Article

Dark Energy versus Modified Gravity

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Abstract

There is now strong observational evidence that the expansion of the universe is accelerating. The standard explanation invokes an unknown "dark energy" component. But such scenarios are faced with serious theoretical problems, which has led to increased interest in models where instead General Relativity is modified in a way that leads to the observed accelerated expansion. The question then arises whether the two scenarios can be distinguished. Here we show that this may not be so easy, demonstrating explicitely that a generalised dark energy model can match the growth rate of the DGP model and reproduce the 3+1 dimensional metric perturbations. Cosmological observations are then unable to distinguish the two cases.

Reference

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INTRODUCTION

The observed accelerated expansion of the late-time universe, as evidenced by a host of cosmological data like type Ia supernovae (SN-Ia) [1], the cosmic microwave background radiation [2] and large scale structure [3] came as a great surprise to cosmologists. Although it is straightforward to explain the effect within the framework of Friedmann-Robertson-Walker cosmology by introducing a cosmological constant or a more general (dynamical) dark energy component, all such explanations give rise to severe coincidence and fine-tuning problems.

An alternative approach postulates that General Relativity is only accurate on small scales and has to be modified on cosmological distances. This in turn leads to the observed late-time acceleration of the expansion of the universe [4, 5, 6, 7]. One of the best-studied examples is the Dvali-Gabadadze-Porrati (DGP) brane-world model [8], in which the gravity leaks off the 4-dimensional Minkowski brane into the 5-dimensional bulk Minkowski space-time. On small scales the gravity is bound to the 4-dimensional brane and the Newtonian gravity is recovered to a good approximation.

One important question is whether such a scenario can be distinguished from one invoking an invisible dark energy component. It is well known that *any* expansion history (as parametrised by the Hubble parameter $H(t)$) can be generated by choosing a suitable equation of state for the dark energy (parameterised by the equation of state parameter $w = p/\rho$ of the dark energy). This can for example be seen from Eq. (1) of [9]. Let us illustrate this explicitly for the DGP model, for which the Hubble parameter evolves as

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho_m \tag{1}$$

where r_c, the crossover scale, separates the 5D and the 4D regimes. It has to be of the order of $1/H_0$ in order to generate late-time acceleration. Since matter is conserved on the brane, ρ_m satisfies the usual conservation equation. Comparing this to the normal Friedmann equation with an additional dark energy component, we see that we can move the crossover term to the right hand side and think of it as a dark energy contribution with $\rho_{\rm DE} = 3H/(8\pi G r_c)$. Looking at the conservation equation we find that it is solved if the effective dark energy has an equation of state given by

$$1 + w_{\rm DE} = -\frac{\dot{H}}{3H^2}. \tag{2}$$

Consequently, it is impossible to rule out "dark energy" based on measurements of the cosmic expansion history (e.g. SN-Ia data).

Recently there have been claims that it is instead possible to use the growth rate of structures for this purpose [6, 10, 11, 12] (but see also [13] for cautionary remarks). This is based on the observation that we can fix the equation of state parameter w of the dark energy from background data and then *predict* the evolution of the dark matter perturbations in a standard cosmological model with dark energy. If the observed growth rate is different from the predictions, then general relativity with dark energy would be ruled out.

However, in this paper we will show that this conclusions makes additional, very strong assumptions about the nature of the dark energy, and that in general the growth rate of structure is not sufficient to distinguish between dark energy models and modifications of gravity. We will show how the dark energy perturbations influence the dark matter and the metric perturbations, and provide an explicit example of a dark energy model which reproduces the 3+1 dimensional metric perturbations of the DGP scenario.

SETTING THE STAGE

We start by discussing the fluid perturbations in standard 3+1 dimensional cosmologies. The perturbations in

the energy density are given by $\delta = \delta\rho/\rho$ and to represent the fluid velocity we use $V = ik_j T_0^j/\rho$. Working in the Newtonian (longitudinal) gauge, the metric can be written as

$$ds^2 = -\left(1 + 2\psi\right)dt^2 + a^2\left(1 - 2\phi\right)dx_i dx^i \qquad (3)$$

with two scalar potentials ϕ and ψ describing the perturbations in the metric. Perturbations in cosmic fluids evolve according to [14]

$$\delta' = 3(1+w)\phi' - \frac{V}{Ha^2} - \frac{3}{a}\left(\frac{\delta p}{\rho} - w\delta\right) \qquad (4)$$

$$V' = -(1-3w)\frac{V}{a} + \frac{k^2}{Ha^2}\left(\frac{\delta p}{\rho} + (1+w)(\psi - \sigma)\right) \qquad (5)$$

where a prime denotes a derivative with respect to the scale factor a. The physical properties of the fluid are given by the anisotropic stress σ and the pressure perturbation δp (in general both can be functions of k). The latter is often parametrised in terms of the rest-frame sound speed c_s^2,

$$\delta p = c_s^2 \delta\rho + 3Ha(c_s^2 - c_a^2)\rho\frac{V}{k^2} \qquad (6)$$

where $c_a^2 = \dot{p}/\dot{\rho}$ is the adiabatic sound speed. Collisionless cold dark matter has zero pressure ($w_m = 0$), vanishing sound speed ($c_{s,m}^2 = 0$) and no anisotropic stress ($\sigma_m = 0$). For the dark energy all these quantities are a priori unknown functions and have to be measured. For the special case of dark energy due to a minimally coupled scalar field we have a variable w (corresponding to the choice of the scalar field potential, and fixed by the expansion history of the universe), $c_{s,\mathrm{DE}}^2 = 1$ and $\sigma = 0$ (see e.g. [15, 16]).

The perturbations in different fluids are linked via the perturbations in the metric ϕ and ψ. Introducing the comoving density perturbation $\Delta \equiv \delta + 3HaV/k^2$, their evolution in the standard cosmology is given by

$$k^2\phi = -4\pi Ga^2 \sum_i \rho_i \Delta_i \qquad (7)$$

$$k^2\left(\phi - \psi\right) = 12\pi Ga^2 \sum_i \left(1 + w_i\right)\rho_i\sigma_i \qquad (8)$$

where the sum runs over matter and dark energy in our case.

The quantity of interest to us is the growth factor $g \equiv \Delta_m/a$ which parameterises the growth of structure in the dark matter. The growth factor is normalised so that $g = 1$ for $a \ll 1$ (using that $\Delta_m \propto a$ during matter domination and on sub-horizon scales). For definiteness we fix $k = 200/H_0$ for the numerical results. We assume that g is an observable quantity (even though of course large scale structure surveys observe luminous baryonic matter, not dark matter, adding yet another layer of complications).
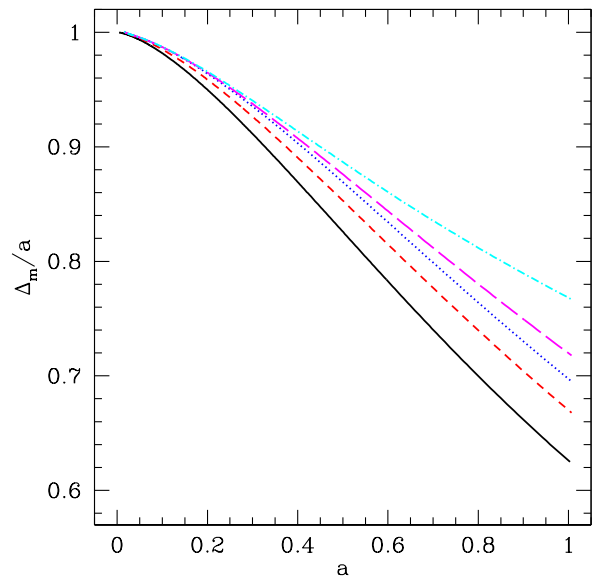

FIG. 1: This figure shows how the growth of the matter perturbations depends on the clustering properties of the dark energy. From the top downward the sound speed is $c_s^2 = -2 \times 10^{-4}$ (cyan dash-dotted line), $c_s^2 = -10^{-4}$ (magenta long dashed line), $c_s^2 = 0$ (blue dotted line) and $c_s^2 = 1$ (red dashed line). For comparison we also plot the growth factor of the DGP model (black solid line).

THE IMPORTANCE OF DARK ENERGY PERTURBATIONS

We start by noticing that the growth factor is *not* uniquely determined by the expansion history of the universe (and hence w_{DE}). Although the main effect of the dark energy is to change H, leading to $g < 1$ at late times, there is an additional link through the gravitational potential ψ. Different dark energy perturbations will lead to a different evolution of ψ, which can modify the behaviour of g. Conventionally one assumes that the dark energy perturbations are unimportant, e.g. [17]. This is a good assumption for scalar field dark energy where the high sound speed prevents clustering on basically all scales. However, a small sound speed $c_{s,\mathrm{DE}}^2 \approx 0$ is not excluded. Indeed, it could even be negative, leading to very rapid growth of the dark energy perturbations. It could also vary in time. We show in Fig. 1 how the growth factor of the dark matter changes in response to large dark energy perturbations [23].

What happens is that, as we decrease the sound speed, the dark energy is able to cluster more and more. The increased dark energy perturbations lead to enhanced metric perturbations. The dark matter in turn falls into the potential wells created by the dark energy, leading to an increase of the growth factor. Although clearly g is not uniquely determined by w_{DE}, we notice that it always

increases as we decrease $c_{s,\mathrm{DE}}^2$ (at least as long as the linearised theory is applicable, see also [18]). Looking at the evolution equations (4) and (5) for $\sigma = 0$ ($\Leftrightarrow \phi = \psi$) we see that the response of the fluids to the metric perturbations is governed by the sign of $1 + w$. Non-phantom dark energy (as required to mimic the DGP expansion history) clusters therefore in fundamentally the same way as the dark matter and can only *increase* the growth of matter relative to the case of negligible dark energy perturbations (excluding highly fine-tuned initial conditions).

So although the dark energy perturbations can influence the growth factor of the dark matter, they only seem capable of enhancing it. But Fig. 1 also shows the prediction for the growth factor in the DGP model from [19], and it is *smaller* than the one of a smooth dark energy component. We therefore need to change something else if we want to mimic DGP with dark energy. For this we need to take a closer look at the DGP model.

ANISOTROPIC STRESS AND MODIFIED GRAVITY MODELS

An important aspect of DGP and other brane-world models is that the dark matter does not see the higher-dimensional aspects of the theory as it is bound to the three-dimensional brane. Its evolution is then *the same as in the standard model*. The modifications appear only in the gravitational sector, represented by the metric perturbations.

The metric perturbation in DGP can be written as [19, 20]

$$k^2 \phi \;=\; -4\pi G\, a^2 \left(1 - \frac{1}{3\beta}\right) \rho_m \Delta_m \qquad (9)$$

$$k^2 \psi \;=\; -4\pi G\, a^2 \left(1 + \frac{1}{3\beta}\right) \rho_m \Delta_m \qquad (10)$$

where the parameter β is defined as:

$$\beta = 1 - 2r_c H \left(1 + \frac{\dot{H}}{3H^2}\right) = 1 + 2r_c H w_{\mathrm{DE}} \qquad (11)$$

The dark matter does not care if the metric perturbations are generated (in addition to its own contribution) by a modification of gravity or by an additional dark energy fluid. Its response to them is identical. Or to put it differently, if the dark energy and dark matter together can create the ϕ and ψ of Eqs. (9) and (10) then the growth factor (and indeed all other cosmological observables) will be the same as in the DGP scenario.

We see immediately that in order to generate these metric perturbations we will need to introduce an anisotropic stress since $\phi \neq \psi$. This seems to be a very generic property of modified gravity that is also present in $f(R)$ models [21] and has been noticed before. We plot in
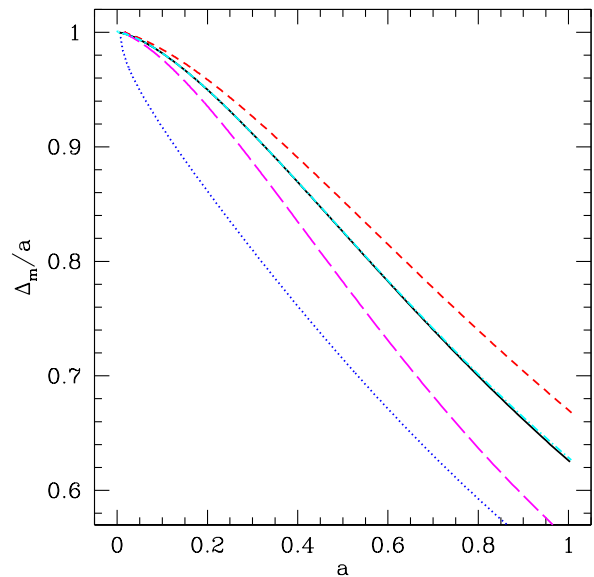

FIG. 2: In this figure we show how the anisotropic stress of the dark energy affects the growth of the dark matter perturbations. The red dashed line corresponds to scalar field dark energy with $c_s^2 = 1$ and $\sigma = 0$. The dotted blue line shows how the dark matter growth factor decreases for a constant $\sigma_{\mathrm{DE}} = -0.1$. The long-dashed magenta line uses the theoretical anisotropic stress of Eq. (12) with $c_s^2 = 1$, which suppresses the growth of the matter perturbations too much. Finally, the dash-dotted cyan line (nearly on top of black solid DGP line) uses the same σ_{DE} but sets the pressure perturbation of the dark energy to $\delta p = (1 + w)\rho\sigma$ in its rest frame.

Fig. 2 again the growth factor for scalar field dark energy and the DGP model, but now also a family of dark energy models with non-vanishing anisotropic stress σ. We notice that these models can easily suppress the growth of perturbations in the dark matter for $\sigma < 0$ and mimic the behaviour of the DGP model.

Formally we can recover the DGP metric perturbations by choosing

$$\sigma_{\mathrm{DE}} = \frac{2}{9\beta(1 + w_{\mathrm{DE}})} \frac{\rho_m}{\rho_{\mathrm{DE}}} \Delta_m. \qquad (12)$$

for the anisotropic stress of the dark energy, if we can also generate dark energy perturbations with

$$\rho_{\mathrm{DE}} \Delta_{\mathrm{DE}} = -\frac{1}{3\beta} \rho_m \Delta_m. \qquad (13)$$

We notice that these are very large dark energy perturbations. Indeed, if we keep $c_s^2 = 1$ and set σ to the expression (12) we suppress the growth of the matter perturbations too much, see Fig. 2. Since $\beta < 0$ the large dark energy perturbations of Eq. (13) then increase the matter clustering back to the DGP value.

The required size of the dark energy perturbations in itself is no problem, as we can lower the sound speed

and even make it negative. However, while for $\sigma = 0$ we were not able to *decrease* Δ_m with the help of the dark energy perturbations, we find that with a large, negative anisotropic stress we are unable to *increase* it. The required anisotropic stress is far larger than the gravitational potential ψ, and it starts to be the dominant source of dark energy clustering in Eq. (5). As it enters with the opposite sign it now leads to anti-clustering of the dark energy with respect to the dark matter which feels only ψ (ie. dark matter overdensities are dark energy voids). There is still enough freedom in the choice of σ to match the growth factor very precisely, but if we could measure both ϕ and ψ separately then we could detect the differences between the two models.

Is it really not possible to match *both* ψ and ϕ of the DGP model within a generalised fluid dark energy model? Yes, it is: The metric perturbations have two degrees of freedom, and we do have two degrees of freedom of the dark energy to adjust, σ and δp. As it turns out, the parametrisation in terms of the rest-frame sound speed is too restrictive. This can happen for example if the dark energy is not composed of a single fluid, see e.g. the discussion in [16]. Allowing free use of the pressure perturbations, we can choose them for example to cancel the direct effect of σ onto the dark energy perturbation in Eq. (5) by setting $\delta p = (1+w)\rho\sigma$. This reverses the sign of $\Delta_{\rm DE}$, and minor adjustments to the pressure perturbations can then provide the required match to Δ_m. For the cyan dash-dotted curve in Fig. 2 we set $\delta p = (1+w)\rho\sigma + 3Hac_a^2\rho V/k^2$, ie. we cancelled the contribution of the anisotropic stress in the dark energy rest frame. This provides a very good solution to Eqs. (12) and (13) during matter domination. It is easy to improve the solution to the point where it is impossible to distinguish observationally between the DGP scenario and this generalised dark energy model.

Is linear perturbation theory still valid with such a large anisotropic stress? Using Eq. (13) we can rewrite Eq. (12) as $\sigma_{\rm DE} = -2/(3(1+w_{\rm DE}))\Delta_{\rm DE}$. The anisotropic stress is therefore comparable in size to Δ_m and $\Delta_{\rm DE}$, and at high redshift DGP approaches GR. It is thus safe to study the dark energy with linear perturbation theory as long as the dark matter perturbations stay in the linear regime, even in the presence of the anisotropic stresses.

CONCLUSIONS

We have shown in this *letter* that the growth factor is not sufficient to distinguish between modified gravity and generalised dark energy, even if the expansion history (and so the effective equation of state of the dark energy) has been fixed by observations. We have also demonstrated that in some cases (notably DGP) the dark energy can match the metric perturbations completely so that cosmological observations cannot distinguish be-

tween the two possibilities.

Although the construction of a matching dark energy model for the DGP case may seem very fine tuned, we are here more concerned with the question to what degree this is possible at all. Just measuring a growth factor that does not agree with scalar field dark energy is not sufficient to rule out "dark energy" and General Relativity. But clearly, if the expansion history *and* the growth of matter perturbations were to match those predicted from a physically motivated and self-consistent modified gravity model, a statistical analysis would rule out a fine tuned dark energy model. However, we should not forget that as observations seem to indicate $w_{\rm DE} \approx -1$ it is rather the modified gravity models that are about to be ruled out [22] or look increasingly fine tuned.

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