

Dark energy with gravitational lens time delays

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Abstract. Strong lensing gravitational time delays are a powerful and cost effective probe of dark energy. Recent studies have shown that a single lens can provide a distance measurement with 6-7% accuracy (including random and systematic uncertainties), provided sufficient data are available to determine the time delay and reconstruct the gravitational potential of the deflector. Gravitational-time delays are a low redshift ($z \sim 0 - 2$) probe and thus allow one to break degeneracies in the interpretation of data from higher-redshift probes like the cosmic microwave background in terms of the dark energy equation of state. Current studies are limited by the size of the sample of known lensed quasars, but this situation is about to change. Even in this decade, wide field imaging surveys are likely to discover thousands of lensed quasars, enabling the targeted study of ~ 100 of these systems and resulting in substantial gains in the dark energy figure of merit. In the next decade, a further order of magnitude improvement will be possible with the 10^4 systems expected to be detected and measured with LSST and Euclid. To fully exploit these gains, we identify three priorities. First, support for the development of software required for the analysis of the data. Second, in this decade, small robotic telescopes (1-4m in diameter) dedicated to monitoring of lensed quasars will transform the field by delivering accurate time delays for ~ 100 systems. Third, in the 2020's, LSST will deliver 1000's of time delays; the bottleneck will instead be the acquisition and analysis of high resolution imaging follow-up. Thus, the top priority for the next decade is to support fast high resolution imaging capabilities, such as those enabled by the James Webb Space Telescope and next generation adaptive optics systems on large ground based telescopes.

1. Executive Summary

Strong gravitational lensing time delays measure distances, and hence the Hubble constant, dark energy, and dark matter. They measure cosmographic distance ratios that are highly complementary to those measured by supernovae and baryon acoustic oscillations and break many of the degeneracies inherent in the interpretation of cosmic microwave background data. They require no new major facilities, simply support for development and execution of sophisticated data analysis techniques to detect, measure and model the lenses, re-purposing and robotization of existing telescopes, and support of adaptive optics technology.

The priorities to support strong lensing time delays as a dark energy probe are:

- (a) Support the development of software and methods to ensure maximal information gain from the planned wide field, cadenced survey imaging data, and accurate inferences from them;
- (b) Transformation of a 2-4m class telescope (or a network of 1m telescopes) into a high cadence, long term dark energy monitoring experiment (non-exclusive use);
- (c) Support development of high performance adaptive optics systems on 8-30m class telescopes.

This program leverages and enhances existing surveys such as Pan-STARRS1, Dark Energy Survey, and Hyper Suprime-Cam which will discover > 100 well-measurable time delay systems this decade, and LSST which will discover > 1000 systems next decade, and involves high performance computing delivering added value to the analysis pipelines of these experiments.

2. Introduction

The acceleration of the universe is one of the most profound mysteries in physics (Riess et al. 1998; Perlmutter et al. 1999). Understanding its origin may lead to a rethinking of the standard paradigm of the cosmological model based on general relativity and cold dark matter. Ongoing and upcoming studies of dark energy rely heavily on the accurate knowledge of distances in the nearby universe (redshift $z \lesssim 1$). This is illustrated very clearly by the recent Planck results shown in Figure 1. The anisotropies of the cosmic microwave background are primarily sensitive to the angular diameter distance to the epoch of recombination ($z \sim 1000$) and therefore contain little information of later time phenomena like dark energy. This results in substantial degeneracies between the equation of state parameter (w), curvature (Ω_k), and the Hubble constant (H_0). The addition of CMB lensing information mitigates but does not solve the problem (Planck Collaboration et al. 2013). Only with the addition of lower redshift probes, like gravitational time delays (or Baryonic Acoustic Oscillations or local distance ladder measurements of H_0) these degeneracies can be broken allowing one to learn about the nature of dark energy.

In order to reach the goal of the next decade’s cosmological experiments, it will be necessary to pin down the accuracy on local distances, and thus equivalently H_0 , below the percent level (Riess et al. 2011; Freedman et al. 2012). Moreover, to identify unknown systematic errors in existing techniques, it is essential to gather several independent measurements (Weinberg et al. 2012; Linder 2011). The tension between previous measurements of H_0 and that recently derived by the Planck team within the assumptions of a six-parameter flat Λ CDM model (including tension with WMAP9) highlights the need for multiple independent measurements. If the tension cannot be resolved by unknown systematics, it will force the rejection of the six-parameter model in favor of a more complex alternative, thus leading to new physics such as a non-trivial dark energy equation of state or alternative theories of gravity. Even if the current tension can be resolved by discovering unknown systematics, adding a lower redshift measurement is essential to probe dark energy when it becomes relevant.

The gravitational time delay technique (Refsdal 1964), applied to a large number of lensed systems, is one of only a few that can lead to subpercent accuracy on low redshift distances and therefore H_0 . By measuring the time delay Δt between pairs of strongly gravitationally lensed images, and modeling the mass distribution of the lens galaxy, the time delay distance $D_{\Delta t}$ can be inferred †. Being a physical distance (as opposed to a relative distance modulus), this quantity is primarily sensitive to H_0 . However, samples of lenses also contain cosmological information in the form of distance ratios (e.g., Oguri 2007; Coe & Moustakas 2009, and references therein). In the past, the technique was plagued by poor time delay measurements, invalid assumptions about the lens mass profile, and systematic errors associated with over-simplistic modelling of the mass distribution in and around the lens. However, times have changed. It has been recently demonstrated that a single gravitational lens with well-measured time delays can be used to measure time delay distances to 6-7% total uncertainty (random *and* systematic Suyu et al. 2010, 2013). The key breakthroughs required to achieve this were: 1) multi-year monitoring to determine time delays (Fassnacht et al. 2002; Kochanek et al. 2006; Tewes et al. 2012, Figure 2), 2) the use of high resolution imaging and stellar kinematics to pinpoint the gravitational potential

† The time delay distance $D_{\Delta t} \equiv (1+z_d)D_d D_s / D_{ds}$ is a ratio of angular diameter distances (D ; s =source, d =deflector) and contains all the cosmological information (see, e.g., Treu 2010, for a description)

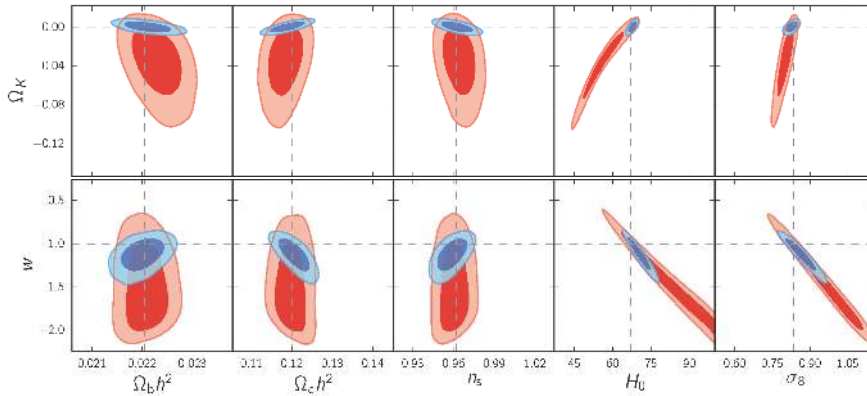


Figure 1. Cosmological inference based on the Planck data alone (red contours) and in combination with a low redshift probe (Baryonic Acoustic Oscillations; in blue). Note how for generic models CMB data alone cannot constrain simultaneously Ω_k , w , and H_0 , because the main observable provided by CMB data is the angular diameter distance to the epoch of recombination. Only when combining with a low redshift probe, like BAO, time delay lenses, or local measurements of the Hubble constant, the degeneracies can be broken and one can determine w and curvature. From Planck Paper XVI (Planck Collaboration et al. 2013).

of the lens using advanced modeling techniques (Treu & Koopmans 2002; Suyu et al. 2010, 2013, Figure 3), and 3) accounting in detail for the mass distribution along the line of sight (Suyu et al. 2010; Fassnacht et al. 2011; Greene et al. 2013; Collett et al. 2013). The analysis of just two systems produced measurements of H_0 , w , and Ω_k that are competitive with and highly complementary to those from established methods such as Cepheids, Baryonic Acoustic Oscillations and Supernovae Ia (Fig. 4). Gravitational lens time delays are not only independent of these other methods, but are also a cost-effective probe of the dark energy equation of state, as described below.

3. How Does Time Delay Lens Cosmography Work?

According to Fermat’s principle, images form at the extrema of the arrival time surface of any lens – including gravitational ones. The time delay between multiple images can be measured by monitoring a variable source like a quasar. In short, the difference in arrival time is given by $\Delta t \propto D_{\Delta t} \Delta \phi$. Here, the time delay distance contains all of the cosmological dependence, while the Fermat potential ϕ depends only on the details of the mass distribution. By measuring a time delay and determining a mass model for the main deflector, one obtains the time delay distance $D_{\Delta t}$ and, thus, a determination of cosmological parameters (Refsdal 1964; Schechter et al. 1997; Treu & Koopmans 2002; Kochanek 2002; Koopmans et al. 2003; Oguri 2007; Vuissoz et al. 2008; Suyu et al. 2010; Paraficz & Hjorth 2010; Suyu et al. 2013). Time delays constrain cosmology in two ways. First, they pin down the Hubble constant and thus remove degeneracies in the interpretation of CMB data in terms of w and its evolution (Linder 2011, Figure 5). Second, a sample of lenses at different redshifts measures angular distance ratios and thus is directly sensitive to other parameters (Coe & Moustakas 2009), especially curvature (Suyu et al. 2013, see Figure 4). The strengths of time delay lens cosmography are that (1) the method is based on simple geometry and well-tested physics (i.e., general relativity) rather than complicated astrophysics (e.g., supernova explosions, structure formation, etc.) that may not be completely understood, and (2) it produces a direct physical measurement of a cosmological distance. We also note that each time delay distance in a sample is largely independent, such that the scatter between measurements provides a self-contained test of unknown systematics. Suyu et al. (2013) show that unknown unknowns are currently negligible with respect to known unknowns. At present, the power of the method is limited by the small sample size of known lensed quasars: there are only a few known lensed quasars suitable for this experiment.

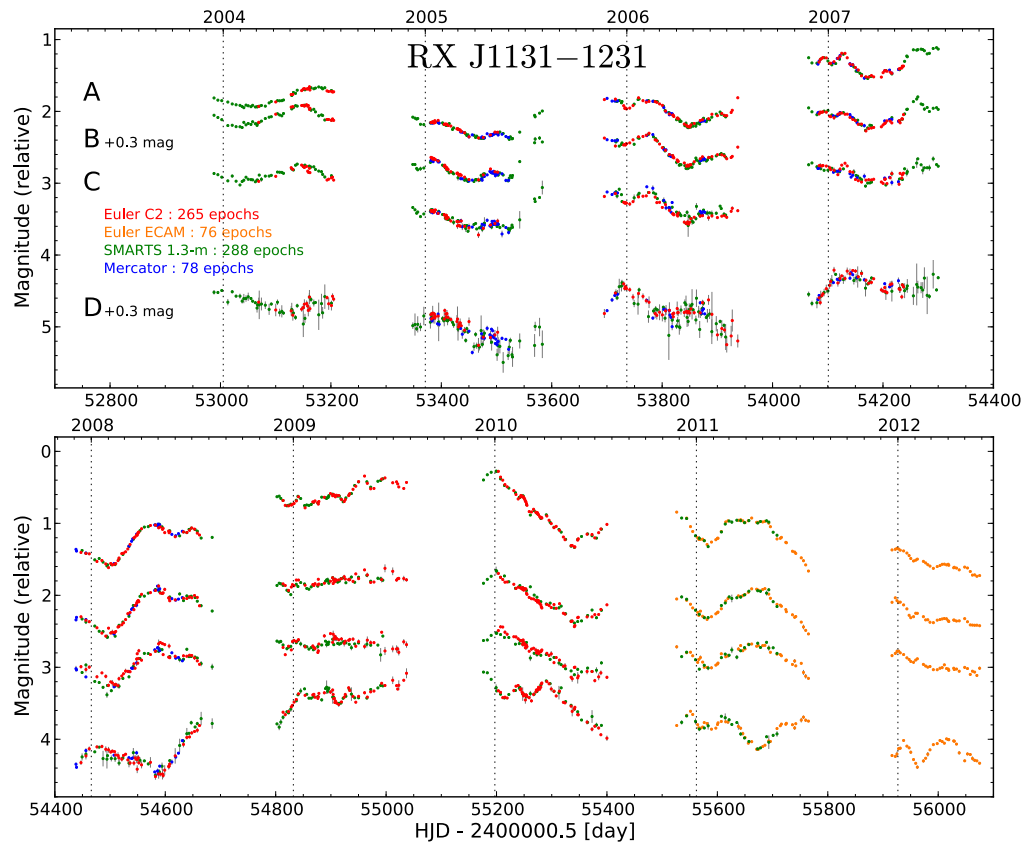


Figure 2. Example of multi-year monitoring of RXJ1131–1231, yielding time delays with 1.5% accuracy. The delay between images A,B,C and D can be clearly seen by eye! Figure from Tewes et al. (2012).

4. Medium Term (< 2020) and Long Term (2020-2030) Goals

A sample of > 100 lensed quasars with well-measured time delays and mass models will be transformative. As nicely summarized by Linder (2011) “Adding time delay data to supernovae plus cosmic microwave background information can improve the dark energy figure of merit by almost a factor of 5, and determine the matter density Ω_m to 0.004, the Hubble constant H_0 to 0.7% and the dark energy equation of state time variation w_a to ± 0.26 , systematics permitting.” Furthermore, the analysis of this large sample of systems will enable a direct check of unknown systematics by comparing blindly the inferred parameters for each system. In this white paper, we argue that this is achievable, through a concerted observational effort, by the end of this decade. Hundreds of lensed quasars are expected to be discovered in ongoing Stage III imaging surveys such as Pan-STARRS-1, the Dark Energy Survey (DES) and the Subaru Hyper Suprime-Cam Survey (Oguri & Marshall 2010). The human and observational resources required to confirm, follow-up and derive cosmological parameters from this sample are described below.

In the longer term, the LSST is expected to discover and provide time delays for thousands of lensed quasars (LSST Dark Energy Science Collaboration 2012). This will provide a further increase in sample size by over an order of magnitude, necessary to continue to complement meaningfully the precision of other future cosmological experiments. As described in the LSST Dark Energy Science Collaboration White Paper (LSST Dark Energy Science Collaboration 2012), a number of challenges must be addressed in this decade in order for us to be able to take full advantage of the power of LSST. This is best achieved through a combination of planned infrastructure work for LSST and analysis of Stage III datasets.

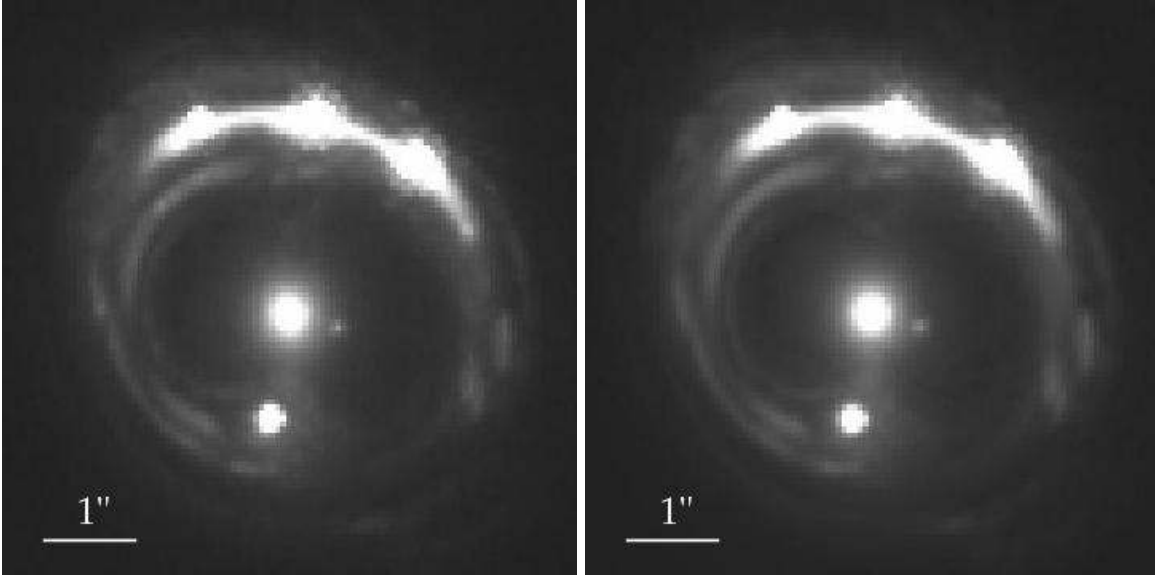


Figure 3. ACS image reconstruction of the lens system RXJ1131–1231 Suyu et al. (2013). Left: observed ACS F814W image. Right: image predicted by the lens model. The lens model reproduces to high fidelity tens of thousands of data points providing extremely tight constraints on the mass model of the deflector and thus on cosmological parameters.

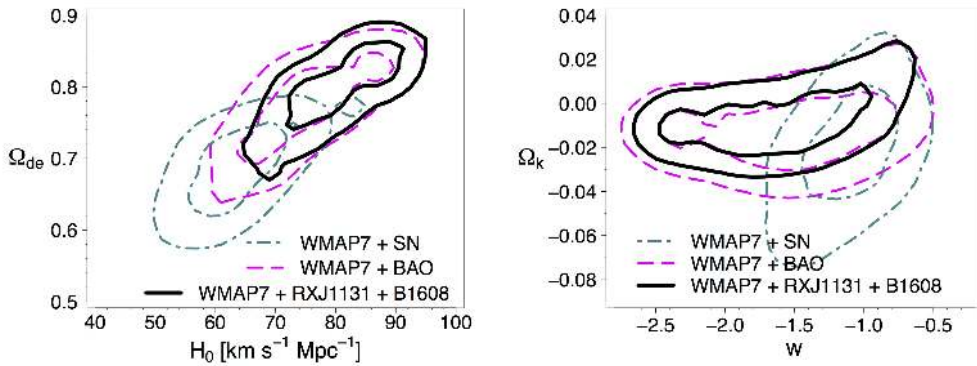


Figure 4. Posterior PDF of H_0 , Ω_{de} , w and Ω_k for BAO (red dashed; Percival et al. (2010)), Supernovae (SN) (blue dot-dashed; Hicken et al. (2009)), time delay lenses (black solid; Suyu et al. 2013) when each is combined with WMAP7 in a curved cosmology with non trivial dark matter ow CDM. Contours/shaded regions mark the 68.3%, and 95.4% credible regions. Time delay lenses are highly complementary to other probes, particularly CMB and SN. From Suyu et al. (2013).

5. Requirements

In order to measure time delay distances for each lensed quasar, the following ingredients are needed: i) precise time delays; ii) deep high resolution images of the lensed quasar host galaxy to model the gravitational potential of the deflector; iii) the redshifts of both the deflector and the source; iv) the stellar velocity dispersion of the main deflector; v) multiband imaging of the field of the lens, and redshifts of nearby companions, to characterize the environment along the full line of sight.

The diversity of observational requirements characterizes time delay cosmography. Whereas some other dark energy probes can be carried out as self-contained experiment with a single facility (eg. BAO), or by coupling a survey facility to a single follow-up facility (eg. SNeIa), time delay lenses require a variety of existing facilities

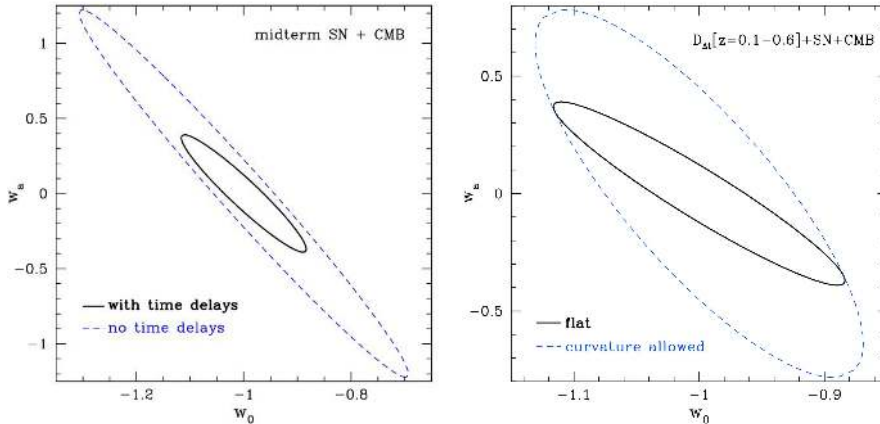


Figure 5. Forecasts for a time delay experiment from Linder (2011) based on 150 time delay distances. **Left:** 68% confidence level constraints on the dark energy equation of state parameters w_0 and w_a using midterm supernova distances and CMB information, and with (solid curve) or without (dashed curve) time delay measurements. The time delay probe demonstrates strong complementarity, tightening the area of uncertainty by a factor 4.8. **Right:** 68% confidence level constraints on the dark energy equation of state parameters w_0 and w_a using time delay, midterm supernovae, and CMB information, assuming spatial flatness (solid curve) or allowing curvature (dashed curve). Although relaxing the flatness constraints worsens the constraints in the (w_0, w_a) plane (there is a factor of 4 increase in the size of the confidence region between the solid and dashed curves), the constraints are much worse (a factor of 20 larger than the solid curve) if the time delay data are not included.

to follow-up each planned survey. We now discuss the facilities required for each observational ingredient. The analysis requires dedicated software, which is described in the last subsection.

5.1. Time delays

Recent work has demonstrated that multi-year monitoring campaigns are needed to determine time delays with the necessary accuracy and precision to carry out this experiment. A 5-year monitoring campaign on a dedicated telescope is needed for each lens (Tewes et al. 2012). Much of the work is currently being carried out with 1m class telescopes (such as Euler and SMARTS; Kochanek et al. 2006; Tewes et al. 2012), but it has been difficult to extend these campaigns, primarily because traditional observational astronomy works on much shorter time scales. Time on general observer telescopes is typically allocated on a semester by semester basis, and then typically scheduled in contiguous blocks, preventing the stability, longevity and flexibility necessary for this experiment.

With several 1-4m class telescopes currently being divested or closed, there is an opportunity to transform some of these general observer facilities into a dedicated dark energy experiment. What is required is a robotic telescope (or network of) capable of delivering single band images with typical seeing $\sim 1''$. After an initial investment for robotization (estimated at the level of a few 100k\$/telescope), the operating costs could be minimal given the extremely simple program involved ($\sim 100\text{k}\$/\text{yr}/\text{telescope}$). Depending on the size of the telescope, only a fraction of the time might be sufficient.

After this decade, a dedicated monitoring system like this one could continue to supplement the cadence provided by the LSST, while planned space-based probes could provide higher-precision time delay measurements in a smaller sample, sufficient to probe the nature of dark matter as described in a companion white paper (see also Moustakas et al. 2008).

5.2. High resolution imaging

The necessary high-resolution imaging, with a stable and well-characterized point-spread function, has so far been carried out with the Hubble Space Telescope (HST). We expect HST to continue to be a major workhorse for this application throughout the duration of its mission. However, as the sample grows in size, obtaining

high resolution images would become a major bottleneck unless additional capabilities are developed. After launch, the James Webb Space Telescope (JWST) will provide some of this capability; in addition, planned advances in adaptive optics technology will enable large ground-based telescopes to complement JWST at bluer wavelengths. High-strehl adaptive optics systems for 8-10m telescopes, and for the next generation of extremely large telescopes, will provide higher resolution than JWST and will be needed to fully exploit this method into the next decade. For example, the Next Generation Adaptive Optics system proposed for the Keck Telescope (Max et al. 2008) and the NFIRAOS system planned for the Thirty Meter Telescope (Herriot et al. 2012) will provide this capability. With these advanced adaptive optics systems it will be feasible to obtain high resolution imaging in a short amount of time. For example, we estimate that complete imaging follow-up of 1000 lensed quasars discovered by LSST would take approximately 50 nights of Keck-NGAO, and 10 nights of Thirty Meter Telescope time.

5.3. *Redshifts of Lens and Source, and Lens Stellar Velocity Dispersions*

Ground based spectroscopy is needed to determine redshifts of the deflector and the source, as well as to measure the stellar velocity dispersion of the main deflector (an additional independent mass measurement important in constraining the normalisation of the density profile). This is currently achieved using modest amounts of time on 8-10m class ground based telescopes (Treu & Koopmans 2002; Suyu et al. 2013). With the continued operation of these telescopes through the next decade we expect this to remain viable, although coordination with large scale spectroscopic surveys, such as those planned for BAO experiments, may be able to provide much of this information. AO systems such as those mentioned above with integral field spectrographs could provide in one shot a clean image of the source emission as well as the necessary redshifts and velocity dispersions.

5.4. *Multiband Imaging of the Field, and Redshifts of Nearby Companions, to Characterize the Lens Environment*

At present, determining the contribution of the environment and the line of sight to the total convergence (Treu 2010) requires dedicated imaging and spectroscopic follow-up (Collett et al. 2013; Greene et al. 2013). As samples grow in size the expectation is that this information will be provided by the finder surveys themselves (e.g. DES in this decade, and LSST in the next).

5.5. *Software and analysis tools*

Time delay cosmography is computationally intensive at the moment, typically requiring months of CPU *and* scientist-time for the full exploration of the model uncertainties of each system. Furthermore, characterization of the line of sight contributions require ray-tracing through state of the art cosmological simulations. Both software requirements will be stressed by the explosion of data expected in the near future. Exploiting this dark energy probe will require advances in lens modeling software and hardware to reduce the computing and investigator time per system to a manageable level. Likewise ray tracing through multiple independent cosmological simulations with different cosmological parameters and treatment of baryonic physics will be needed to quantify residual systematic uncertainties below the 1% level.

6. Conclusions

Recent studies have shown that gravitational time delays are a viable tool for cosmography. The power of this method is currently limited by the small size of current samples of known lensed quasars. However, large samples will become available in the coming decade thanks to dedicated surveys such as DES and LSST. The successful application of this tool to incoming Stage III and Stage IV datasets thus hinges on the development of follow-up capabilities. In turn, this requires the continued support of a number of existing and approved facilities like the James Webb Space Telescope and large ground based telescopes. In addition to supporting these activities, we advocate the following three priorities for new activities in the next two decades.

- Support the development of software for the analysis of time delay systems and its application to existing and stage III datasets.
- In this decade, we propose to convert a 2-4m class telescope (or a network of 1m telescopes) into a dark energy monitoring experiment. This would be transformative and cost effective when compared with dedicated surveys or space missions, requiring funding to cover only a fraction of the operating cost of a ground based imaging telescope.
- In the 2020 decade, the top priority will be to support the development of diffraction-limited high performance adaptive optics systems and their instrumentation on 8-30m class ground based telescopes to enable high fidelity lens mass modeling.

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