Dark matter produced from right-handed neutrinos

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ABSTRACT: Right-handed neutrinos (RHNs) provide a natural portal to a dark sector accommodating dark matter (DM). In this work, we consider that the dark sector is connected to the standard model only via RHNs and ask how DM can be produced from RHNs. Our framework concentrates on a rather simple and generic interaction that couples RHNs to a pair of dark particles. Depending on whether RHNs are light or heavy in comparison to the dark sector and also on whether one or both of them are in the freeze-in/out regime, there are many distinct scenarios resulting in rather different results. We conduct a comprehensive and systematic study of all possible scenarios in this paper. For illustration, we apply our generic results to the type-I seesaw model with the dark sector extension, addressing whether and when DM in this model can be in the freeze-in or freeze-out regime. Some observational consequences in this framework are also discussed.

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1 Introduction

Right-handed neutrinos (RHNs) are a highly motivated extension to the Standard Model (SM), often considered to be responsible for the generation of neutrino masses [1-5] and the matter-antimatter asymmetry of the universe [6, 7]. It is thus tempting to ask whether they are related to dark matter (DM), the existence of which has been evidenced by a variety of cosmological and astrophysical observations [8].

Perhaps the simplest answer is that RHNs themselves could be DM. Indeed, given an appropriate mass (~keV) and small mixing, RHNs have long been considered as a popular DM candidate, known as sterile neutrino DM (see Refs. [9–12] for reviews). Typically being produced via the Dodelson–Widrow [13] or Shi–Fuller [14] mechanism, sterile neutrino DM is not absolutely stable, albeit long-lived compared to the age of the universe. Its decay could cause observable X-rays which, together with recent Lyman- α observations [15–18], have ruled out the original scenario proposed by Dodelson and Widrow.

Going beyond the simplest answer, RHNs due to their singlet nature under the SM gauge symmetries could readily couple to a dark sector. There has been rising interest in a rather simple interaction, $\nu_R \chi \phi$, which couples a RHN ν_R to a dark fermion χ and a dark scalar ϕ [19–39]. This is the minimal setup for RHN-portal DM with absolute stability.

Given the interaction $\nu_R \chi \phi$, DM could be produced from RHNs in the early universe, and if being sufficiently produced, it could also annihilate and return energy and entropy to the SM thermal bath via RHNs at a late epoch. The thermal evolution of the dark sector depends on the abundance of RHNs. In Refs. [36, 37], RHNs are produced via freeze-in and dominantly decay to χ and ϕ . The freeze-in is achieved with a tiny Yukawa coupling in the type-I seesaw Lagrangian. In this setup, the DM relic abundance depends on the amount of RHN particles being produced, almost independent of the dark sector coupling. In Ref. [38], the type-I seesaw model is also assumed but RHNs are in thermal equilibrium, which is a natural consequence if the seesaw Yukawa couplings are not fine tuned. From RHNs to the dark sector, it is a freeze-in process, assuming the dark sector coupling is sufficiently small. Therefore, depending on parameter settings, different scenarios (e.g. freeze-in from the SM to RHNs versus freeze-in from RHNs to the dark sector) can be achieved in the same model.

Since various scenarios might be possible for RHN-portal DM, we aim to conduct a comprehensive and systematic analysis of all possible scenarios. We investigate, case by case, the thermal evolution and relic abundance of DM, assuming that thermal equilibrium may or may not be reached between the SM and RHNs, and between RHNs and the dark sector. The criteria for whether the equilibrium can be reached are derived for all cases. Depending on whether the dark sector particles are lighter or heavier than RHNs, the production of DM is dominated by decay or scattering processes, respectively. The former if kinematically allowed is usually much more efficient than the latter, leading to very different results for heavy and light RHNs. Thus we believe that a comprehensive investigation to cover various possibilities is necessary and might be useful for more extensive studies.

Despite that many scenarios in this framework have been studied before, we would like to point out that there are still some scenarios that have not been considered in the literature and might have interesting observational consequences. For example, when ν_R is in the freeze-in regime and lighter than DM, a considerably large amount of energy budget could be stored via ν_R into the dark sector and then returned to ν_R at a relatively late epoch. This would be particularly interesting if neutrinos are Dirac particles because the returned energy budget could lead to a potentially large contribution to the effective number of relativistic species, N_{eff} .

Our work is structured as follows. In Sec. 2 we introduce the framework of RHN-portal DM and propose four generic cases to be investigated. In addition, by briefly reviewing the Boltzmann equation, we also set up the necessary formalism for calculations. Then in Sec. 3, we present detailed analyses. Readers solely interested in the results are referred to Tab. 2 for a summary. For illustration, we apply our results to type-I seesaw DM in Sec. 4. A few observational consequences are discussed in Sec. 5. Finally we conclude in Sec. 6 and relegate some calculations to the appendix.

2 Framework

2.1 Lagrangian and conventions

Our framework assumes that the dark sector is connected to the SM content only via a RHN, ν_R . As aforementioned, the minimal setup for RHN-portal DM with absolute stability requires a pair of dark sector particles, a fermion χ and a scalar ϕ . The Lagrangian reads:

$$\mathcal{L} \supset y\chi\nu_R\phi + \text{h.c.},\tag{2.1}$$

where y is a Yukawa coupling. Throughout we adopt the Weyl spinor notation for all fermions so the product of χ and ν_R is simply written as $\chi\nu_R$. The particle masses of ν_R , χ , and ϕ are denoted by m_{ν_R} , m_{χ} , and m_{ϕ} , respectively. For simplicity, we assume

$$m_{\chi} < m_{\phi} \,, \tag{2.2}$$

so that only χ is a DM candidate while ϕ produced in the early universe would eventually decay to χ . Since in many models the dark sector may have a symmetry responsible for the DM stability, we assume ϕ is a complex scalar. Switching to a real scalar would change some of our results by a factor of 1/2.

For all particles considered in this work, we assume that there is no asymmetry in the thermal dynamics between particles and anti-particles:

$$n_X = n_{\overline{X}} \quad (\text{for } X = \chi, \ \phi, \ \nu_R, \ \nu_L, \ \cdots), \tag{2.3}$$

where \overline{X} denotes the antiparticle partner of X and $n_{X(\overline{X})}$ denotes the number density of $X(\overline{X})$, respectively. Under the assumption of Eq. (2.3), in this work we use notations of particles and anti-particles interchangeably. For instance, the process $\nu_R + \overline{\nu_R} \to \chi + \overline{\chi}$ will be written as $2\nu_R \to 2\chi$ for brevity. In case of potential confusions, one can always recover the notations of particles and anti-particles explicitly.

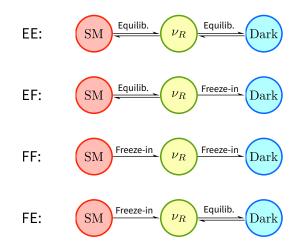


Figure 1. Four different cases for the entropy transfer among the SM thermal bath, ν_R , and the dark sector.

2.2 Four generic cases of the RHN portal

We aim at a comprehensive study of various possible scenarios arising from the framework of Eq. (2.1). Depending on the coupling y and also on how RHNs interact with the SM, there are four possible cases we may encounter, as illustrated in Fig. 1 and elucidated as follows.

- Equilibrium-Equilibrium (EE): both ν_R and the dark sector are in thermal equilibrium with the SM thermal bath. In this case, if ν_R is lighter than DM, then it is essentially in the standard WIMP paradigm. The DM abundance is determined by the moment when $2\chi \rightarrow 2\nu_R$ freezes out. If ν_R is heavier than DM, it becomes more complicated since $2\chi \rightarrow 2\nu_R$ is kinematically suppressed in the non-relativistic regime.
- Equilibrium-Freeze-in (EF): ν_R is in thermal equilibrium with the SM whereas the dark sector is frozen-in from ν_R . In this case, the DM abundance is determined by the integrated production rates of χ and ϕ from ν_R . The production rates crucially depend on whether $\nu_R \to \chi \phi$ is kinematically allowed or not. In any case, the production process $2\nu_R \to 2\chi$ is always kinematically allowed at high temperatures.
- Freeze-in-Freeze-in (FF): no thermal equilibrium is reached between ν_R and the SM thermal bath, or between the dark sector and ν_R . DM is produced via a two-step freeze-in mechanism.
- Freeze-in-Equilibrium (FE): ν_R is frozen-in from the SM and keeps thermal equilibrium with the dark sector. In this case, the DM abundance depends on how many ν_R particles have been produced through the thermal history, if ν_R is heavier than $m_{\chi} + m_{\phi}$ and dominantly decays to χ and ϕ . If ν_R is lighter than m_{χ} , the DM abundance also relies on the freeze-out of $2\chi \to 2\nu_R$.

2.3 Boltzmann equations

For a generic species X, the number density n_X is governed by the following Boltzmann equation:

$$\frac{dn_X}{dt} + 3Hn_X = C_X \,, \tag{2.4}$$

where H is the Hubble parameter and C_X is the collision term including contributions of all processes that create or annihilate X particles. For $X = \nu_R$, χ , or ϕ in our framework, each collision term may receive contributions from several reaction processes:

$$C_{\chi} = C_{2\nu_R \leftrightarrow 2\chi} + C_{\nu_R \leftrightarrow \chi\phi} + C_{\phi \leftrightarrow \chi\nu_R} + C_{2\phi \leftrightarrow 2\chi}, \qquad (2.5)$$

$$C_{\phi} = C_{2\nu_R \leftrightarrow 2\phi} + C_{\nu_R \leftrightarrow \chi\phi} + C_{\chi\nu_R \leftrightarrow \phi} + C_{2\chi \leftrightarrow 2\phi}, \qquad (2.6)$$

$$C_{\nu_R} = C_{\mathrm{SM}\leftrightarrow\nu_R} + C_{2\chi\leftrightarrow2\nu_R} + C_{2\phi\leftrightarrow2\nu_R} + C_{\phi\leftrightarrow\chi\nu_R} + C_{\chi\phi\leftrightarrow\nu_R} \,. \tag{2.7}$$

Here the subscripts of the C's on the right-hand side indicate the specific processes, except for SM $\leftrightarrow \nu_R$ which generically stands for unspecified processes connecting the SM and ν_R . In practice, not all the processes above have to be taken into account; some may be suppressed or kinematically forbidden.

For a generic two-to-two process, $1 + 2 \leftrightarrow 3 + 4$, the collision term is formulated as

$$C_{1+2\leftrightarrow 3+4} = C_{1+2\to 3+4} - C_{3+4\to 1+2}, \qquad (2.8)$$

with

$$C_{1+2\to3+4} = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 [f_1 f_2 (1\pm f_3)(1\pm f_4)] |\mathcal{M}|^2 (2\pi)^4 \delta^4, \qquad (2.9)$$

where $d\Pi_i \equiv \frac{d^3 \mathbf{p}_i}{2E_i(2\pi)^3}$; f_i is the momentum distribution function of particle *i*; \mathcal{M} denotes the matrix element of the process (see Tab. 1); and δ^4 denotes the delta function responsible for momentum conservation. The "±" sign takes "+" for bosons and "–" for fermions.

For one-to-two or two-to-one processes, the collision terms are similar except that one of the initial or final particles in Eq. (2.9) is removed.

Since the SM thermal bath has much more degrees of freedom than the dark sector, the total energy density of the early universe is dominated by the SM content. The Hubble parameter is determined by

$$H = \sqrt{\frac{8\pi}{3m_{\rm pl}^2} \left(\rho_{\rm SM} + \rho_{\rm dark}\right)} = g_H \frac{T_{\rm SM}^2}{m_{\rm pl}}, \qquad (2.10)$$

where $\rho_{\rm SM}$ and $T_{\rm SM}$ denote the SM energy density and temperature, $\rho_{\rm dark}$ denotes the energy density of the $\nu_R - \chi - \phi$ sector, and $m_{\rm pl} = 1.22 \times 10^{19}$ GeV. For convenience, we have defined $g_H \equiv \sqrt{8\pi^3 g_\star/90}$ where g_\star is the effective number of degrees of freedom in $\rho_{\rm SM}$ and $\rho_{\rm dark}$. For the SM g_\star , we adopt the results in Figure 2.2 of Ref. [40]. For additional species that are produced via freeze-in and stay in the freeze-in regime, their energy densities are low. For additional species that undergo non-relativistic freeze-out, their contributions to g_\star are also suppressed at the moment of freeze-out. Therefore, only for Cases EF and EE with light ν_R , g_\star receives a sizable contribution from $\rho_{\rm dark}$.

Table 1. Matrix elements for relevant processes considered in this work. Two-to-two processes are expressed in terms of the Mandelstam variables (s, t, u). The matrix elements are computed assuming either $m_{\nu_R} \ll m_{\chi}$, m_{ϕ} (light- ν_R approx.) or $m_{\nu_R} \gg m_{\chi}$, m_{ϕ} (heavy- ν_R approx., assuming Majorana mass). The mass splitting parameter δm is defined by $\delta m^2 \equiv m_{\phi}^2 - m_{\chi}^2$.

Processes	$ \mathcal{M} ^2$	Processes	$ \mathcal{M} ^2$
ν_R χ	$y^4 \left(\frac{t-m_{\chi}^2}{t-m_{\phi}^2}\right)^2$ light- ν_R approx.	ν_R	$y^2 m_{\nu_R}^2$ heavy- ν_R approx.
ν_R χ ν_R ϕ	$y^4 rac{tu-m_{\phi}^4}{\left(t-m_{\chi}^2\right)^2}$ light- ν_R approx.	$\phi \longrightarrow \phi \longrightarrow \phi$	$y^2 \left(m_{\phi}^2 - m_{\chi}^2 \right)$ light- ν_R approx.
ϕ	$y^{4} \frac{2tm_{\phi}^{2} - \delta m^{4} - ts - t^{2}}{t^{2}}$ light- ν_{R} approx. $y^{4} \frac{s - 2m_{\chi}^{2}}{m_{\nu_{R}}^{2}}$ heavy- ν_{R} approx.	χ ν_R F ϕ	$\begin{array}{l} y^2 y'^2 \frac{s+t-m_{\phi}^2}{s} \\ \text{light-} \nu_R \text{ approx.} \\ y^2 y'^2 \frac{m_{\chi}^2 - t}{m_{\nu_R}^2} \\ \text{heavy-} \nu_R \text{ approx.} \end{array}$

The SM entropy density, $s_{\rm SM} = 2\pi^2 g_{\star}^{(s)} T_{\rm SM}^3/45$, depends on a slightly different effective number of degrees of freedom, $g_{\star}^{(s)}$. In most cases, we neglect the small difference and assume $g_{\star}^{(s)} \approx g_{\star}$.

The total entropy of the SM thermal bath in a comoving volume is approximately conserved, i.e. $d(s_{\rm SM}a^3)/dt \approx 0$ where a is the scale factor in the FRW metric. Using $H = a^{-1}da/dt$, we obtain

$$\frac{ds_{\rm SM}}{dt} + 3Hs_{\rm SM} \approx 0.$$
(2.11)

In the freeze-in regime, n_X can be computed as follows. Using Eq. (2.11) together with Eq. (2.4) one gets $dn_X/ds_{\rm SM} = (C_X - 3Hn_X)/(-3Hs_{\rm SM})$ which can be written as

$$d\left(\frac{n_X}{s_{\rm SM}}\right) = -\frac{C_X}{3Hs_{\rm SM}^2} ds_{\rm SM}, \quad \text{or} \quad n_X = s_{\rm SM} \int_{s_{\rm SM}}^{\infty} \frac{C_X}{3H\tilde{s}_{\rm SM}^2} d\tilde{s}_{\rm SM}, \tag{2.12}$$

where H in the integral depends on the integration variable \tilde{s}_{SM} . If g_{\star} does not change significantly during the relevant epoch, the integral in Eq. (2.12) can be written in terms of T_{SM} :

$$n_X \approx T_{\rm SM}^3 \frac{m_{\rm pl}}{g_{\rm Hf.i.}} \int_{T_{\rm SM}}^{\infty} C_X \tilde{T}_{\rm SM}^{-6} d\tilde{T}_{\rm SM} , \qquad (2.13)$$

where $g_{Hf,i}$ denotes the freeze-in value of g_H , i.e. it denotes the value of g_H at the point when the integrand peaks. Taking a decay process for example, it should be when the decaying particle becomes non-relativistic.

After freeze-in, the comoving number density of X is conserved and $n_X/T_{\rm SM}^3$ would remain as a constant if $T_{\rm SM}$ scales as $T_{\rm SM} \propto 1/a$. However, due to many subsequent annihilation of SM species, $T_{\rm SM}$ actually scales as $a^{-1}g_{\star}^{-1/3}$. Therefore, as the universe cools down, the variation of g_{\star} leads to the following correction to n_X :

$$n_X \to n_X \frac{g_\star}{g_{\star \text{f.i.}}} \,, \tag{2.14}$$

where $g_{\star f.i.}$ denotes the freeze-in value of g_{\star} , similar to $g_{Hf.i.}$ introduced above.

Let us finally comment on the use of the integrated Boltzmann equation (2.4) and the momentum distribution functions, denoted by f_X for $X = \nu_R$, χ , or ϕ . For later convenience, we also denote the momentum distribution function of X in thermal equilibrium by f_X^{eq} . If X is not in thermal equilibrium, f_X can be very different from f_X^{eq} .

Strictly speaking, for non-thermal f_X , one needs to solve the more fundamental Boltzmann equation for f_X rather than the integrated one for n_X but under certain circumstances the latter suffices for accurate calculations. For non-thermal species being produced (frozenin) from a thermal sector, the back-reaction is negligible and those factors due to Fermi-Dirac or Bose-Einstein statistics [such as $(1 \pm f_3)(1 \pm f_4)$ in Eq. (2.9)] are also negligible. So the collision term mainly depends on quantities of thermal species and hence can be well determined. When studying the connection between two thermal or almost thermal sectors (such as the freeze-out scenario), the integrated form could also be approximately used because the collision term can be written in terms of number densities.

For non-thermal species being produced from another non-thermal species, however, the spectral shape of the latter can significantly affect the result. In this case, one needs more elaborated treatments involving numerical techniques and analytical approximations, as we will elucidate in Sec. 3.1.2.

3 DM relic abundance

With the framework set up in Sec. 2, we now start to investigate the four generic cases proposed in Fig. 1. The goal is to derive generic formulae for the DM relic abundance and also the criteria for identifying the four cases. The results are summarized in Tab. 2.

anging from $\mathcal{O}(0.1)$ to $\mathcal{O}(10)$ for GeV-1eV particles, are defined in Eq. (5.3) and Eq. (5.21).								
	Cases	light ν_R limit $(m_{\chi,\phi} \gg m_{\nu_R})$		heavy ν_R limit $(m_{\nu_R} \gg m_{\chi,\phi})$				
		Valid range of y	$\Omega_\chi h^2/0.12$	Valid range of y	$\Omega_{\chi}h^2/0.12$			
	\mathbf{EE}	$y > 2.7 \times 10^{-4} R_1$	Eq. (3.17)	$y > 1.7 \times 10^{-7} R_2$	Eq. (3.26)			
	\mathbf{EF}	$y < 2.7 \times 10^{-4} R_1$	Eq. (3.4) or (3.7)	$y < 1.7 \times 10^{-7} R_2$	Eq. (3.20)			
	\mathbf{FF}	$y < 2.7 \times 10^{-4} R_1$	Eq. (3.11)	$y < 1.7 \times 10^{-7} R_2$	Eq. (3.28)			
	FE	$y>2.7\times 10^{-4}R_1$	Eq. (3.17)	$y > 1.7 \times 10^{-7} R_2$	Eq. (3.28)			

Table 2. Results of our analyses for the four generic cases. The quantities R_1 and R_2 , typically ranging from $\mathcal{O}(0.1)$ to $\mathcal{O}(10)$ for GeV-TeV particles, are defined in Eq. (3.5) and Eq. (3.21).

The results crucially depend on whether $\nu_R \to \chi \phi$ is kinematically allowed $(m_{\nu_R} > m_{\chi} + m_{\phi})$ or not $(m_{\nu_R} < m_{\chi} + m_{\phi})$ because the collision term $C_{\nu_R \to \chi \phi}$ is much larger, roughly by a factor of $4\pi^2/y^2$, than $C_{2\nu_R \to 2\chi}$ and $C_{2\nu_R \to 2\phi}$. For simplicity, we assume $m_{\nu_R} \gg m_{\chi,\phi}$ in the decay-allowed case, and $m_{\nu_R} \ll m_{\chi,\phi}$ in the decay-forbidden case. In what follows, they are referred to as the heavy and light ν_R limit, respectively. Increasing or decreasing m_{ν_R} to a level comparable to $m_{\chi,\phi}$ without crossing the threshold $(m_{\chi} + m_{\phi})$ would lead to qualitatively similar results, though the calculations would be much more complicated.

3.1 Light ν_R limit

In the light ν_R limit, i.e., $m_{\chi,\phi} \gg m_{\nu_R}$, the analyses and results below are almost independent of m_{ν_R} , which in principle can vary freely from any scales well below $m_{\chi,\phi}$ down to zero. If sufficiently light, ν_R might contribute to N_{eff} and hence be constrained by the precision measurement of N_{eff} .

3.1.1 Case EF in the light ν_R limit

Let us start with a sufficiently small y so that the dark sector is not thermalized, while the ν_R sector is in thermal equilibrium. This corresponds to the EF case in Fig. 1. The dark sector particles are produced from ν_R via $2\nu_R \rightarrow 2\chi$ and $2\nu_R \rightarrow 2\phi$. The latter contributes indirectly to the production of χ via ϕ decay. Let us first concentrate on the $2\nu_R \rightarrow 2\chi$ process. The collision term is calculated in Appendix A and can be approximately written as

$$C_{2\nu_R \to 2\chi} \approx \frac{y^4}{128\pi^5} \left[m_{\chi} T_{\rm SM} K_1 \left(\frac{m_{\chi}}{T_{\rm SM}} \right) \right]^2, \qquad (3.1)$$

where K_1 is the modified Bessel functions of order 1. In the derivation of this result, we have assumed $\delta m^2 \equiv m_{\phi}^2 - m_{\chi}^2 \ll T_{\text{SM}}^2$, $m_{\chi} \gg (\delta m, m_{\nu_R})$, and the Boltzmann statistics.

Given the collision terms, the number densities of χ and ϕ can be computed by solving the Boltzmann equations which in the freeze-in regime have solutions of the integral forms in Eqs. (2.12) and (2.13). Substituting Eq. (3.1) into Eq. (2.13) and integrating it down to a temperature well below m_{χ} , we obtain the number density of n_{χ} after freeze-in. In addition, we also take the g_{\star} correction in Eq. (2.14) into account. The result reads:

$$n_{\chi} = \frac{3y^4 m_{\rm pl}}{2^{12} \pi^3 g_{Hf,i.} m_{\chi}} \frac{g_{\star}^{(s)}}{g_{\star f,i.}^{(s)}} T_{\rm SM}^3 \,. \tag{3.2}$$

Apart from the direct production of χ particles via $2\nu_R \to 2\chi$, there is also an indirect production channel via $2\nu_R \to 2\phi$ followed by $\phi \to \nu_R + \chi$ decay. The lifetime of ϕ is much shorter than the age of the universe if $y \gtrsim 10^{-20} \cdot (m_{\phi}/\text{GeV})^{-1/2}$. Therefore, almost all ϕ particles produced in the early universe will eventually decay to ν_R and χ . Since both $C_{2\nu_R\to 2\phi}$ and $C_{2\nu_R\to 2\chi}$ are proportional to y^4 , the production rates of ϕ should be comparable to that of χ . Indeed, according to the calculation in Appendix A [see Eqs. (A.10) and (A.11)], the number density of ϕ after freeze-in assuming ϕ does not decay would be

$$n_{\phi} \approx 1.87 n_{\chi} \,. \tag{3.3}$$

Taking ϕ decay into account, each ϕ particle produces one χ particle via $\phi \to \nu_R + \chi$. Hence the number density of χ is increased by $n_{\chi} \to 2.87 n_{\chi}$. It is conventional to write the result in terms of $\Omega_{\chi}h^2$ where $\Omega_{\chi} = \rho_{\chi}/\rho_{\rm cri.}$ is the ratio of ρ_{χ} to the critical energy density $\rho_{\rm cri.}$ and $h \equiv H_0/(100 \text{ km/sec/Mpc})$ with H_0 the Hubble constant today. In terms of $\Omega_{\chi}h^2$, the result reads (including the contribution of ϕ decay):

$$\Omega_{\chi} h^2 \approx 0.12 \left(\frac{y}{3.8 \times 10^{-6}}\right)^4 \left(\frac{106.75}{g_{\star \text{f.i.}}}\right)^{\frac{3}{2}}.$$
(3.4)

Note that, as mentioned below Eq. (2.10), g_{\star} in this case receives an addition contribution from light ν_R . So the value of $g_{\star f.i.}$ should be the SM value at freeze-in plus 7/8, assuming a single flavor of ν_R .

Eq. (3.4) implies that to generate $\Omega_{\chi}h^2 = 0.12$ in the EF case, one needs $y \sim 10^{-6}$ for thermalized ν_R and $10 \leq g_{\star f.i.} \leq 10^2$. This is consistent with the result in Ref. [41], see Fig. 3 therein.

The validity of the above analysis for the EF case is based on the assumption that y is sufficiently small. In this regime, the back-reactions $2\chi \rightarrow 2\nu_R$ is negligible due to the low density of χ particles. Now let us increase y, which will eventually lead to a high back-reaction rate comparable to the production rate. Then the equilibrium between ν_R and χ will be established, leading to $n_{\chi} = n_{\nu_R}$. Substituting $n_{\chi} = n_{\nu_R}$ into Eq. (3.2) and solving it for y, we obtain the following solution:

$$y_{\rm eq} = 2.7 \times 10^{-4} R_1, \quad R_1 \equiv \left(\frac{g_H}{17.2} \frac{m_\chi}{\rm GeV}\right)^{1/4}.$$
 (3.5)

For $y \leq y_{eq}$, the freeze-in calculation is approximately valid. For $y \geq y_{eq}$, it becomes the EE case, which will be discussed in Sec. 3.1.3.

Finally we would like to comment on a potentially important contribution from the off-shell ν_R decay. Although ν_R is lighter than χ and ϕ , off-shell ν_R can be produced from SM particle scattering and then decay to χ and ϕ . This part of contribution depends on the coupling of ν_R to the SM. In particular, in the presence of a relatively strong ν_R -SM interaction, the effective thermal mass of ν_R might actually exceed $m_{\chi} + m_{\phi}$ so that the production rate of the forbidden channel $\nu_R \to \chi \phi$ can be considerably large. This is very similar to the plasmon decay $(\gamma^* \to 2\nu)$ used to constrain neutrino magnetic moments [42–44]. A dedicated treatment requires taking finite-temperature effects into account consistently in both scattering and decay processes [45, 46], which will be studied in our future work.

Here we provide a straightforward estimate of this contribution assuming that the finite-temperature effects are negligible and that ν_R couples to a pair of SM particles with a different coupling y'. The collision term for two light SM particles scattering to produce χ and ϕ is

$$C_{\text{SM}\to\nu_R^*\to\chi\phi} = \frac{y^2 y'^2}{256\pi^5} \left[m_{\chi} T_{\text{SM}} K_1\left(\frac{m_{\chi}}{T_{\text{SM}}}\right) \right]^2.$$
(3.6)

Comparing it to Eq. (3.1), one can see that this channel dominates when $y' > \sqrt{2}y$. For $y' \gg y$, the DM relic abundance is given by

$$\Omega_{\chi}h^2 \approx 0.12 \left(\frac{y}{2.5 \times 10^{-6}}\right)^2 \left(\frac{y'}{10^{-5}}\right)^2 \left(\frac{106.75}{g_{\star \text{f.i.}}}\right)^{\frac{3}{2}}.$$
(3.7)

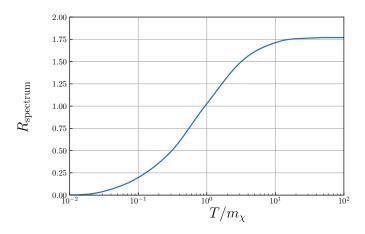


Figure 2. The spectral distortion factor R_{spectrum} defined in Eq. (3.9) as a function of T.

For $y' \ll y$, one should use Eq. (3.4) instead.

3.1.2 Case FF in the light ν_R limit

In the FF case, ν_R is not in thermal equilibrium so the result depends on the specific form of the momentum distribution function of ν_R . For the FF case, we modify Eq. (3.1) as follows:

$$C_{2\nu_R \to 2\chi} \approx \frac{y^4}{128\pi^5} \left\langle \frac{f_{\nu_R}}{f_{\nu_R}^{\rm eq}} \right\rangle^2 \left[m_{\chi} T_{\rm SM} K_1 \left(\frac{m_{\chi}}{T_{\rm SM}} \right) \right]^2, \tag{3.8}$$

where $\langle f_{\nu_R}/f_{\nu_R}^{\text{eq}} \rangle$ is, by definition, to absorb the difference between FF and EF collisions terms for $2\nu_R \to 2\chi$. Since $(n_{\nu_R}, n_{\nu_R}^{\text{eq}}) = \int (f_{\nu_R}, f_{\nu_R}^{\text{eq}}) \frac{d^3p}{(2\pi)^3}$, we expect $\langle f_{\nu_R}/f_{\nu_R}^{\text{eq}} \rangle$ to be around the same order of magnitude as $n_{\nu_R}/n_{\nu_R}^{\text{eq}}$. Hence we introduce a spectral distortion factor defined as

$$R_{\rm spectrum} \equiv \left\langle \frac{f_{\nu_R}}{f_{\nu_R}^{\rm eq}} \right\rangle \left(\frac{n_{\nu_R}}{n_{\nu_R}^{\rm eq}} \right)^{-1}.$$
 (3.9)

In the absence of spectral distortion $(f_{\nu_R} \propto f_{\nu_R}^{\text{eq}})$, we have $R_{\text{spectrum}} = 1$. For light ν_R produced from heavy particle decay, the spectral shape of f_{ν_R} in the freeze-in regime can be computed analytically [47, 48]—see also Appendix B for a brief review. With the analytical expression for f_{ν_R} , we can compute $C_{2\nu_R \to 2\chi}$ numerically¹, extract $\langle f_{\nu_R}/f_{\nu_R}^{\text{eq}} \rangle$ in Eq. (3.8), and then obtain the spectral distortion factor R_{spectrum} , which is presented in Fig. 2.

Using the numerical value of R_{spectrum} , we repeat the analysis in Sec. 3.1.1 and find that the non-thermal f_{ν_R} results in the following correction to the DM relic abundance in Eq. (3.4):

$$\Omega_{\chi} h^2 \to 1.74 \left(\frac{n_{\nu_R}}{n_{\nu_R}^{\rm eq}}\right)^2 \Omega_{\chi} h^2 \,, \tag{3.10}$$

Therefore, for the FF case, the DM relic abundance abundance can be written as

$$\Omega_{\chi} h^2 \approx 0.12 \left(\frac{n_{\nu_R}}{n_{\nu_R}^{\rm eq}}\right)^2 \left(\frac{y}{3.3 \times 10^{-6}}\right)^4 \left(\frac{106.75}{g_{\star \rm f.i.}}\right)^{\frac{3}{2}}.$$
(3.11)

¹Here we have used the code available at https://github.com/xunjiexu/Thermal_Monte_Carlo.

Here we have assumed that $n_{\nu_R}/n_{\nu_R}^{\rm eq}$ is approximately a constant during the epoch of the second freeze-in of FF. This is a good approximation if the two freeze-in processes of FF occur at well separated temperature scales. If the two freeze-in temperatures are close to each other, then Eq. (3.11) is not valid but it can be used as a crude estimate. The exact result in this case can be obtained by numerical integration.

3.1.3 Cases EE and FE in the light ν_R limit

The EE and FE cases in Fig. 1 require that the dark sector keeps thermal equilibrium with ν_R . The criteria for such cases are just mentioned above.

In these two cases, we have $n_{\chi} \approx n_{\nu_R}$ at temperatures well above both masses. As the temperature falls below $\sim m_{\chi}$, a large amount of χ particles will annihilate to ν_R , leaving only a small portion of χ particles due to the well-known freeze-out mechanism. Since the freeze-out mechanism has been extensively studied, we refer to textbooks [49, 50] and lectures [51–53] for a full calculation starting from the Boltzmann equation. Here one can adapt the known result of the standard freeze-out calculation to our framework, with the key difference being that the ν_R -dark sector may have a different temperature compared to the SM one. The details are presented in Appendix C and the results are summarized below.

For convenience, we define two ratios

$$\epsilon \equiv T_{\chi}/T_{\rm SM} \,, \ x \equiv m_{\chi}/T_{\chi} \,, \tag{3.12}$$

and denote their corresponding freeze-out values by $\epsilon_{\text{f.o.}}$ and $x_{\text{f.o.}}$. In the EE case, we have $\epsilon_{\text{f.o.}} = 1$. In the FE case, $\epsilon_{\text{f.o.}}$ depends on the abundance of ν_R produced from the SM sector. The value of $x_{\text{f.o.}}$ according to Appendix C is approximately given by

$$x_{\rm f.o} = 35.5 + 9.5 \log_{10} \left[y \cdot \epsilon_{\rm f.o}^{1/2} \cdot \left(\frac{m_{\chi}}{\text{GeV}} \frac{g_H}{17.2} \right)^{-1/4} \right].$$
(3.13)

In the standard WIMP paradigm, $x_{\rm f.o.}$ typically varies around $20 \sim 25$.

The relic abundance of DM is given by

$$\Omega_{\chi}h^2 = 0.12 \frac{x_{\text{f.o.}} \epsilon_{\text{f.o.}}}{\sqrt{g_{\star \text{f.o.}}}} \cdot \frac{1.4 \times 10^{-9} \text{ GeV}^{-2}}{\langle \sigma v \rangle}, \qquad (3.14)$$

where $\langle \sigma v \rangle$ is the velocity-averaged cross section, widely used in the WIMP paradigm. It is related to the collision term by

$$\langle \sigma v \rangle \equiv \frac{1}{n_{\chi}^2} C_{2\chi \to 2\nu_R} \,, \tag{3.15}$$

and in the non-relativistic regime given by

$$\langle \sigma v \rangle \approx \frac{y^4}{32\pi m_\chi^2} \,.$$
 (3.16)

Substituting Eq. (3.16) into Eq. (3.14), we obtain

$$\Omega_{\chi} h^2 = 0.12 \left(\frac{m_{\chi}}{100 \text{ GeV}}\right)^2 \left(\frac{0.19}{y}\right)^4 \frac{x_{\text{f.o.}} \epsilon_{\text{f.o.}}}{\sqrt{g_{\star \text{f.o.}}}} \,. \tag{3.17}$$

Similar to the discussion below Eq. (3.4), g_{\star} in this case receives an addition contribution from light ν_R . So the value of $g_{\star f.o.}$ should be the SM value at freeze-out plus $7\epsilon_{f.o.}^4/8$, assuming a single flavor of ν_R .

Note that in the EE and FE cases, the influence of $\phi \to \nu_R \chi$ on the DM abundance should be small if $\delta m^2 x_{\text{f.o.}} > 2m_{\chi}^2$ because $n_{\phi}/n_{\chi} \approx \exp\left[\left(m_{\chi}-m_{\phi}\right)/T\right]$ is small at the freeze-out temperature. Taking $x_{\text{f.o.}} = 25$ and $\delta m^2 = 0.2m_{\chi}^2$ for example, we get $n_{\phi}/n_{\chi} =$ 9.2%. Although the observed value of $\Omega_{\chi}h^2$ has been determined very precisely (~1%), a 9% variation of $\Omega_{\chi}h^2$ can be easily absorbed into a small variation of y, which is a rather free parameter in the model. So here we neglect the contribution of $\phi \to \chi \nu_R$ to the DM abundance.

A particularly interesting feature of the FE case is that due to the DM annihilation at a relatively late epoch, the abundance of ν_R might be substantially enhanced, causing observable changes to N_{eff} if ν_R are sufficiently light.

3.2 Heavy ν_R limit

In the heavy ν_R limit $(m_{\nu_R} \gg m_{\chi,\phi})$, $\nu_R \to \chi \phi$ as well as its back-reaction $\chi \phi \to \nu_R$ is kinematically allowed. In the presence of $\nu_R \leftrightarrow \chi \phi$, the energy conversion between the two sectors via such two-to-one or one-to-two processes is much more efficient than that via two-to-two processes considered in Sec. 3.1. Therefore, the contributions of $2\nu_R \leftrightarrow 2\chi$ and $2\nu_R \leftrightarrow 2\phi$ will be neglected in what follows.

3.2.1 Case EF in the heavy ν_R limit

Similar to the analysis in Sec. 3.1, we start with a sufficiently small y so that the dark sector is not thermalized. On the other hand, the ν_R sector may or may not be in thermal equilibrium with the SM thermal bath, depending on the interaction of ν_R with the SM. Let us first consider that the ν_R -SM coupling is sufficiently large so that the ν_R sector has been thermalized. This leads to the EF case. The collision term for the dominant DM production process $\nu_R \to \chi \phi$ reads

$$C_{\nu_R \to \chi \phi} = y^2 \frac{m_{\nu_R}^3 T_{\rm SM}}{32\pi^3} K_1 \left(\frac{m_{\nu_R}}{T_{\rm SM}}\right).$$
(3.18)

Now substituting Eq. (3.18) into Eq. (2.13) and taking the g_{\star} correction in Eq. (2.14) into account, we obtain the values of n_{χ} and n_{ϕ} after freeze-in:

$$n_{\chi} = n_{\phi} = \frac{3y^2 m_{\rm pl}}{64\pi^2 m_{\nu_R} g_{Hf.i.}} \frac{g_{\star}^{(s)}}{g_{\star f.i}^{(s)}} T_{\rm SM}^3.$$
(3.19)

As we have assumed $m_{\nu_R} > m_{\phi} + m_{\chi}$, ϕ cannot decay to χ and ν_R . However, this does not mean that ϕ would be stable. As long as the ν_R -SM coupling is not extremely suppressed, ϕ can eventually decay to χ and some SM final states with ν_R as an intermediate state. And the lifetime of ϕ is generally expected to be much shorter than the age of the universe. So for today's n_{χ} , we should take $n_{\chi} \to n_{\chi} + n_{\phi} = 2n_{\chi}$. Consequently, today's DM relic abundance is given by

$$\Omega_{\chi}h^2 = 0.12 \left(\frac{y}{1.25 \times 10^{-11}}\right)^2 \left(\frac{m_{\chi}/m_{\nu_R}}{0.01}\right) \left(\frac{106.75}{g_{\star \text{f.i.}}}\right)^{\frac{3}{2}}.$$
(3.20)

Note that there is an interesting exception known as the SuperWIMP mechanism [54– 56] in which ν_R had been in thermal equilibrium but froze out before the freeze-in process $\nu_R \to \chi \phi$ started. Strictly speaking, this does not belong to the EF case because ν_R has already decoupled from the thermal bath when the majority of DM is being produced via $\nu_R \to \chi \phi$. We will discuss this exception in Sec. 3.2.3.

Similar to Eq. (3.5), there is also an upper limit of y for the validity of the freeze-in calculation. This is determined by equating n_{χ} to its equilibrium value. By solving it for y, we obtain the equilibrium criterion:

$$y_{\rm eq} = 1.7 \times 10^{-7} R_2, \ R_2 \equiv \left(\frac{m_{\nu_R}}{1 \text{ TeV}} \cdot \frac{g_H}{17.2}\right)^{1/2}.$$
 (3.21)

For $y \leq y_{eq}$, the above calculation for the EF case is valid. Otherwise, it becomes the EE case, as we will discuss below.

3.2.2 Case EE in the heavy ν_R limit

For y exceeding the limit in Eq. (3.21), χ and ϕ are in thermal equilibrium with the SM thermal bath, entering the EE case. As the universe cools down to a temperature below m_{χ} and m_{ϕ} , the energy and entropy stored in the ϕ - χ sector will be released via off-shell ν_R (virtual) to the SM thermal bath. The calculation for this scenario requires assumptions on the ν_R -SM interaction. To proceed, we assume that ν_R is coupled to a pair of SM particles (*F* and *B*) similar to Eq. (2.1) with a different coupling y':

$$\mathcal{L} \supset y' B \nu_R F \,. \tag{3.22}$$

Here F is a chiral fermion and B a scalar boson, with masses well below m_{ν_R} . With the interaction in Eq. (3.22), the most efficient channel converting particles in the dark sector particles to SM species is co-annihilation—see e.g. [57–59]. Through the *s*-channel tree-level process $\chi\phi \to FB$, each pair of χ and ϕ can be efficiently converted to a pair of F and B when the dark sector becomes non-relativistic. The process stops when χ and ϕ particles are too rare to meet each other, i.e., when $n_{\chi} = n_{\phi} \sim H/\langle \sigma v \rangle$. Hence similar to the standard WIMP freeze-out, there is also freeze-out in the co-annihilation scenario.

The co-annihilation collision term in the non-relativistic regime and in the small mass splitting limit reads:

$$C_{\chi\phi\to FB} \approx \frac{m_{\chi}^3 T^3 y^2 y'^2}{128\pi^4 m_{\nu_R}^2} e^{-2m_{\chi}/T} \,. \tag{3.23}$$

Correspondingly, the velocity-averaged cross section is

$$\langle \sigma v \rangle = \frac{1}{n_{\chi}^2} C_{\chi \phi \to FB} \approx \frac{y^2 y'^2}{32x} \left[\frac{e^{-x}}{m_{\nu_R} K_2(x)} \right]^2 \approx \frac{y^2 y'^2}{16\pi m_{\nu_R}^2}.$$
 (3.24)

The freeze-out temperature can be determined by solving $n_{\chi} \langle \sigma v \rangle = H$. Similar to Eq. (3.13), here we also have an approximate solution (see Appendix C):

$$x_{\rm f.o} \approx 22.2 + 4.88 \log_{10} \left[yy' \cdot \left(\frac{m_{\chi}}{\text{GeV}}\right)^{1/2} \cdot \left(\frac{m_{\nu_R}}{1 \text{ TeV}}\right)^{-1} \cdot \left(\frac{g_H}{17.2}\right)^{-1/2} \right].$$
(3.25)

The DM relic abundance can be computed using Eq. (3.14) with $x_{f,o}$ from Eq. (3.25) and $\langle \sigma v \rangle$ from Eq. (3.24). The result reads

$$\Omega_{\chi}h^{2} = 0.12 \left(\frac{m_{\nu_{R}}}{1 \text{ TeV}}\right)^{2} \cdot \left(\frac{0.39}{yy'}\right)^{2} \cdot \left(\frac{x_{\text{f.o}}}{22.2}\right) \cdot \left(\frac{106.75}{g_{\star\text{f.o.}}}\right)^{\frac{1}{2}}.$$
(3.26)

3.2.3 Cases FF and FE in the heavy ν_R limit

In these two cases, after a certain amount of ν_R particles are produced via freeze-in, they will eventually decay to ϕ and χ particles, if we assume that $\nu_R \to \chi \phi$ is the dominant decay mode of ν_R . This is expected in the FE case because ν_R should be more tightly coupled to the dark sector than to the SM sector. As for the FF case, a considerably large part of the parameter space should meet the assumption. Hence for both cases under this assumption, the number density of χ is determined by the number of ν_R accumulated during the first freeze-in:

$$n_{\chi} = 2 \frac{g_{\star}}{g_{\star \text{f},\text{i}}} T_{\text{SM}}^3 \frac{m_{\text{pl}}}{g_{\text{H}\text{f},\text{i}}} \int_{T_{\text{SM}}}^{\infty} C_{\text{SM} \to \nu_R} \tilde{T}_{\text{SM}}^{-6} d\tilde{T}_{\text{SM}} \,.$$
(3.27)

Here we have added a factor of 2 to account for the contribution of ϕ decay—see the discussion below Eq. (3.19).

Assuming ν_R is produced via Eq. (3.22), the integral in Eq. (3.27) gives

$$\Omega_{\chi}h^2 = 0.12 \left(\frac{m_{\chi}/m_{\nu_R}}{0.01}\right) \cdot \left(\frac{y'}{1.25 \times 10^{-11}}\right)^2 \cdot \left(\frac{106.75}{g_{\star f.i.}}\right)^{\frac{3}{2}}.$$
(3.28)

Note that the result is independent of y. Although y can be used to tell whether it belongs to the FF or FE case, the difference between the two cases becomes unimportant as long as ν_R dominantly decays to χ and ϕ . The dominance can be guaranteed if $y \gg y'$ which is plausible if one compares the benchmark value of $y' = 1.25 \times 10^{-11}$ in Eq. (3.28) to 1.7×10^{-7} in Eq. (3.21).

As previously mentioned in Sec. 3.2.1, heavy ν_R could allow for the SuperWIMP mechanism [54–56] in which DM is produced from frozen-out ν_R . This scenario is technically similar to the FF case because ν_R has decoupled from the SM thermal bath when it becomes responsible for DM production. Assuming that ν_R dominantly decays to the dark sector, the abundance of DM can be estimated from the number density of ν_R after the freeze-out. In the SuperWIMP mechanism, the dominance could be achieved by setting $m_B \geq m_{\nu_R}$ or $y \gg y'$. Under this assumption, the DM relic abundance is given by

$$\Omega_{\chi}h^2 = 0.12 \frac{m_{\nu_R}}{10 \text{ GeV}} \frac{m_{\chi}}{50 \text{ MeV}} \left(\frac{0.019}{y'}\right)^4 \frac{x_{\text{f.o.}}}{\sqrt{g_{\star\text{f.o.}}}}.$$
(3.29)

Here we would like to compare the FF and FE cases in the heavy ν_R limit with the corresponding ones in the light ν_R limit. In the FF case with $m_{\nu_R} \ll m_{\chi}$, the second

freeze-in $(2\nu_R \to 2\chi)$ stops when the ν_R kinetic energy drops below the mass of χ . In particular, after this freeze-in stops, a large amount of ν_R may still be present. In the FF case with $m_{\nu_R} \gg m_{\chi}$, the second freeze-in can only be stopped by complete depletion of ν_R . Hence the production rate $C_{\nu_R \to \chi \phi}$ does not need to compete with the Hubble expansion, rendering y unimportant as we have just mentioned.

As for the FE case, the $\nu_R \cdot \chi \cdot \phi$ coupled sector acquires a certain amount of energy and entropy injected from the SM thermal bath. The key difference between $m_{\nu_R} \ll m_{\chi}$ and $m_{\nu_R} \gg m_{\chi}$ is that the majority of this portion of energy and entropy will eventually be stored in the lightest species, which is ν_R (for $m_{\nu_R} \ll m_{\chi}$) or χ (for $m_{\nu_R} \ll m_{\chi}$).

Finally, we would like to comment on a potentially important two-to-two process in the heavy ν_R limit. For heavy ν_R , DM could be directly produced by the scattering of two SM particles, through an off-shell ν_R as the intermediate state, to ϕ and χ . At $T_{\rm SM} \ll m_{\nu_R}$, this is the dominant channel for DM production. The collision term of such a process assuming Eq. (2.6) is proportional to $y^2 y'^2 / m_{\nu_R}^2$, suppressed by $m_{\nu_R}^{-2}$ in the heavy ν_R limit, whereas the production via on-shell ν_R decay is exponentially suppressed. Despite the dominance of the two-to-two process at $T_{\rm SM} \ll m_{\nu_R}$, the overall contribution after integrating over $T_{\rm SM}$ is still subdominant compare to that of on-shell ν_R decay.

3.3 Discussions

The calculations presented above are all based on the assumption that the dark sector only interacts with the SM via ν_R . However, in this framework, the singlet scalar ϕ can be readily coupled to the SM Higgs via the Higgs portal. Assuming that the Higgs portal interaction is $\mathcal{L} \supset \frac{1}{4}\lambda |H|^2 |\phi|^2$, to avoid a significant amount of dark sector particles being produced via the Higgs portal, the coupling λ needs to sufficiently small.

Here let us briefly estimate the production rate of ϕ via $2H \to 2\phi$ in the early universe. The matrix element for this process is simply an energy-independent constant, $|\mathcal{M}| \sim \lambda$. For $m_{\phi} \gg m_{H}$, we obtain the following collision term:

$$C_{2H\to 2\phi} = \frac{\lambda^2}{128\pi^5} \left[m_{\phi} T_{\rm SM} K_1 \left(\frac{m_{\phi}}{T_{\rm SM}} \right) \right]^2, \qquad (3.30)$$

which is similar to the $2\nu_R \rightarrow 2\chi$ collision term. So it implies that for a sufficiently small λ , it should be in the freeze-in regime. Following the usual freeze-in calculation, we obtain that the abundance of ϕ (in the absence of ϕ decay) would be comparable to $\Omega_{\chi}h^2 = 0.12$ if $\lambda \approx 2.5 \times 10^{-11}$. Therefore, we conclude that for λ well below 10^{-11} , the Higgs portal interaction can be neglected.

4 Application: Type-I seesaw dark matter

In this section, we apply our framework to the type-I seesaw model, i.e., we take the type-I seesaw model and extend it by a pair of dark particles, χ and ϕ , with the interaction given by Eq. (2.1). Such an extension can accommodate an absolutely stable DM candidate and meanwhile still retain the virtue of being responsible for light neutrino masses via the type-I seesaw mechanism.

The type-I seesaw Lagrangian reads:

$$\mathcal{L} \supset y_{\nu} \tilde{H}^{\dagger} L \nu_R + \frac{1}{2} m_{\nu_R} \nu_R \nu_R + \text{h.c.}, \qquad (4.1)$$

where $H = \frac{1}{\sqrt{2}}(0, h + v)^T$ is the SM Higgs doublet in the unitarity gauge, $\tilde{H} \equiv i\sigma_2 H^*$, and $L = (\nu_L, e_L)^T$ is a lepton doublet. After the electroweak symmetry breaking, the above Yukawa interaction gives rises to the Dirac mass term, $\mathcal{L} \supset m_D \nu_L \nu_R$ with $m_D = y_{\nu} v / \sqrt{2}$. Then at low energy scales, ν_L acquires an effective Majorana mass via the seesaw mechanism:

$$m_{\nu_L} = m_D^2 / m_{\nu_R} \,. \tag{4.2}$$

For simplicity, we ignore the flavor structure and regard m_{ν_L} , m_D , and m_{ν_R} all as single-value quantities rather than 3×3 matrices.

It is noteworthy that the type-I seesaw model itself in principle allows for one of the ν_R 's to be a DM candidate in the keV regime—see e.g. the so-called ν MSM [60, 61]. However, since the simplest scenario where the keV sterile neutrino DM is produced by the Dodelson–Widrow mechanism [13] has been excluded by X-ray and Lyman- α observations, we do not consider the possibility that ν_R servers as a DM candidate in this work.

Given the approximately known scale of m_{ν_L} , we make use of the seesaw mass relation (4.2) to determine the Yukawa coupling:

$$y_{\nu} = \frac{1}{v} \sqrt{2m_{\nu_L} m_{\nu_R}} \approx 0.057 \cdot \left(\frac{m_{\nu_R}}{10^{12} \text{ GeV}}\right)^{1/2}$$
(4.3)

$$\approx 5.7 \times 10^{-8} \cdot \left(\frac{m_{\nu_R}}{1 \text{ GeV}}\right)^{1/2},$$
 (4.4)

where we have assumed $m_{\nu_L} \approx 0.1$ eV.

The seesaw mechanism also leads to the active-sterile neutrino mixing with the mixing angle given by

$$\theta \approx \sqrt{\frac{m_{\nu_L}}{m_{\nu_R}}} \approx 5.7 \times 10^{-13} y_{\nu}^{-1}$$
 (4.5)

Below we will show that y_{ν} and θ determined by the type-I seesaw relation are sufficiently large to thermalize ν_R . Hence the type-I seesaw DM is always in the EE or EF case.

4.1 Production rates of ν_R via Yukawa and gauge interactions

4.1.1 Above the electroweak scale

For m_{ν_R} well above the electroweak scale, the dominant process for ν_R production is $h + \nu_L \rightarrow \nu_R$. The collision term is

$$C_{h+\nu_L\to\nu_R} \approx y_{\nu}^2 \frac{m_{\nu_R}^3 T_{\rm SM}}{32\pi^3} K_1\left(\frac{m_{\nu_R}}{T_{\rm SM}}\right).$$
 (4.6)

Using $C_{h+\nu_L\to\nu_R}$, we can estimate [similar to the derivation of Eq. (3.21)] the criterion for ν_R reaching thermal equilibrium:

$$y_{\nu} \gtrsim 1.7 \times 10^{-7} \cdot \left(\frac{m_{\nu_R}}{1 \text{ TeV}}\right)^{1/2},$$
 (4.7)

which is always satisfied according to Eq. (4.3). Therefore, ν_R must have been in thermal equilibrium if y_{ν} is determined by the seesaw relation and m_{ν_R} is well above the electroweak scale.

4.1.2 Below the electroweak scale

For m_{ν_R} well below the electroweak scale, $h + \nu_L \rightarrow \nu_R$ is kinematically forbidden but $h \rightarrow \nu_L + \nu_R$ is allowed. The collision term is similar:

$$C_{h\to\nu_L+\nu_R} \approx y_{\nu}^2 \frac{m_h^3 T_{\rm SM}}{32\pi^3} K_1\left(\frac{m_h}{T_{\rm SM}}\right),\tag{4.8}$$

where $m_h \approx 125$ GeV is the Higgs mass.

In addition to the Higgs decay, ν_R can also be produced via Z and W^{\pm} decays. One can compute the decay widths either in the mass-eigenstate basis or in the chiral basis. The results from the two different approaches are the same. Taking Z decay as an example, in the mass-eigenstate basis, it is straightforward to obtain $C_{Z\to\nu_L+\nu_R} \approx \theta^2 C_{Z\to\nu_L+\nu_L}$, though strictly speaking one should replace ν_L and ν_R with their respective dominant mass eigenstates. In the chiral basis, one treats the Dirac mass term $m_D\nu_L\nu_R$ as a perturbative term so that the Feynman diagram contains a mass insertion proportional to m_D and an intermediate ν_L propagator proportional to $1/\not{p}$. The propagator effectively contributes a factor of $1/m_{\nu_R}$ after imposing the on-shell condition of ν_R . So the diagram is suppressed by a factor of $\theta \approx m_D/m_{\nu_R}$, and $C_{Z\to\nu_L+\nu_R}$ is suppressed by θ^2 . Therefore, in either way, one obtains

$$C_{Z \to \nu_L + \nu_R} \approx \frac{y_{\nu}^2 g^2 m_Z^3 T_{\rm SM}}{128\sqrt{2}\pi^3 \cos^2 \theta_W G_F m_{\nu_R}^2} K_1\left(\frac{m_Z}{T_{\rm SM}}\right),\tag{4.9}$$

$$C_{W \to e_L + \nu_R} \approx \frac{y_{\nu}^2 g^2 m_W^3 T_{\rm SM}}{64\sqrt{2}\pi^3 G_F m_{\nu_R}^2} K_1\left(\frac{m_W}{T_{\rm SM}}\right),\tag{4.10}$$

where g is the SM SU(2) gauge coupling and θ_W is the Weinberg angle. Compared to Eq. (4.8), the rates of ν_R production via Z and W^{\pm} decays are enhanced by $\sim 1/(G_F m_{\nu_R}^2)$. This conclusion is consistent with the previous calculation in Ref. [36].

Note that below the electroweak scale ν_R can also be produced via neutrino oscillations which, as we will show below, are much more efficient than the decay processes.

4.2 Production rates of ν_R via oscillations

Neutrino oscillations in the early universe have been extensively studied as they play a key role in sterile neutrino DM—see [9–12] for recent reviews. In the non-resonant production (NRP) regime with negligible lepton asymmetries, the production rate of ν_R reads [62–64]:

$$\Gamma_{\nu_R} \approx \theta_{\text{eff}}^2 \Gamma_{\nu_L} \,, \tag{4.11}$$

with

$$\Gamma_{\nu_L} \approx 1.27 G_F^2 \epsilon T_{\rm SM}^5 \,, \tag{4.12}$$

$$\sin^2 2\theta_{\rm eff} \approx \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - 2\epsilon T_{\rm SM} V_T / m_{\nu_R}^2)^2}, \qquad (4.13)$$

where $\epsilon \approx 3.15$ and θ_{eff} is the effective mixing angle which is related to but different from θ due to the thermal MSW potential, $V_T \approx -10.88 \times 10^{-9} \text{ GeV}^{-4} \times \epsilon T^5$. Unlike the conventional MSW potential proportional to G_F , V_T in the cosmological thermal plasma is proportional to G_F^2 . This is because the contributions of particles and anti-particles in the background cancel out at the leading order. According to Eq. (4.13), at low temperatures $(2\epsilon T_{\text{SM}}|V_T|/m_{\nu_R}^2 \ll \cos 2\theta)$, we have $\theta_{\text{eff}} \approx \theta$ while at high temperatures $\sin^2 2\theta_{\text{eff}}$ would be suppressed by T_{SM}^{-12} .

By comparing $C_{\text{osc.}} \equiv \Gamma_{\nu_R} n_{\nu_L}$ with Γ_{ν_R} given in Eq. (4.11) to Eq. (4.8), we obtain

$$\frac{\int C_{h \to \nu_L + \nu_R} T_{\rm SM}^{-6} dT_{\rm SM}}{\int C_{\rm osc.} T_{\rm SM}^{-6} dT_{\rm SM}} \lesssim 10^{-3} \frac{m_{\nu_R}}{1 \text{ GeV}} \ll 1, \qquad (4.14)$$

where as a crude approximation we have taken $\theta_{\text{eff}} \approx \theta$ for $2\epsilon T_{\text{SM}}|V_T|/m_{\nu_R}^2 \leq 10^{-1}$ and only integrated $C_{\text{osc.}}T_{\text{SM}}^{-6}$ below the corresponding temperature. The actual integral in the denominator is larger, leading to a more suppressed ratio. Eq. (4.14) implies that the production of ν_R via neutrino oscillations is much more efficient than that from Higgs decay, as long as m_{ν_R} is below the electroweak scale.

To check whether neutrino oscillations could bring ν_R into thermal equilibrium, we compare Γ_{ν_R} with the Hubble parameter:

$$\frac{\Gamma_{\nu_R}}{H} \sim 10^4 \gg 1\,,\tag{4.15}$$

where the ratio is estimated at the temperature when Γ_{ν_R} peaks, i.e., when $2\epsilon T_{\rm SM} V_T / m_{\nu_R}^2$ in the denominator of Eq. (4.13) is equal to $\cos 2\theta \approx 1$.

Eq. (4.15) implies that the production rate of ν_R via oscillation is sufficiently high to render ν_R thermal, provided that its mass is below the electroweak scale and the mixing angle $\theta \approx \sqrt{0.1 \text{ eV}/m_{\nu_R}}$ is determined by the seesaw relation. This conclusion is consistent with Ref. [65] which obtained $m_{\nu_R}^2 \sin^4 2\theta \gtrsim 10^{-5} \text{ eV}^2$ as the condition for ν_R to reach equilibrium—see Eqs. (38) and (39) therein.

4.3 Benchmarks

As we have shown in Sec. 4.1 and Sec. 4.2, the production rate of ν_R in the early universe is sufficiently high for ν_R to reach thermal equilibrium, provided that the interaction of ν_R with the SM is determined by the type-I seesaw mass relation². Therefore, the type-I seesaw DM is either in the EE or EF case, depending on the dark sector coupling y introduced in Eq. (2.1).

²This is only true when we neglect the flavor structure, which in principle could accommodate three rather hierarchical ν_R 's. In fact, one could have two ν_R 's responsible for the observed light neutrino masses and one ν_R with much more suppressed y_{ν} and θ than those given in Eqs. (4.4) and (4.5) so as to circumvent the restriction of the seesaw mass relation—see, e.g., Refs. [36, 37].

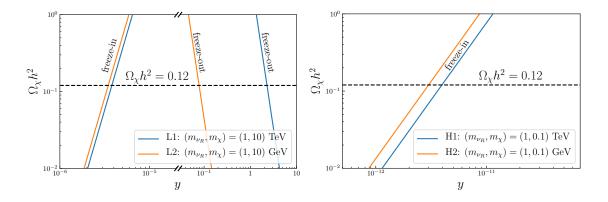


Figure 3. The relic density of type-I seesaw DM for benchmarks L1/2 and H1/2.

Now let us consider some specific benchmarks for the type-I seesaw DM:

Benchmark L1: $(m_{\nu_R}, m_{\chi}, m_{\phi}) = (1, 10, 12)$ TeV; Benchmark L2: $(m_{\nu_R}, m_{\chi}, m_{\phi}) = (1, 10, 12)$ GeV; Benchmark H1: $(m_{\nu_R}, m_{\chi}, m_{\phi}) = (1, 0.1, 0.12)$ TeV; Benchmark H2: $(m_{\nu_R}, m_{\chi}, m_{\phi}) = (1, 0.1, 0.12)$ GeV.

Here L/H indicates that ν_R is much lighter/heavier than χ . Two of the benchmarks are at TeV scales and the other two at GeV scales. We set a small but sizable mass splitting between m_{χ} and m_{ϕ} so that eventually all ϕ particles produced in the early universe decay to χ and other particles.

According to Tab. 2, Benchmarks L1 and L2 would be in the EE case if $y \gg 10^{-4}$ or in the EF case if $y \ll 10^{-4}$. For Benchmarks H1 and H2, the criterion is similar except that 10^{-4} is changed to 10^{-7} .

Let us first concentrate on Benchmarks L1 and L2, for which the DM candidate χ is produced via $2\nu_R \rightarrow 2\chi$. When y is small $(y \ll 10^{-4})$, this is a freeze-in process and, according to Tab. 2, we take Eq. (3.4) with $n_{\nu_R}/n_{\nu_R}^{\rm eq} = 1$ to compute $\Omega_{\chi}h^2$. The results are presented in the left panel of Fig. 3 as the increasing lines (labeled "freeze-in") which go across $\Omega_{\chi}h^2 = 0.12$ at

$$y = 3.8 \times 10^{-6} \text{ (for L1) or } 3.5 \times 10^{-6} \text{ (for L2)}.$$
 (4.16)

The small difference is caused by the value of g_{\star} at freeze-in [see $g_{\star f.i.}$ in Eq. (3.4)] which is evaluated at $T_{\rm SM} \sim m_{\chi}$. For L1, the freeze-in temperature is well above the electroweak scale, hence $g_{\star f.i.} \approx 106.75$. At 10 GeV (the case of L2), $g_{\star f.i.}$ falls to 83.51 [40].

If we increase y to a sufficiently large value, it will enter the EE regime. The DM candidate χ keeps in thermal equilibrium via $2\nu_R \leftrightarrow 2\chi$ until it becomes non-relativistic and freezes out from the thermal bath. According to Tab. 2, the relic abundance $\Omega_{\chi}h^2$ is determined by Eq. (3.17), corresponding to the decreasing lines (labeled "freeze-out") in the left panel of Fig. 3. The two lines go across $\Omega_{\chi}h^2 = 0.12$ at

$$y = 2.3 \text{ (for L1) or } 0.078 \text{ (for L2)}.$$
 (4.17)

Note that in the EE regime, larger m_{χ} implies larger y in order to account for the correct DM relic abundance. If one sets m_{χ} to higher values ~ $\mathcal{O}(100)$ TeV, then y would exceed the unitarity bound (~ $\sqrt{4\pi}$). This is known as the Griest-Kamionkowski bound, which dictates that thermal freeze-out DM should be lighter than 340 TeV [66], otherwise the annihilation cross section would be too large and violate the unitarity bound.

Next, let us turn to Benchmarks H1 and H2. Due to $m_{\nu_R} \gg m_{\chi}$, the DM candidate χ is directly produced via ν_R decay, $\nu_R \to \chi \phi$. Therefore, the critical value of y separating the EE and EF cases is much smaller than that for L1 and L2. For $y \ll 10^{-7}$, Benchmarks H1 and H2 are in the EF regime. Hence we take Eq. (3.20) to compute $\Omega_{\chi}h^2$. The results are presented in the right panel of Fig. 3 as the increasing lines which go across $\Omega_{\chi}h^2 = 0.12$ at

$$y = 3.9 \times 10^{-12} \text{ (for H1) or } 3.0 \times 10^{-12} \text{ (for H2)}.$$
 (4.18)

For larger y, in principle it could enter the EE regime. However, it turns that for Benchmarks H1 and H2 there are no freeze-out solutions if the neutrino-Higgs coupling y_{ν} is determined by the seesaw mass relation. This is because ν_R due to its heavy masses in these two benchmarks cannot be the final states of DM (co-)annihilation processes. The dark sector particles χ and ϕ have to annihilate or co-annihilate to lighter SM species. Possible processes could be $\chi \phi \to h \nu_L$ mediated by an off-shell ν_R , $\chi \chi \to \nu_L \nu_L$ or $e_L e_L$ via box diagrams, $\chi \phi \to 3\nu_L$ with ν_R and Z as intermediate states, etc. If $\chi \phi \to h \nu_L$ is not kinematically suppressed, it is the dominant process for DM freeze-out. Using Eq. (3.26) with $y' = y_{\nu}$ determined by Eq. (4.4) and neglecting the Higgs mass, one would obtain $y = 2.3 \times 10^5$ for Benchmark H1. This obviously violates the unitarity bound. In other words, if y is limited within the unitarity bound ($y \leq \sqrt{4\pi}$), then the cross section of the dominant freeze-out process would be too small to deplete the overproduced DM. For Benchmark H2 where m_{χ} is well below the electroweak scale, $\chi \phi \to h \nu_L$ is kinematically suppressed. One would have to consider other processes that are further suppressed by additional vertices in the diagrams.

There is, however, an interesting scenario that could potentially allow freeze-out solutions for ν_R heavier than χ . According to Eq. (3.26) where $\Omega_{\chi}h^2$ is roughly [neglecting all $\mathcal{O}(1 \sim 10)$ quantities] proportional to $m_{\nu_R}^2/y'^2 = \frac{v^2}{2}m_{\nu_R}/m_{\nu_L}$, if ν_R is sufficiently light (e.g., ~keV), then $\Omega_{\chi}h^2$ could be suppressed by m_{ν_R} , possibly leading to a freeze-out solution. Using Eqs. (3.26) and (3.25) with $yy' = 4\pi v^{-1}\sqrt{2m_{\nu_L}m_{\nu_R}}$ and $m_{\chi}/m_{\nu_R} = 0.1$, we find that the freeze-out solution is at $m_{\nu_R} \approx 5$ keV. By varying yy' and m_{χ}/m_{ν_R} one can obtain different values, but the mass scale cannot be much higher than the keV scale. Despite the existence of such freeze-out solutions, this scenario is ruled out by BBN observations due to additional thermal species at the MeV scale. If we only consider that these particles are well above the MeV scale, and if the magnitude of y_{ν} is subject to the seesaw constraint, then the type-I seesaw DM with ν_R heavier than χ cannot have a freeze-out solution.

In Fig. 4, we show the parameter space of the type-I seesaw DM model assuming two different mass ratios, one with $m_{\nu_R} \ll m_{\chi,\phi}$ and the other with $m_{\nu_R} \gg m_{\chi,\phi}$. As previously discussed, the heavy ν_R scenario does not allow for freeze-out solutions. So the blue curve in the right panel of Fig. 4 is a freeze-in solution for $\Omega_{\chi}h^2 = 0.12$. The value of y of this

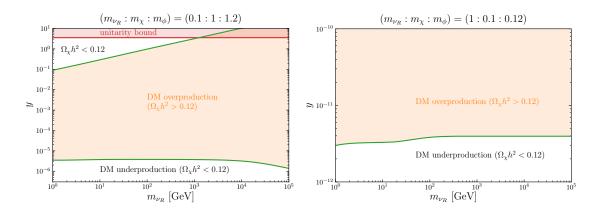


Figure 4. The parameter space of the type-I seesaw DM model for two different mass ratios, one with $m_{\nu_R} \ll m_{\chi,\phi}$ (left panel) and the other with $m_{\nu_R} \gg m_{\chi,\phi}$ (right panel).

solution varies in a limited range from 3×10^{-12} to 4×10^{-12} due to the variation of $g_{\star f.i.}$. In the left panel, the variation of the freeze-in solution due to $g_{\star f.i.}$ is insignificant but it bends down significantly at $m_{\nu_R} \gtrsim 10^4$ GeV because the *s*-channel contribution in Eq. (3.7) becomes dominant. In addition, there is also a freeze-out solution (the top green line) which would exceed the unitarity bound for $m_{\nu_R} \gtrsim 1$ TeV.

5 Observational consequences

DM produced via the RHN portal is difficult to probe via direct detection experiments because it does not directly interact with normal matter consisting of electrons and quarks. Nevertheless, we would like to point out an interesting process, $\chi + e_L^- \rightarrow \phi + \overline{\nu}_L + e_L^-$, with ν_R and W^- as intermediate states. This process might be possible if the active-sterile neutrino mixing is not too small. The major concern is that the non-relativistic χ might not have sufficient energy to cause an observable electron recoil, unless χ has been boosted.

Regarding indirect detection, the annihilation process $\chi \overline{\chi} \to \nu_R \overline{\nu_R}$ produces two RHNs which may subsequently decay to stable particles (γ , e^{\pm} , ν , etc.) in the SM, as has recently been studied comprehensively in Ref. [67]. Alternatively, e^{\pm} can be more straightforwardly produced from $\chi \overline{\chi}$ annihilation at the one-loop level with ϕ , ν_R , and W^{\pm} appearing in a box diagram. We make a crude estimation and find that that the annihilation cross section is too small to to be relevant to the current indirect detection experiments. In addition to annihilation, one might consider the ϕ decay as a source of indirect detection signals. As estimated below Eq. (3.2), if ν_R is light, $\phi \to \chi + \nu_R$ is possible but the coupling or the mass splitting has to be extremely suppressed in order to render ϕ long-lived at a cosmological time scale. If ν_R is heavy, ϕ may decay to χ and, via an off-shell ν_R , to some SM states. In this scenario, the lifetime of ϕ can be substantially longer but whether there is viable parameter space for observable indirect detection signals remains an open question.

Perhaps the most interesting observational consequence is the cosmological effective number of relativistic species, N_{eff} . If RHNs are sufficiently light, they may contribute to N_{eff} . As is well known, neutrinos can be Dirac particles and this possibility motivates extensive studies on the potential contribution to $N_{\rm eff}$ [68–72]. In our framework, a particularly noteworthy scenario is the FE case with very light ν_R . Through the freeze-in from the SM to ν_R , a considerably large amount of energy and entropy can be transferred to the dark sector, which after undergoing freeze-out at a relatively late epoch will release almost all the entropy stored in the dark sector to ν_R . In this scenario, the contribution to $N_{\rm eff}$ can be very significant without overproducing DM. In contrast, DM frozen-in from ν_R or more directly from ν_L usually cannot change $N_{\rm eff}$ significantly [41, 73] because the amount of energy transferred to DM is limited by $\Omega_{\chi}h^2 = 0.12$, corresponding to $n_{\chi} = 9.74 \times 10^{-12} {\rm eV}^{-4}/(2m_{\chi}) \approx 6.3 \times 10^{-4} {\rm cm}^{-3} \times ({\rm MeV}/m_{\chi})$ which is much lower than the neutrino number density. The potentially significant contribution to $N_{\rm eff}$ in the FE case calls for a more dedicated study in light of current and upcoming precision measurements of $N_{\rm eff}$.

6 Conclusion

In this paper we have presented a comprehensive investigation of a generic framework in which DM is produced from the RHN portal. As formulated in Eq. (2.1), our framework assumes that a RHN (ν_R) is coupled to a dark fermion (χ , which serves as DM with absolute stability) and a dark scalar (ϕ). Since the analyses crucially depend on whether the equilibrium between ν_R and the SM, and between the dark sector and ν_R , can be established, we propose four basic cases, namely EE, EF, FF, and FE, as illustrated in Fig. 1. Our results of the DM relic density and the criteria for identifying the four cases have been summarized in Tab. 2. Note that the results also depend on whether ν_R is heavy or light compared the mass scale of the dark sector, because the dominant production processes are decay and annihilation for heavy and light ν_R , respectively.

As a simple application, we considered the type-I seesaw model extended by the dark sector in our framework. We found that when the neutrino-Higgs Yukawa coupling is determined by the seesaw mass relation, the thermal history of the dark sector can only be in the EE or EF case. For light ν_R , both have viable solutions accounting for the correct DM relic abundance, while for heavy ν_R , only EF is viable—see Fig. 3.

Finally, we discussed a few observational consequences in this framework. Although it seems difficult to cause observable signals in direct detection experiments, the RHNportal DM may have important implications for indirect detection and future precision measurements of N_{eff} .

Acknowledgments

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A Collision terms

The analytical calculations presented below assume the Maxwell-Boltzmann statistics, which implies not only that the Maxwell-Boltzmann distribution $f = e^{-E/T}$ is used for bosonic and fermionic species, but also that $(1 \pm f)$ in Eq. (2.9) are neglected. With this assumption, the number density can be computed as follows:

$$n = \int_0^\infty f(p) \frac{dp^3}{(2\pi)^3} = \int_0^\infty \exp\left(-\frac{\sqrt{p^2 + m^2}}{T}\right) \frac{dp^3}{(2\pi)^3} = \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right).$$
(A.1)

The collision terms for two-to-two, two-to-one, and one-to-two processes take the following forms:

$$C_{1+2\to3+4} = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 f_1 f_2 |\mathcal{M}|^2 (2\pi)^4 \delta^4 , \qquad (A.2)$$

$$C_{1+2\to3} = \int d\Pi_1 d\Pi_2 d\Pi_3 f_1 f_2 |\mathcal{M}|^2 (2\pi)^4 \delta^4 , \qquad (A.3)$$

$$C_{1\to 3+4} = \int d\Pi_1 d\Pi_3 d\Pi_4 f_1 |\mathcal{M}|^2 (2\pi)^4 \delta^4 \,. \tag{A.4}$$

The collision terms computed assuming the Maxwell-Boltzmann statistics typically deviate from the exact values by $\sim 10\%$ —see e.g. Tab. III in Ref. [69].

A.1 Two-to-two collision terms

Let us first consider a two-to-two process, generically denoted by $1 + 2 \rightarrow 3 + 4$.

Assuming $m_1 = m_2 \equiv m$ and $f_{1,2} = e^{-E_{1,2}/T}$, the collision term can be computed analytically [74]:

$$C_{1+2\to3+4} \approx \frac{T}{32\pi^4} \int_{4m^2}^{\infty} s^{1/2} (s-4m^2) \sigma_{1+2\to3+4} K_1\left(\frac{s^{1/2}}{T}\right) ds \,, \tag{A.5}$$

where $\sigma_{1+2\rightarrow 3+4}$ is the total cross section of this process.

For $\nu_R + \overline{\nu_R} \to \chi + \overline{\chi}$, the cross section reads

$$\sigma_{2\nu_R \to 2\chi} = \frac{y^4}{16\pi s} \sqrt{1 - \frac{4m_\chi^2}{s}} + \mathcal{O}(\delta m^2/s) \,, \tag{A.6}$$

where $\delta m^2 \equiv m_{\phi}^2 - m_{\chi}^2$. Substituting Eq. (A.6) into Eq. (A.5), we obtain the result in Eq. (3.1). In particular, $C_{2\nu_R \to 2\chi}$ has the following high- and low-temperature limits (assuming ν_R in thermal equilibrium):

$$\lim_{T \to \infty} C_{2\nu_R \to 2\chi} \approx \frac{y^4}{128\pi^5} T^4 \,, \tag{A.7}$$

$$\lim_{T \to 0} C_{2\nu_R \to 2\chi} \approx \frac{y^4}{256\pi^4} T^3 m_{\chi} e^{-2m_{\chi}/T} \,. \tag{A.8}$$

For $\nu_R + \overline{\nu_R} \to \phi + \phi^*$, the cross section reads

$$\sigma_{2\nu_R \to 2\phi} = \frac{y^4}{8\pi s^2} \left[s \coth^{-1}\left(s/\overline{s}\right) - \overline{s} \right] + \mathcal{O}(\delta m^2/s) \,, \tag{A.9}$$

where $\overline{s} \equiv \sqrt{s^2 - 4m_{\phi}^2 s}$. Substituting Eq. (A.9) into Eq. (A.5), we find that the integral of s can only be calculated numerically but to a good approximation (~ 10% within 0.3 < $T/m_{\phi} < 30$) it can be approximated as

$$C_{2\nu_R \to 2\phi} \approx \alpha \log\left(\frac{T}{m_{\phi}} + e^{-\beta T/m_{\phi}}\right) C_{2\nu_R \to 2\chi},$$
 (A.10)

where $\alpha = 1.77$ and $\beta = 0.84$ are obtained via fitting. When the collision term is used in Eq. (2.13), we are mainly concerned with the integral $\int C_{2\nu_R \to 2\phi} T^{-6} dT$. Numerically performing the integral, we find

$$\lim_{m_{\phi} \to m_{\chi}} \frac{\int_{0}^{\infty} C_{2\nu_{R} \to 2\phi} T^{-6} dT}{\int_{0}^{\infty} C_{2\nu_{R} \to 2\chi} T^{-6} dT} \approx 1.87, \qquad (A.11)$$

which implies that with the same coupling y and the small mass splitting limit ($\delta m^2 \ll m_{\chi}^2$), the integrated production rate of ϕ is almost twice as large as that of χ .

For the co-annihilation process $\chi \phi \to BF$ appeared in Sec. 3.2.2, we only consider cases with approximately equal masses of χ and ϕ . In addition, the process is only important at $T \ll m_{\nu_R}$ so we assume that m_{ν_R} is sufficiently heavy in our calculation. Under these assumptions, the cross section is given by

$$\sigma_{\chi\phi\to BF} = \frac{sy^2 y'^2}{32\pi m_{\nu_R}^2 \sqrt{s\left(s - 4m_{\chi}^2\right)}} \,. \tag{A.12}$$

Substituting Eq. (A.12) into Eq. (A.5), we obtain the corresponding collision term. For a general value of T, there is no simple expression for the result. Technically one can express the integral in terms of the Meijer G-function, but the expression is not useful in practice. We are only interested in the approximate result at $T \ll m_{\chi}$, for which we have

$$C_{\chi\phi\to BF} \approx \frac{m_{\chi}^3 T^3 y^2 y'^2}{128\pi^4 m_{\nu_R}^2} e^{-2m_{\chi}/T} \,. \tag{A.13}$$

A.2 Two-to-one or one-to-two collision terms

For $C_{1+2\rightarrow3}$ and $C_{1\rightarrow3+4}$ in Eqs. (A.3) and (A.4), we assume the initial and final state masses are negligible, respectively. Then we adopt the results from Appendix A of Ref. [72] and obtain

$$C_{1+2\to3} \approx \frac{|\mathcal{M}|^2}{32\pi^3} K_1\left(\frac{m_3}{T}\right) m_3 T ,$$
 (A.14)

$$C_{1\to3+4} \approx \frac{|\mathcal{M}|^2}{32\pi^3} K_1\left(\frac{m_1}{T}\right) m_1 T$$
. (A.15)

It is worth mentioning that $C_{1\to 3+4}$ has the following high and low temperature limits:

$$\lim_{T \to \infty} C_{1 \to 3+4} \approx \frac{|\mathcal{M}|^2}{32\pi^3} T^2 \,, \tag{A.16}$$

$$\lim_{T \to 0} C_{1 \to 3+4} \approx \frac{|\mathcal{M}|^2}{32\pi^3} m_1^2 \sqrt{\frac{\pi}{2}} \left(\frac{T}{m_1}\right)^{3/2} e^{-m_1/T} \,. \tag{A.17}$$

For $C_{1+2\rightarrow 3}$, the limits are the same, except that m_1 is replaced by m_3 .

When dealing with the Boltzmann equation of a momentum distribution function [see Eq. (B.1)], one needs the following collision terms:

$$C_{1+2\to3}^{(f)} = \frac{1}{2E_3} \int d\Pi_1 d\Pi_2 f_1 f_2 |\mathcal{M}|^2 (2\pi)^4 \delta^4 , \qquad (A.18)$$

$$C_{1\to3+4}^{(f)} = \frac{1}{2E_3} \int d\Pi_1 d\Pi_4 f_1 |\mathcal{M}|^2 (2\pi)^4 \delta^4 , \qquad (A.19)$$

where 3 in $1 + 2 \rightarrow 3$ or $1 \rightarrow 3 + 4$ denotes the particle of which the distribution is to be computed.

Let us first compute Eq. (A.19), with the same assumptions as aforementioned. Integrating out $d\Pi_4$, we obtain

$$C_{1\to3+4}^{(f)} = \frac{1}{2E_3} \int \frac{2\pi p_1^2 dp_1 dc_{13}}{(2\pi)^3 2E_1} \frac{f_1 |\mathcal{M}|^2}{2E_4} (2\pi) \delta(E_1 - E_3 - E_4) \big|_{E_4 \to \sqrt{p_1^2 + p_3^2 - 2p_1 p_3 c_{13}}}, \quad (A.20)$$

where $c_{13} \equiv (\mathbf{p}_1 \cdot \mathbf{p}_3)/(p_1 p_3)$. One can further integrate out c_{13} together with the δ function, provided that the argument in the δ function can reach zero when c_{13} scans from -1 to 1. It is straightforward to find out that this requires

$$p_1 > p_1^{\min}, \quad p_1^{\min} = \left| \frac{m_1^2 - 4p_3^2}{4p_3} \right|.$$
 (A.21)

Integrating out c_{13} , we obtain

$$C_{1\to3+4}^{(f)} = \frac{1}{2E_3} \int_{p_1^{\min}}^{\infty} dp_1 \frac{f_1 |\mathcal{M}|^2 p_1^2}{8\pi E_1 (E_1 - E_3)} \frac{1}{\Delta}, \qquad (A.22)$$

where $\Delta = p_1 p_3 / |E_1 - E_3|$ arising from $\int \delta(E_1 - E_3 - E_4) dc_{13}$. Finally, integrating out p_1 with $f_1 = e^{-E_1/T}$, we obtain

$$C_{1\to3+4}^{(f)} = \frac{T|\mathcal{M}|^2}{16\pi p_3^2} e^{-\frac{m_1^2 + 4p_3^2}{4p_3 T}}.$$
(A.23)

One can check that $\int C_{1\to 3+4}^{(f)} d^3p_3/(2\pi)^3$ reproduces the result in Eq. (A.15).

As for $C_{1+2\to3}^{(f)}$, the calculation is similar except that p_1 has both upper and lower bounds, $(E_3 - p_3)/2 < p_1 < (E_3 + p_3)/2$, and the result is given as follows:

$$C_{1+2\to3}^{(f)} = \frac{|\mathcal{M}|^2}{16\pi E_3} e^{-\frac{E_3}{T}}.$$
 (A.24)

Again, one can check that $\int C_{1+2\rightarrow 3}^{(f)} d^3p_3/(2\pi)^3$ reproduces the result in Eq. (A.14).

B Calculation of the freeze-in momentum distribution

The momentum distribution function, f, is governed by the following Boltzmann equation:

$$\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right]f(t, p) = C^{(f)}, \qquad (B.1)$$

where $C^{(f)}$ is the collision term for f. In the freeze-in regime where the back-reaction is negligible, Eq. (B.1) can be written as an integral of $C^{(f)}$. Below we show the details.

First, we introduce a transformation of variables from (t, p) to (a, x_p) where a is the scale factor $(\dot{a}/a = H)$ and $x_p = p/T$. Then the Jacobian matrix of this transformation reads

$$\begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial p} \end{pmatrix} = \begin{pmatrix} Ha \ Hx_p \\ 0 \ T^{-1} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial a} \\ \frac{\partial}{\partial x_p} \end{pmatrix}.$$
(B.2)

Next, we express Eq. (B.1) in terms of (a, x_p) . As we will adopt (a, x_p) as new variables, we define

$$F(a, x_p) = f(t, p),$$
 (B.3)

and write

$$(1, -Hp) \begin{pmatrix} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial p} \end{pmatrix} f = (Ha, 0) \begin{pmatrix} \frac{\partial}{\partial a} \\ \frac{\partial}{\partial x_p} \end{pmatrix} F.$$
(B.4)

The zero entry in (Ha, 0) above allows us to write the left-hand side of Eq. (B.1) as a total derivative:

$$\frac{d}{da}F = \frac{C^{(f)}}{Ha}.$$
(B.5)

Replacing $da \to dT = Ta^{-1}da$, we obtain

$$F(a, p/T) = \int_{T}^{\infty} \frac{C^{(f)}(T')}{H(T')T'} dT'.$$
 (B.6)

For a given collision term, one can use Eq. (B.6) to compute the momentum distribution function f. For example, taking the collision term $C_{1\to3+4}^{(f)}$ in Eq. (A.23), we obtain

$$f = \frac{m_{\rm pl} e^{-x_p} |\mathcal{M}|^2}{8\pi g_H m_1^3 x_p} \left(\sqrt{\pi x_p} \operatorname{erf}\left(\frac{x_m}{2\sqrt{x_p}}\right) - y e^{-\frac{x_m^2}{4x_p}} \right), \tag{B.7}$$

where $x_m \equiv m_1/T$. In the limit of $x_m \gg \sqrt{x_p}$, the erf function reduces to unity and Eq. (B.7) reduces to

$$f = \frac{m_{\rm pl} |\mathcal{M}|^2}{8g_H m_1^3} \frac{1}{\sqrt{\pi x_p}} e^{-x_p} \,. \tag{B.8}$$

From Eq. (B.8), one can readily obtain the average value of x_p , $\langle x_p \rangle \equiv \int f x_p^3 dx_p / \int f x_p^2 dx_p = 2.5$, which is lower than the well-known values $(\langle x_p^{\text{MB}} \rangle, \langle x_p^{\text{FD}} \rangle, \langle x_p^{\text{BE}} \rangle) = (3, 3.15, 2.70)$ for Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein distributions, respectively.

For two-to-two processes, the integral in Eq. (B.6) generally cannot be analytically integrated but the numerical evaluation is straightforward.

C Calculation of the secluded freeze-out

In this appendix, we derive Eq. (3.14) for the secluded freeze-out scenario by revisiting the calculation for the standard freeze-out mechanism, with some steps modified in order to take into account that the dark sector has a different temperature as the SM sector.

Let us first inspect the freeze-out temperature, which is determined by

$$n_{\chi} \langle \sigma v \rangle = H \,,$$
 (C.1)

where H approximately depends on $T_{\rm SM}$ while n_{χ} depends on the dark sector temperature T_{χ} . Before freeze-out, n_{χ} in the non-relativistic regime is given by

$$n_{\chi} = \int e^{-E/T_{\chi}} \frac{d^3 p}{(2\pi)^3} = \frac{m_{\chi}^3}{2\pi^2 x} K_2(x) \approx e^{-x} m_{\chi}^3 \left(\frac{1}{2\pi x}\right)^{3/2} .$$
 (C.2)

where x has been defined in Eq. (3.12).

The freeze-out value of x, denoted by $x_{\text{f.o.}}$, is determined by substituting Eq. (C.2) and Eq. (2.10) into Eq. (C.1):

$$e^{-x_{\rm f.o}} = \frac{(2\pi)^{3/2} g_H}{\epsilon_{\rm f.o}^2 m_\chi m_{\rm pl} x_{\rm f.o}^{1/2} \langle \sigma v \rangle}, \qquad (C.3)$$

which can be solved numerically to determined $x_{f.o.}$

For the DM annihilation process, $2\chi \to 2\nu_R$, $\langle \sigma v \rangle$ is given by Eq. (3.15):

$$\langle \sigma v \rangle \approx \frac{y^4}{32\pi} \left[\frac{K_1(x_{\rm f.o})}{m_\chi K_2(x_{\rm f.o})} \right]^2 \approx \frac{y^4}{32\pi m_\chi^2} \left[1 - 3x_{\rm f.o}^{-1} + 6x_{\rm f.o}^{-2} + \mathcal{O}(x_{\rm f.o}^{-3}) \right].$$
 (C.4)

Using Eq. (C.4), we find that the solution of $x_{f.o.}$ in Eq. (C.3) can be approximately fitted by

$$x_{\rm f.o} = 35.5 + 9.52 \log_{10} \left[y \cdot \epsilon_{\rm f.o.}^{1/2} \cdot \left(\frac{m_{\chi}}{\text{GeV}} \frac{g_H}{17.2} \right)^{-1/4} \right].$$
(C.5)

For the DM co-annihilation process, $\chi \phi \to BF$, we have $\langle \sigma v \rangle = C_{\chi \phi \to BF}/(n_{\chi} n_{\phi})$ with $C_{\chi \phi \to BF}$ given by Eq. (A.13). The approximate solution reads:

$$x_{\rm f.o} \approx 22.2 + 4.88 \log_{10} \left[yy' \cdot \left(\frac{m_{\chi}}{\rm GeV}\right)^{1/2} \cdot \left(\frac{m_{\nu_R}}{1 \text{ TeV}}\right)^{-1} \cdot \left(\frac{g_H}{17.2}\right)^{-1/2} \right].$$
(C.6)

In Fig. 5, we show how well the above approximate expressions can fit the exact results. In the figure, \tilde{y} denotes the quantities in the square brackets of Eqs. (C.5) and (C.6).

Once $x_{f.o}$ is obtained from solving Eq. (C.3), the number density at the moment of freeze-out can be computed by

$$n_{\chi \text{f.o.}} = \frac{m_{\chi}^2 g_{H\text{f.o.}}}{m_{\text{pl}} \langle \sigma v \rangle x_{\text{f.o}}^2 \epsilon_{\text{f.o.}}^2} \,. \tag{C.7}$$

After freeze-out, the comoving number density $n_{\chi}a^3$ is conserved while in the SM sector, many species will eventually annihilate and inject their energy into lighter species. Nevertheless, the SM comoving entropy density $s_{\rm SM}a^3$ is conserved. Hence the ratio $n_{\chi}/s_{\rm SM}$ after freeze-out is a constant:

$$\frac{n_{\chi}}{s_{\rm SM}} = \left. \frac{n_{\chi}}{s_{\rm SM}} \right|_{\rm f.o.}.$$
 (C.8)

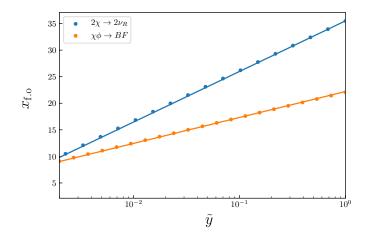


Figure 5. The approximate solutions (solid lines) for $x_{f,o}$ compared to the exact ones (dots). The blue and orange solid lines are obtained using Eqs. (C.5) and (C.6) respectively.

Using Eqs. (C.8), (C.7), (2.10), and $s_{\rm SM} = 2\pi^2 g_{\star}^{(s)} T_{\rm SM}^3 / 45$, we obtain

$$n_{\chi} = \frac{2\pi^{3/2} \epsilon x_{\text{f.o.}} g_{\star}^{(s)} T_{\text{SM}}^3}{3\sqrt{5}m_{\chi}m_{\text{pl}} \langle \sigma v \rangle \sqrt{g_{\star \text{f.o}}}} \,. \tag{C.9}$$

Eq. (C.9) remains valid even in relatively late eras dominated by matter or by vacuum energy, provided that $s_{\rm SM}$ in these eras are interpreted as the entropy density of radiation (photons and neutrinos) and $T_{\rm SM}$ in Eq. (C.9) is interpreted as the photon temperature. Although the universe is dominated by non-radiation content in these eras, the comoving entropy of photons and neutrinos is conserved. Taking $g_{\star}^{(s)} = 3.938$ and $T_{\rm SM} = 2.7$ K, we obtain from Eq. (C.9) today's energy density of DM:

$$\rho_{\chi+\overline{\chi}}|_{\text{today}} = 2m_{\chi}n_{\chi} = 1.35 \times 10^{-11} \text{ eV}^4 \cdot \frac{x_{\text{f.o.}}\epsilon}{\sqrt{g_{\star\text{f.o}}}} \cdot \frac{10^{-9} \text{ GeV}^{-2}}{\langle \sigma v \rangle} \,. \tag{C.10}$$

It is conventional to write the result in terms of $\Omega_{\rm DM}h^2$ which is defined by

$$\Omega_{\rm DM} \equiv \frac{\rho_{\rm DM}}{\rho_{\rm cri.}} \,, \ \rho_{\rm cri.} \equiv \frac{3H_0^2 m_{\rm pl}^2}{8\pi} \,, \ h \equiv \frac{H_0}{100 \ \rm km/sec/Mpc} \,, \tag{C.11}$$

where H_0 is the Hubble constant today and $\rho_{\rm cri.}$ is the critical energy density. Note that $\Omega_{\rm DM}h^2$ is independent of h (or H_0) so that it is not affected by the long-standing Hubble tension problem—see [75–77] for latest reviews. Combining Eqs. (C.10) and (C.11), we obtain

$$\frac{\Omega_{\rm DM}h^2}{0.12} = \frac{\rho_{\rm DM}}{9.74 \times 10^{-12} \text{ eV}^4} = \frac{x_{\rm f.o.}\epsilon}{\sqrt{g_{\star f.o.}}} \cdot \frac{1.4 \times 10^{-9} \text{ GeV}^{-2}}{\langle \sigma v \rangle} \,. \tag{C.12}$$

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