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Dark energy from backreaction

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Abstract.

We consider the effect of inhomogeneities on the expansion of the Einstein-de Sitter universe. We find that the backreaction of linear scalar metric perturbations results in apparent dark energy with a mixture of equations of state between 0 and $-4/3$. We discuss the possibility that backreaction could account for present-day acceleration.

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1. Introduction

The concordance model. Perhaps the most surprising observation in recent cosmology is that the expansion of the universe seems to be accelerating. This all the more puzzling since the acceleration has apparently started in the recent past, at a redshift of probably less than one. These conclusions are based on data from the cosmic microwave background, large scale structure and supernovae [1, 2, 3, 4, 5, 6]. Though the only significant direct evidence for acceleration comes from the supernova observations, data from different sources seem to fit together. There is also some evidence for acceleration from correlation of the CMB with large scale structure [7, 8, 9].

The preferred framework for interpreting the observations is the ‘concordance model’ in which the universe is spatially flat, and cold dark matter and baryons contribute $\Omega_{\text{m}} \approx 1/3$ to the energy density and vacuum energy contributes the rest, $\Omega_{\Lambda} \approx 2/3$.

Most alternatives to the ‘concordance model’ replace the cosmological constant with some more complicated component with negative pressure, called ‘dark energy’. Indeed, the relation $\Omega_{\text{de}} \approx 2\Omega_{\text{cdm}}$ suggests a connection between dark energy and dark matter, and motivates the construction of models of unified, or coupled, dark matter and dark energy [10, 11, 12, 13, 14, 15, 16].

However, the conclusion that some component with negative pressure is needed at all is prior-dependent [17, 18]: the possibility that a model with no exotic ingredients could fit the data as well is not excluded. As a notable example, a model with $\Omega_{\text{m}} = 0.88$ and neutrino energy density $\Omega_{\nu} = 0.12$ can fit the CMB and large scale structure data even better than the ‘concordance model’, though it does not fit the data from

supernovae [18]. The neutrino component is needed, since in spatially flat models with $\Omega_m = 1$ the density perturbation amplitude σ_8 seems to be generically too high.

There are two main motivations for looking for alternatives. First, the ‘concordance model’ does not fit all of the data very well. In particular, the prediction for the amplitudes of the quadrupole and octopole of the CMB is too high[‡]. The probability of getting the observed CMB amplitudes depends on the method of evaluation, but it seems that in about 95% of its realisations, the ‘concordance model’ does not reproduce the observed CMB spectrum [22, 23]. Another phrasing is that the model is ruled out at about 2σ level. (In [21] the discrepancy was evaluated to be much lower, about 70%, or 1σ .) The second motivation is not observational but theoretical.

The coincidence problem. The most unattractive feature of the ‘concordance model’ is the coincidence problem: why has the acceleration started in the recent past? Or, to phrase it differently, why has the energy density of dark energy become comparable to the energy density of matter only recently? There are three possible answers to this question.

The first possibility is that this is just a coincidence. If the theory of quantum gravity determines the unique value of vacuum energy, perhaps this simply happens to be of the order of the matter energy density today. Since $(\rho_m)^{1/4} \approx 10^{-3}\text{eV} \approx (\text{TeV})^2/M$, where M is the (reduced) Planck mass, this may not be unreasonable [24]. This is also the scale of neutrino mass splittings, so there might be a relation [25].

The second possibility is that there is an anthropic reason. If various vacua with different vacuum energies are realised in different parts of the universe, then the value of vacuum energy in the part of the universe we observe is naturally so low as not to prevent the formation of galaxies [26]. Also, it will naturally not be so negative as to cause the universe to collapse very early. This argumentation provides only a window of values, and one then needs to have some principle or a specific model to end up with the value apparently observed today, or at least to narrow the window sufficiently [27].

The third possibility is that there is a dynamical reason for the acceleration to have started recently. If so, then this is presumably related to the dynamics observed in the recent universe (in principle the dark energy component can of course have its own dynamics, only weakly related to those of the visible universe). The important events in recent cosmic history, meaning within the latest few thousand redshifts, are transition from radiation to matter domination at around $z = 3500$, radiation-matter decoupling at around $z = 1088$ and the growth of structure and related phenomena at around $z \sim 10$ and below.

Models where the dark energy component is sensitive to the transition from radiation to matter domination have been constructed [28]. In these models the contribution of the dark energy tracks the radiation density during the radiation dominated era, and starts to rise after the transition to the matter dominated era.

[‡] The low multipoles are susceptible to contamination from the Galaxy, but this is thought to be under control [19, 20, 21, 22].

However, one still generically has to explain why the dark energy component has started to dominate at a redshift of at most a few, and not earlier or later. Since the matter-radiation equality and the start of the acceleration are far away both in time and in redshift, one could naturally have the dark energy dominate much earlier or much later.

As for the radiation-matter decoupling, it is nearer to the start of the acceleration than radiation-matter equality, but does not seem provide a promising trigger mechanism.

Structure formation occurs around the same redshift as the acceleration starts, and so the possibility that the acceleration is related to the growth of inhomogeneities in the universe seems natural. One way to implement this is to use the growth of inhomogeneities as a trigger for a dark energy component [13]. However, one can also look at the effect of the growth of structure itself, rather than introducing new fundamental physics that is sensitive to structure formation (outside the fitting problem to be discussed, this has been suggested in [11]).

The fitting problem. The cosmological observations leading to the conclusion that there is a dark energy component have been interpreted in the context of a homogeneous and isotropic model for the universe. The reasoning is that since the universe appears to be homogeneous and isotropic on large scales[§], taking the metric and the energy-momentum tensor to be isotropic and homogeneous should be a good approximation.

In the usual approach, one *first* takes the average of the metric and the energy-momentum tensor, and then plugs these averaged quantities into the Einstein equation. Observables such as the expansion rate are then calculated from this equation. Physically, the correct thing to do is to plug the full inhomogeneous metric and energy-momentum tensor into the Einstein equation, and *then* take the average. Also, observables should be expressed directly in terms of the inhomogeneous metric and sources and then averaged.

Since the Einstein equation is non-linear, the equations for the quantities which have been averaged before plugging them in (that is, the usual Friedmann-Robertson-Walker equations) will in general not be the same as the average of the equations for the inhomogeneous quantities. We may equivalently say that the averaged quantities do not satisfy the Einstein equation. This is the fitting problem discussed in [30]. When one fits the parameters of a Friedmann-Robertson-Walker model to observational data, is one fitting the right model? The difference between the equations for the quantities averaged beforehand and the average of the equations for the real inhomogeneous quantities is also known as backreaction.

The fitting problem has been approached from two directions. One may try to solve the full problem to obtain the equations satisfied by the averaged quantities, without assuming a given background [31, 32, 33, 34, 35, 36]. A more modest approach is to

[§] Though it has been argued that the homogeneity has in fact not been established, on the basis that the standard statistical tools used to measure deviation from homogeneity assume homogeneity on large scales [29].

assume a homogeneous and isotropic background and study the effect of perturbations on this background [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53]. (A brief overview of some of the averaging procedures that have been used is given in [54].) Sometimes the term backreaction is used to refer only to the second, more limited, approach.

The mathematical problem of obtaining the average metric and the equations satisfied by the average quantities has not been solved, though some progress has been made [33, 34]. The issue of the metric is particularly complicated, since in order to integrate a tensor one has to parallel transport its components with respect to some background, but this background is precisely what one is trying to determine. From a physical point of view one might expect that when deviations from homogeneity and isotropy are small, the homogeneous and isotropic metric should be a good description. However, the study of backreaction on inflationary backgrounds has shown that even this is not necessarily true, since the large number of perturbative modes can compensate for their small amplitude [40, 41, 46, 47, 48, 49, 50, 51, 52, 53].

We will take the more modest approach. We assume that there is a given homogeneous and isotropic background with perturbations on it and that both satisfy the Einstein equation. We will then study the effect of these perturbations on the local expansion rate and see how it differs from the background expansion rate.

The most straightforward way to consider the impact of perturbations is to expand the equations in a perturbative series and solve them in a consistent manner. For cosmological perturbations this is an involved task. The second order solutions that are known [52, 55, 56] have been built order by order, assuming that the equations for higher order terms have no impact on the equations for lower order terms, so they are not fully consistent second order calculations.

We will not solve the second order equations, but will simply assume the background and the perturbations to be given by first order perturbation theory. Obviously, if the impact of the perturbations on the background turns out to be large, this is no longer a good approximation, and a consistent second order calculation would be needed.

In section 2 we calculate the local expansion rate for an observer in a perturbed FRW universe. We evaluate this for the Einstein-de Sitter case and take the average. We find a non-vanishing correction to the expansion rate from the perturbations. In section 3 we discuss the relation to dark energy and summarise our results.

2. The backreaction calculation

2.1. The local expansion rate

The metric and the Einstein equation. We are interested in the expansion rate measured by a comoving observer. We will expand this observable in terms of the perturbations around homogeneity and isotropy, and take the average. Our approach closely follows that of [50].

We take the homogeneous and isotropic background spacetime to be spatially flat. We take the source to be a single fluid with no anisotropic stress, and we will not consider vector or tensor perturbations. To first order in perturbations, the metric can then be written as [57]

$$ds^2 = -(1 + 2\Phi(t, \mathbf{x}))dt^2 + (1 - 2\Phi(t, \mathbf{x}))a(t)^2d\mathbf{x}^2 . \quad (1)$$

The perturbation Φ coincides with a gauge-invariant quantity in first order perturbation theory, and is identified as the gravitational potential in the Newtonian limit. We choose the background scale factor to be normalised to unity today, $a(t_0) = 1$. An overdot will be used to denote derivative with respect to the time t . Note that because of the perturbations, t is not the proper time measured by a comoving observer, and one has to be careful to recast the time-dependence of observables in terms of the proper time, as emphasised in [50].

The Einstein equation reads

$$G_{\mu\nu} = \frac{1}{M^2}T_{\mu\nu} = \frac{1}{M^2}((\rho + p)u_\mu u_\nu + pg_{\mu\nu}) , \quad (2)$$

where $M = 1/\sqrt{8\pi G_{\text{N}}}$ is the (reduced) Planck mass, ρ and p are the energy density and pressure of matter, respectively, and u^μ is the velocity of the matter fluid, with $u_\mu u^\mu = -1$.

The expansion rate. The observable of interest, the expansion rate measured by an observer comoving with the matter fluid, is given by

$$\theta(t, \mathbf{x}) = u^\mu{}_{;\mu} , \quad (3)$$

where $;$ stands for the covariant derivative.

In order to evaluate θ for a typical comoving observer, we will expand to second order in Φ and take the average. For this purpose, let us look at the $0i$ -component of (2):

$$G^{0i} = \frac{1}{M^2}(\rho + p)u^0 u^i = \frac{1}{3}(4G_{\mu\nu}u^\mu u^\nu + G_{\mu\nu}g^{\mu\nu})u^0 u^i , \quad (4)$$

where we have used the Einstein equation (2) again. Writing $G_{\mu\nu}$ in terms of the metric, we have an iterative equation from which u^μ can be solved to any desired order in Φ . Given the initial condition that for the background spacetime $u^\mu = (1, \mathbf{0})$, we get to second order

$$\begin{aligned} u^0 &\simeq 1 - \Phi + \frac{3}{2}\Phi^2 + \frac{1}{2a^2\dot{H}^2}\partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) \\ u^i &\simeq \frac{1}{a^2\dot{H}}\partial_i(\dot{\Phi} + H\Phi) + \frac{1}{a^2\dot{H}}(\dot{\Phi}\partial_i\Phi + 5\Phi\partial_i\dot{\Phi} + H\Phi\partial_i\Phi) \\ &\quad + \frac{1}{a^2\dot{H}^2}\partial_i(\dot{\Phi} + H\Phi)\left(\ddot{\Phi} + H\dot{\Phi} + \frac{1}{a^2}\nabla^2\Phi\right) , \end{aligned} \quad (5)$$

where $H = \dot{a}/a$ is the background expansion rate.

It is noteworthy that u^μ contains terms with more than two derivatives (specifically spatial derivatives). This perhaps surprising feature can be understood from (4) as

follows. Let us assume that u^i contains exactly one spatial derivative (a non-zero u^i must have at least one spatial derivative to give the index). Then the right-hand side of (4) contains products of G_{ij} and three powers of u^i , which in general contain five spatial derivatives (and do not cancel). But this makes the equation inconsistent, since the left-hand side only contains terms with at most two derivatives. Obviously, the conclusion also holds for two, or any other finite number of, spatial derivatives in u^i : the only possibilities for the number of derivatives in u^μ that are consistent with (4) are zero and infinite. So, the number of derivatives in an iterative solution for u^μ depends on the power of Φ at which the iteration is stopped. Since the derivative terms arise by combining components of $G_{\mu\nu}$ algebraically, there are no terms with more than two derivatives, either spatial or temporal, acting on a given Φ . Therefore the maximum number of derivatives at order Φ^N is $2N$.

Plugging the expression (5) for u^μ into (3) we have, to second order,

$$\theta \simeq 3H - 3(\dot{\Phi} + H\Phi) - 3\Phi\dot{\Phi} + \frac{9}{2}H\Phi^2 + \frac{3}{2}\frac{H}{a^2\dot{H}^2}\partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) - 2\frac{1}{a^2\dot{H}}\partial_i(\dot{\Phi} + H\Phi)\partial_i\Phi + \partial_t\left(\frac{1}{2}\frac{1}{a^2\dot{H}^2}\partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi)\right) + \partial_i u^i, \quad (6)$$

where the total gradient $\partial_i u^i$ has not been written explicitly. Since u^μ contains (at second order) terms with four derivatives, θ contains terms with five derivatives. The next to last term has two spatial and three temporal derivatives and the last term, $\partial_i u^i$, has a contribution with two spatial and three temporal derivatives and a contribution with four spatial derivatives and one temporal derivative.

The proper time. In order to find the physical expansion rate, we should recast (6) in terms of the proper time τ of a comoving observer. The derivative in the direction orthogonal to the hypersurface defined by the velocity u^μ is $\partial_\tau = u^\mu\partial_\mu$. From the condition $\partial_\tau\tau = 1$ we obtain, using (5), an iterative equation for τ . Given the initial condition that for the background spacetime $\tau = t$, we get to second order

$$\tau \simeq t + \int^t dt' \left(\Phi - \frac{1}{2}\Phi^2 - \frac{1}{2}\frac{1}{a^2\dot{H}^2}\partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) - \frac{1}{a^2\dot{H}}\partial_i(\dot{\Phi} + H\Phi)\int^{t'} dt''\partial_i\Phi \right). \quad (7)$$

If we neglected the gradient terms, we would get (to all orders in Φ) $\tau = \int dt\sqrt{|g_{00}|}$, in agreement with [50], where gradients were dropped.

From (5), (6) and (7) we can calculate the expansion rate in terms of the proper time. For general functions a and Φ the expression is cumbersome and not very illuminating. However, for the Einstein-de Sitter universe things simplify considerably.

The Einstein-de Sitter universe. As discussed earlier, we take a and Φ from the first order formalism and calculate θ to second order with these expressions. If the effect of

the perturbations on θ is large, then this approach has reached its limit of validity, and a consistent second order calculation would be needed.

The matter is taken to be pure cold dark matter, $\Omega_{\text{cdm}} = 1$. When relevant, we will mention what effect a realistic baryon content of $\Omega_{\text{b}} = 0.05$ [5, 58] would have; at our level of approximation, the difference is minimal. The perturbations are taken to be purely adiabatic, Gaussian (with zero mean), and to have a scale-invariant spectrum, $n = 1$. We will also consider only the growing mode of the perturbations.

The background expansion rate is given by the FRW solution for pressureless matter, so $a = (t/t_0)^{2/3}$ and $H = 2/(3t)$. We take the value of H today to be $H_0 = h$ 100 km/s/Mpc, with $h = 0.7$; our results are not sensitive to the precise value of h . For $a = (t/t_0)^{2/3}$, the local expansion rate is, from (5), (6) and (7),

$$\begin{aligned} \theta \simeq & 3H_\tau - 3(\dot{\Phi} + H\Phi) + 3H\frac{1}{t} \int dt\Phi - 3\Phi\dot{\Phi} + \frac{9}{2}H\Phi^2 - 3H \left(\frac{1}{t} \int dt\Phi \right)^2 \\ & - \frac{3}{2}H\frac{1}{t} \int dt\Phi^2 + \frac{2}{3}\frac{1}{a^2H^3} \partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) \\ & + \frac{4}{3}\frac{1}{(aH)^2} \partial_i(\dot{\Phi} + H\Phi)\partial_i\Phi + \frac{2}{9}\partial_t \left(\frac{1}{(aH)^2H^2} \partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) \right) \\ & - \frac{2}{3}H\frac{1}{t} \int dt \frac{1}{(aH)^2H^2} \partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) \\ & + 2H\frac{1}{t} \int^t dt' \left(\frac{1}{(aH)^2} \partial_i(\dot{\Phi} + H\Phi) \int^{t'} dt'' \partial_i\Phi \right) + \partial_i u^i, \end{aligned} \quad (8)$$

where we have defined $H_\tau = 2/(3\tau)$, and $\partial_i u^i$ has again not been written explicitly. The terms after $3H_\tau$ are the backreaction contribution.

2.2. The average expansion rate

Taking the average. To evaluate the backreaction, we should take the average of (8) over the hypersurface of constant τ . The backreaction has been expressed in terms of the background coordinates t and x^i . We have to rewrite it in terms of the proper time τ and spatial coordinates orthogonal to τ , denoted by y^i . We also have to take into account the integration measure on the hypersurface of constant τ . After a somewhat lengthy calculation, we get

$$\begin{aligned} \langle \theta \rangle \simeq & 3H_\tau - \partial_\tau \langle 3\Phi + 2\phi + \frac{2}{3} \frac{1}{(a_\tau H_\tau)^2} \nabla^2 \Phi \rangle_0 \\ & - \frac{1}{2} \partial_t \left(\langle 3\Phi + 2\phi + \frac{2}{3} \frac{1}{(aH)^2} \nabla^2 \Phi \rangle_0 \right)^2 \\ & + 2\frac{1}{H} \partial_t^2 \langle \phi\Phi \rangle_0 + 14\partial_t \langle \phi\Phi \rangle_0 + \partial_t \langle \phi^2 \rangle_0 - \frac{3}{2} \partial_t \langle \Phi^2 \rangle_0 \\ & + \frac{3}{2} H \frac{1}{t} \int dt t \partial_t \langle \Phi^2 \rangle_0 - \frac{4}{9} \frac{1}{H} \partial_t^2 \left(\frac{1}{(aH)^2} \langle \partial_i \phi \partial_i \Phi \rangle_0 \right) \\ & - \frac{24}{9} \partial_t \left(\frac{1}{(aH)^2} \langle \partial_i \phi \partial_i \Phi \rangle_0 \right) + \frac{2}{3} \partial_t \left(\frac{1}{(aH)^2} \langle \partial_i \Phi \partial_i \Phi \rangle_0 \right) \end{aligned}$$

$$\begin{aligned}
& +\frac{2}{9}\partial_t \left(\frac{1}{(aH)^2 H^2} < \partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) >_0 \right) \\
& +2H\frac{1}{t} \int dt t \partial_t \left(\frac{1}{(aH)^2} < \partial_i\Phi\partial_i\Phi >_0 \right) \\
& +\frac{2}{3}H\frac{1}{t} \int dt t \partial_t \left(\frac{1}{(aH)^2 H^2} < \partial_i(\dot{\Phi} + H\Phi)\partial_i(\dot{\Phi} + H\Phi) >_0 \right) \\
& +\frac{2}{9}\frac{1}{(aH)^2} < \partial_i \left[8\dot{\Phi}\partial_i\Phi - 6\Phi\partial_i\dot{\Phi} + 15H\Phi\partial_i\Phi + 2\frac{1}{H}\dot{\Phi}\partial_i\dot{\Phi} \right. \\
& \left. +2\frac{1}{H^2}\ddot{\Phi}\partial_i(\dot{\Phi} + H\Phi) + 2\frac{1}{H}\phi\partial_i\ddot{\Phi} + 10\phi\partial_i\dot{\Phi} - 4H\phi\partial_i\Phi \right] >_0 \\
& \left. +\frac{4}{9}\partial_t \left(\frac{1}{(aH)^4} < \partial_i [\nabla^2\Phi\partial_i\Phi] >_0 \right) , \tag{9}
\end{aligned}$$

where we have defined $\phi = t^{-1} \int dt \Phi$ and $a_\tau = (\tau/t_0)^{2/3}$. (Note that when $\dot{\Phi} = 0$, we have $\phi = \Phi$.) Here the arguments of Φ and its derivatives are understood to be (τ, \mathbf{y}) rather than (t, \mathbf{x}) , and $< A >_0$ denotes $(\int d^3y)^{-1} \int d^3y A(\tau, \mathbf{y})$. Since the calculation is only to second order, we have substituted t for τ in the terms with two powers of Φ .

The backreaction terms can be divided into three groups: linear terms, quadratic terms which are not total gradients and quadratic terms which are total gradients (and therefore reduce to boundary terms). It is noteworthy that all terms apart from the first total gradient term are total time derivatives of dimensionless expectation values: for example, there are no terms of the form $H(aH)^{-2} < \partial_i\Phi\partial_i\Phi >_0$.

The linear terms. It might seem that the average of the linear backreaction terms must be zero since the perturbations are assumed to be Gaussian (with zero mean). However, we have not specified the hypersurface with respect to which this holds. Are the perturbations Gaussian with respect to the (unphysical) hypersurface of constant t of the background spacetime, or with respect to the (physically meaningful) perturbed hypersurface of constant τ ? Obviously, if the perturbations are distributed according to Gaussian statistics on the background spacetime, they are not Gaussian with respect to the perturbed spacetime, and vice versa.

The situation is ambiguous because this question only arises at second order, and we are plugging in perturbations from first order perturbation theory. The issue would probably have to be resolved by a consistent second order calculation. Note that this non-Gaussianity related to the choice of hypersurface is distinct from the intrinsic non-Gaussianity of second order perturbations found in [55, 56].

It seems more physically meaningful to take the perturbations to be Gaussian with respect to the hypersurface of constant τ . However, we keep in the simple approximation of using first order results for the perturbations, and take them to be Gaussian with respect to the background. Then the linear terms, and the squares of linear terms, in (9) vanish. Under another assumption about the statistics, this may not be true.

The quadratic non-total gradient terms. In the Einstein-de Sitter universe, the non-decaying mode of Φ is constant in time, $\dot{\Phi} = 0$, in first order perturbation theory. The expansion rate (8) then simplifies to

$$\theta \simeq 3H_\tau + \frac{118}{45} \frac{1}{a^2 H} \partial_i \Phi \partial_i \Phi + \partial_i u^i. \quad (10)$$

It is noteworthy that all non-gradient correction terms have disappeared: only gradients contribute to the backreaction, in agreement with [50]. The average expansion rate (9) simplifies to (neglecting the linear terms and their squares),

$$\begin{aligned} \langle \theta \rangle \simeq 3H_\tau \left(1 - \frac{22}{135} \frac{1}{(aH)^2} \langle \partial_i \Phi \partial_i \Phi \rangle_0 + \frac{22}{27} \frac{1}{(aH)^2} \langle \partial_i (\Phi \partial_i \Phi) \rangle_0 \right. \\ \left. + \frac{8}{27} \frac{1}{(aH)^4} \langle \partial_i (\nabla^2 \Phi \partial_i \Phi) \rangle_0 \right). \end{aligned} \quad (11)$$

Postponing discussion of the total gradient terms, let us evaluate the first correction term:

$$\begin{aligned} \frac{1}{(aH)^2} \langle \partial_i \Phi \partial_i \Phi \rangle_0 &= \frac{1}{(aH)^2} \int_0^\infty \frac{dk}{k} k^2 \Delta_\Phi^2(k) \\ &\approx \frac{9}{4} \frac{(aH)^4}{(aH)^2} \int_0^\infty \frac{dk}{k} k^{-2} \Delta_\delta^2(k, a) \\ &= \frac{9}{4} \frac{1}{(aH)^2} \int_0^\infty \frac{dk}{k} A^2 k^2 T(k)^2 \\ &\approx 8 \cdot 10^{-5} a, \end{aligned} \quad (12)$$

where Δ_Φ^2 and Δ_δ^2 are the power spectra of the metric and density perturbations, respectively, and we have used $\Phi_{\mathbf{k}} \approx -3(aH)^2 \delta_{\mathbf{k}} / (2k^2)$ (the low- k part of the spectrum where this is not a good approximation gives negligible contribution). The density power spectrum is taken to be $\Delta_\delta^2(k, a) = A^2 k^4 T(k)^2 / (aH)^4$, where $A = 1.9 \cdot 10^{-5}$ and $T(k)$ is the CDM transfer function, for which we use the BBKS fitting formula [59]. Taking into account a baryon contribution of $\Omega_b = 0.05$ with the shape parameter $\Gamma = h \exp(-\Omega_b)$ [60] would only change the prefactor from 8 to 7 in (12).

The magnitude of the effect can be understood as follows. Due to the transfer function, the main contribution comes from around k_{eq} , so a rough estimate is $H_0^{-2} \langle k^2 \Phi^2 \rangle \sim (k_{\text{eq}}/H_0)^2 \langle \Phi^2 \rangle \sim (150)^2 (2 \cdot 10^{-5})^2 \sim 10^{-5}$. Note that the backreaction is enhanced by the large factor $(k_{\text{eq}}/H_0)^2$ and so could be large even with Φ much below unity.

Taking into account the numerical factor from (11), we get $-1 \cdot 10^{-5}$ for the relative correction. The negative sign may seem surprising, since from (10) it might appear that the backreaction definitely increases the expansion rate. This is true when evaluated over the background surface of constant t , but taking into account the perturbations in the hypersurface changes the sign. To appreciate the importance of the choice of hypersurface, it may be helpful to note that evaluating the expansion rate $3H_\tau$ over the background hypersurface of constant t gives an apparent backreaction contribution. The choice of hypersurface has been discussed in [50, 53].

The backreaction seems to increase proportional to the scale factor a , and would seem to be more important in the future, eventually dominating the expansion rate. However, this is not true, because the linear regime of perturbations does not extend to infinitely small scales, due to the process of structure formation. We will take this into account by introducing a time-dependent cut-off, denoted by k_{\perp} , at the scale at which the mean square of the density perturbations becomes unity, $\sigma^2 = 1$ ^{||}. The end of the linear regime of density perturbations is today at around $k_{\perp} = 0.1 \text{ Mpc}^{-1}$. Putting a cut-off in the integral (12) at this k_{\perp} only changes the prefactor from 8 to 2 (at this level of accuracy, the baryons make no difference). However, since we are cutting the perturbations off at some time-dependent scale in momentum space, the perturbations in position space are no longer time-independent. Therefore, the simple result (11) applies only before the formation of bound structures, that is, before $\sigma^2 = 1$ on any scale.

Note that we are applying the cut-off to the perturbations themselves, not to the integration range. This seems to be more physically correct for the high- k cut-off (though not for the horizon cut-off, were we to introduce one). If we applied the cut-off to the integration range instead, we would have $\dot{\Phi} = 0$ and the simple result (11) would still hold[¶]. The treatment of the onset of non-linearity will be discussed in more detail in section 3.

For a realistic transfer function, it is not possible to calculate the full backreaction (9) analytically in the presence of a time-dependent cut-off in momentum space. However, we can determine the time behaviour of the backreaction by looking at its asymptotic behaviour.

The asymptotic future value of (9) can be found by a simple dimensional argument. There are three scales in the problem, aH , k_{\perp} and (from the transfer function) k_{eq} ; the scale factor a enters only via these scales. Therefore the non-gradient expectation values such as $\langle \Phi^2 \rangle_0$ must be some function of $k_{\perp}/(aH)$ and $k_{\text{eq}}/(aH)$, and the gradient expectation values such as $\langle \partial_i \Phi \partial_i \Phi \rangle_0$ must be $(aH)^2$ (or $(aH)^2 H^2$ in the case of expectation values involving $\dot{\Phi} + H\Phi$) times some function of the same two variables.

In the future, structure formation will have proceeded so that all scales smaller than $1/k_{\text{eq}} \approx 30 \text{ Mpc}$ will have gone non-linear. The CDM transfer function $T(k)$ is essentially unity for scales much smaller than k_{eq} , and for $T(k) = 1$ the condition $\sigma^2 = 1$ gives $k_{\perp} = \sqrt{2/A}(aH) \propto aH^+$. The scale k_{eq} has disappeared, and k_{\perp} is proportional to aH . It follows that the non-gradient expectation values are constant, and the gradient ones are simply constants times $(aH)^2$ (or $(aH)^2 H^2$, as appropriate). The terms we are considering contribute to the average expansion rate (9) only via total time derivatives of dimensionless expectation values. Therefore, the backreaction vanishes.

^{||} We should also introduce a lower cut-off at the horizon scale $(aH)/2$, to take into account that the average is properly over the horizon volume and not over all space. However, for the terms considered here, the effect of this cut-off is negligible.

[¶] Note that in (9) the integration range has been assumed not to depend on time.

⁺ For simplicity, we are using a top hat in momentum space as the window function.

So, the backreaction from the quadratic terms which are not total gradients behaves as follows. Before $\sigma^2 = 1$ on any scale, the backreaction is given by (12). It slows down the expansion rate, its relative contribution rises with time and is of the order $10^{-5}a$. When the limit $\sigma^2 = 1$ is reached, two things happen. First, the integral (12) receives a cut-off at k_{\perp} , which decreases as a increases and larger scales become non-linear. Second, time derivatives of the perturbations start contributing. The net effect is to reduce the magnitude of the backreaction, so that it is asymptotically zero in the future.

It should be noted that for the scale-invariant spectrum that we have considered, the limit $\sigma^2 = 1$ is actually reached at all times on sufficiently small scales. Even taking into account collisional damping and free streaming, the limit is reached quite early for a pure adiabatic spectrum of CDM perturbations. In [61], structure formation was estimated to start at $z \approx 40 - 60$ in the case of supersymmetric CDM. For light dark matter, the power spectrum would be more damped, and the limit would be reached later [62].

The conclusion of asymptotically vanishing backreaction depends on the absence of a scale other than aH : if there is another scale present, the backreaction will in general not vanish. In particular, the backreaction will be non-zero if the power spectrum of density perturbations is not scale-invariant. If the spectrum is given by a power-law with a constant spectral index n , the integrand in (12) will be multiplied by $(k/k_P)^{n-1}$, where k_P is some constant scale. As discussed above, in the future the transfer function will be unity for all modes in the linear regime. Then the integral (12) will be proportional to $k_{\perp}^{n-1} \propto (aH)^{n-1}$. Therefore, the total time derivatives in (9) will not vanish, and the relative backreaction will be proportional to $(aH)^{n-1} \propto a^{(1-n)/2}$. For a red spectrum, $n < 1$, the backreaction will grow and eventually dominate the expansion of the universe. The impact on the expansion rate will be discussed in more detail in section 3.

The quadratic total gradient terms. The backreaction terms previously considered could be readily evaluated using the standard methods of cosmological perturbation theory. The remaining total gradient terms are more problematic. In the standard treatment, cosmological perturbations are assumed to be periodic on some large scale, so that one can decompose them as a Fourier series. It is assumed that the periodicity has no impact on observables as long as the periodicity scale is large enough. However, the average of a total gradient yields a boundary term which is of course sensitive to boundary conditions: the periodicity forces the average of a total gradient to vanish. (The conditions imposed by the existence of a Fourier transform are less straightforward, but they also imply the vanishing of averages of total gradients.) There is no obvious physical reason for the quadratic total gradient terms in (9) to vanish, so this seems to be just a mathematical artifact.

As noted earlier, we should properly take the average over only the present horizon, while the periodicity scale should be taken to be much larger than the horizon. Therefore, the proper way to evaluate the total gradient terms in the standard treatment of cosmological perturbations is to take a box much bigger than the horizon and consider

the effect of these terms on horizon-sized subsamples of space. If the hypothesis that the periodicity has no impact on observables (for a sufficiently large box) is correct, then the result should be the same that we would get by evaluating the total gradient terms with realistic boundary conditions, not imposing periodicity. Note that the time development of the backreaction terms (and therefore the apparent equation of state, to be discussed in section 3), is fixed by the time-development of the perturbations, so that only the magnitude remains to be determined.

Such a calculation has been done in a different, Newtonian, backreaction formalism, where backreaction vanishes completely for periodic boundary conditions [34]. The box was taken to be of about the horizon size, and it was found that the backreaction can have a substantial impact on the expansion rate on scales smaller than about 50 Mpc, either increasing or decreasing it.

We will not here embark on such a numerical computation, but will naively estimate the magnitude of the total gradient terms. The total gradient terms in (9) have the form $\langle \partial_i(f\partial_i g) \rangle_0 = \langle \partial_i f \partial_i g \rangle_0 + \langle f \nabla^2 g \rangle_0$. Evaluated over all space, the two parts cancel (in the standard treatment of perturbations). Evaluated over a volume smaller than the box size, the cancellation will not be perfect (or the terms may even have the same sign). We will therefore crudely estimate the magnitude of the backreaction by looking at the parts evaluated individually over all space.

The first total gradient term in (9) contains two spatial derivatives and is thus similar to the terms previously evaluated. When this term does not average to zero, it is expected to modify the numerical coefficients of the previously considered gradient terms by factors of at most order one, possibly changing their sign. It could also make the asymptotic value of the backreaction different from zero.

The second total gradient term in (9) is qualitatively different from any others in that it contains four spatial derivatives. Since

$$\frac{1}{(aH)^4} \langle \nabla^2 \Phi \nabla^2 \Phi \rangle_0 = \frac{9}{4} \langle \delta^2 \rangle_0 \equiv \frac{9}{4} \sigma^2, \quad (13)$$

the first part of the backreaction term is simply $-\partial_t \sigma^2$ and the second is obviously $\partial_t \sigma^2$. Before the limit $\sigma^2 = 1$ is reached on any scale, the first part of the backreaction term (relative to $3H_\tau$) is given by $-\frac{2}{3} \sigma^2$ and the second by $\frac{2}{3} \sigma^2$, since $\sigma^2 \propto a^2$. After the limit $\sigma^2 = 1$ is reached, we should introduce a cut-off at k_\perp , meaning that σ^2 saturates at unity and both parts of the backreaction vanish independently, $\partial_t \sigma^2 = 0$.

So, the contribution of the total gradient term with four spatial derivatives grows like a^2 until the limit $\sigma^2 = 1$ is reached, at which point it drops to zero. (The expansion rate thus jumps discontinuously when σ^2 reaches unity, due to the unrealistic sharp cut-off at k_\perp .) The average over all space is always zero, and so the backreaction before the start of the formation of bound objects will lead to slower expansion in some regions and to faster expansion in others. A reasonable estimate of the relative contribution to the expansion rate might be of the order $\frac{2}{3} \sigma^2$ or so, though in some regions it will be much smaller or larger. The probability distribution of magnitudes should be properly evaluated, for example by numerical computation. Also, the conclusions here

are sensitive to the sharp cut-off at k_{L} . For example, clearly the physical expansion rate will not jump discontinuously when the limit $\sigma^2 = 1$ is reached, so the backreaction could be large for at least some time afterwards.

3. Discussion

The effective dark energy. To appreciate the impact of backreaction, let us look at the Hubble law. Let us parametrise the quadratic backreaction terms in (9) as follows:

$$\begin{aligned} \left(\frac{1}{3} < \theta >\right)^2 &\simeq H_{\tau}^2(1 + \lambda_1 a^{m_1} + \lambda_2 a^{m_2})^2 \\ &= H_{\tau}^2 + H_{\tau}^2(2\lambda_1 a^{m_1} + 2\lambda_2 a^{m_2} + 2\lambda_1 \lambda_2 a^{m_1+m_2} + \lambda_1^2 a^{2m_1} + \lambda_2^2 a^{2m_2}), \end{aligned} \quad (14)$$

where the subscripts 1 and 2 refer to terms with one and two powers of $k^2/(aH)^2$, respectively. The coefficient λ_1 is of the order 10^{-5} , while λ_2 may give a contribution of order one before the limit $\sigma^2 = 1$ is reached, and is expected to drop to zero afterwards. Before the limit $\sigma^2 = 1$, we have $m_1 = 1$ and $m_2 = 2$. After the threshold $\sigma^2 = 1$ is reached, m_1 slowly approaches zero, while m_2 is expected to go to zero rapidly. (The power-law behaviour in (14) is obviously only valid piecewise.)

To someone fitting the observed expansion rate to the homogeneous and isotropic FRW equation $(\theta/3)^2 = \rho/(3M^2)$ it would seem that there is a mysterious energy component which affects the expansion rate but which is nowhere to be seen. The apparent equation of state w of the ‘dark energy’ is easy to determine from the FRW relations $H^2 \propto a^{-3}$, $\rho_{\text{de}} \propto a^{-3(1+w)}$. These give the equations of state $w = -m/3$, where m is m_1 , m_2 , or one of the combinations in (14). Before structure formation, we would have a mixture of the equations of state $-\frac{1}{3}$, $-\frac{2}{3}$, -1 and $-\frac{4}{3}$. (Note that it is perfectly natural to get an equation of state which is more negative than -1 .) Today, the equation of state would be between $-\frac{2}{3}$ and 0 . The relative dark energy density today would seem to be $\Omega_{\text{de}} = 1 - 1/(1 + \lambda_1 + \lambda_2)^2$.

What does this imply for the expansion history of the universe? Neglecting λ_1 as probably small, we are left with λ_2 . Early on, before $\sigma^2 = 1$ on any scale, its relative contribution is related to σ^2 and rises like a^2 and a^4 , corresponding to the mixture of equations of state $-\frac{2}{3}$ and $-\frac{4}{3}$. After the limit $\sigma^2 = 1$ is reached, the contribution drops to zero. This term would induce a period of acceleration if its sign is positive and its contribution is comparable to that of dark matter, which could be the case around the beginning of the formation of bound structures. While the period of acceleration would drive the Hubble parameter up from the FRW Einstein-de Sitter value, this sort of an expansion history probably could not account for the supernova data.

After λ_2 goes to zero, the only backreaction comes from λ_1 . The contribution from this term has probably always been negligible, but for a red spectrum it grows in time. As discussed earlier, for a constant spectral index n the relative backreaction will grow like $a^{(n-1)/2}$ in the future, giving a combination of the equations of state $(n-1)/6$ and $(n-1)/3$. Even for the quite red spectrum with $n = 0.8$ in [18], we

would only get a mixture of $w \approx -0.03$ and $w \approx -0.07$. The backreaction would grow very slowly and eventually cause the Einstein-de Sitter universe to collapse. (Of course, the approximation that the perturbations behave according to first order perturbation theory would break down before that, and a consistent second order calculation would be needed.)

One should be careful with the Hubble law, because the square of $\langle \theta \rangle$ is not the same as the average of θ^2 . From (5), (7) and (8) we find (to second order) the following relation between the average of the square and the square of the average:

$$\langle \theta^2 \rangle - \langle \theta \rangle^2 = \partial_\tau \langle \theta \rangle - \langle \partial_\tau \theta \rangle . \quad (15)$$

The above relation has previously been found (in an exact form) in the different backreaction formalism of [33, 34]. The physical content of (15) is that the relative change of the integration measure on the hypersurface of constant proper time with respect to the proper time gives the expansion rate. Denoting the integration measure by J , we have $J^{-1}\partial_\tau J = \theta$. For example, for a spatially flat FRW spacetime we have $a^{-3}\partial_t(a^3) = 3H$.

One should be careful to identify the observable actually measured by an experiment and compare the theoretical average of that observable with the experimental data. For the supernova data, it seems more correct to take the average first and then square it. Averaging the square instead would lead to a positive (non-total gradient) contribution of $\frac{1}{9}\sigma^2$ to the square of the Hubble rate before structure formation (with the equations of state $-\frac{2}{3}$ and $-\frac{4}{3}$), and a correction of order one afterwards (with the equation of state 0). Note that (15) indicates that the deduced value of $\partial_\tau\theta$ is larger if one takes the average first, in contrast to what happens with θ^2 .

As an aside, we note that with the relation $J^{-1}\partial_\tau J = \theta$ the average expansion rate can be written in the simple form

$$\langle \theta \rangle = \langle J \rangle_0^{-1} \langle J\theta \rangle_0 = \partial_\tau(\ln \langle J \rangle_0) . \quad (16)$$

The form (16) is more suited to a systematic study of higher order terms than the straightforward calculation used to arrive at (9). From (16) it seems transparent that the backreaction is given by total time derivatives of dimensionless expectation values, something that was not obvious earlier. This may seem to be at odds with the fact that one of the backreaction terms in (9) did not seem to be a total time derivative. The resolution is that any term can be written as a total time derivative simply as $\langle f \rangle_0 = \partial_\tau \langle \int d\tau f \rangle_0$. Backreaction from most terms in (9) was found to vanish asymptotically because they contained no such integrals, but this is not true for all terms. So, while the form of (16) looks suggestive, it in fact contains no information about whether the backreaction can be written in such a total derivative form as would vanish asymptotically in the future.

Transition to non-linearity. The calculation has been made for perturbations in the linear regime, and we have treated the onset of non-linearity by simply introducing a sharp cut-off for the perturbations at a transition scale k_{NL} . A more realistic treatment of

the transition would include a description of the perturbation modes smoothly becoming time-dependent and breaking away from the general expansion. The effect of this on the backreaction is not clear, but since the magnitude of Φ increases, one would expect the backreaction to increase, and it could also change sign. Another reason to expect a large effect is that high- k modes give the main contribution to the backreaction integrals (12) and (13).

Backreaction from the non-linear regime has been considered in [43], in an approach where the Einstein equation was averaged, yielding a backreaction term like (12). The result for the backreaction from the linear regime of density perturbations was of the same order, 10^{-5} , but with a positive sign. The calculation was extended into the non-linear regime of density perturbations using the relation $\Phi_{\mathbf{k}} \approx -3(aH)^2\delta_{\mathbf{k}}/(2k^2)$, with the result that backreaction from the non-linear regime is also negligible. However, this result is not reliable.

As discussed above and in section 2, one has to be careful to identify the correct observable, to recast its time-dependence in terms of the physical proper time τ and to take the average over the hypersurface of constant τ . In [43] the quantity considered was $(\dot{a}/a)^2$, which does not give the expansion rate measure by a comoving observer, as seen from (6). Also, the time coordinate used was t and the average was taken over the hypersurface of constant t . The sign difference between the present calculation and the result of [43] for the linear regime arises from these factors. (As mentioned after (12), if one took the average of (10) over the hypersurface of constant t , the contribution of the non-total gradient terms would be positive.)

For the non-linear perturbations, there are two other important issues. First, the linear relation between $\Phi_{\mathbf{k}}$ and $\delta_{\mathbf{k}}$ is not valid in the non-linear regime, even though $\Phi_{\mathbf{k}}$ calculated from this equation is perturbatively small (since $k \gg aH$). For example, the gravitational potential inside stabilised collapsed objects is constant, whereas the relation $\Phi_{\mathbf{k}} \propto \delta_{\mathbf{k}}/a$ would give $\Phi_{\mathbf{k}} \propto a^{1/2}$ in the stable clustering approximation.

Second and more relevant, the calculation of [43] does not take into account that very non-linear structures have broken away from the expansion. We are interested in the locally measured expansion rate, and should not include contributions from inside the structures whose relative motion we are considering. This is not to say that any effect of the gravitational fields inside bound structures on the average expansion is ruled out, simply that such effects cannot be captured by the perturbative formalism of [43] or the present paper.

However, the effect of perturbations in the process of breaking away from the expansion can be calculated in a perturbative framework. Such perturbations could significantly change the results for the apparent dark energy, since their absolute value rises by orders of magnitude in the process of breaking away. Apart from boosting the magnitude, one would expect the growth of the perturbations to also make the equation of state more negative.

Since the sign of the global backreaction we calculated for the linear perturbations turned out to be negative, one might think that increased inhomogeneity in general

globally slows down the expansion. However, this is not the case. This can be easily seen by considering the average expansion rate (9) with the power-law behaviour $\Phi \propto a^l$, with constant l . The contribution of the non-gradient quadratic backreaction terms in (9) to the expansion rate is positive for all $l > 0$, while the contribution of the quadratic (non-total) gradient terms turns out to be positive for $l > \frac{1}{2}$. This sort of power-law behaviour is not physically relevant (except possibly piecewise in momentum space), but it shows that increased inhomogeneity can lead to faster expansion.

The answer that the backreaction of perturbations breaking away from the general expansion might give to the dark energy question would be simple: there appears to be a dark energy component because the observations are fitted to a model that does not take into account the impact of inhomogeneities on the expansion rate. This would also naturally solve the coincidence problem: the backreaction would be small until perturbations start becoming large.

Conclusion. The essence of the results is that there is a non-vanishing backreaction from inhomogeneities in an Einstein-de Sitter background, and that its magnitude is boosted by powers of $k^2/(aH)^2 = a(k/H_0)^2$ compared to the naive expectation of powers of Φ . This is in contrast to backreaction in inflationary backgrounds [40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53], where gradients are negligible and the effect is boosted by the large phase space of infrared terms. However, in both cases, the backreaction can be large though the magnitude of each individual mode $\Phi_{\mathbf{k}}$ is small.

For linear perturbations, backreaction of order $(k/aH)^2\Phi^2$ was found to have a magnitude of at most 10^{-5} today. Backreaction of order $(k/aH)^4\Phi^2$ reduces to a boundary term which is zero when evaluated over the whole space with periodic boundary conditions. The magnitude over the horizon volume with realistic boundary conditions is unknown, but could be of order one, at least before structure formation. At second order in Φ there are no higher orders in momentum. However, at higher orders there could be terms such as $(k/aH)^8\Phi^4$ that would not reduce to boundary terms and that would straightforwardly contribute corrections of order one.

That perturbations might be important today is perhaps not surprising. A measure of the inhomogeneity and anisotropy of spacetime is given by the Weyl tensor. For the metric (1), the ratio of the square of the Weyl tensor to the square of the scalar curvature is (for $a \propto t^{2/3}$)

$$\frac{C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}}{R^2} \simeq \frac{8}{9} \frac{1}{(aH)^4} \left(\partial_i\partial_j\Phi\partial_i\partial_j\Phi - \frac{1}{3}\nabla^2\Phi\nabla^2\Phi \right), \quad (17)$$

which, when averaged, is essentially the integral (13), in other words σ^2 . That this ratio is not small suggests that the impact of inhomogeneities can be large.

A few words about the backreaction framework are in order. If the backreaction is sizeable, then we have clearly exceeded the range of validity of the approximation of taking the background and the perturbations from first order perturbation theory. What would change in a consistent second order calculation?

First of all, first order scalar perturbations will in general give rise to second order vector and tensor perturbations [52, 55, 63], so the metric could not be written in the simple form (1). Second, we should check higher order terms of the type $(\partial_i \Phi \partial_i \Phi)^2$ and $(\nabla^2 \Phi \nabla^2 \Phi)^2$ to verify that the truncation to second order is consistent*.

Ignoring these complications and assuming that θ and τ would still be given by (5), (6) and (7), the main change would be to couple the development of the perturbations to their effect on the background expansion rate. If the backreaction led to faster expansion, this would be expected to cause the perturbations in the linear regime to decay, which would in turn slow down the expansion (as well as affect the apparent equation of state). As in the ‘concordance model’, this would alleviate the problem of too high σ_8 in models with $\Omega_m = 1$, so that neutrinos might not be needed to damp the power spectrum. The backreaction would also be expected to have an impact on the low multipoles of the CMB. Naively, one would expect the decay of the gravitational potential to increase the amplitude of the low multipoles just as in the ‘concordance model’. However, without considering the second order calculation in detail, it is not clear what the effect on the low multipoles would actually be.

As noted earlier, a consistent second order calculation would probably also answer the question of whether the perturbations are Gaussian with respect to the physical perturbed spacetime or the background spacetime.

It is interesting that the backreaction naturally gives the magnitudes $(k_{\text{eq}}/H_0)^2 \Phi^2 H_0 \sim 10^{-5} H_0$ and H_0 . Even the first one, though too small to explain the apparent acceleration, is closer to H_0 than what would be expected from particle physics motivated models of dark energy. Indeed, it seems more natural for the scale of present-day acceleration to emerge from cosmology and astrophysics rather than particle physics. In the present perturbative backreaction framework, the impact of the perturbations which are breaking away from the general expansion seems promising. These could have a large effect on the expansion rate, with a negative apparent equation of state, at the right time to solve the coincidence problem.

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* Note that the last three terms in (14) are strictly speaking quartic in Φ .

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