Data Dependence of Noor and SP Iterative Schemes when dealing with Quasi-Contractive Operators

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ABSTRACT

We prove results concerning data dependence of Noor and SP iterative schemes using certain quasi-contractive operators in real Banach spaces. Our results reveal that by choosing an approximate quasi-contractive operator (for which it is possible to compute the fixed point); we can approximate the fixed point of the given operator. An example is also provided to explain the validity of our results.

General Terms

Computational Mathematics

Keywords

SP iteration, Noor iteration, Quasi-Contractive Operators

1. INTRODUCTION

Let *E* be a real Banach space and *B* be a nonempty closed, convex subset of *E*. Let *T, S* be two self operators on *B*.

In a complete metric space, the Picard iterative process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n \; n = 0, 1, \dots \tag{1.1}$$

has been employed to approximate the fixed points of mappings satisfying the inequality

$$d(Tx, Ty) \leq \alpha\, d(x, y) \tag{1.2}$$

for all $x, y \in X$ and $\alpha \in [0, 1)$.

Condition (1.2) is called the Banach's contraction condition. Any operator satisfying (1.2) is called a strict contraction.

In 1953, W.R. Mann defined the Mann iteration [8] as

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n Tu_n\,, \tag{1.3}$$

where $\{\alpha_n\}$ is a sequences of positive numbers in [0,1].

In 1974, S. Ishikawa defined the Ishikawa iteration [7] as

$$s_{n+1} = (1-\alpha_n)s_n + \alpha_n Tt_n$$
$$t_n = (1-\beta_n)s_n + \beta_n Ts_n\,, \tag{1.4}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in [0,1].

For a given $x_0 \in B$ and $u_0 \in B$, we consider the Noor iteration [9] for operators *S* and *T* as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n\,, \quad y_n = (1-\beta_n)x_n + \beta_n Tz_n\,,$$
$$z_n = (1-\gamma_n)x_n + \gamma_n Tx_n\,, \tag{1.5}$$

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n Sv_n\,, \; v_n = (1-\beta_n)u_n + \beta_n Sw_n\,,$$
$$w_n = (1-\gamma_n)u_n + \gamma_n Su_n\,, \tag{1.6}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in [0,1] satisfying $\lim_{n\to\infty}\alpha_n = \lim_{n\to\infty}\beta_n = 0, \quad \sum_{n=0}^{\infty}\alpha_n = \infty$.

Also, for a given $x_0 \in B$ and $u_0 \in B$, we consider SP iteration [12] for operators *S* and *T* as

$$x_{n+1} = (1-\alpha_n)y_n + \alpha_n Ty_n\,, \quad y_n = (1-\beta_n)z_n + \beta_n Tz_n\,,$$
$$z_n = (1-\gamma_n)x_n + \gamma_n Tx_n\,, \tag{1.7}$$

$$u_{n+1} = (1-\alpha_n)v_n + \alpha_n Sv_n\,, \quad v_n = (1-\beta_n)w_n + \beta_n Sw_n\,,$$
$$w_n = (1-\gamma_n)u_n + \gamma_n Su_n\,, \tag{1.8}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in [0,1] satisfying $\beta_n \leq \alpha_n, \gamma_n \leq \alpha_n, \sum_{n=0}^{\infty}\alpha_n = \infty$.

Remarks

1. If $\gamma_n = 0$, then Noor iteration (1.5) reduces to the Ishikawa iteration (1.4).

2. If $\beta_n = \gamma_n = 0$, then Noor iteration (1.5) reduces to the Mann iteration (1.3).

3. If $\beta_n = \gamma_n = 0$, then SP iteration (1.7) reduces to the Mann iteration (1.3).

Several authors [1, 4, 12-20] have studied the convergence of various iterative schemes. In 1972, Zamfirescu [20] obtained the following interesting fixed point theorem:

Theorem 1.1.[20] Let (*E, d*) be a complete metric space and $T : E \longrightarrow E$ a mapping for which there exists real numbers *a, b* and c satisfying $a \in (0,1)$, $b, c \in (0, \frac{1}{2})$ such that for each pair $x, y \in E$ at least one of the following conditions hold

(i) $d(Tx,Ty) \le a\, d(x,y)$

(ii) $d(Tx,Ty) \le b[d(x,Tx) + d(y,Ty)]$

(iii) $d(Tx,Ty) \le c[d(x,Ty) + d(y,Tx)]$ (1.9)

Then T has a unique fixed point p and the Picard iteration $\{x_n\}$ defined by

$$x_{n+1} = Tx_n,\ n = 0, 1, \ldots$$

converges to p for any arbitrary but fixed $x_0 \in E$.

The operators satisfying the condition (1.9) are called Zamfirescu operators.

Berinde[1] introduced a new class of operators on an arbitrary Banach space satisfying

$$d(Tx,Ty) \le 2\delta\, d(x,Tx) + \delta d(x,y) \qquad (1.10)$$

$\forall\, x, y \in E$ and $\delta \in [0,1)$.

He proved that this class is wider than the class of Zamfirescu operators and used the Ishikawa iteration process to approximate fixed points of this class of operators in an arbitrary Banach space given in the form of following theorem.

Theorem 1.2[1] Let K be a nonempty closed convex subset of an arbitrary Banach space E and $T : K \longrightarrow K$ a mapping satisfying (1.9). Let $\{s_n\}_{n=0}^{\infty}$ be defined through the Ishikawa iteration (1.4) and $x_0 \in K$, where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences of positive numbers in [0,1] with $\{\alpha_n\}$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{s_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.

Rafiq [14] studied the convergence of the three step iteration process for quasi-contractive operators. Osilike [10] generalized and extended some of the results of Rhoades [15] by using the following more general contractive definition than (1.10): there exist $q \in [0,1)$, $L \ge 0$ such that

$$d(Tx, Ty) \le Ld(x, Tx) + q\, d(x, y),\ \forall\ x, y \in E \qquad (1.11)$$

He established the stability of Picard, Mann, Ishikawa and Noor iterative schemes using (1.11).

Imoru and Olatinwo [6] proved the stability of the Picard and the Mann iteration process for the following operator which is more general than the one introduced by Osilike [10] .The operator satisfies the following contractive definition : there exist $q \in [0, 1)$ and a monotone increasing function $\varphi : R^+ \longrightarrow R^+$ with $\varphi(0) = 0$, such that

$$d(Tx, Ty) \le \varphi(d(x, Tx)) + q\, d(x, y),\ \forall\ x, y \in E$$

Since the metric is induced by norrn, above contractive condition can be written as

$$\|Tx - Ty\| \le \varphi(\|x - Tx\|) + q\|x - y\|,\ \forall x, y \in X \qquad (1.12)$$

Olaleru and Akewe [11] proved the convergence of Jungck type iterations for generalized contractive-like operators in Banach space. Renu Chugh and Vivek Kumar [4] proved the convergence of Jungck-SP iterative scheme using quasi-contractive operators satisfying (1.12).

Remarks

4. Putting $X = Y$ and $S=I$ (identity mapping) in Corollary 2 of Theorem 2 [11], convergence of Noor iteration to a fixed point of quasi-contractive operators satisfying (1.12) can be obtained easily.

5. Putting $X = Y$, $L=0$ and $S=I$ (identity mapping) in Theorem 4.1 [4], convergence of SP iteration to a fixed point of quasi-contractive operators satisfying (1.12) can be obtained easily.

2. PRELIMINARIES

The results on data dependence for Picard iteration are in [2, 16]. Data dependence for Mann and Ishikawa iterations using contraction condition (1.2) was proved by Solutz in [17,18]. Data dependence for Ishikawa iterations when dealing with contractive like operators satisfying (1.12) was proved by Solutz in [19].

In 2010, by providing an example Ciric et al.[5] proved that Noor iteration can have a better covergence rate as compared to Mann and Ishikawa iterative schemes. In 2011, W. Pheungrattana and S. Suantai [12] defined SP iterative scheme and proved that this iterative scheme converges faster than Mann, Ishikawa and Noor iterative schemes for increasing functions. This is the main reason for considering Noor and SP iterative schemes in this paper.

Motivated by the work of Solutz[17-19], in this paper we prove the data dependence results for Noor and SP iterative schemes using the quasi-contractive operators satisfying (1.12). An example is also provided to explain the results.

We shall need the following Lemma to prove our results.

Lemma 2.1.[19] Let $\{a_n\}_{n=0}^{\infty}$ be a nonnegative sequence for which one suppose there exist $n_0 \in N$ such that for all $n \geq n_0$ one has satisfied the following inequality :

$$a_{n+1} \leq (1-r_n)a_n + r_n t_n ,$$

where $r_n \in (0,1)$, for all $n \in N$, $\sum_{n=1}^{\infty} r_n = \infty$ and $t_n \geq 0 \ \forall n \in N$.

Then ,

$$0 \leq \lim_{n\to\infty}\sup a_n \leq \lim_{n\to\infty}\sup t_n .$$

3. MAIN RESULTS

First we prove the result on data dependence for Noor iterative scheme.

Theorem 3.1. Let K be a nonempty convex, closed subset of a real Banach space E and $T: K \longrightarrow K$ a quasi-contractive operator satisfying (1.12). Let S be an approximate operator of T i.e.

$$\| Tx - Sx \| \leq \varepsilon$$

for all $x \in K$, $\varepsilon > 0$ and $\{x_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ be the Noor iterations associated to T, respectively to S, starting from x_0.

If $Tx^* = x^*$ and $Su^* = u^*$ (u^* taken nearest to x^*), then we have

$$\| x^* - u^* \| \leq \frac{\varepsilon}{1-q}$$

Proof. It follows from (1.5) and (1.6) that

$$\begin{aligned}
\| x_{n+1} - u_{n+1} \| &\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n \| Ty_n - Sv_n \| \\
&\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n \| Ty_n - Tv_n \| \\
&\quad + \alpha_n \| Tv_n - Sv_n \| \\
&\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n q \| y_n - v_n \| \\
&\quad + \alpha_n \varphi(\| y_n - Ty_n \|) + \alpha_n \varepsilon \\
&\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n \varepsilon + q\alpha_n(1-\beta_n)\| x_n - u_n \| \\
&\quad + q\alpha_n\beta_n \| Tz_n - Sw_n \| + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n \varepsilon + q\alpha_n(1-\beta_n)\| x_n - u_n \| \\
&\quad + q\ \alpha_n\beta_n \| Tz_n - Tw_n \| + q\alpha_n\beta_n \| Tw_n - Sw_n \| \\
&\quad + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\leq (1-\alpha_n)\| x_n - u_n \| + \alpha_n \varepsilon + \alpha_n q(1-\beta_n)\| x_n - u_n \| \\
&\quad + q^2\alpha_n\beta_n \| z_n - w_n \| + q\alpha_n\beta_n \varphi(\| z_n - Tz_n \|) \\
&\quad + q\alpha_n\beta_n\varepsilon \ + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\leq [1-\alpha_n + \alpha_n q(1-\beta_n)]\| x_n - u_n \| + \alpha_n \varepsilon \\
&\quad + q^2\alpha_n\beta_n(1-\gamma_n)\| x_n - u_n \| \\
&\quad + q^2\alpha_n\beta_n\gamma_n \| Tx_n - Su_n \| + q\alpha_n\beta_n \varphi(\| z_n - Tz_n \|) \\
&\quad + q\ \alpha_n\beta_n\varepsilon + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\leq [1-\alpha_n + \alpha_n q(1-\beta_n)]\| x_n - u_n \| + \alpha_n \varepsilon \\
&\quad + q^2\alpha_n\beta_n(1-\gamma_n)\| x_n - u_n \| \\
&\quad + q^2\alpha_n\beta_n\gamma_n \| Tx_n - Tu_n \| + q^2\alpha_n\beta_n\gamma_n\varepsilon \\
&\quad + q\ \alpha_n\beta_n \varphi(\| z_n - Tz_n \|) + q\ \alpha_n\beta_n\varepsilon + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\leq [1-\alpha_n + \alpha_n q(1-\beta_n)]\| x_n - u_n \| + \alpha_n \varepsilon \\
&\quad + q^2\alpha_n\beta_n(1-\gamma_n)\| x_n - u_n \| \\
&\quad + q^3\alpha_n\beta_n\gamma_n \| x_n - u_n \| + q^2\alpha_n\beta_n\gamma_n \varphi(\| x_n - Tx_n \| \\
&\quad + q^2\alpha_n\beta_n\gamma_n\varepsilon + +q\ \alpha_n\beta_n \varphi(\| z_n - Tz_n \|) \\
&\quad + q\ \alpha_n\beta_n\varepsilon + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&= [1-\alpha_n(1-q) - \alpha_n\beta_n q(1-q) - \alpha_n\beta_n\gamma_n q^2(1-q)]\| x_n - u_n \| \\
&\quad + \frac{\alpha_n(1-q)[q^2\beta_n\gamma_n\varphi(\| x_n - Tx_n \|) + q\beta_n\varphi(\| z_n - Tz_n \|)]}{1-q} \\
&\quad + \frac{\alpha_n(1-q)[\varphi(\| y_n - Ty_n \|) + q^2\beta_n\gamma_n\varepsilon + q\ \beta_n\varepsilon + \varepsilon]}{1-q} \qquad (3.1)
\end{aligned}$$

Now, φ is a continuous mapping and $\{x_n\}$,$\{y_n\}$,$\{z_n\}$, converges to a fixed point of T (using Remark 4), hence

$$\lim_{n\to\infty}\varphi(\|x_n - Tx_n\|) = \lim_{n\to\infty}\varphi(\|y_n - Ty_n\|) = \lim_{n\to\infty}\varphi(\|z_n - Tz_n\|) = 0.$$

Putting $r_n = \alpha_n(1-q)$ and

$$t_n = \frac{q^2\beta_n\gamma_n\varphi(\| x_n - Tx_n \|) + q\beta_n\varphi(\| z_n - Tz_n \|)}{1-q} + \frac{\varphi(\| y_n - Ty_n \|) + q^2\beta_n\gamma_n\varepsilon + q\ \beta_n\varepsilon + \varepsilon}{1-q}$$

in (3.1) and using Lemma (2.1) , we get

$\| x^* - u^* \| \leq \frac{\varepsilon}{1-q}$. Hence the result.

Now, we prove the result on data dependence for SP iterative scheme.

Theorem 3.2. Let K be a nonempty convex, closed subset of a real Banach space E and $T: K \longrightarrow K$ a quasi-contractive operator satisfying (1.12). Let S be an approximate operator of T i.e.

$$\| Tx - Sx \| \leq \varepsilon$$

for all $x \in K$, $\varepsilon > 0$ and $\{x_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ be the SP iterations associated to T, respectively to S, starting from x_0.

If $Tx^* = x^*$ and $Su^* = u^*$(u^* taken nearest to x^*), then we have

$$\| x^* - u^* \| \leq \frac{3\varepsilon}{1-q}$$

Proof. It follows from (1.7) and (1.8) that

$$\begin{aligned}
\| x_{n+1} - u_{n+1} \| &\le (1-\alpha_n)\| y_n - v_n \| + \alpha_n \| Ty_n - Sv_n \| \\
&\le (1-\alpha_n)\| y_n - v_n \| + \alpha_n \| Ty_n - Tv_n \| \\
&\qquad + \alpha_n \| Tv_n - Sv_n \| \\
&\le (1-\alpha_n)\| y_n - v_n \| + \alpha_n q \| y_n - v_n \| \\
&\qquad + \alpha_n \varphi(\| y_n - Ty_n \|) + \alpha_n \varepsilon \\
&\le (1-\alpha_n(1-q))\| y_n - v_n \| + \alpha_n \varphi(\| y_n - Ty_n \|) \\
&\qquad + \alpha_n \varepsilon \qquad (3.2)
\end{aligned}$$

Now, we have the following estimates :

$$\begin{aligned}
\| y_n - v_n \| &\le (1-\beta_n)\| z_n - w_n \| + \beta_n \| Tz_n - Sw_n \| \\
&\le (1-\beta_n)\| z_n - w_n \| + \beta_n \| Tz_n - Tw_n \| \\
&\qquad + \beta_n \| Tw_n - Sw_n \| \\
&\le (1-\beta_n)\| z_n - w_n \| + \beta_n q \| z_n - w_n \| \\
&\qquad + \beta_n \varphi(\| z_n - Tz_n \|) + \beta_n \varepsilon \\
&\le (1-\beta_n(1-q))\| z_n - w_n \| + \beta_n \varphi(\| z_n - Tz_n \|) \\
&\qquad + \beta_n \varepsilon \qquad (3.3)
\end{aligned}$$

and

$$\begin{aligned}
\| z_n - w_n \| &\le (1-\gamma_n)\| x_n - u_n \| + \gamma_n \| Tx_n - Su_n \| \\
&\le (1-\gamma_n)\| x_n - u_n \| + \gamma_n \| Tx_n - Tu_n \| \\
&\qquad + \gamma_n \| Tu_n - Su_n \| \\
&\le (1-\gamma_n)\| x_n - u_n \| + \gamma_n q \| x_n - u_n \| \\
&\qquad + \gamma_n \varphi(\| x_n - Tx_n \|) + \gamma_n \varepsilon \\
&\le (1-\gamma_n(1-q))\| x_n - u_n \| + \gamma_n \varphi(\| x_n - Tx_n \|) \\
&\qquad + \gamma_n \varepsilon \qquad (3.4)
\end{aligned}$$

Substituting (3.3) and (3.4) in (3.2), we get

$$\begin{aligned}
&\le [1-\alpha_n(1-q)][1-\beta_n(1-q)][1-\gamma_n(1-q)]\| x_n - u_n \| \\
&\qquad + \alpha_n \varphi(\| y_n - Ty_n \|) + \alpha_n \varepsilon \\
&\qquad + [1-\alpha_n(1-q)]\beta_n\, \varphi(\| z_n - Tz_n \|) \\
&\qquad + [1-\alpha_n(1-q)]\beta_n \varepsilon \\
&\qquad + [1-\alpha_n(1-q)][1-\beta_n(1-q)]\gamma_n \varphi(\| x_n - Tx_n \|) \\
&\qquad + [1-\alpha_n(1-q)][1-\beta_n(1-q)]\gamma_n \varepsilon
\end{aligned}$$

$$\begin{aligned}
&\le [1-\alpha_n(1-q)]\| x_n - u_n \| + \alpha_n \varphi(\| y_n - Ty_n \|) + \alpha_n \varepsilon \\
&\quad + \beta_n\, \varphi(\| z_n - Tz_n \|) + \beta_n \varepsilon + \gamma_n \varphi(\| x_n - Tx_n \|) + \gamma_n \varepsilon \\
&\le [1-\alpha_n(1-q)]\| x_n - u_n \| \\
&\quad + \frac{\alpha_n (1-q)[\varphi(\| y_n - Ty_n \|) + \varphi(\| z_n - Tz_n \|)]}{(1-q)} \\
&\qquad + \frac{\alpha_n (1-q)[\varphi(\| x_n - Tx_n \|) + 3\varepsilon]}{(1-q)}
\end{aligned}$$

(3.5)

Now, φ is a continuous mapping and $\{x_n\}$,$\{y_n\}$, $\{z_n\}$ converges to a fixed point of T (using Remark 5), hence $\lim_{n\to\infty} \varphi(\|x_n - Tx_n\|) = \lim_{n\to\infty} \varphi(\|y_n - Ty_n\|) = \lim_{n\to\infty} \varphi(\|z_n - Tz_n\|) = 0.$

Putting $r_n = \alpha_n (1-q)$,

$$t_n = \frac{\varphi(\| y_n - Ty_n \|) + \varphi(\| z_n - Tz_n \|) + \varphi(\| x_n - Tx_n \|) + 3\varepsilon}{(1-q)}$$

in (3.5) and using Lemma (2.1) ,we get

$\| x^* - u^* \| \le \dfrac{3\varepsilon}{1-q}$. Hence the result.

Remark 6. Since Mann and Ishikawa iterative schemes are special cases of Noor iterative scheme , data dependence results of these iterative schemes can be obtained similarly.

The following example follows from [19].

Example 3.1 Let $T : R \longrightarrow R$ be defined by

$T(x) = 0$ if $x \in (-\infty, 2]$

$= -0.5$ if $x \in (2, +\infty)$

with unique fixed point 0. Consider the mapping S: $R \longrightarrow R$ defined by

$S(x) = 1$ if $x \in (-\infty, 2]$

$= -1.5$ if $x \in (2, +\infty)$

with unique fixed point 1.

Take $\varepsilon = 1$ such that $\| Tx - Sx \| \le 1$.

Set $u_0 = x_0 = 0$ and $\alpha_n = \beta_n = \gamma_n = \dfrac{1}{\sqrt{n+1}}$.

By using computer programs in C++ , Noor and SP iterative schemes leads to the following table .

Number of Iterations	Noor Iteration	Number of Iterations	SP Iteration
1	0.707107	1	0.974874
2	0.876209	2	0.998103
3	0.938104	3	0.999763
-	-	-	-
6	0.987406		0.999998
7	0.991858		0.999999
8	0.994572		**1**
9	0.996289		**1**
-	-	-	-
47	0.999999		**1**
48	0.999999		**1**
49	0.999999		**1**
50	0.999999		**1**
51	**1**		**1**
52	**1**		**1**
53	**1**		**1**

Hence Noor and SP iterative schemes when applied to S converges to the fixed point $u^* = x^*$ =1. Obviously distance between the fixed points is of *S* and *T* is 1. Without computing the fixed point of *S* (and without knowing it), from Theorem (3.1) , we have the following estimate :

$$\| x^* - u^* \| \le \frac{1}{1-q} = \frac{1}{1-0.2} = 1.2$$

Also if $\varepsilon = \frac{1}{3}$, then using Theorem (3.2) we have the following estimate :

$$\| x^* - u^* \| \le \frac{1}{1-q} = \frac{1}{1-0.2} = 1.2$$

4. CONCLUSION

From Example 3.1, we conclude that instead of computing fixed points of *S*, if we choose *T* more close to *S*, the distance between the fixed points of *S* and *T* will shrink too.

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