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Data-driven multivariable controller design using Ellipsoidal Unfalsified Control

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Abstract— Ellipsoidal Unfalsified Control is a data-driven, plant-model-free control design method. In this work, this framework is extended to cover full-block multivariable controllers. A new controller structure and a sequential update procedure are proposed. A simulation example shows the effectiveness of the method.

I. Introduction

Unfalsified Control is an emerging, data-driven control design method that uses merely measured input/output data of the system to be controlled, without any plant model. As a consequence, the approximations and assumptions introduced in the plant modeling step are omitted. Additional motives to support data-driven control design are, e.g., a priori definition of controller complexity and adaptation to specific disturbances.

In recent work, the control design method of Ellipsoidal Unfalsified Control (EUC) is introduced [1], [2]. EUC originates from the Unfalsified Control paradigm, as is introduced in [3]. In this method, the ability of controllers to meet a given performance criterium is evaluated by employment of a fictitious reference signal. This mathematical "trick" enables the evaluation of controllers without the need to actually implement them. If this ability is achieved for a specific controller, that controller is "unfalsified" for the current measurement data. By considering the intersection of the sets of unfalsified controllers for all time instances, a recursive algorithm is constructed.

In early works on Unfalsified Control, the control parameter space was gridded, resulting in a finite, but often large, set of candidate controllers. This restriction is lifted by applying a quadratic performance requirement to a control law, where the control parameters appear affine [4]. As a result, the region of unfalsified control parameters is continuous and can be described by an ellipsoid, resulting in simple algebraic equations to describe the entire set.

In Ellipsoidal Unfalsified Control [1], the ellipsoidal description of the region of unfalsified controllers is combined with an l_∞ performance requirement. This combination enables the analytic update of the region of unfalsified controllers with new measurement data. Real-time online implementation of the control design method is feasible, even on systems with a high sample rate.

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In this paper, the extension of the EUC algorithm to multi-input multi-output (MIMO) plants is considered. The current algorithm is only suited for single-input single-output (SISO) plants. A diagonal controller can be constructed by applying the current EUC algorithm to several inputs and outputs (decentralized control). However, the performance with decentralized control may be poor because no attempt is made to counteract the interactions [5]. To overcome this shortcoming, an extension to the EUC framework is proposed to cover general, full-block multivariable controllers.

Section II recalls the procedure for general Ellipsoidal Unfalsified Control for SISO plants. In Section III the algorithm for the control of MIMO plants is derived. Section IV presents an example application and Section V contains the conclusions.

II. Ellipsoidal Unfalsified Control

In [1], the Ellipsoidal Unfalsified Control approach is introduced. In this section, the data-driven, plant-model-free controller design method is recalled. It is shown, how this algorithm selects the region of controllers, for which the ability to meet the performance requirement is not falsified by measurement data. Furthermore, the selection of a controller from this region is handled. In the next section, the extension to MIMO plants will be discussed.

A. Candidate Controllers

A "cloud" of candidate controllers is selected, the candidate controller set. When no measurement data is available yet, no controllers have been falsified, and the candidate controller set is, trivially, equal to the initial candidate controller set. When measurement data is available, though, candidate controllers might get falsified.

Definition 1: The *True Unfalsified set* is the set of controllers, which are currently unfalsified by all available measurement data.

In Ellipsoidal Unfalsified Control, the True Unfalsified set is approximated by an ellipsoid. This ellipsoid is used as a representation of the True Unfalsified set and is denoted as the Unfalsified set. The Unfalsified set is used as the candidate controller set.

The need for gridding of the candidate controller set is overcome by describing the Unfalsified set with a continuous region. The description of the Unfalsified set with an ellipsoid allows for the evaluation of the entire set with simple algebraic equations, see [4].

The Unfalsified set at time t_{k-1} is described by

$$\mathcal{E}(t_{k-1}) = \{\underline{\theta} \,|\, \big(\underline{\theta} - \underline{\theta}_c(t_{k-1})\big)^T \Sigma^{-1}(t_{k-1}) \big(\underline{\theta} - \underline{\theta}_c(t_{k-1})\big) \leq 1\} \tag{1}$$

with $\underline{\theta} \in \mathbb{R}^p$ the controller parameters, $\underline{\theta}_c(t_k) \in \mathbb{R}^p$ the center of the ellipsoid and $\Sigma(t_k) \in \mathbb{R}^{p \times p}$ the symmetric, positive definite matrix that describes the shape of the ellipsoid.

B. Fictitious Reference

For a given controller in the candidate controller pool, a "fictitious reference" r^{fict} can be constructed. The "fictitious reference" is an abstract notion, but it can be thought of as a *controller parameter dependent* reference that would have resulted in exactly the measured input and output, if that controller would have been in the loop during the measurements.

Let the controller structure be chosen such, that $r^{\text{fict}}(\underline{\theta}, t_k)$ is affine in the controller parameters $\underline{\theta}$.

$$r^{\text{fict}}(\underline{\theta}, t_k) = w(u(t_k), y(t_k), q^{-1})^T \underline{\theta} \tag{2}$$

Here, q^{-1} is the discrete time backward shift operator $\big(q^{-1}x(t_k) \overset{\text{def}}{=} x(t_{k-1})\big)$. Let $\check{\theta}(t_k)$ denote the actually implemented controller parameter set at time t_k. For $\underline{\theta} = \check{\theta}(t_k)$, $r^{\text{fict}}(\underline{\theta}, t_k)$ exactly results in the actual reference $r(t_k)$, provided that $w(\cdot)$ is stably invertible for $u(t_k)$ (Stably-Causally-Left-Invertible (SCLI), [6, Def. 9]). Of course, the restriction that $w(\cdot)$ is SCLI limits the selection of candidate controllers.

The control action $u(t_k)$ is computed from (2), with $\underline{\theta} = \check{\theta}(t_k)$ and given the reference $r(t_k)$ and measured input and output data.

The concept of a fictitious reference enables the evaluation of controllers even if they were not in the loop at the time of the measurement, since the actual (measured) output $y(t_k)$ can be compared with the desired output $G_m(q^{-1})r^{\text{fict}}(\underline{\theta}, t_k)$ for an arbitrary controller parameter set. Here, $G_m(q^{-1})$ is a reference model, which defines the desired closed loop dynamics.

C. Unfalsification

Given a desired performance specification, and exploiting the fictitious reference, a region can be constructed of controller parameters which are unfalsified by current measurement data.

Let the performance requirement be defined as a (time-dependent) bound on the tracking error $0 < \Delta(t_k) < \infty$ [1] plus a κ-weighted control effort, $\kappa(t_k) > 0$. Then, the region of controller parameters that is unfalsified by current measurement data at time t_k is given by

$$\mathcal{U}(t_k) = \{\underline{\theta} \,|\, |e^{\text{fict}}(\underline{\theta}, t_k)| + \kappa(t_k)|u(t_k)| \leq \Delta(t_k)\} \tag{3}$$

$$= \{\underline{\theta} \,|\, -\hat{\Delta}(t_k) \leq e^{\text{fict}}(\underline{\theta}, t_k) \leq \hat{\Delta}(t_k)\} \tag{4}$$

with

$$e^{\text{fict}}(\underline{\theta}, t_k) = G_m(q^{-1})r^{\text{fict}}(\underline{\theta}, t_k) - y(t_k) \tag{5}$$

$$\hat{\Delta}(t_k) = \Delta(t_k) - \kappa(t_k)|u(t_k)| \tag{6}$$

It should be noted that $\mathcal{U}(t_k)$ is empty for $\hat{\Delta}(t_k) < 0$. The introduction of the κ-weighted control effort therefore limits the control effort to $|u(t_k)| \leq \Delta(t_k)/\kappa(t_k)$.

Note that with (3) not only controllers are falsified that, ultimately, do not meet the performance requirement, but also those that are not able to do that starting from the current controller states. This implies that switching controller parameters $\underline{\theta}$ does not need any accompanying measures, like resetting the controller states, to guarantee a suitable transient.

From the combination of (2) and (4) through (6), it is clear that $\mathcal{U}(t_k)$ defines two parallel half-spaces in the controller parameter space:

$$\mathcal{U}(t_k) = \{\underline{\theta} \,|\, -1 \leq \frac{G_m(q^{-1})w(\cdot)^T}{\hat{\Delta}(t_k)}\underline{\theta} - \frac{y(t_k)}{\hat{\Delta}(t_k)} \leq 1\}$$

D. Update Unfalsified set

The region of controllers that is unfalsified by all available measurement data (hence, including all past and present measurement data) is given by the intersection of the candidate controllers $\mathcal{E}(t_{k-1})$ from section II-A (the Unfalsified Set) and the controllers $\mathcal{U}(t_k)$ from section II-C (the controllers that are unfalsified by the present measurement data).

To maintain an ellipsoidal Unfalsified set, the intersection $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ is approximated by a minimum-volume outer-bounding ellipsoid $\mathcal{E}(t_k)$. Since $\mathcal{U}(t_k)$ defines two parallel half-spaces, this approximation can be computed analytically, as is shown in [7]. To compute $\mathcal{E}(t_k)$, define the variables

$$y_k = \frac{y(t_k)}{\hat{\Delta}(t_k)} \tag{7}$$

$$\phi_k = \frac{G_m(q^{-1})w(u(t_k), y(t_k), q^{-1})}{\hat{\Delta}(t_k)} \tag{8}$$

$$g = \phi_k^T \Sigma(t_{k-1})\phi_k \tag{9}$$

$$a_+ = \max\left(\frac{y_k - \phi_k^T \underline{\theta}_c(t_{k-1}) - 1}{\sqrt{g}}, -1\right) \tag{10}$$

$$a_- = \max\left(\frac{-y_k + \phi_k^T \underline{\theta}_c(t_{k-1}) - 1}{\sqrt{g}}, -1\right) \tag{11}$$

If $a_+ a_- \geq 1/p$ (Recall from (1) that p is the number of controller parameters), $\mathcal{E}(t_{k-1})$ is the minimum-volume outer-bounding ellipsoid of the intersection, hence, $\mathcal{E}(t_k) = \mathcal{E}(t_{k-1})$. Consequently, $\Sigma(t_k) = \Sigma(t_{k-1})$ and $\underline{\theta}_c(t_k) = \underline{\theta}_c(t_{k-1})$, with $\Sigma(t_k)$ and $\underline{\theta}_c(t_k)$ as in (1). To guarantee a limited number of distinctive ellipsoids, as is required for the stability analyses of [2], also for $1/p > a_+ a_- \geq \epsilon/p$, $0 < \epsilon < 1$ the current ellipsoidal region is maintained.

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For $a_+a_- < \epsilon/p$ and $a_+ \neq a_-$, $\mathcal{E}(t_k)$ is defined by [7]:

$$\Sigma(t_k) = \delta\left(\Sigma(t_{k-1}) - \frac{\sigma}{g}\Sigma(t_{k-1})\phi_k\phi_k^T\Sigma(t_{k-1})\right) \quad (12)$$

$$\underline{\theta}_c(t_k) = \underline{\theta}_c(t_{k-1}) + \frac{\sigma(a_+ - a_-)}{2\sqrt{g}}\Sigma(t_{k-1})\phi_k \quad (13)$$

with

$$\delta = \frac{p^2}{p^2-1}\left(1 - \frac{a_+^2 + a_-^2 - \rho/p}{2}\right) \quad (14)$$

$$\sigma = \frac{1}{p+1}\left[p + \frac{2}{(a_+ - a_-)^2}\left(1 - a_+a_- - \frac{\rho}{2}\right)\right] \quad (15)$$

$$\rho = \sqrt{4(1-a_+^2)(1-a_-^2) + p^2(a_+^2 - a_-^2)^2} \quad (16)$$

If $a_+ = a_-$, (15) becomes unbounded. Therefore, for $a_+a_- < \epsilon/p$ and $a_+ = a_- = a$, $\mathcal{E}(t_k)$ is defined by

$$\Sigma(t_k) = \frac{p(1-a^2)}{p-1}\left(\Sigma(t_{k-1})\right.$$
$$\left. - \frac{1 - pa^2}{(1-a^2)g}\Sigma(t_{k-1})\phi_k\phi_k^T\Sigma(t_{k-1})\right) \quad (17)$$

$$\underline{\theta}_c(t_k) = \underline{\theta}_c(t_{k-1}) \quad (18)$$

E. Controller Selection

A controller that is unfalsified by the available measurement data is to be inserted in the loop. Or in other words, one controller inside $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ is to be implemented.

The selection of the controller that is to be implemented can depend on several criteria or might even be chosen randomly within $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$. Here, a deterministic selection is presented.

Consider the controller selection algorithm

$$\check{\underline{\theta}}(t_k) = \begin{cases} \check{\underline{\theta}}(t_{k-1}) & \text{if } -1 \leq \gamma \leq 1 \\ \alpha\frac{-1-\gamma_c}{\gamma-\gamma_c}\check{\underline{\theta}}(t_{k-1}) + \\ \quad \left(1 - \alpha\frac{-1-\gamma_c}{\gamma-\gamma_c}\right)\underline{\theta}_c(t_k) & \text{if } \gamma < -1 \\ \alpha\frac{1-\gamma_c}{\gamma-\gamma_c}\check{\underline{\theta}}(t_{k-1}) + \\ \quad \left(1 - \alpha\frac{1-\gamma_c}{\gamma-\gamma_c}\right)\underline{\theta}_c(t_k) & \text{if } \gamma > 1 \end{cases} \quad (19)$$

with $\check{\underline{\theta}}(t_{k-1})$ the controller parameters implemented at time t_{k-1}, and

$$\alpha \in [0,1) \quad (20)$$

$$\gamma = \phi_k^T\check{\underline{\theta}}(t_{k-1}) - y_k \quad (21)$$

$$\gamma_c = \phi_k^T\underline{\theta}_c(t_k) - y_k \quad (22)$$

Note that for $|\gamma| > 1$, $\check{\underline{\theta}}(t_{k-1})$ is falsified by current measurement data (see (II-C)).

The parameter α, (20), determines the stepsize of the switching algorithm. Choosing $\alpha = 0$ corresponds to switching to the center of the Unfalsified set, which is the point furthest from the bound of the Unfalsified set, but which

might be considered as aggressive switching. To decrease aggressiveness, a larger α might be chosen. However, to guarantee a limited number of switches, α should be chosen strictly smaller then 1 (see [8]).

F. Extension to MIMO

The theory of EUC has successfully been applied to several SISO systems [1], [2]. However, for MIMO systems both the controller structure (2) has to be adapted to treat multiple inputs, outputs, and references, and the update of the Unfalsified Set (Section II-D) has to be addressed such that the same arithmetics can be applied, which enables real-time implementation [1].

III. MULTIVARIABLE EUC

In this section, the methodology of Ellipsoidal Unfalsified Control is extended to cover general multivariable controllers. The conditions that are imposed on the control law are analyzed, and a controller structure that fulfills these conditions is proposed. It is shown how the new controller structure fits in the EUC framework and how the same arithmetics as in EUC can be used to update the Unfalsified Set.

A. Plant properties

Consider the general MIMO plant in Fig. 1. Plant $P(q^{-1})$ has inputs $\underline{u}(t_k) \in \mathbb{R}^m$, outputs $\underline{y}(t_k) \in \mathbb{R}^n$ and performance channels $\underline{z}(t_k) \in \mathbb{R}^l$.
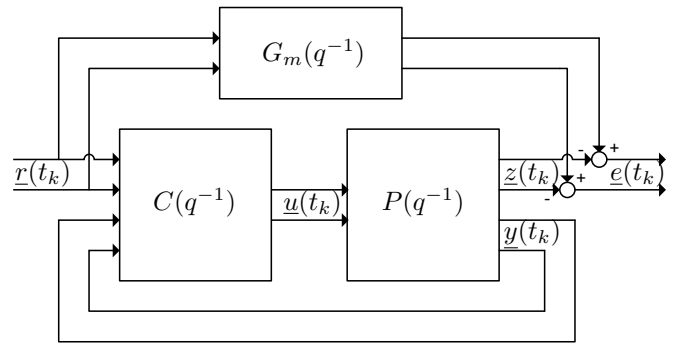

Fig. 1: Schematic representation of general MIMO plant $P(q^{-1})$ with controller $C(q^{-1})$ and reference model $G_m(q^{-1})$.

$G_m(q^{-1})$ is the desired multivariable closed loop dynamics of the controlled system. $G_m(q^{-1})$ might for instance be diagonal, if a decoupled closed-loop system is desired. $G_{m,i}(q^{-1})$ denotes the ith row of $G_m(q^{-1})$.

It is assumed that the controller $C(q^{-1})$ has access to all plant outputs $\underline{y}(t_k)$ and references $\underline{r}(t_k) \in \mathbb{R}^l$. The error $e(t_k)$ is defined as $e(t_k) = G_m(q^{-1})r(t_k) - z(t_k); \underline{e}(t_k) \in \mathbb{R}^l$.

B. Controller structure

If we consider a full-block multivariable controller, all individual plant inputs depend on all plant outputs and references.

$$u_i(t_k) = f_i(\underline{r}(t_k), \underline{y}(t_k), q^{-1}), \text{ for } i = 1, \ldots, m \quad (23)$$

with $u_i(t_k)$ the ith element of vector $\underline{u}(t_k)$. In the EUC framework, a restriction applies to (23) so that it can be converted to a controller structure as (24), i.e., the inverse to $\underline{r}(t_k)$ of (23) has to be linear in the parameters:

$$\underline{r}(t_k) = W(\underline{u}(t_k), \underline{y}(t_k), q^{-1})\underline{\check{\theta}} \quad (24)$$

A further restriction is that $\underline{u}(t_k)$ has to be defined by (24), for given $\check{\theta}$, $\underline{r}(t_k)$ and measured inputs and outputs. Therefore, consider the controller structure

$$\underline{r}(t_k) = \check{\Theta}_u \underline{u}(t_k) + \Lambda(\underline{u}(t_{k-1}), \underline{y}(t_k), q^{-1})\underline{\check{\theta}}_l \quad (25)$$

$$= \begin{bmatrix} \mathbb{I} \otimes \underline{u}^T(t_k) & \Lambda(\underline{u}(t_{k-1}), \underline{y}(t_k), q^{-1}) \end{bmatrix} \begin{bmatrix} \underline{\check{\theta}}_u \\ \underline{\check{\theta}}_l \end{bmatrix} \quad (26)$$

Here, $\otimes$ denotes the Kronecker product. The matrix $\check{\Theta}_u$ and vector $\underline{\check{\theta}}_u$ contain the elements of $\underline{\check{\theta}}$ which correspond to $\underline{u}(t_k)$. The matrix Λ contains (stably filtered) measured input/output data. From (25), it can be seen that $\underline{u}(t_k)$ is well defined, if $\check{\Theta}_u$ is invertible. The resulting controller has an ARMA structure, with basis functions defined by the elements of Λ.

From (24) the fictitious reference $\underline{r}^{\text{fict}}(\underline{\theta}, t_k)$ is easily derived, by considering general parameters $\underline{\theta}$

$$\underline{r}^{\text{fict}}(\underline{\theta}, t_k) = W(\underline{u}(t_k), \underline{y}(t_k), q^{-1})\underline{\theta} \quad (27)$$

C. Performance Requirement

Consider the l_∞ performance requirement (3), as prescribed by EUC. For multivariable systems, this translates to the vector requirement

$$|G_m(q^{-1})\underline{r}^{\text{fict}}(t_k) - \underline{z}(t_k)| + |K(t_k)\underline{u}(t_k)| \leq \underline{\Delta}(t_k) \quad (28)$$

where $\underline{\Delta} \in \mathbb{R}^l$ defines the maximum allowed tracking errors and $K(t_k)$ is a full rank matrix of appropriate size $\forall t_k$.

D. Unfalsified Set

The Unfalsified Set at time t_{k-1} is described by the ellipsoid $\mathcal{E}(t_{k-1})$, (1). The region of parameters that is unfalsified by the current measurement data, $\mathcal{U}(t_k)$, is defined by

$$\mathcal{U}(t_k) = \Big\{ \underline{\theta} \mid -\underline{\hat{\Delta}}(t_k) \leq$$

$$G_m(q^{-1})W(\underline{u}(t_k), \underline{y}(t_k), q^{-1})\underline{\theta} - \underline{z}(t_k) \leq \underline{\hat{\Delta}}(t_k) \Big\} \quad (29)$$

with $\underline{\hat{\Delta}}(t_k) = \underline{\Delta}(t_k) - |K(t_k)\underline{u}(t_k)|$.

Equation (29) has to hold for all rows and, therefore, is evaluated element-wise. As a consequence, (29) defines l sets of parallel half-spaces, i.e., a polygon.

The Unfalsified Set at time t_k, $\mathcal{E}(t_k)$, is constructed from the intersection $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$. However, current analytical results to approximate the intersection only exist for the intersection of an ellipsoid with 1 set of parallel half-spaces. Therefore, the intersection is approximated by considering the parallel half-spaces (29) sequentially. The ordering in this sequential procedure can be chosen arbitrarily.

The sequential update of the Unfalsified Set is in many ways analogous to the update in consecutive time-steps, where only the parallel half-spaces of the last measurement are regarded. However, with time-steps the ordering is fixed.

Lemma 1: The condition $\mathcal{E}(t_k) \supseteq \big(\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)\big)$ holds with the sequential update of $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$

Proof: Let $\mathcal{U}_i$ be defined as the set of ith parallel half-spaces of $\mathcal{U}(t_k)$ for $i = 1, \ldots, l$ and let $\mathcal{E}_i$ be the ellipsoid after the sequential update with $\mathcal{U}_i$. From Section II-D it follows that

$$\mathcal{E}_i \supseteq (\mathcal{E}_{i-1} \cap \mathcal{U}_i) \quad (30)$$

since $\mathcal{E}_i$ is the outer-bounding ellipsoidal approximation of the intersection. Consequently, it holds that

$$\mathcal{E}_{i+1} \supseteq (\mathcal{E}_i \cap \mathcal{U}_{i+1}) \supseteq (\mathcal{E}_{i-1} \cap \mathcal{U}_i \cap \mathcal{U}_{i+1}) \quad (31)$$

By expanding (30) and (31) for $i = 1, \ldots, l$, it follows that

$$\mathcal{E}_l \supseteq (\mathcal{E}_0 \cap \mathcal{U}_1 \cap \cdots \cap \mathcal{U}_l) \quad (32)$$

Next, consider that at some time t_k, $\mathcal{E}_0 = \mathcal{E}(t_{k-1})$ and $\mathcal{E}_l = \mathcal{E}(t_k)$. Furthermore, $\mathcal{U}(t_k) = \mathcal{U}_1 \cap \cdots \cap \mathcal{U}_l$. The lemma follows by substitution of $\mathcal{E}(t_{k-1})$, $\mathcal{E}(t_k)$ and $\mathcal{U}(t_k)$ in (32). ∎

The sequential update procedure results in a sub-optimal approximation of the Unfalsified Set, since consecutive outer-bounding approximation are made. Nevertheless, the volume of the Unfalsified Set decreases monotonically, as follows from Section II-D and the observation of the analogy with consecutive time-steps.

E. Controller Selection

For MIMO EUC, the controller selection (19) is maintained. However, the controller update is applied after every sequential update with $\mathcal{U}_i$, $i = 1, \ldots, l$. Then, $\underline{\check{\theta}}(t_k)$ is the controller after the last sequential update at time t_k.

F. Stability

In [8] sufficient conditions are derived for the stability of SISO EUC. The conditions can be summarized as: 1) feasibility of the adaptive control problem, 2) discarding of demonstrably destabilizing controllers, and 3) a maximum number of controller switches. It was shown that conditions 2) and 3) are fulfilled for SISO EUC. Hence, with the assumption of feasibility, SISO EUC is stable.

For MIMO EUC, condition 2) and 3) are also fulfilled. The controller structure (26) is Stably-Causally-Left-Invertible if $\check{\Theta}_u$ is invertible and $\Lambda(\cdot)$ contains only stable filters, and the cost-function (28) is an l_∞ performance requirement, hence, MIMO EUC discards demonstrably destabilizing controllers. Furthermore, the number of distinctive ellipsoids is limited and the number of controller switches per ellipsoids is limited, thereby limiting the number of overall controller switches. For proofs see [8].

To guarantee that the final controller is chosen from the region with controllers that fulfill the performance requirement, a controller from $\mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ should be chosen. However, due to the sequential update of $\mathcal{E}(t_k)$, it can no longer be guaranteed that $\check{\underline{\theta}}(t_k) \in \mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ $\forall t_k$ with controller selection (19). Nevertheless, the following lemma can be derived.

Lemma 2: If $\mathcal{E}(t_{k-1}) = \mathcal{E}(t_k)$, it can be guaranteed that $\check{\underline{\theta}}(t_k) \in \mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ with sequential application of controller selection (19).

Proof: First, suppose $\underline{\theta}_c(t_{k-1}) \notin \mathcal{U}(t_k)$. Then for some $i \leq l$, $|G_{m,i}(q^{-1})W(\cdot)\underline{\theta}_c(t_{k-1}) - z_i(t_k)| \not\leq \hat{\Delta}_i(t_k)$ and $a_+a_- < 0$ (7)-(11), (27), (28). For $a_+a_- < 0 < \epsilon/p$, $\mathcal{E}(t_k) \neq \mathcal{E}(t_{k-1})$ (12), (13). Hence, if $\mathcal{E}(t_k) = \mathcal{E}(t_{k-1})$, then $\underline{\theta}_c(t_{k-1}) \in \mathcal{U}(t_k)$.

Second, from controller selection (19) it follows that $\check{\underline{\theta}}(t_k) \in \left[\check{\underline{\theta}}(t_{k-1}), \underline{\theta}_c(t_k)\right]$, if $\mathcal{E}(t_k) = \mathcal{E}(t_{k-1})$. Since $\alpha < 1$ (20), $\check{\underline{\theta}}(t_k)$ is selected inside the bound $\mathcal{U}_i$ that is closest to $\underline{\theta}_c(t_k)$ on $\left[\check{\underline{\theta}}(t_{k-1}), \underline{\theta}_c(t_k)\right]$. Consequently, $\check{\underline{\theta}}(t_k) \in \mathcal{U}(t_k)$. Furthermore, by applying the controller selection (19) sequentially, it is guaranteed that $\check{\underline{\theta}}(t_k) \in \mathcal{E}(t_k)$. Hence, it can be concluded that $\check{\underline{\theta}}(t_k) \in \mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ if $\mathcal{E}(t_{k-1}) = \mathcal{E}(t_k)$. ∎

With Lemma 2, it can be guaranteed that $\check{\underline{\theta}}(t_k) \in \mathcal{E}(t_{k-1}) \cap \mathcal{U}(t_k)$ if the Unfalsified Set approaches the region with controllers that satisfy the performance requirement at all times. The existence of this region follows from the feasibility assumption.

Concluding, with the assumption of feasibility, MIMO EUC is stable.

IV. Example Application

Ellipsoidal Unfalsified Control has been applied to the MIMO plant shown in Fig. 2. The plant is sampled at 1 kHz with a zero-order-hold and the EUC algorithm is applied every sample time. The complexity of the algorithm is such that an online implementation would be feasible at this sample rate. An uncorrelated, bounded noise with power 10^{-10} [m^2] and a maximum of 10^{-3} [m] is added to the plant outputs $\underline{y}$. The plant inputs $\underline{u}$ are in [kN]. The performance
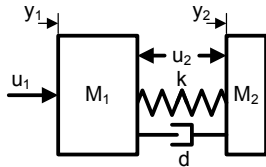
Fig. 2: Schematic representation of dual-stage plant, with M_1 = 1 [kg], M_2 = 0.1 [kg], k = 0.9 [N/m] and d = 0.1 [Ns/m].

outputs $\underline{z}$ are the displacements of both masses: $\underline{z} = \underline{y}$. The trajectory for M_1 is a square wave of amplitude 5 [m] every 5 [s], starting at $t_k = 0.5$ [s]. The trajectory for M_2 is a square wave of amplitude 1 [m] every 2 [s], starting at $t_k = 0$ [s]. The reference model G_m is given by

$$G_m(s) = \begin{bmatrix} \frac{10^2}{s^2+2\cdot 10s+10^2} & 0 \\ 0 & \frac{40^2}{s^2+2\cdot 40s+40^2} \end{bmatrix} \quad (33)$$
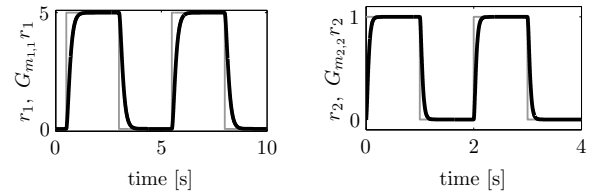
Fig. 3: Plots of trajectories (grey) and filtered trajectories (black) (Note the different scales).

It should be noted that G_m is given here in continuous time solely for ease of perception.

The bounds on the tracking errors are given by

$$\underline{\Delta}(t_k) = \begin{bmatrix} 10e^{-0.15t_k} + 0.005 \\ 10e^{-0.20t_k} + 0.0012 \end{bmatrix} \quad (34)$$

An offset is included in $\underline{\Delta}(t_k)$ to guarantee feasibility in the presence of output noise, whereas the exponential decay is included to allow for transients. The matrix $K(t_k)$ is chosen as a constant matrix such that $|\underline{u}(t_k \to \infty)| \leq [5, 5]^T$:

$$K = \frac{1}{5}\begin{bmatrix} 0.005 & 0 \\ 0 & 0.0012 \end{bmatrix} \quad (35)$$

The controller structure is given by

$$W(u,y,q^{-1}) = \mathbb{I}_2 \otimes$$
$$\begin{bmatrix} \underline{u}^T(t_k) & \frac{1}{1-0.8q^{-1}}\underline{u}^T(t_{k-1}) & \underline{y}^T(t_k) & \underline{y}^T(t_{k-1}) \end{bmatrix} \quad (36)$$

The controller is initiated with the parameter set

$$\check{\underline{\theta}}(0) = \begin{bmatrix} 1\,0\,0\,0\,1\,0\,0\,0 & 0\,1\,0\,0\,0\,1\,0\,0 \end{bmatrix} \quad (37)$$

which is equivalent to $\underline{u}(t_k) = \underline{r}(t_k) - \underline{y}(t_k)$. This initial controller is destabilizing the plant. The initial ellips $\mathcal{E}(0)$ is defined by $\underline{\theta}_c(0) = \check{\underline{\theta}}(0)$, $\Sigma(0) = 1e4\, \mathbb{I}_{16}$.

In Fig. 4, the errors of both performance channels are shown as a function of time, together with the performance bounds.
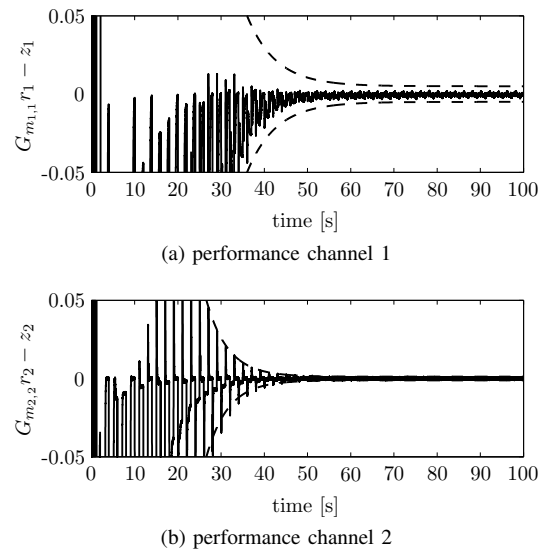
(a) performance channel 1

(b) performance channel 2

Fig. 4: Plot of tracking error (solid) of controlled system and bounds $\pm\Delta$ (dashed) as a function of time.

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When the performance requirement is not met, the current controller parameter set is falsified and replaced by a new controller parameter set. In Fig. 5, $\check{\theta}_2$ is shown as a function of time, as an example of the evolution of the implemented controller parameters. Synchronously, the Unfalsified Set can change even if the currently implemented controller parameter set meets the performance requirement, as follows from the theory of EUC (29).
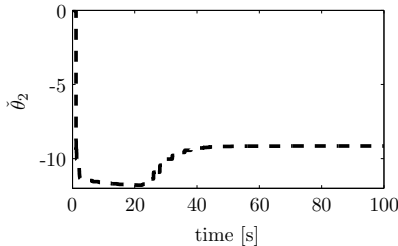

Fig. 5: Plot of $\check{\theta}_2$ as a function of time.

The final values of the controller parameters are shown in Table I. As already mentioned in Section II-E, these parameters are just one selection from the Unfalsified Set. However, since it is in the Unfalsified set, this set fulfills the performance requirement.

TABLE I: Parameter values at $t_k = 100$

$\check{\theta}_1$	10.1442	$\check{\theta}_5$	200.4127	$\check{\theta}_9$	-0.0001	$\check{\theta}_{13}$	0.6815
$\check{\theta}_2$	-9.1482	$\check{\theta}_6$	1.4528	$\check{\theta}_{10}$	6.6789	$\check{\theta}_{14}$	50.7923
$\check{\theta}_3$	0.0108	$\check{\theta}_7$	-199.4222	$\check{\theta}_{11}$	0.0001	$\check{\theta}_{15}$	-0.6754
$\check{\theta}_4$	-0.2962	$\check{\theta}_8$	-1.4442	$\check{\theta}_{12}$	0.0142	$\check{\theta}_{16}$	-49.7983

Since in the simulation the plant-model is known, the frequency response functions of the closed loop system are investigated a posteriori. In Fig. 6, the closed loop transfer function is shown from r_i to z_i. The diagonal terms resemble the reference model (33), whereas the non-diagonal terms are close to zero. The influence of the tight error bound on performance channel 2 is visible from the small (2,1)-component of the closed loop transfer function, which implies that the undesired coupling from r_1 to z_2 is reduced to a factor 10^{-5} (–100 dB). From the output sensitivity frequency response function, shown in Fig. 7, it follows that low-frequent output-noise is suppressed by approximately 40 dB. The non-diagonal terms maintain this suppression in the entire frequency range.

V. CONCLUSION

In this paper, the EUC framework as introduced in [1] is extended to cover multivariable controllers. With the extension, general full-block multivariable controllers can be obtained via this data-driven, plant-model-free control design method.

The extensions, as proposed in Section III, cover a general multivariable controller, with an arbitrary number of inputs, outputs and performance channels. Inherent to the EUC method, the controllers have a fixed, predefinable structure. A reference model can be prescribed to enforce a desired closed
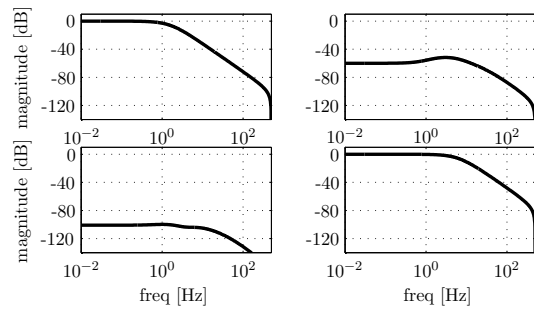

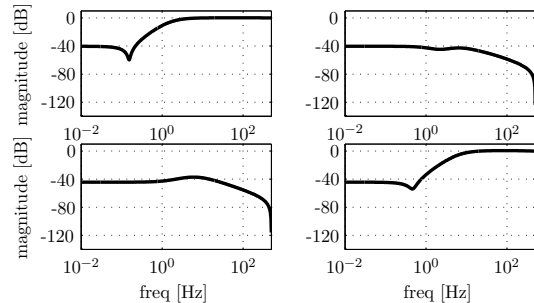Fig. 6: Bode magnitude-plot of the closed loop frequency response function.

Fig. 7: Bode magnitude-plot of the output sensitivity frequency response function.

loop behavior (e.g., decoupling). As in previous works, an l_∞ performance requirement is imposed on the tracking performance of the performance channel. The extensions consist of a proposal for the controller structure (26) and an update procedure of the ellipsoidal Unfalsified Set that considers the intersection with the parallel half-spaces of $\mathcal{U}(t_k)$ sequentially. The effectiveness of the proposed method to find a decoupling controller is shown in a simulation example.

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