at least two exactly degenerate phases. The first phase is associated with the physical vacuum whereas the second one is identified with the supersymmetric Minkowski vacuum in which the vacuum energy density vanishes in the leading approximation. However non-perturbative effects may give rise to the breakdown of SUSY in the second phase resulting in a small vacuum energy density. This small energy density should be then transferred to our vacuum by the assumed degeneracy.

If SUSY breaking takes place in the second vacuum, it can be caused by the strong interactions in the observable sector. Indeed, the SM gauge couplings g_1 , g_2 and g_3 , which correspond to $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$ gauge interactions respectively, change with the energy scale. Their evolution obeys the renormalization group equations (RGEs) that in the one-loop approximation can be written as

$$\frac{d \log \alpha_i(Q)}{d \log Q^2} = \frac{b_i \alpha_i(Q)}{4\pi}, \quad (13)$$

where Q is a renormalization scale, $i = 1, 2, 3$ and $\alpha_i(Q) = g_i^2(Q)/(4\pi)$. In the pure MSSM $b_3 < 0$ and the gauge coupling $g_3(Q)$ of the $SU(3)_C$ interactions increases in the infrared region. Thus although this coupling can be rather small at high energies it becomes rather large at low energies and one can expect that the role of non-perturbative effects is enhanced.

To simplify our analysis we assume that the SM gauge couplings at high energies are identical in the physical and supersymmetric Minkowski vacua. Consequently for $Q > M_S$, where M_S is a SUSY breaking scale in the physical vacuum, the renormalization group (RG) flow of these couplings is also the same in both vacua. When $Q < M_S$ all superparticles in the physical vacuum decouple. Therefore the $SU(3)_C$ beta function in the physical and supersymmetric Minkowski vacua become very different. Because of this, below the scale M_S the values of $\alpha_3(Q)$ in the physical and second vacua ($\alpha_3^{(1)}(Q)$ and $\alpha_3^{(2)}(Q)$) are not the same. For $Q < M_S$ the $SU(3)_C$ beta function in the physical phase \tilde{b}_3 coincides with the corresponding SM beta function, i.e. $\tilde{b}_3 = -7$. Using the value of $\alpha_3^{(1)}(M_Z) \approx 0.1184$ and the matching condition $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, one can estimate the value of the strong

gauge coupling in the second vacuum

$$\frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2}. \quad (14)$$

In the supersymmetric Minkowski vacuum all particles of the MSSM are massless and the EW symmetry is unbroken. So, in the second phase the $SU(3)_C$ beta function b_3 remains the same as in the MSSM, i.e. $b_3 = -3$. Since the MSSM $SU(3)_C$ beta function exhibits asymptotically free behaviour, $\alpha_3^{(2)}(Q)$ increases in the infrared region. The top quark is massless in the supersymmetric phase and its Yukawa coupling also grows in the infrared with the increasing of $\alpha_3^{(2)}(Q)$. At the scale

$$\Lambda_{SQCD} = M_S \exp \left[\frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)} \right], \quad (15)$$

where the supersymmetric QCD interactions become strong in the supersymmetric Minkowski vacuum, the top quark Yukawa coupling is of the same order of magnitude as the $SU(3)_C$ gauge coupling. So a large value of the top quark Yukawa coupling may give rise to the formation of a quark condensate. This condensate breaks SUSY, resulting in a non-zero positive value for the dark energy density

$$\rho_\Lambda \simeq \Lambda_{SQCD}^4. \quad (16)$$

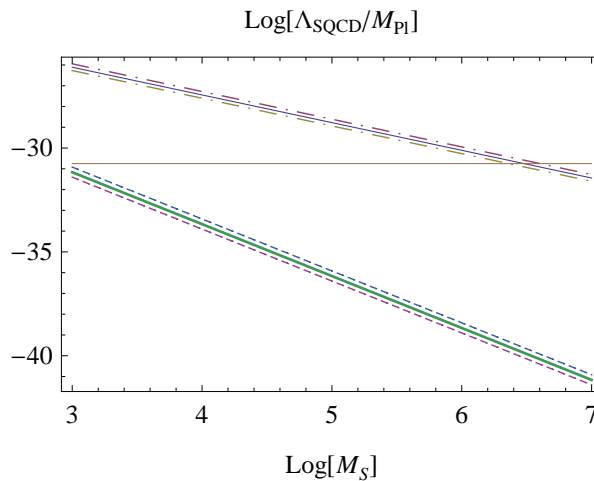


Fig. 1. The value of $\log [\Lambda_{SQCD}/M_{Pl}]$ versus $\log M_S$. The thin and thick solid lines correspond to the pure MSSM and the MSSM with an additional pair of $5 + \bar{5}$ supermultiplets, respectively. These lines are obtained for $\alpha_3(M_Z) = 0.1184$. The dashed and dash-dotted lines represent the uncertainty in $\alpha_3(M_Z)$. The upper dashed and dash-dotted lines are obtained for $\alpha_3(M_Z) = 0.121$, while the lower ones correspond to $\alpha_3(M_Z) = 0.116$. The horizontal line represents the observed value of $\rho_\Lambda^{1/4}$. The SUSY breaking scale, M_S , is measured in GeV.

The dependence of Λ_{SQCD} on the SUSY breaking scale M_S is shown in Fig. 1. Since $\tilde{b}_3 < b_3$ the value of the QCD gauge coupling below M_S is larger in the physical phase than in the second one. Therefore the scale Λ_{SQCD} is substantially lower than the QCD scale in the SM and decreases with increasing M_S . When the supersymmetry breaking scale in the physical phase is of the order of 1 TeV, we get $\Lambda_{SQCD} = 10^{-26} M_{Pl} \simeq 100$ eV^{10,12,13}. This leads to the value of the cosmological constant $\rho_\Lambda \simeq 10^{-104} M_{Pl}^4$, which is enormously suppressed as compared with $v^4 \simeq 10^{-67} M_{Pl}^4$. The measured value of the dark energy density is reproduced when $\Lambda_{SQCD} = 10^{-31} M_{Pl} \simeq 10^{-3}$ eV. The appropriate values of Λ_{SQCD} can be obtained only if $M_S \simeq 10^3 - 10^4$ TeV^{10,12,13}. However models with such a large SUSY breaking scale do not lead to a suitable dark-matter candidate and also spoil the unification of the SM gauge couplings.

3.2. Split SUSY scenario

The problems mentioned above can be addressed within the Split SUSY scenario of superymmetry breaking^{14,15}. In other words, let us now assume that in the physical vacuum SUSY is broken so that all scalar bosons gain masses of order of $M_S \gg 10$ TeV, except for a SM-like Higgs boson, whose mass is set to be around 125 GeV. The mass parameters of gauginos and Higgsinos are protected by a combination of an R-symmetry and Peccei Quinn symmetry so that they can be many orders of magnitude smaller than M_S . To ensure gauge coupling unification all neutralino, chargino and gluino states are chosen to lie near the TeV scale in the Split SUSY scenario¹⁵. Also a TeV-scale lightest neutralino can be an appropriate dark matter candidate¹⁵⁻¹⁸.

Thus in the Split SUSY scenario supersymmetry is not used to stabilize the EW scale^{14,15}. This stabilization is expected to be provided by some other mechanism, which may also explain the tiny value of the dark energy density. Therefore in the Split SUSY models M_S is taken to be much above 10 TeV. In the Split SUSY scenario some flaws inherent to the MSSM disappear. The ultra-heavy scalars, whose masses can range from hundreds of TeV up to 10^{13} GeV¹⁵, ensure the absence of large flavor changing and CP violating effects. The stringent constraints from flavour and electric dipole moment data, that require $M_S > 100 - 1000$ TeV, are satisfied and the dimension-five operators, which mediate proton decay, are also suppressed within the Split SUSY models. Nevertheless, since the sfermions are ultra-heavy the Higgs sector is extremely fine-tuned, with the understanding that the solution to both the hierarchy and cosmological constant problems might not involve natural cancellations, but follow from anthropic-like selection effects¹⁹. In other words galaxy and star formation, chemistry and biology, are basically impossible without these scales having the values found in our Universe¹⁹⁻²¹. In this case supersymmetry may be just a necessary ingredient in a fundamental theory of Nature like in the case of String Theory.

It has been argued that String Theory can have a huge number of long-lived

metastable vacua^{22–31} which is measured in googles ($\sim 10^{100}$)^{25–31}. The space of such vacua is called the “landscape”. To analyze the huge multitude of universes, associated with the “landscape” of these vacua a statistical approach is used^{26–31}. The total number of vacua in String Theory is sufficiently large to fine-tune both the cosmological constant and the Higgs mass, favoring a high-scale breaking of supersymmetry^{30,31}. Thus it is possible for us to live in a universe fine-tuned in the way we find it simply because of a cosmic selection rule, i.e. the anthropic principle¹⁹.

The idea of the multiple point principle and the landscape paradigm have at least two things in common. Both approaches imply the presence of a large number of vacua with broken and unbroken SUSY. The landscape paradigm and MPP also imply that the parameters of the theory, which results in the SM at low energies, can be extremely fine-tuned so as to guarantee a tiny vacuum energy density and a large hierarchy between M_{Pl} and the EW scale. Moreover the MPP assumption might originate from the landscape of string theory vacua, if all vacua with a vacuum energy density that is too large are forbidden for some reason, so that all the allowed string vacua, with broken and unbroken supersymmetry, are degenerate to very high accuracy. If this is the case, then the breaking of supersymmetry at high scales is perhaps still favored. Although this scenario looks quite attractive it implies that only a narrow band of values around zero cosmological constant would be allowed and the surviving vacua would obey MPP to the accuracy of the width w of this remaining band. However such accuracy is not sufficient to become relevant for the main point of the present article, according to which MPP “transfers” the vacuum energy density of the second vacuum to the physical vacuum.

In order to estimate the value of the cosmological constant we again assume that the physical and second phases have precisely the same vacuum energy densities and the gauge couplings at high energies are identical in both vacua. This means that the renormalization group flow of the SM gauge couplings down to the scale M_S is the same in both vacua as before. For $Q < M_S$ all squarks and sleptons in the physical vacuum decouple and the beta functions change. At the TeV scale, the corresponding beta functions in the physical phase change once again due to the decoupling of the gluino, neutralino and chargino. Assuming that $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, one finds

$$\frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_g^2}{M_Z^2} - \frac{b'_3}{4\pi} \ln \frac{M_S^2}{M_g^2}, \quad (17)$$

where M_g is the mass of the gluino and $b'_3 = -5$ is the one-loop beta function of the strong gauge coupling in the Split SUSY scenario. The values of Λ_{SQCD} and ρ_Λ can be estimated using Eqs. (15) and (16), respectively.

In Fig. 2 we explore the dependence of Λ_{SQCD} in the second phase on the SUSY breaking scale M_S in the physical vacuum. In our analysis we set $M_g = 3 \text{ TeV}$. As before Λ_{SQCD} diminishes with increasing M_S . The observed value of the dark en-

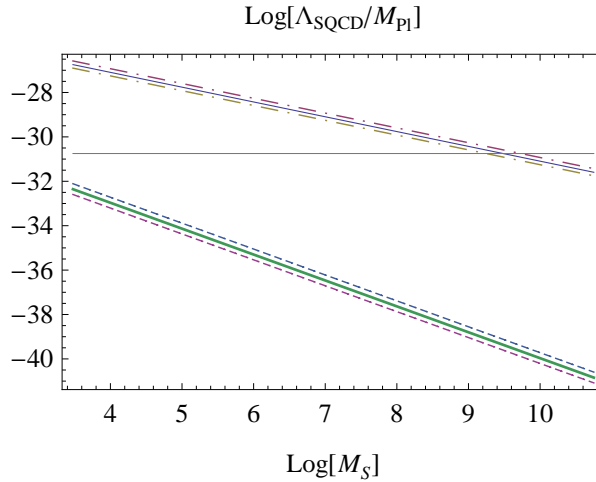


Fig. 2. The value of $\log [\Lambda_{SQCD}/M_{Pl}]$ versus $\log M_S$ for $M_q = M_g = 3 \text{ TeV}$. The thin and thick solid lines correspond to the Split SUSY scenarios with the pure MSSM particle content and the MSSM particle content supplemented by an additional pair of $5 + \bar{5}$ supermultiplets respectively. The dashed and dash-dotted lines represent the uncertainty in $\alpha_3(M_Z)$. The thin and thick solid lines are obtained for $\alpha_3(M_Z) = 0.1184$, the upper (lower) dashed and dash-dotted lines correspond to $\alpha_3(M_Z) = 0.116$ ($\alpha_3(M_Z) = 0.121$). The horizontal line represents the observed value of $\rho_\Lambda^{1/4}$. The SUSY breaking scale M_S is measured in GeV.

energy density can be reproduced when $M_S \sim 10^9 - 10^{10} \text{ GeV}^{32-34}$. The value of M_S , which results in the measured cosmological constant, depends on $\alpha_3(M_Z)$ and the gluino mass. However this dependence is rather weak. In particular, with increasing M_g the value of M_S , which leads to an appropriate value of the cosmological constant, decreases. When $\alpha_3(M_Z) = 0.116 - 0.121$ and $M_g = 500 - 2500 \text{ GeV}$, the corresponding value of the SUSY breaking scale varies from $2 \cdot 10^9 \text{ GeV}$ up to $3 \cdot 10^{10} \text{ GeV}^{32-34}$.

The obtained prediction for M_S can be tested. A striking feature of the Split SUSY model is the extremely long lifetime of the gluino. The gluino decays through a virtual squark to a quark antiquark pair and a neutralino $\tilde{g} \rightarrow q\bar{q} + \chi_1^0$. The large squark masses give rise to a long lifetime for the gluino. This lifetime can be estimated as^{35,36}

$$\tau \sim 8 \left(\frac{M_S}{10^9 \text{ GeV}} \right)^4 \left(\frac{1 \text{ TeV}}{M_g} \right)^5 s. \quad (18)$$

From Eq. (18) it follows that the supersymmetry breaking scale in the Split SUSY models should not exceed 10^{13} GeV^{15} . Otherwise the gluino lifetime becomes larger than the age of the Universe. When M_S varies from $2 \cdot 10^9 \text{ GeV}$ ($M_g = 2500 \text{ GeV}$) to $3 \cdot 10^{10} \text{ GeV}$ ($M_g = 500 \text{ GeV}$) the gluino lifetime changes from 1 sec. to $2 \cdot 10^8 \text{ sec.}$ (1000 years). Thus the measurement of the mass and lifetime of gluino should allow one to estimate the value of M_S in the Split SUSY scenario.

3.3. Models with low SUSY breaking scale

The observed value of the dark energy density can be also reproduced when the SUSY breaking scale is around 1 TeV. This can be achieved if the MSSM particle content is supplemented by an extra pair of $5 + \bar{5}$ supermultiplets which are fundamental and antifundamental representations of the supersymmetric $SU(5)$ GUT. The additional bosons and fermions would not affect gauge coupling unification in the leading approximation, since they form complete representations of $SU(5)$. In the physical phase states from $5 + \bar{5}$ supermultiplets can gain masses around M_S . The corresponding mass terms in the superpotential can be induced because of the presence of the bilinear terms $[\eta(5 \cdot \bar{5}) + h.c.]$ in the Kähler potential of the observable sector^{37,38}. In the Split SUSY scenario we assume that new bosonic states from $5 + \bar{5}$ supermultiplets gain masses around the supersymmetry breaking scale, whereas their fermion partners acquire masses of the order of the gluino, chargino and neutralino masses. In our numerical studies we set the masses of extra quarks to be equal to the gluino mass, i.e. $M_q \simeq M_g$. In the supersymmetric Minkowski vacuum new bosons and fermions from $5 + \bar{5}$ supermultiplets remain massless. As a consequence they give a substantial contribution to the β functions in this vacuum. Indeed, the one-loop beta function of the strong interaction in the second phase changes from $b_3 = -3$ (the $SU(3)_C$ beta function in the MSSM) to $b_3 = -2$. This leads to a further reduction of Λ_{SQCD} . At the same time, extra fermion states from $5 + \bar{5}$ supermultiplets do not affect much the RG flow of gauge couplings in the physical phase below the scale M_S . For example, in the Split SUSY scenario the one-loop beta function that determines the running of the strong gauge coupling from the SUSY breaking scale down to the TeV scale changes from -5 to $-13/3$. As follows from Figs. 1 and 2 in the case of the SUSY model with extra $5 + \bar{5}$ supermultiplets the measured value of the dark energy density can be reproduced even for $M_S \simeq 1$ TeV^{10,12,13,32-34}. Nevertheless, the Split SUSY scenario which was discussed in the previous subsection has the advantage of avoiding the need for any new particles beyond those of the MSSM, provided that $M_S \simeq 10^9 - 10^{10}$ GeV. On the other hand, the MPP scenario with extra $5 + \bar{5}$ supermultiplets of matter and SUSY breaking scale in a few TeV range is easier to verify at the LHC in the near future.

3.4. The breakdown of SUSY in the hidden sector

The non-zero value of the vacuum energy density can be also induced if supersymmetry in the second phase is broken in the hidden sector. This can happen if the SM gauge couplings are sufficiently small in the supersymmetric Minkowski vacuum and by one way or another, only vector supermultiplets associated with unbroken non-Abelian gauge symmetry remain massless in the hidden sector. Then these vector supermultiplets, that survive to low energies, can give rise to the breakdown of SUSY in the second phase. Indeed, at the scale Λ_X , where the gauge interactions that correspond to the unbroken gauge symmetry in the hidden sector become

strong in the second phase, a gaugino condensate can be formed. This gaugino condensate does not break global SUSY. Nonetheless if the gauge kinetic function $f_X(z_m)$ has a non-trivial dependence on the hidden sector superfields z_m then the corresponding auxiliary fields F^m can acquire non-zero VEVs

$$F^{z_m} \propto \frac{\partial f_X(z_k)}{\partial z_m} \bar{\lambda}_a \lambda_a + \dots, \quad (19)$$

which are set by $\langle \bar{\lambda}_a \lambda_a \rangle \simeq \Lambda_X^3$. Thus it is only via the effect of a non-renormalisable term that this condensate causes the breakdown of supersymmetry. Therefore the SUSY breaking scale in the SUSY Minkowski vacuum is many orders of magnitude lower than Λ_X , while the scale Λ_X is expected to be much lower than M_{Pl} . As a result a tiny vacuum energy density is induced

$$\rho_\Lambda \sim \frac{\Lambda_X^6}{M_{Pl}^2}. \quad (20)$$

The postulated exact degeneracy of vacua implies then that the physical phase has the same energy density as the second phase where the breakdown of local SUSY takes place near Λ_X . From Eq. (20) it follows that in order to reproduce the measured cosmological constant the scale Λ_X has to be somewhat close to Λ_{QCD} in the physical vacuum, i.e.

$$\Lambda_X \sim \Lambda_{QCD}/10. \quad (21)$$

Although there is no compelling reason to expect that Λ_X and Λ_{QCD} should be related, one may naively consider Λ_{QCD} and M_{Pl} as the two most natural choices for the scale of dimensional transmutation in the hidden sector.

In the one-loop approximation one can estimate the value of the energy scale Λ_X using the simple analytical formula

$$\Lambda_X = M_{Pl} \exp \left[\frac{2\pi}{b_X \alpha_X(M_{Pl})} \right], \quad (22)$$

where $\alpha_X(M_{Pl}) = g_X^2(M_{Pl})/(4\pi)$, g_X and b_X are the gauge coupling and one-loop beta function of the gauge interactions associated with the unbroken non-Abelian gauge symmetry that survive to low energies in the hidden sector. For the $SU(3)$ and $SU(2)$ gauge groups $b_X = -9$ and -6 , respectively. In Fig. 3 we show the dependence of Λ_X on $\alpha_X(M_{Pl})$. As one might expect, the value of the energy scale Λ_X diminishes with decreasing $\alpha_X(M_{Pl})$. The observed value of the dark energy density is reproduced when $\alpha_X(M_{Pl}) \simeq 0.051$ in the case of the SUSY model based on the $SU(2)$ gauge group and $\alpha_X(M_{Pl}) \simeq 0.034$ in the case of the $SU(3)$ SUSY gluodynamics. It is worth noting that in the case of the $SU(3)$ SUSY model the value of the gauge coupling $g_X(M_{Pl}) \simeq 0.654$, that leads to $\alpha_X(M_{Pl}) \simeq 0.034$, is just slightly larger than the value of the QCD gauge coupling at the Planck scale in the SM, i.e. $g_3(M_{Pl}) = 0.487^8$.

In this scenario SUSY can be broken at any scale in the physical vacuum. In particular, the breakdown of local supersymmetry can take place near the Planck

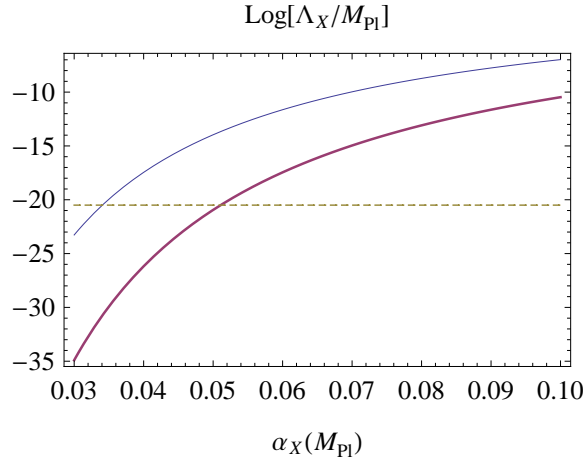


Fig. 3. The value of $\log[\Lambda_X/M_{Pl}]$ as a function of $\alpha_X(M_{Pl})$. The thin and thick solid lines are associated with the $SU(3)$ and $SU(2)$ gauge symmetries, respectively. The horizontal line corresponds to the value of Λ_X that results in the measured cosmological constant.

scale. If this is the case, one can explain the small values of $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ by postulating the existence of a third degenerate vacuum. In this third vacuum local SUSY can be broken near the Planck scale while the EW symmetry breaking scale can be just a few orders of magnitude lower than M_{Pl} . Since now the Higgs VEV is somewhat close to M_{Pl} one must take into account the interaction of the Higgs and hidden sector fields. Thus the full scalar potential takes the form

$$V = V_{hid}(z_m) + V_0(H) + V_{int}(H, z_m) + \dots, \quad (23)$$

where $V_{hid}(z_m)$ is the part of the full scalar potential associated with the hidden sector, $V_0(H)$ is the part of the scalar potential that depends on the SM Higgs field only and $V_{int}(H, z_m)$ describes the interactions of the SM Higgs doublet with the fields of the hidden sector. Although in general $V_{int}(H, z_m)$ should not be ignored the interactions between H and hidden sector fields can be quite weak if the VEV of the Higgs field is substantially smaller than M_{Pl} (say $\langle H \rangle \lesssim M_{Pl}/10$) and the couplings of the SM Higgs doublet to the hidden sector fields are suppressed. Then the VEVs of the hidden sector fields in the physical and third vacua can be almost identical. As a consequence, the gauge couplings and $\lambda(M_{Pl})$ in the first and third phases should be basically the same and the value of $|m^2|$ in the Higgs effective potential can be still much smaller than M_{Pl}^2 and $\langle H^\dagger H \rangle$ in the third vacuum. In this limit $V_{hid}(z_m^{(3)}) \ll M_{Pl}^4$ and the requirement of the existence of the third vacuum with vanishingly small energy density again implies that $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ are approximately zero in the third vacuum. Because in this case the couplings in the third and physical phases are basically identical, the presence of such a third vacuum should result in the predictions (2) for $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ in the physical

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vacuum.

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