Data-Mining Synthesised Schedulers for Hard Real-Time Systems

Christos Kloukinas *

VERIMAG, Centre Équation, 2 avenue de Vignate, 38610 Gières, France
http://www-verimag.imag.fr/PEOPLE/Christos.Kloukinas
E-mail: Christos.Kloukinas @ imag.fr

Abstract

The analysis of hard real-time systems, traditionally performed using RMA/PCP or simulation, is nowadays also studied as a scheduler synthesis problem, where one automatically constructs a scheduler which can guarantee avoidance of deadlock and deadline-miss system states. Even though this approach has the potential for a finer control of a hard real-time system, using fewer resources and easily adapting to further quality aspects (memory/energy consumption, jitter minimisation, etc.*), synthesised schedulers are usually extremely large and difficult to understand. Their big size is a consequence of their inherent precision, since they attempt to describe exactly the frontier among the safe and unsafe system states. It nevertheless hinders their application in practise, since it is extremely difficult to validate them or to use them for better understanding the behaviour of the system.*

In this paper we show how one can adapt data-mining techniques to decrease the size of a synthesised scheduler and force its inherent structure to appear, thus giving the system designer a wealth of additional information for understanding and optimising the scheduler and the underlying system. We present, in particular, how it can be used for obtaining hints for a good task distribution to different processing units, for optimising the scheduler itself (sometimes even removing it altogether in a safe manner) and obtaining both per-task and per-system views of the schedulability of the system.

Keywords : *Software engineering, data-mining, hard real-time systems, schedulability analysis, scheduler synthesis, decision-tree induction.*

1. Introduction

The ongoing advances of the model-checking community have made possible a third possibility to Rate Monotonic Analysis (RMA) [6]/Priority Ceiling Protocol (PCP) [9] and simulation for the analysis of hard real-time systems - that of *scheduler synthesis* [1]. The basic idea behind it is that one will model the system without performing any of the model "normalisations" needed by RMA/PCP, just as is done in the case of system simulation. However, instead of simulating the system, one uses automated tools for examining the entire state space of this model and synthesise a scheduler for it which can *guarantee* that all deadlock and deadline-miss states will be unreachable from now on. Of course the computational cost of performing scheduler synthesis is a lot higher than that of the RMA/PCP analysis. This is somewhat offset by the fact that scheduler synthesis, unlike RMA/PCP, can be applied to any kind of system and that, unlike simulation (itself a computationally expensive method), scheduler synthesis *can prove (better yet,* ***render****) systems correct*. Another incentive for using scheduler synthesis is that the models used with it are usually more detailed than the ones used for RMA/PCP, which means that systems declared to be unschedulable by RMA/PCP due to their model over-approximations may well be proved to be schedulable when using synthesis. So, by imposing the RMA/PCP policy upon the system model, one could well use scheduler synthesis as a more expensive but also more accurate method to perform RMA/PCP schedulability analysis. Finally, unlike RMA/PCP which imposes a static set of task priorities that are difficult to combine with scheduling policies for extra qualities, synthesis can produce schedulers which are easier to combine with quality scheduling policies of this sort. This is because synthesis imposes the minimum set of constraints upon the system for rendering it safe and, thus, leaves many choices, among which we can choose those which best optimise the behaviour of the system with respect to further qualities (memory/energy consumption, jitter minimisation, *etc.*).

In order to evaluate the practical usefulness of the scheduler synthesis method, we have investigated its application to *real-time Java programmes* [5, 4] obtaining promising results concerning its applicability. We investigated in particular a synthesis methodology [5] for obtaining sched-

* Funded by the Grenoble Network Initiative.

uling constraints which can be directly linked to: *(i)* different safety properties (*e.g.*, deadlock avoidance, respect of deadlines) and *(ii)* underlying assumptions concerning the system execution model (*e.g.*, non-preemptive execution versus preemptive execution) or *(iii)* different assumptions about the scheduler itself (*e.g.*, ability to observe the system clocks or lack of it). Our aim was to obtain schedulers which are easier to understand and analyse in practise, than what one would normally obtain when applying scheduler synthesis. At the same time, this methodology has the advantage of decreasing the problem of state space explosion, since it is based on examining successive refinements of the system model, thus paying the cost of extra accuracy only when this is absolutely necessary and beneficial. The schedulers thus synthesised are a set of predicates on the current system state, each declaring a system thread to be safe to execute or not.

Nevertheless, these schedulers still suffer from a number of problems, which hinder their practical use. First of all, they tend to be big, since we had refrained from any sort of optimisation on the predicates. In fact, synthesised schedulers are by nature considerably bigger than the set of static priorities the RMA/PCP analysis produces, since this approach attempts to construct an exact description of the frontier among the safe and unsafe system states. This precision allows for an easier control of the system, hopefully using fewer resources, and for an easier consideration of further quality aspects of the system during scheduling [5].However, their big size is particularly problematic in the case of embedded systems, where we have stringent memory constraints. In addition to memory, the big size of our schedulers has also repercussions upon its execution time, which we would like to render as small as possible; indeed, in our models we make the assumption that the scheduler executes "instantly", *i.e.*, that its execution time is negligible when compared to that of the system sub-tasks. Another problem with the schedulers we synthesise is that they are completely unstructured. That is, for each system task we synthesise a long predicate, as a disjunction of the conjunctions which characterise each unsafe state for this task, *i.e.*, the states where the scheduler should not allow this task to execute so as to keep the whole system in the set of states where the safety properties can be guaranteed. This form makes it extremely difficult for a human to understand the scheduling constraint that should be imposed upon a system task, thus rendering the validation of the scheduler problematic. At the same time, it does not allow the designer to obtain any kind of reasonable feedback concerning the behaviour of the system, which could help in seeding further system optimisations.

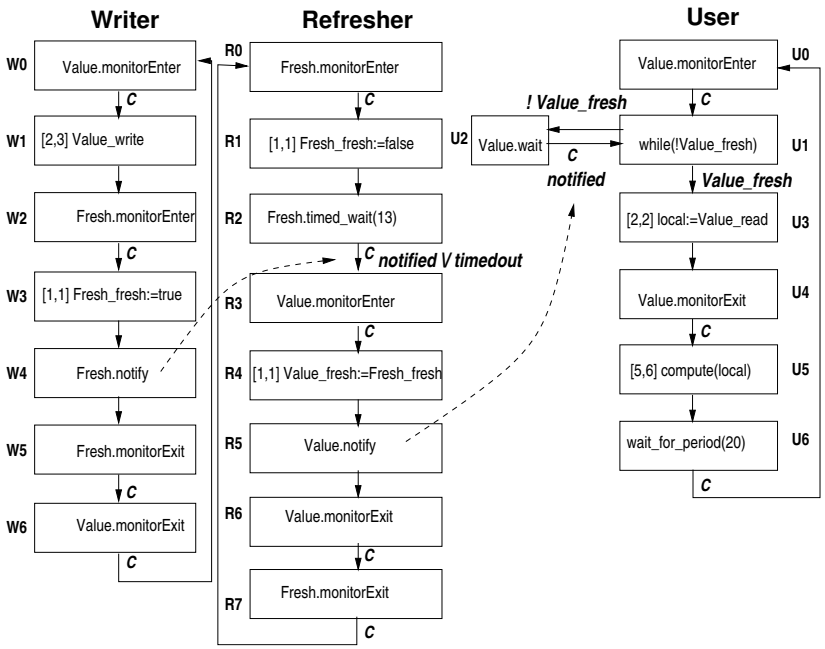
When referring to minimisation of some predicate, one may naturally think of *binary decision diagrams (BDDs)* [2]. Nevertheless, BDDs' primary role is to provide a *canonical* representation which allows for fast computations with predicates. It is well known that what plays a crucial role in obtaining a small BDD is the choice of the variable ordering along the diagram. BDDs do not offer any substantial help in this respect and the fact that they impose a *single variable ordering* across all the decision tree branches sometimes makes it difficult to obtain a small decision tree.

In our context, where we do not need to perform any further computations with our scheduling constraints and where we would also like their final representation to be *easily readable by humans*, we can obtain a much better representation by using *machine-learning* techniques for *data-mining*. Indeed, there exist data-mining methods such as ID3 [7] which can produce a small representation of our scheduling constraints as a decision tree, using *information-theory heuristics* for choosing the order of variables at each sub-tree. This last feature allows data-mining methods to produce a decision tree which is more useful to a *human*, highlighting the system variables which are *more important* at different stages of the system's execution, at least as far as the schedulability of the system is concerned. As usual with applying data-mining techniques, we will have to *adapt the basic data-mining algorithm to our particular domain*, in order to reap the most benefits out of it.

In the following we present the basic idea behind scheduler synthesis and the scheduler we synthesise for a simple case study. We then show how one can adapt ID3 to better structure synthesised scheduler constraints and analyse/optimise a real-time system.

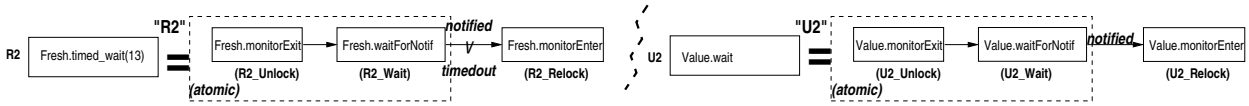
2. Synthesising Scheduler Constraints

The basic idea behind scheduler synthesis for safety properties (*e.g.*, deadlock freedom, guaranteeing deadlines) is to explore the complete state space graph of the system and find the states where the system scheduler must not allow some application tasks to execute, in order to be able to *guarantee* that the system will avoid undesired states later on (*e.g.*, states where the system is deadlocked, or misses some deadline). If at some state the scheduler is left with no safe choice, we add it to the set of bad states, so that we can avoid it as well (otherwise, the scheduler itself may cause the system to deadlock). The scheduler takes its decisions based on a set of observation variables, which form its model of the current state of the system. In our setting, these variables are the programme counters of the application tasks and the global clock of the system. Note, that in our models the scheduler is not allowed to exert control to the system at all states [5]. It can only control the application at states where the application attempts to lock (or unlock) some resource, or when an application task becomes ready to execute after some alarm has expired (*e.g.*, a new



*(Each computation is annotated with its execution duration interval. Synchronisation is done using monitors and communication using condition variables. The dotted arrows show the correspondence of notifications to wait actions. Controllable transitions are annotated with a "**C**".)*

Figure 1. Control-flow graphs of a small real-time database system's tasks



(Since the internal states `X_Unlock` & `X_Wait` are atomic, we consider them as a single state, and use `X` to refer to them, in contrast to `X_Relock`. So, it is `R2`, `R2_Relock` & `U2`, `U2_Relock` respectively.)

Figure 2. The behaviour of the `timed_wait` & `wait` primitives

task period has been signalled, or the task has timed-out while waiting for some notification). This effectively means that the programme counters of the tasks need not refer to real programme instructions; they can instead be abstract locations of these programmes signifying the start (respectively end) of some block of non-controllable actions. For example, a mutually exclusive block of code can be described with three abstract positions: one modelling the demand of the mutually exclusive access to the block (*i.e.*, the lock or the action of entering a monitor), another modelling the execution inside the mutually exclusive block and a final one, modelling the end of the block (*i.e.*, the unlock action, or the monitor exit one). In our model, tasks can be in either one of three basic states: *blocked*, *ready* & *executing*. Tasks which are *ready* can either be *safe* to execute or not. Note here that the *safe* task predicate *is not defined upon all system states*.

2.1. Case Study: A Simple R-T DB

Figure 1 gives an example of an abstract control-flow graph of a simple real-time database system consisting of three tasks. In this system, the `Writer` task is producing a series of values (*e.g.*, by reading some sensor) and the `Refresher` task is ensuring that the values the periodic `User` task is using are fresh enough, *i.e.*, have been produced sometime during the last 13ms time frame. Synchronisation among the system tasks is achieved through the use of monitors, using primitives `monitorEnter` and `monitorExit`, which basically lock/unlock the mutex associated with the object they are called with. Communication among tasks is done through condition variables, using primitives `wait`, `timed_wait`, `notify` and `notifyAll` (not used in our example). Finally, the `wait_for_period` primitive blocks a task until its next period, which is given as an argument. In the ex-

```
IS_UNSAFE(Writer):=
   ((User=U0)        ⋀ (Refresher=R3) ⋀ (Writer=W0))
 ⋁ ((User=U2)        ⋀ (Refresher=R3) ⋀ (Writer=W0))
 ⋁ ((User=U2_Relock) ⋀ (Refresher=R3) ⋀ (Writer=W0))
 ⋁ ((User=U5)        ⋀ (Refresher=R3) ⋀ (Writer=W0))
 ⋁ ((User=U6)        ⋀ (Refresher=R3) ⋀ (Writer=W0))

IS_UNSAFE(Refresher):=
   ((User=U0)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W1))
 ⋁ ((User=U0)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W2))
 ⋁ ((User=U2)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W2))
 ⋁ ((User=U2)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W1))
 ⋁ ((User=U2_Relock) ⋀ (Refresher=R2_Relock) ⋀ (Writer=W1))
 ⋁ ((User=U2_Relock) ⋀ (Refresher=R2_Relock) ⋀ (Writer=W2))
 ⋁ ((User=U5)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W1))
 ⋁ ((User=U5)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W2))
 ⋁ ((User=U6)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W1))
 ⋁ ((User=U6)        ⋀ (Refresher=R2_Relock) ⋀ (Writer=W2))
```

Table 1. Scheduler against deadlocks

(a) Writer-Unsafe decision-tree

```
Attribute: Writer [Gain-Ratio : 0.080892585]
--> W0
     \
      Attribute: Refresher [Gain-Ratio : 0.3615773]
      --> R3             Class is: Writer-Unsafe
```

(b) Refresher-Unsafe decision-tree

```
Attribute: Refresher [Gain-Ratio : 0.103661135]
--> R2_Relock
     \
      Attribute: Writer [Gain-Ratio : 0.50555944]
      +-> W1             Class is: Refresher-Unsafe
      --> W2             Class is: Refresher-Unsafe
```

Table 2. Structured deadlock-free scheduler

ample of Figure 1, the `User` is the only periodic task, with a period of 20ms. Figure 2 shows how the `wait` primitive functions internally - it is in fact two states, the first of which atomically exits the monitor and blocks the thread waiting for a notification, while the second one (*i.e.*,`R2_Relock` or `U2_Relock`) is the state where the thread has been notified and thus wakes up and attempts to re-enter the monitor. In the following, we will use the location of the `wait` primitive (*i.e.*, `R2`, `U2`) to refer to the first atomic state where the task blocks, using the same location with the suffix `_Relock` (*i.e.*, `R2_Relock`, `U2_Relock`) to refer to the state where the task attempts to re-enter the monitor once notified.

Table 1 shows the synthesised constraints for avoiding deadlocks in this system. Even though there are only a few constraints in this case, the problem is still obvious. One cannot easily identify which variables play the major role in establishing that a certain task will deadlock if allowed to execute, nor is it easy to extract any further information about the inter-dependencies of the tasks. Consider, however, the reformulation of the above constraints in Table 2. Immediately it becomes obvious that, as far as the `Writer` is concerned, the problem arises when it finds itself at the `W0` location (where it wants to lock object `Value` and will subsequently ask to lock object `Fresh`), in states where the `Refresher` is at the `R3` location (where it has already acquired `Fresh` and will next try to lock `Value`). Similarly, the `Refresher` is unsafe to execute at location `R2_Relock` (where it tries to acquire the object `Fresh`), when the `Writer` is at locations `W1` or `W2` (where it has already locked the object `Value`). Indeed, this situation will lead to a deadlock, since the `Refresher` will subsequently ask for object `Value` while the `Writer` will ask for object `Fresh`. It is also clearly evident that the `User` task has no effect on either the `Writer` or the `Refresher` tasks being safe or not. Structuring the synthesised constraints in such a manner can greatly help when their number increases. Indeed, for the case where we are trying to guarantee the deadlines of the tasks under a non-preemptive execution model (*i.e.*, that `User` never misses its period), we synthesise 61 constraints in total, while the decision-tree structured constraints are only 19 [1].

3. Inducing Schedulers with ID3

Let us start with a brief introduction of Ross Quinlan's ID3 data-mining algorithm [7]. The basic idea for structuring the scheduler constraints as a decision tree, is that we can consider the *controllable states* in the state-space graph of the system as a set of classified examples, where the class of a state is exactly the set of currently safe tasks. Given this and a representation of each state as the vector of the values of the *observable* system variables, we can use the *top-down induction of decision trees (TDIDT)* algorithm ID3 so as to extract the underlying structure of our scheduler constraints. ID3 performs a *greedy heuristic* search on the set of possible variable observations, having as a goal the minimisation of the height of the final decision tree. The skeleton of the ID3 tree-induction algorithm, *as adapted to our domain*, is shown in Figure 3 (adaptations are typeset using an ***italic bold*** face). Faced with a set of training examples (*i.e.*, controllable states), we select the attribute which maximises the *gain ratio*, defined as the ratio of the *information gain* over the *information value* of an attribute. The main idea behind this heuristic is to try to maximise the mutual entropy of the attribute and class random variables, correcting for the fact that many-valued attributes will have a higher mutual entropy than few-valued ones. The definitions of these information-theory metrics are shown in equations 1–5, where C is the ensemble of classes, A the ensemble of the values v of an attribute, and $H(X)$ the entropy of

1 There are 45 for the Writer, 14 for the Refresher and 2 for the User. The respective trees have 11, 7 & 1 leaves, for a total of 19 constraints.

```
1  Adapted-ID3(examples, attributes, attribute_observation_costs)
2  Let
3     C-C  := COMPOSED  class of examples  // M-C := Majority class of examples
4  In
5   If all examples in class C Then
6     return a leaf labelled as C
7   Else If no more attributes Then
8     return a leaf labelled as C-C   // ...labelled as M-C
9   Else
10    Select an attribute, A, for the root, among the least cost attributes
11    For each possible value v_i of A
12      examples_i := subset of examples having value v_i for A
13      Add a branch for the test A=v_i
14      If examples_i is empty Then
15        create a leaf labelled as SAFE  // ...labelled as M-C
16      Else
17        new subtree := Adapted-ID3(examples_i, attributes - {A}, attribute_observation_costs)
```

*(Adaptations are shown using an **italic bold** font; original code has been commented out.)*

Figure 3. Adapted ID3 algorithm

an ensemble X (*i.e.*, the average Shannon information content of an outcome), see Quinlan [7] for more details.

$$\text{Gain\_Ratio}(A, E) = \text{Info\_Gain}(A, E)/\text{Info\_Value}(A, E) \quad (1)$$
$$\text{Info\_Value}(A, E) = H(A) \quad (2)$$
$$\text{Info\_Gain}(A, E) = \text{Info}(E) - \text{Exp\_Info}(A, E) \quad (3)$$
$$\text{Info}(E) = H(C) \quad (4)$$
$$\text{Exp\_Info}(A, E) = -H(A) + H(C, A) \quad (5)$$

3.1. Applying ID3 on Scheduler Constraints

Having introduced the ID3 basics, let us now describe how it can be adapted for structuring and mining our synthesised scheduler constraints. As aforementioned, the training examples we have are the states at which the scheduler is called to exert control. Our attributes are the PC vector of the tasks and the value of the global system clock, since these are the system variables we allow our scheduler to observe. The classes of the examples/states belong to the set describing which tasks are safe. Given our specific data domain, we can *adapt the basic ID3 algorithm* so that it uses our knowledge of the domain and thus produces better results.

3.1.1. Adapting ID3 First, we can use the fact that our classes are not *independent*, as is generally assumed, but can be *composed*. The composed class of two examples having different classes is the most conservative of the two (*i.e.*, the task-unsafe one). So, when the ID3 algorithm is faced with a set of examples of different classes which it cannot classify for lack of further observation attributes (line 8 of Figure 3), then we can take the *composition* of these classes as the label of the tree leaf, instead of the most common class in the current example set. ID3 would have chosen the most common class because it considers the other examples as *outliers*, since it aims to increase the overall classification accuracy of the final decision-tree. In our case, these examples/states are simply a set of unsafe/safe states which the scheduler cannot distinguish in its system abstraction, not noise due to some badly performed experiment/measure. Since we aim for *safety*, instead of maximising the overall classification accuracy, *we must always err on the conservative side*, thus label the leaf with the composed class (*i.e.*, unsafe).

Then, at line 15 of Figure 3 we do not have to label a leaf lacking examples with the most common class of its parent node, but can simply label it as ***SAFE***. Here again, ID3 assumed that the examples were missing because our experiments/measurements failed to cover the state space. Lacking any domain knowledge, it makes sense to assign the leaf the majority class of its parent node. However, in our domain, missing examples represent the case where we do not have any control state whose attributes have these values, *i.e.*, it is an unreachable state of the system (or one where the task is not ready and thus its safety is not defined). Since we do not want to over-constraint the system, we explicitly assign to this leaf the ***SAFE*** class, knowing that our decision will not have any impact on the schedulability of the system.

Attributes with an Info_Value (see equation 2) equal to zero are not considered by our tree induction algorithm, since a zero value means that there is only one value for this attribute in the examples, so this attribute is adequately characterised by the attributes which have been observed higher in the decision-tree branch we are currently examining.

3.1.2. Continuous Variables In order to produce trees which are easier to understand, we have also changed the

treatment of continuous attributes, that is, the system clock. So, instead of treating each clock value as a distinct one, we transform them into those intervals which correspond to an unsafe task. This value to interval transformation is performed in the following manner. For each task we compute the list of examples/states for which the task is unsafe, sort the examples according to the values of the clock and then construct intervals of the sorted values. For example, if for task T the clock values at the set of examples/states for which this task is unsafe are $\{0, 3, 4, 6, 7, 8\}$, then we construct the intervals $[0, 0], [3, 4], [6, 8]$. Then, we combine the intervals constructed for each task, in order to obtain those intervals which can best characterise all the unsafe tasks. So, if we had another task T$'$ whose unsafe interval was $[4, 5]$, the combined intervals would be $[0, 0], [3, 3], [4, 4], [5, 5], [6, 8]$, where we have broken up the $[3, 4]$ and the $[4, 5]$ intervals, since at 3 only the first task is unsafe, at 5 only the second task is unsafe, while at 4 both of them are unsafe. This greatly reduces both the depth and the breadth of the induced trees, since we avoid performing consecutive tests/splits on specific midpoint values (which is one way of handling continuous attributes, see [8]) or splitting into as many sub-trees as possible clock values.

3.1.3. Variable Observation Costs Finally, we have introduced *observation costs* for each variable but we do not treat them as weights, which is the usual way of using them, *i.e.*, only as guidelines which can be ignored sometimes. Instead, we always choose the next attribute to observe among the set of attributes having the smallest cost, see line 10 of Figure 3. This allows us to postpone the observation of the clock attribute, since this has a much larger observation cost than the programme counters of the different tasks. Indeed, in [5] we explicitly aimed for obtaining time-independent schedulers, exactly because of the high cost of observing the system and task execution clocks and the great decrease of the scheduler size thus obtained. In addition, variable observation costs allow us to induce trees where we have *imposed* the choice of the first attribute. In this way, we can choose a particular programme counter as the top attribute and obtain a tree which gives us a *view of the system's behaviour according to that particular task.*

3.2. Types of Decision Trees

Having set the framework, we now examine the two basic ways we can use our adapted ID3 algorithm. First, we can induce separate decision trees for each task, effectively computing the predicates IS_UNSAFE(T_i), for all $i \leq N$, where N is the number of tasks. In this case, we would use binary classes {`Safe, Unsafe`} for all tasks. The second alternative is to induce a decision tree for all tasks at the same time, that is, compute *in parallel* the safe/unsafe bit-vector with one decision tree. In this case, we will have 2^N classes [2]. For example, for the system of Figure 1, the classes would be {`All_Safe, Writer_Unsafe, Refresher_Unsafe, User_Unsafe, Writer_Refresher_Unsafe, Writer_User_Unsafe, Refresher_User_Unsafe, Writer_Refresher_User_Unsafe`}. The *"parallel"* decision-tree form of the scheduler obtained for the example of Figure 1 is shown in Table 3. According to this decision-tree, the system task which seems to be most problematic for the system's schedulability is the `Writer`. It is also clear that the `User` never causes the system to move towards an unsafe (here deadlocked) state, since its programme counter does not even appear in the tree.

In both cases, as we have already seen, we set the clock attributes to have the highest cost, so as to observe them last. In this way, the attribute order of trees will start with the PCs of the tasks and, thus, we will obtain an order of *importance* for the tasks of the system, with respect to our ability to schedule the system safely. The task whose behaviour has the highest effect on system schedulability will be placed first, the next one second, *etc.*, and this will be done *separately* for each branch of the decision tree.

3.3. Preprocessing of the Training Examples

Preprocessing of the training examples is another way we can introduce our knowledge of the particular domain, for obtaining better decision trees. In the case of the binary class trees, the preprocessing consists of removing from the example set all these examples which correspond to states where the respective task is not ready, since in these states the safety of the task is undefined (remember that a task needs to be ready to be characterised as safe or unsafe). This allows ID3 to use them in its convenience, as either safe or unsafe examples, depending on which characterisation will help it construct a smaller tree. For example, the tree for `Writer` in Table 2 was obtained by considering that the *missing* examples `W0`, `R3`, {`U1`,`U3`,`U4`} are all unsafe, even though in these states `Writer` is blocked (since `User` has the lock on object `Value` which is needed by the `Writer`). To see why, imagine having chosen the attributes/values `Writer`/`W0` and `Refresher`/`R3`. At that point, since we have removed all examples/states where `Writer` is blocked, all remaining states belong to the `Writer_Unsafe` class, and the value of attribute `User` does not offer any further increase in the classification precision of the decision-tree. Thus, ID3 will choose to directly construct a leaf labelled as `Writer_Unsafe`, executing line 6 of Figure 3.

---

2 In reality, there will be 2^{N+1} classes, since we also have the IDLE task in timed models of the system and the scheduler needs to decide at each control point whether idling is safe or not.

```
Attribute: Writer [Gain-Ratio : 0.16776516]
+-> W0
|   \
|    Attribute: Refresher [Gain-Ratio : 0.3319911]
|    --> R3                          Class is: Writer-Unsafe     (Refresher & User are Safe)
+-> W1
|   \
|    Attribute: Refresher [Gain-Ratio : 0.5633241]
|    --> R2_Relock                   Class is: Refresher-Unsafe  (Writer & User are Safe)
--> W2
    \
     Attribute: Refresher [Gain-Ratio : 0.38146517]
     --> R2_Relock                   Class is: Refresher-Unsafe  (Writer & User are Safe)
```

Table 3. Scheduler against deadlocks, structured as a "parallel" decision tree

3.3.1. Preprocessing Examples for "Parallel" Trees Unfortunately, we cannot do the same kind of preprocessing for the tree with the 2^N classes, which attempts to compute in parallel the N different safety predicates. This is because a state may well contain some tasks which are not ready (so their safety is undefined) and some tasks which are unsafe. Removing these states from the training examples could make us take a wrong decision where the scheduler considers certain unsafe tasks as safe to execute, with possibly catastrophic consequences. Therefore, in this case the preprocessing of the examples is performed in the following manner. For each example we use the N binary-class trees to classify it as either safe or unsafe for each task and then use the composition of these classifications as its global class. This allows us to classify certain states where a task is blocked as a state where this task is safe, while other blocked states are classified as unsafe for that task. This different treatment of non-ready tasks helps in obtaining a smaller 2^N-classes "parallel" tree, than we would have obtained if we considered all non-ready tasks to always be either safe or unsafe. Therefore, the binary-class trees help us in finding a convenient characterisation for non-ready tasks, whose class (*i.e.*, safety) is undefined.

4. System Analysis & Optimisation

The N binary-class decision trees allow us to look at the system from the *view-point* of the particular task, *i.e.*, find out, for each of its possible locations, what is the task that has the *highest effect* on its execution being safe or not. This knowledge can help the designer find *strong inter-dependencies among tasks* and thus construct sets of *strongly dependent tasks*. These task sets can then be used for *selecting distribution of tasks to processing units*, *e.g.*, placing dependent tasks together on the same processing unit in order to avoid an increased inter-processor communication for their synchronisation. For example, by examining the trees in Table 2 we can see that `Writer` and `Refresher` are strongly dependent upon each other and thus it might be better to place them on the same processing unit, while `User` could be placed in another one. They can also be used for *highlighting potential design problems*. Imagine, for example, the case where a highly critical task is strongly dependent on a low criticality task; it is evident that we would like to re-design the system so as to decrease such a dependency or avoid it altogether.

Another observation we can make by examining in Table 4 the (partial) decision-tree of the scheduler against deadline-misses in a non-preemptive model, is that the most important task is the `Refresher`, not the `User` as we might have expected. Indeed, this is logical since it is the `Refresher` which in the end will allow the `User` to advance from waiting for a fresh sensor value. We also see that when the `User`, the `Refresher` and the `Writer` all compete for the same resource (*i.e.*, `Value`) at locations `R7`, `W0` and `U0` or `U2_Relock`, then the scheduler gives `User` the priority, so that it does not miss its deadline.

4.1. Counterexample Analysis

By data-mining the synthesised scheduler, we effectively get a *succinct view of all the counterexamples for the property we want to guarantee*, highlighting just the first states where the system execution went wrong. Table 3, for example summarises all the system execution counterexample traces reaching a deadlock state in our case study, that is the counterexamples we would obtain from a model-checker if checking our system for deadlock-freedom. Table 3 tells us that the root of the erroneous behaviour in these counterexample traces is the states where the system finds itself in one of the configurations $(*, \texttt{W0}, *, \texttt{R3}, *)$, $(*, \texttt{W1}, *, \texttt{R2\_Relock}, *)$, $(*, \texttt{W2}, *, \texttt{R2\_Relock}, *)$. In this manner, it clearly specifies the tasks which initiate an erroneous behaviour, showing us where we should focus our attention, as well as, from what moment/state on the erroneous behaviour starts. Therefore, unlike current practise, we do not have to simulate and study the execution of the system at all the states of a counterexample trace leading to the problem.This is a particularly interesting feature when the erroneous behaviour needs a great number of steps before it becomes possi-

ble [3]. It also allows us to "group" all the counterexample traces one can obtain, into a small set of the *root* problematic states. By keeping just the variables which are important in the decision tree, data-mining also allows us to identify and subsequently ignore the tasks which are not really part of the problem when examining a counterexample (as is the case for the `User` in our example). Thus, the combination of scheduler synthesis and data-mining can also be used to more easily *understand* the results of a model-checker. Instead of model-checking against a property ϕ and examining all the different counterexample traces trying to figure out when & why things started going wrong, one can synthesise a scheduler which guarantees ϕ and use this synthesised scheduler as a starting point for understanding the reasons for which ϕ does not hold in the system.

4.2. Optimising Scheduler Implementation

Obtaining N decision trees can also be useful for the *implementation* of the synthesised scheduler. Normally, we implement the scheduler as a large function (containing the "parallel" decision tree), which the application tasks call at their control points to compute the next task to execute. Now, we can implement it as *small, embedded* code at the control points of each task, effectively copying there the sub-tree for the particular task position, as extracted from that task's "personal" decision tree. An evident optimisation we can do with this *distributed* scheduler is at the control points where we know that a task is safe to execute; there we can remove the scheduling code altogether, effectively giving it priority over other tasks. In our case study, the only point in the `Writer` where we would need to ask our synthesised scheduler the permission to proceed so as to avoid deadlocks is the `W0` location and the only ones where `Refresher` should ask for permission are the `R3` and `R2_Relock` ones. This means that we can avoid the scheduling decisions for deadlock avoidance at the locations `W2`, `W5`, `W6`, `R0`, `R6`, `R7`, `U0`, `U2_Relock` and `U4`. That is, we can remove 9 out of the 12 control points of our system, since there we do not need to execute any particular synchronisation protocol – the simple action of locking/unlocking the mutex which is performed at these locations suffices to keep the system deadlock-free.

5. Pruning Decision Trees

Once we have synthesised a scheduler for safely controlling the system, we might be interested in exploring the case where we do not allow this scheduler to observe certain system variables, for example the clocks. This way, we can obtain a *smaller*, more *robust*, *time-independent* scheduler, which is also *easier* to implement and *faster* to execute (no need to use timers, *etc.*), see [5]. However, if we remove the clock observations from the synthesised scheduler constraints we may introduce new deadlock/deadline-miss states in the system, which means that we have to do again the synthesis, using the new, pruned constraints as our initial scheduler, so as to be sure that the system remains safe. Now, however, that we have available the "parallel" decision tree description of the scheduler constraints, we can substantially simplify this step.

Indeed, it suffices to examine the composition of the classes labelling the leafs which are reachable from the subtrees corresponding to the clock attribute in the "parallel" decision tree. If the composed class assigns at least one application task to the safe class, then we can *safely prune* the tree at that point and replace all the sub-tree starting from the observation on the clock by a leaf labelled with the composed class, without needing to validate the new, pruned scheduler. As an example, let us consider a case where when testing on the global clock, we obtain 3 leaves with classes `Writer_Refresher_Unsafe`, `Refresher_Unsafe`, and `Writer_Unsafe`. The composed class being `Writer_Refresher_Unsafe`, we can substitute the test on the global clock and the subsequent 3 leaves, with a single leaf labelled as `Writer_Refresher_Unsafe`, *i.e.*, allow only the `User` task to execute (even though the `Writer` and the `Refresher` tasks could have being safely executed for certain values of the global clock).

In this manner, we can see if we need to do the synthesis again. Apparently, this is only needed if at some point the composed class turns out to be `All_Unsafe`. Note that when the composed class of some attribute's sub-tree is `All_Unsafe` it doesn't mean that pruning will render the system non-schedulable. This is because, it is possible that we can synthesise further scheduling constraints for the system, or that we have already rendered the newly problematic states unreachable, as a side-effect of our having pruned this attribute away at some other node of the tree.

We can continue applying this decision-tree pruning until we obtain a decision-tree/scheduler which is as small as we want. Since the decision tree has placed the least important variables last, we have a clear indication of which variables should be candidates for pruning - we always start pruning from the leaves, moving towards the root of the tree.

5.1. Pitfall: Pruning & Blocked Tasks

We should note here that our argument holds only if while synthesising the scheduler we had allowed the system to idle even at states where there existed safe tasks to

3 One can imagine "watchpointing" a counterexample simulation, instead of manually stepping over each state.

execute. In the opposite case, the pruned decision tree of the scheduler may well forbid certain tasks to execute once they reach a certain location, because we are essentially forcing them to idle and this may not be safe. In this case, one must make sure that the tree ensures tasks' liveness. That is, at each leaf the tasks which are classified as unsafe do not contain all the *ready* tasks which have been used for deciding upon the class of the current state, thus ensuring that some task will execute and change the current state. Let us consider an example. If we synthesise a scheduler to avoid deadline-misses for our case study under a non-preemptive execution model and always execute one of the safe tasks (*i.e.*, idling is allowed only when we have no safe choice), the decision tree of the scheduler constraints will contain the sub-tree shown in Table 4(a). The pruned decision sub-tree shown in Table 4(b) is evidently correct, because once `User` changes location we will leave this sub-tree and thus allow `Writer` & `Refresher` to execute again. If, however, we prune even further (*e.g.*, remove the observation of `User`'s location, see Table 4(c)) then this sub-tree will force `Refresher` and `Writer` to stop executing for ever, once they reach their respective locations `R7` and `W0`, which will eventually cause `User` itself to miss its deadline. The difference between the two cases is that in 4(b) the set of tasks classified as unsafe {`Writer`, `Refresher`} is a true subset of the set of *ready* tasks used for the classification {`Refresher`, `Writer`, `User`}, while in 4(c) this is no longer the case. If we had allowed the system to idle even when there are other safe choices, then the scheduler (& therefore its corresponding decision tree) would have also included the situations under which it is not safe to idle and, in that case, pruning at `User` as was done in Table 4(c) would have resulted in an `All_Unsafe` class.

(a) Original sub-tree

```
Attribute: Refresher [Gain-Ratio : 0.1261203]
......
--> R7
    \
     Attribute: Writer [Gain-Ratio : 0.085499726]
     --> W0
          \
           Attribute: User [Gain-Ratio : 0.17595403]
           +-> U0
           |    \
           |     Attribute: Global_Clock [Gain-Ratio : 0.89254934]
           |     +-> in [7 , 7]   Class is: Writer_Refresher-Unsafe
           |     +-> in [8 , 10]  Class is: Writer-Unsafe
           |     --> in [11 , 11] Class is: Writer_Refresher-Unsafe
           --> U2_Relock
                \
                 Attribute: Global_Clock [Gain-Ratio : 1.0]
                 +-> in [8 , 10]  Class is: Writer-Unsafe
                 --> in [11 , 11] Class is: Writer_Refresher-Unsafe
```

(b) Pruned sub-tree

```
Attribute: Refresher [Gain-Ratio : 0.1261203]
......
--> R7
    \
     Attribute: Writer [Gain-Ratio : 0.085499726]
     --> W0
          \
           Attribute: User [Gain-Ratio : 0.17595403]
           +-> U0             Class is: Writer_Refresher-Unsafe
           --> U2_Relock      Class is: Writer_Refresher-Unsafe
```

(c) Excessively pruned sub-tree

```
Attribute: Refresher [Gain-Ratio : 0.1261203]
......
--> R7
    \
     Attribute: Writer [Gain-Ratio : 0.085499726]
     --> W0                   Class is: Writer_Refresher-Unsafe
```

Table 4. Sub-tree against deadline-misses

5.2. Tree Pruning and Schedulability

As we have seen in the previous discussion, the tree induction process will optimise the representation of the constraints by making decisions only on the variables which can indeed change the class of a task and selecting them in an order which reflects their importance for the system's schedulability. For example, this allowed us to minimise the 15 constraints used for deadlock-freedom in our case study to just 3, and the 61 constraints used for respecting deadlines under a non-preemptive execution model, to just 19 [4]. Note, however, that the set of the important variables for the class of tasks is computed with respect to the currently available training set.

5.2.1. When to Prune? When we move from a constrained system, *e.g.*, one where the system uses a non-preemptive execution model, to a less constrained one, where we do allow computations to be preempted, we introduce more states and thus new training examples which were not taken into account during the decision tree induction. These are the states where a task has preempted another one. If we use the pruned decision tree form of the constraints in the intermediate steps of the synthesis process, then we may loose accuracy in the latter steps. This is because in the latter steps we introduce extra states in the state space and thus extra training examples, which were not available before.

However, this loss of accuracy cannot lead us to declare that there is no possible scheduler for a system, when indeed there is one. This is due to the fact that if we were able to schedule a more constrained system with the pruned scheduler, then we will also be able to eventually schedule the less constrained system as well. In the worst case, we will need to synthesise constraints which re-impose the initial constrained system model, *e.g.*, non-preemption. The only problem which we may have is to not be able to meet additional quality aspects of the system, because our pruned scheduler has over-constrained the state space and, therefore, has also over-constrained its choices. For this reason, it may be beneficial to use the induced decision-trees at each synthesis step *only for analysing the resulting synthesised*

4 Constraints and respective trees not shown due to lack of space.

scheduler. Once we have performed the synthesis for the most refined model of our system and the additional quality aspects we wish to meet, we can use the decision-trees for implementing the scheduler as well, without fear of over-constraining the system.

During analysis, we could use more "aggressive" pruning algorithms, like ID3's successor C4.5 [8]. C4.5 [5] uses the gain-ratio attribute selection criterion as we do but prunes the induced tree in a way which, even though not safe for our domain, can nevertheless help to better highlight the major problematic situations.

6. Conclusions

In [5] we presented a methodology for performing scheduler synthesis for hard real-time systems, which allows a designer to obtain fine-grain scheduling constraints corresponding to different safety properties (*e.g.*, deadlock or deadline-miss avoidance), different underlying assumptions concerning the system execution model (*e.g.*, non-preemptive execution versus preemptive execution) or different assumptions about the scheduler (*e.g.*, ability to observe the system clocks or lack of it). In this paper, we have presented a method, based on a classic Machine Learning algorithm for data-mining (ID3 [7]), which allows to extract the underlying structure of the scheduler constraints synthesised with our methodology. Since these synthesised constraints are usually too numerous and unstructured, their reformulation is imperative for being able to understand and validate them.

We have shown how one can adapt ID3 to the task of structuring synthesised schedulers for safety properties and how one can use it to look at the controllability of the system *from the viewpoint of each task* comprising it, as well as, *from a global, system-wide view*. This allows to more easily identify the tasks which have the *highest impact upon the controllability of the system*, to which the designer should focus his attention, as well as, identify *undesired dependencies between critical and non-critical tasks*. Another information we can more easily derive from the decision-tree form of the synthesised schedulers is the *classes of strongly interdependent tasks*, which would be *good candidates for being mapped to the same processing unit*, if the system is distributed. In addition to this kind of information, we showed how synthesised schedulers (especially in their succinct decision-tree form) can be used for *explaining the (usually too numerous) counterexamples of a model-checker*, directly pinpointing the (usually much fewer) states where the problematic behaviours become possible.

We have also shown how to *decrease the number of constraints* needed to implement a safe scheduler by carefully pruning the decision-tree, so as to obtain a *scheduler which is smaller and faster to execute*.

Finally, we showed how a *safe scheduler can be distributed through the different tasks* and be *directly embedded in their code*, possibly *eliminating altogether the scheduling protocol where this is not needed*, as a further optimisation.

The only other work we are aware of where data-mining (indeed ID3) was used in a model-checking context is the work of Edmund Clarke *et al.* [3]. There, the authors used ID3 for discovering the model variables which could better dissociate a number of concrete states belonging to the same abstract state, in order to check whether a counterexample obtained in an abstracted system is also present in the more concrete one or whether it is a spurious one, caused by a coarse abstraction. However, this is the first time that data mining has been applied in a scheduler synthesis context, to the best of our knowledge.

Acknowledgements : The implementation of the adapted tree induction method (ID3) presented herein was based on code written by Raymond Joseph Mooney [6].

References

[1] K. Altisen, G. Gößler, and J. Sifakis. Scheduler modeling based on the controller synthesis paradigm. *Real-Time Systems*, 23(1):55–84, July 2002.

[2] R. E. Bryant. Graph-based algorithms for Boolean function manipulation. *IEEE Trans. on Comp.*, C-35(8):677–691, Aug. 1986.

[3] E. Clarke, A. Gupta, J. Kukula, and O. Strichman. SAT based abstraction-refinement using ILP and machine learning techniques. In *CAV02*, LNCS 2404, pages 265–279, July 2002.

[4] C. Kloukinas, C. Nakhli, and S. Yovine. A methodology and tool support for generating scheduled native code for real-time Java applications. In *EMSOFT'03*, LNCS 2855, pages 274–289, Oct. 2003.

[5] C. Kloukinas and S. Yovine. Synthesis of safe, QoS extendible, application specific schedulers for heterogeneous real-time systems. In *ECRTS'03*, pages 287–294. IEEE Computer Society Press, July 2003.

[6] C. L. Liu and J. W. Layland. Scheduling algorithms for multiprogramming in a hard-real-time environment. *JACM*, 20(1):46–61, Jan. 1973.

[7] J. R. Quinlan. Induction of decision trees. *Machine Learning*, 1(1):81–106, 1986.

[8] J. R. Quinlan. *C4.5: Programs for Machine Learning*. Series in Machine Learning. Morgan Kaufmann, 1993.

[9] L. Sha, R. Rajkumar, and J. P. Lehoczky. Priority inheritance protocols: An approach to real-time synchronization. *IEEE Trans. on Comp.*, C-39(9):1175–1185, Sept. 1990.

5 See J48 in: `http://www.cs.waikato.ac.nz/~ml/weka/`

6 Original code: www.cs.utexas.edu/users/ml/ml-progs.html.