Data normalisation techniques in decision making: case study with TOPSIS method

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Abstract: Data normalisation is essential for decision-making methods because data has to be numerical and comparable to be aggregated into a single score per alternative. In multi-criteria decision-making (MCDM), normalisation must convert criteria values into a common scale, thus, enabling rating and ranking of alternatives. Therefore, it is a challenge to select a suitable normalisation technique to represent an appropriate mapping from source data to a common scale. There are some attempts in the literature to address the subject of normalisation, but it is still an open question which technique is more appropriate for any MCDM method. Our research contribution is an assessment approach for evaluating normalisation techniques. Here, we focus on six well-known normalisation techniques and on TOPSIS method. The proposed assessment process provides a more robust evaluation and selection of the best normalisation technique for usage in TOPSIS.

Keywords: normalisation; TOPSIS; decision making; correlation; consistency; multi-criteria decision-making; MCDM; data fusion.

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1 Introduction

In most multi-criteria decision-making (MCDM) problems, criteria have different scales (e.g., comfort, fuel consumption, design, etc. in selecting a car problem). As such, we should use some pre-processing to obtain a common scale, which will enable aggregation of numerical and comparable criteria to obtain a final score for each alternative.

In general, MCDM models (sometimes also called multiple attribute decision-making, MADM) consist of a set of alternatives A_i ($i = 1, \ldots, m$), a set of criteria C_j ($j = 1, \ldots, n$) and their corresponding weights W_j. Further, r_{ij} is the cell value in the decision matrix, which rates alternative i with respect to criteria j. By normalising the decision matrix values, r_{ij}, we obtain dimensionless elements, therefore enabling aggregation to obtain ratings per alternative (Jahan and Edwards, 2014; Triantaphyllou, 2000). Summarising, the first step for modelling and applying MCDM methods, to solve decision problems, is to choose a suitable normalisation technique for the problem at hand.

There are many performance metrics to assess classification problems (see for example, Eftekhary et al., 2012) but unfortunately, there are very few studies on assessing normalisation techniques for MCDM methods and the question of how to choose the appropriate one is still an open one. In classification problems of the type 'finding features' or 'classifying objects', we can have access to ground-truth results for comparison, however, in MCDM, we only obtain a rating for the candidate alternatives and this rating depends both on the method and normalisation technique used.

Furthermore, if the normalisation technique is not suitable for the decision problem or for the chosen MCDM method, the best decision solution may be overlooked (Chatterjee and Chakraborty, 2014). As Chatterjee and Chakraborty (2014) say "In fact, while the normalisation process scales the criteria values to be approximately of the same magnitude, different normalisation techniques may yield different solutions and, therefore, may cause deviation from the originally recommended solutions". These considerations are the motivation for this article.

In this work, we propose an assessment approach for evaluating six common normalisation techniques (see Table 1), using an illustrative example solved with TOPSIS method (Triantaphyllou, 2000). We chose TOPSIS because it is a well-known and widely used MCDM method (Tzeng and Huang¸2011; Hwang and Yoon, 1981; Yoon and Hwang, 1981), but we plan to perform the same study for other MCDM methods in the future.

Our novel assessment approach combines important aspects of Celen (2014) and Chakraborty and Yeh (2009) processes, such as calculating the ranking consistency index (RCI) to rank normalisation technique; evaluating normalisation techniques with four conditions borrowed from Celen (2014); calculate consistency of techniques with statistical methods such as descriptive statistics and Kolmogorov-Smirnov test; and calculating Pearson and Spearman correlation for both rank of alternatives and closeness of alternatives. Specifically, we propose three steps:

a determining the RCI from Chakraborty and Yeh (2009)

b analysis and evaluation of normalisation techniques consistency with three conditions, borrowed from Celen (2014)

c comparative study between ranking of alternatives using Pearson [as proposed in Celen (2014)] and Spearman correlation (Wang and Luo, 2010) to determine the mean value.

At the current stage, we focused on the TOPSIS method to ensure that our assessment approach is robust, because we compare it with results from Celen (2014). Further, here we only used an illustrative example to check the robustness of the assessment method. This study is a first contribution for finding the most suitable normalisation technique for MCDM methods. In the near future, we will extend this study for other MCDM methods (Triantaphyllou, 2000), such as the simple additive method, ELECTRE, and so forth and we will use simulations for generalising the conclusion about the most suitable technique for each method.

The paper is organised as follows. The next section provides the background and related work for normalisation techniques. Section 3 provides the background for the MCDM method, TOPSIS, used in this work. Section 4 presents a numerical example that is used for illustrating the results obtained with six normalisation techniques, using the TOPSIS method. Section 5 presents a new assessment method that is used for comparing the results and answers the question of which normalisation is more suitable for usage with TOPSIS. Finally, Section 6 summarises this work and points future directions for research on the topic.

2 Normalisation techniques

Normalisation is a transformation process to obtain numerical and comparable input data by using a common scale. After collecting input data, we must do some pre-processing to ensure comparability of data, thus making it useful for decision modelling (Etzkorn, 2015). This pre-processing should consider two important points:

1 all non-numeric data should first be converted into numerical data to allow normalisation (standardisation) (Etzkorn, 2015)

2 choosing a suitable normalisation technique to ensure a common scale and appropriate modelling representation (benefit or cost criteria) as well as comparability on criteria aggregation to obtain alternative ratings (Etzkorn, 2015).

In general, data normalisation and data standardisation means mapping the data values to a common scale, usually within the unity interval [0, 1]. Since some models collapse at the value of zero, sometimes an arbitrary range such as 0.1 to 0.9 is chosen (Etzkorn, 2015), but in this work, we only consider common unity-based normalisation techniques.

Jahan and Edwards (2014) published an interesting paper with an exhaustive survey on normalisation techniques. These authors identified 31 normalisation methods for transforming raw data into dimensionless criteria and classified and discussed some specific pros and cons for each technique. Further, Jahan and Edwards (2014) inspected shortcomings of normalisation techniques for engineering design, which include techniques to handle both cost and benefit criteria.

Pavlicic (2011) analysed the effects of simple (divided by max), linear and vector normalisation techniques on simulations results of TOPSIS, ELECTRE, and simple additive weight (SAW) MCDM methods. Specifically, he showed that results depend on the initial measurement units (e.g., temperature measured in Celsius or Fahrenheit) when using vector or simple normalisation techniques. It should be noted that Pavlicic (2011) provided a motivation to explore other suitable normalisation techniques for the TOPSIS and also to elaborate a more robust assessment method about shortcomings of normalisation techniques for MDCM methods.

Celen (2014) analysed the impact of vector normalisation and three linear normalisations (max-min, max and sum) techniques in TOPSIS method. They used a consistency process for assessing banks performance in Turkey, which included using Pearson correlation. The conclusion was that vector normalisation is the best technique for TOPSIS, in the proposed application. Here, we advance this work by testing other normalisation techniques and also discussing a more general assessment approach to select the best normalisation technique for TOPSIS.

Chakraborty and Yeh (2009) analysed four normalisation techniques (vector normalisation and three linear ones: max-min, max and summation) in the MCDM SAW method. Those authors proposed a RCI to assess which is the best normalisation technique for the SAW method. Here, we tested the same four normalisation techniques plus other two important ones such as the logarithmic and the fuzzification. In addition we performed the assessment for the TOPSIS method.

Among well-known normalisation techniques (Jahan and Edwards, 2014; Celen, 2014; Patro and Sahu, 2015), in this work, we focus on the class of linear ones (N_1, N_2 and N_3 in Table 1) and sum-based ones (N_4 and N_5 in Table 1) and then included the fuzzification technique (Ribeiro, 1996) (N_6 in Table 1). Using fuzzification as a normalisation technique is a novelty of this work and highlights its versatility as a normalisation technique. Most normalisation techniques are divided in two formulas, one for benefit and another for cost criteria, to ensure that the final decision objective (rating) is correct, i.e., when it is a benefit criterion for high values there will correspond high normalised values (maximisation-benefit) and when it is a cost criterion high values will correspond to low normalised values (minimisation-cost). The same logic applies to fuzzification techniques, i.e. memberships functions can be monotonically increasing or decreasing to represent, respectively, benefit or cost.

Table 1 Normalisation techniques

Normalisation technique	*Condition of use*	*Formula*
Linear: Max (N_1) (Celen, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{r_{\max}}$
	Cost criteria	$n_{ij} = 1 - \frac{r_{ij}}{r_{\max}}$
Linear: Max-Min (N_2) (Patro and Sahu, 2015)	Benefit criteria	$n_{ij} = \frac{r_{ij} - r_{\min}}{r_{\max} - r_{\min}}$
	Cost criteria	$n_{ij} = \frac{r_{\max} - r_{ij}}{r_{\max} - r_{\min}}$
Linear: sum (N_3) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$
	Cost criteria	$n_{ij} = \frac{1/r_{ij}}{\sum_{i=1}^{m} 1/r_{ij}}$
Vector normalisation (N_4) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}}$
	Cost criteria	$n_{ij} = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}}$
Logarithmic normalisation (N_5) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{\ln(r_{ij})}{\ln\left(\prod_{i=1}^{m} r_{ij}\right)}$
	Cost criteria	$n_{ij} = \frac{1 - \frac{\ln(r_{ij})}{\ln\left(\prod_{i=1}^{m} r_{ij}\right)}}{m-1}$
Fuzzification (N_6) (Ribeiro, 1996)	Benefit and cost criteria	Using membership function (e.g., trapezoid)

Notes: *r_{ij} is the rating of alternative *i* with respect to criterion *j*.
**n_{ij} is the normalised value of r_{ij}.

Max-min normalisations (N_2 in Table 1) are quite useful for relative comparison between alternatives, i.e., a normalised value provides either the distance from the best candidate (benefit criteria) or from the worst candidate (cost criteria) (Patro and Sahu, 2015). This technique provides normalised values by linear transformation and keeps relationships between original data. A good example of the useful usage of max-min normalisation technique is for classification of candidates in a contest, (each candidate normalised criterion value provides a relative measure of how distant his/her candidacy is from the best candidate and how far away from the worst) (Patro and Sahu, 2015). Vector normalisation techniques (N_4 in Table 1) are symmetric and computationally efficient. Normalisation techniques three (N_3) are not symmetric for cost and benefit ones and the normalised values of alternatives are lower for the benefit criteria and greater for the cost criteria (Jahan and Edwards, 2014). Logarithmic normalisation technique (N_5 in Table 1) is good for situations where we want more discrimination between alternatives (Jahan and Edwards, 2014). Finally, fuzzification (N_6 in Table 1) is related to the intrinsic nature of criteria and objectives (Ribeiro et al., 2014) and it is well suited to represent composed linguistic concepts (criteria) such as 'low prices', 'high temperatures'. The next subsection presents more details about this type of normalisation.

2.1 Fuzzification as a normalisation technique

Fuzzification is the process of converting crisp values into linguistic terms by using membership functions (Schmid, 2005). This process is a mechanism for transforming raw data into fuzzy sets (functions), which appropriately represent concepts understandable for decision makers. This mechanism enables dealing with alternatives and criteria of decision-making problems (Ribeiro et al., 1995, 2014). Lee (1990) introduced the main role of fuzzification as a transformation of data to a suitable form of fuzzy set theory and Ribeiro et al. (2014) proposed this technique for data fusion to obtain a single composite value for alternatives.

An important issue on the variables 'fuzzification' is to select suitable membership functions since we need to consider the context and objective (Ribeiro et al., 2014). There are various proposals in the literature on how to fuzzify concepts/criteria (data). Here, we selected the simplest trapezoidal membership because it is an initial study on the normalisation with fuzzifiction. There are many applications using fuzzification techniques to normalise and allow comparable data (Ross, 2004; Tzeng and Huang, 2011), however, they did not formally recognised fuzzification as another normalisation technique. For example, Pires et al. (1996) and Ribeiro and Varela (2003) used fuzzification for solving fuzzy set optimisation problems. Many other authors also applied fuzzification as a normalisation technique in order to deal with dimensionless data in fuzzy MCDM problems (see for example, Ribeiro et al., 2014; Tzeng and Huang, 2011; Zhang et al., 2014).

3 Background on TOPSIS: a MCDM method

TOPSIS is a technique for order performance by similarity to ideal solution, developed by Hwang and Yoon (1981) and it is one of the well-known MCDM methods. It ranks alternatives based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). PIS is the most beneficial and lowest cost of alternatives and NIS is the lowest benefit and highest cost (Cheng et al., 1999). The general TOPSIS process has the following steps (Joshi et al., 2011):

Step 1 Defining decision matrices that can be expressed as follows:

$$D = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

where $i = 1, .., m$ denotes the alternatives and $j = 1, \ldots, n$ refers to the attributes; r_{ij} represents the j^{th} attribute related to i^{th} alternative.

Step 2 Normalising the value of decision matrices as follows:

$$n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{n} r_{ij}^2}}$$

where $j = 1, \ldots, n$; and $i = 1, \ldots, m$.

Step 3 Calculating the weighted normalised decision matrix by multiplying the normalised decision matrix by its associated weights:

$$W_{ij} = w_{ij} * n_{ij}$$

where w_{ij} represents the weight of the j^{th} attribute related to i^{th} alternative.

Step 4 Determining the PIS (A^+) and NIS (A^-).

$$A^+ = \{W_1^+, \ldots, W_n^+\} = \{(Max\, W_{ij} \mid j \in J), (Min\, W_{ij} \mid j \in J')\}$$

$$A^- = \{W_1^-, \ldots, W_n^-\} = \{(Min\, W_{ij} \mid j \in J), (Max\, W_{ij} \mid j \in J')\}$$

where J represents the positive factors and J' is the negative factors. (e.g., in car selection example, fuel consumption and price are negative factors or criteria and comfort and safety are positive criteria.)

In this work, we used the maximum and minimum as the positive ideal and negative ideal, however, when the data is normalised in scale [0, 1] we also have the option of using 1 for the ideal and 0 for the negative ideal.

Step 5 Calculating the distance of all alternatives to the PIS (D_i^+) and the negative ideal (D_i^-) solution.

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(W_{ij} - W_j^+\right)^2}, \quad i = 1, \ldots, m$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(W_{ij} - W_j^-\right)^2}, \quad i = 1, \ldots, m$$

Step 6 Calculating the relative closeness of each alternative as follow:

$$C_i^* = \frac{D_i^-}{D_i^+ + D_i^-}$$

where C_i^* relies between 0 and 1 and the higher value corresponds to better performance.

TOPSIS is one of the classical MCDM methods used in many different areas such as supply chain management and logistics; design, engineering and manufacturing systems; business and marketing management; health, safety and environment management, and so forth (Behzadian et al., 2012; Kwong and Tam, 2002; Khorshidi and Hassani, 2013; Kahraman et al., 2009; Alimoradi et al., 2011; Mahdavi et al., 2008; Krohling and Campanharo, 2011).

In this paper, we selected TOPSIS for demonstrating how normalisation techniques can affect the results of MCDM methods. Further, we followed the steps described above for determining the results of the illustrative example described next. In future work, we plan to generalise the importance of normalisation with other MCDM methods to demonstrate the generality of our assessment approach to select appropriate normalisation techniques for MCDM methods.

4 Numerical example

As mentioned before, in this paper we discuss the importance of normalisation with a numerical example based on a real project of autonomous landing of drones with hazard avoidance, where the criteria are partial hazard maps, used in the project (http://www.ca3-uninova.org/project_iluv), to reduce the illustrative case to three criteria (C_1, C_2, C_3), which correspond to illumination, reachability, and land texture, and 16 alternatives (A_1, A_2, …, A_{16}), which correspond to candidate location sites for landing.

Table 2 shows the data used for discussing the normalisation techniques, where C_1 and C_2 are criteria benefit, i.e., the higher the raw values the better they should be on the normalisation and C_3 is a cost criteria where low normalised values are desirable.

As in any other MCDM method, TOPSIS Step 1 is defining the decision matrix (see Table 2).

Table 2 Decision matrix for landing drones

	C_1 *(illumination)*	C_2 *(reachability)*	C_3 *(land texture)*
A_1	138.6090	0.3349	6.4543
A_2	154.7214	0.3395	11.4244
A_3	158.3081	0.3441	11.4244
A_4	157.3082	0.3487	6.8542
A_5	144.5976	0.3301	11.2616
A_6	138.5982	0.3346	11.2616
A_7	131.5989	0.3391	11.1988
A_8	132.5988	0.3437	11.1988
A_9	144.5976	0.3252	11.2616
A_{10}	138.5982	0.3297	11.2616
A_{11}	132.5988	0.3342	11.1988
A_{12}	135.9513	0.3387	6.8974
A_{13}	119.7141	0.3204	11.2616
A_{14}	112.7148	0.3248	11.1988
A_{15}	112.7148	0.3292	11.1988
A_{16}	128.9520	0.3337	6.8974

In Step 2, we calculate the normalised decision matrix. To illustrate this process, we use one alternative, A_3, with the six tested normalisation techniques from Table 1. We illustrate the numerical calculations and after the results are summarised in Table 3.

$$n_{Max,31} = \frac{158.3081}{158.3081} = 1$$

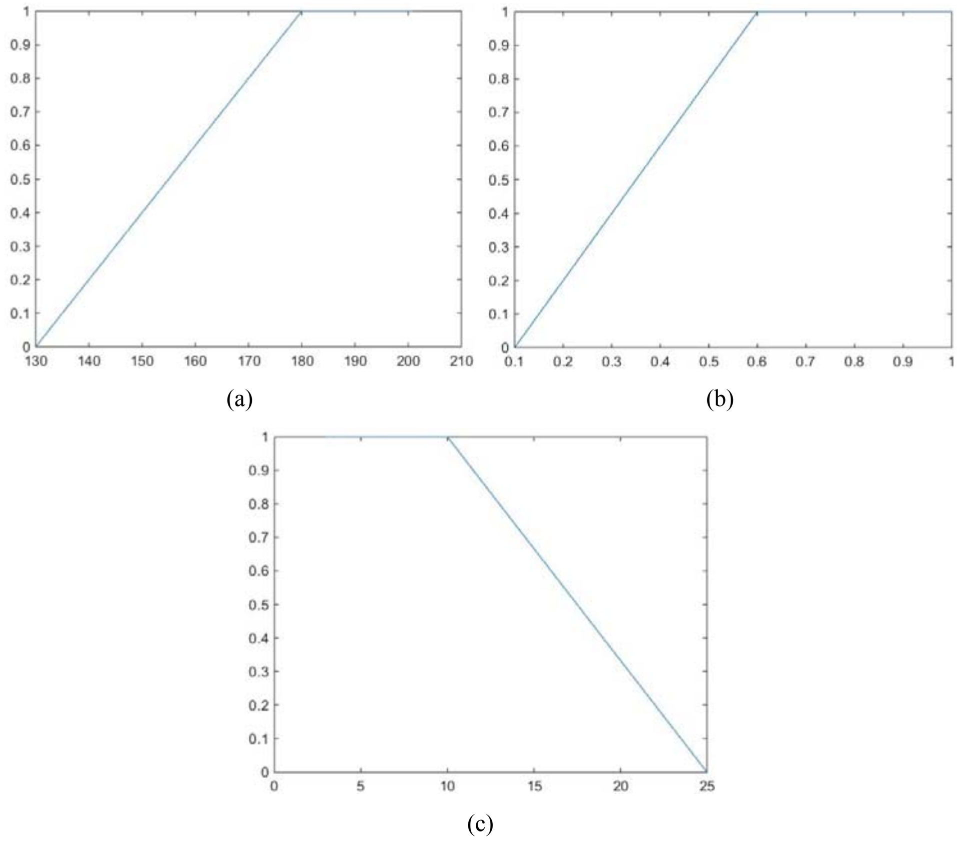
$$n_{Max-Min,31} = \frac{158.3081 - 112.7148}{158.3081 - 112.7148} = 1$$

$$n_{Linear,31} = \frac{158.3081}{138.609 + 154.7214 + \cdots + 128.952} = 0.0725$$

$$n_{Vector,31} = \frac{158.3081}{\sqrt{138.609^2 + 154.7214^2 + \cdots + 128.952^2}} = 0.2887$$

$$n_{Logarithmic,31} = \frac{\ln(158.3081)}{\ln(138.609 * 154.7214 * \cdots * 128.952)} = 0.0645$$

Figure 1 Fuzzification of criteria for landing drones, (a) C_1 domain [130, 180, 201, 201] (b) C_2 domain [0.1, 0.6, 1, 1] (c) C_3 domain [3, 3, 10, 25] (see online version for colours)



For fuzzification, we used trapezoidal membership functions as illustrated in Figure 1 for all criteria. We used trapezoidal membership function because it is the simplest and more common membership function, other membership functions could have been used (Ross, 2004).

C_3 is a cost criteria, so, we should use the trapezoidal membership function that gives higher membership values the lower values and lower rating (membership value) for higher values (see Figure 1).

In Step 3, we calculate the weighted normalised decision matrix by multiplying weights to the criteria of the normalised decision matrix. For simplicity purposes, in this example, we consider all criteria of equal importance, hence, for the three criteria the weights are 0.333.

In Step 4, after choosing the maximum and minimum criteria value from the weighted normalised matrix we calculate the positive-ideal and negative-ideal solutions based on the nature of the criterion (cost or benefit ones). Illustrating for A_3 as follows:

$$D^{+}_{Max,3} = \sqrt{(0.333-0.333)^2 + (0.3285-0.333)^2 + (0-0.334)^2} = 0.0043$$

$$D^{-}_{Max,3} = \sqrt{(0.333-0.2370)^2 + (0.3286-0.3059)^2 + (0-0.1453)^2} = 0.1755$$

For performing Step 5 and Step 6 of TOPSIS, we calculate the relative closeness and the rank of alternatives using positive and NIS values. Table 3 shows the results for alternative A_3 and those results are illustrated for linear normalisation, as follows:

$$C^{*}_{Max,3} = \frac{0.1756}{0.1756-0.0044} = 0.9756$$

Table 3 Result of TOPSIS steps for A_3

		N_1	N_2	N_3	N_4	N_5	N_6
Normalised decision matrix	C_1	1	1	0.0725	0.2888	0.0645	1
	C_2	0.9868	0.8375	0.0643	0.2572	0.0932	0.4882
	C_3	0	0	0.0528	0.7234	0.0622	0.9050
Positive and negative ideal solutions	D^+	0.0044	0.0541	0.0003	0.0011	0.0021	0.0031
	D^-	0.1756	0.5479	0.0153	0.0492	0.0015	0.3349
Relative closeness values	C_3^*	0.9756	0.9101	0.9817	0.9772	0.4244	0.9908
Rank of normalisation techniques	Rank	1	1	1	1	12	1

Now, we will discuss which normalisation technique is more suitable to rate and rank the 16 alternatives. As mentioned before, we compared the following six techniques: max, max-min, sum, vector, logarithmic, and fuzzification normalisation techniques (see Table 1). We performed the same six steps as done for the illustrative case of alternative A_3.

The relative closeness values and comparison of the ranking results for 16 alternatives with six normalisation techniques are shown in Table 4. As expected, the ranking of alternatives differs when using different normalisation techniques. It is interesting to observe alternatives A_3 and A_9 because they are the best alternatives for different criteria. A_3 is considered the best candidate for max (N_1), max-min (N_2), linear (N_3), vector (N_4) and fuzzification (N_6) techniques, while A_9 (N_5) is the best for the logarithmic normalisation.

Table 4 Relative closeness (RC) values and ranking

	N_1		N_2		N_3		N_4		N_5		N_6	
A_1	0.2704	15	0.3901	15	0.2252	15	0.2772	15	0.5031	8	0.8345	8
A_2	0.9366	2	0.8196	2	0.9501	2	0.9380	2	0.4851	10	0.9820	2
A_3	0.9756	1	0.9101	1	0.9817	1	0.9772	1	0.4244	12	0.9908	1
A_4	0.4239	13	0.6030	8	0.3816	13	0.4325	13	0.3629	15	0.9131	5
A_5	0.8202	3	0.6323	6	0.8553	3	0.8189	3	0.6718	3	0.9626	3
A_6	0.7753	5	0.6508	4	0.8160	5	0.7703	5	0.5245	6	0.8577	6
A_7	0.7173	9	0.6454	5	0.7668	9	0.7096	9	0.3751	14	0.1786	12
A_8	0.7283	7	0.6927	3	0.7756	7	0.7204	7	0.2864	16	0.2715	10
A_9	0.8087	4	0.5775	10	0.8470	4	0.8089	4	0.7999	1	0.9540	4
A_{10}	0.7684	6	0.5944	9	0.8111	6	0.7645	6	0.6517	4	0.8557	7
A_{11}	0.7213	8	0.6031	7	0.7710	8	0.7145	8	0.5026	9	0.2695	11
A_{12}	0.2754	14	0.4305	12	0.2592	14	0.2806	14	0.3967	13	0.5888	9
A_{13}	0.6225	10	0.4284	13	0.6850	10	0.6147	10	0.6784	2	0.0777	13
A_{14}	0.5841	12	0.4255	14	0.6502	12	0.5749	12	0.5835	5	0.0745	15
A_{15}	0.5866	11	0.4531	11	0.6520	11	0.5770	11	0.5098	7	0.0758	14
A_{16}	0.2086	16	0.3253	16	0.2092	16	0.2120	16	0.4830	11	0.0259	16

5 Assessment approach of normalisation techniques for TOPSIS

Since it is difficult to assess which is the best normalisation technique just by looking at the results obtained because several seemed appropriate for instance for A_3, we looked at other metrics in the literature to define a consistent assessment approach for selecting the best normalisation technique.

There are few metrics, proposed in the literature, to perform the assessment of normalisation techniques in MCDM methods (Celen, 2014; Chakraborty and Yeh, 2009). Celen (2014) used a consistency process for assessing banks performance in Turkey, which included using Pearson correlation, as a metric to assess normalisation techniques. Chakraborty and Yeh (2009) used a RCI. In our assessment, we used these two plus another correlation metric called Spearman's rank correlation (Wang and Luo, 2010).

Chakraborty and Yeh (2009) mention that "Ranking consistency is used to indicate how well a particular normalization procedure produces rankings similar to other procedures". They calculated the RCI for each normalisation method by using the total number of times that these normalisations have similarity or dissimilarity in the problem with the total number of times that simulation was run (10,000 times in their example). After that, they analysed the RCI for different normalisation techniques by drawing a diagram of the results. Their diagram shows that the best normalisation technique is vector normalisation because of highest RCI. The higher value of RCI has better rank in the example.

Therefore, in our study, we propose a new assessment approach by combining important aspects of the Celen (2014) and Chakraborty and Yeh (2009) processes, and then we compared Pearson correlation (used by Chakraborty and Yeh, 2009) with Spearman's rank correlation (used by Wang and Luo, 2010), to ensure a more robust and consistent evaluation and selection of the best normalisation technique for TOPSIS. Specifically, our novel assessment approach includes three steps:

a Determining the RCI from Chakraborty and Yeh (2009).

b Analysis and evaluation of normalisation techniques consistency with three conditions, borrowed from Celen (2014).

c Comparative study between ranking of alternatives using Pearson (as proposed in Celen, 2014) and Spearman correlation (Wang and Luo, 2010) to determine the mean value.

5.1 Step A

For step A, we do not use simulation, but instead, we calculate the RCI (from Chakraborty and Yeh, 2009) with the number of similarity or dissimilarity for the tested normalisations. Since we have six normalisation techniques we start by defining the consistency weight (CW) as follows:

1 if a technique is consistent with all other five techniques, then CW = 5/5 = 1

2 if a technique is consistent with four of the five techniques, then CW = 4/5

3 if a technique is consistent with three of the five techniques, then CW = 3/5

4 if a technique is consistent with two of the five techniques, then CW = 2/5

5 if a technique is consistent with one of the five techniques, then CW = 1/5

6 if a technique is not consistent with any other five techniques, then CW = 0/5 = 0.

And then the RCI, for instance for N_1, is calculated as (Chakraborty and Yeh, 2009):

$$
\begin{aligned}
RCI(N_1) = \Big[& \left(T_{123456} * (CW = 1)\right) + \left(T_{12345} * (CW = 4/5)\right) + \left(T_{13456} * (CW = 4/5)\right) \\
& + \left(T_{12456} * (CW = 4/5)\right) + \left(T_{12346} * (CW = 4/5)\right) + \left(T_{12356} * (CW = 4/5)\right) \\
& + (T_{1234} * (CW = 3/5)) + \left(T_{1235} * (CW = 3/5)\right) + \left(T_{1236} * (CW = 3/5)\right) \\
& + \left(T_{1245} * (CW = 3/5)\right) + \left(T_{1246} * (CW = 3/5)\right) + \left(T_{1256} * (CW = 3/5)\right) \\
& + \left(T_{1346} * (CW = 3/5)\right) + \left(T_{1356} * (CW = 3/5)\right) + \left(T_{1345} * (CW = 3/5)\right) \\
& + \left(T_{1456} * (CW = 3/5)\right) + \left(T_{123} * (CW = 2/5)\right) + \left(T_{124} * (CW = 2/5)\right) \\
& + \left(T_{125} * (CW = 2/5)\right) + \left(T_{126} * (CW = 2/5)\right) + \left(T_{134} * (CW = 2/5)\right) \\
& + \left(T_{12} * (CW = 1/5)\right) + \left(T_{13} * (CW = 1/5)\right) + \left(T_{14} * (CW = 1/5)\right) \\
& + \left(T_{15} * (CW = 1/5) + \left(T_{16} * (CW = 1/5)\right) + \left(TD_{123456} * (CW = 0)\right)\right) / TS \Big]
\end{aligned}
$$

where

$RCI(X)$	RCI for normalisation procedure ($X = N_1, N_2, \ldots, N_6$)
TS	total number of times the simulation was run (in this study $TS = 1$)
TD_{123456}	total number of times N_1, N_2, N_3, N_4, N_5 and N_6 produced different rankings
T_{123456}	total number of times N_1, N_2, N_3, N_4, N_5 and N_6 produced the same ranking
T_{12345}	total number of times N_1, N_2, N_3, N_4 and N_5 produced the same ranking
T_{1234}	total number of times N_1, N_2, N_3 and N_4 produced the same ranking
T_{123}	total number of times N_1, N_2 and N_3 produced the same ranking
T_{12}	total number of times N_1 and N_2 produced the same ranking.

The RCI for the other normalisation techniques is calculated similarly to the above formula and the results are depicted in Table 5. As shown, RCI points to vector normalisation (N_4) as the best normalisation technique for TOPSIS method and the worst one is logarithmic (N_5).

Table 5 RCI of normalisation techniques

	RCI	*Rank*
Linear: Max (N_1)	36.8	2
Linear: max-min (N_2)	24.2	5
Linear: Sum (N_3)	34.8	3
Vector normalisation (N_4)	37.4	1
logarithmic normalisation (N_5)	8.6	6
Fuzzification (N_6)	28.6	4

5.2 Step B

For step B of our assessment approach, we analyse and evaluate the consistency of all normalisation techniques with three conditions borrowed from Celen (2014). The chosen conditions are defined as follows (Celen, 2014):

- Condition 1: The result should consider similarity (closeness) metrics in distributional properties such as means, standard deviations, minimum and maximum values.
- Condition 2: Check for normal distributions to ensure consistency using Kolmogorov- Smirnov test.
- Condition 3: Comparison of best and worst ranking three results for robustness purposes. When normalisation techniques rank alternatives mostly in the same order we can say the results are more robust.

For the first condition of the assessment approach, we determine the descriptive statistics (Celen, 2014) for the six normalisation techniques (see Table 6). By just looking at Table 6, we cannot determine similarity in distributional properties, so, we also applied the Kolmogorov-Smirnov test (Celen, 2014) to check the consistency of normalisation techniques about normal distribution.

For condition 2, we used the Kolmogorov-Smirnov test (see Table 6). The amount of Skewness and Kurtosis is between (–2, 2) so, we can say there is the possibility for our data to have normal distributions. However, to be sure about normal distributions we also need to calculate the statistic test and significant level test (Sig) (Field, 2000; Trochim and Donnelly, 2006). The amount of statistical test should be less than 1 and the amount of significant level test (Sig) should be more than 0.05 (sig > 0.05) (Field, 2000; Trochim and Donnelly, 2006). In Table 6, for all normalisation techniques in Kolmogorov-Smirnov test, the amount of statistic test are less than 1 but the significant level test (Sig) for N_1, N_2, N_4 and N_5 is not higher than 0.05. Therefore, for those cases their normal distribution consistency is not proven. N_3 and N_6 both fulfil condition 2 and they are consistent regarding normal distribution. The Sig test should be further investigated because we have some doubts about the requirement for normal distribution in the normalisation techniques.

Table 6 Condition 1 and 2 – descriptive statistics, Kolmogorov-Smirnovtest for normalisation techniques

		N_1	N_2	N_3	N_4	N_5	N_6
Statistics of closeness coefficient values	Mean	0.6389	0.5739	0.6648	0.6370	0.5149	0.5571
	Std. deviation	0.2346	0.1588	0.2551	0.2321	0.1355	0.3966
	Minimum	0.2086	0.3253	0.2092	0.2120	0.2864	0.0259
	Maximum	0.9756	0.9101	0.9817	0.9772	0.7999	0.9908
Kolmogorov-Smirnov test statistics	Skewness	–0.2164	0.4168	–0.5719	–0.8760	0.4381	–0.6273
	Kurtosis	–1.9673	–0.1026	–0.5665	–0.5441	–0.0649	–0.5574
	Statistic	0.193	0.152	0.227	0.185	0.159	0.258
	Sig.	0.112	0.200	0.027	0.145	0.200	0.006

After, we evaluate the consistency for condition 3 (Celen, 2014). For this evaluation, we examined the result of TOPSIS by choosing the highest three and the lowest three ranked alternatives for each normalisation technique (see Table 7). As it is shown, the logarithmic normalisation technique (N_5) has very different scoring from other techniques. Also, max-min (N_2) and fuzzification (N_6) have some different scores from the others (they are highlighted in Table 7 with the grey colour). The other three techniques (N_1, N_3 and N_4) have the same results, i.e., these normalisation techniques generate rather the more robust results.

Table 7 Condition 3 – comparison of best and worst normalisation techniques

	Rank	N_1	N_2	N_3	N_4	N_5	N_6
Three highest rank	1	A_3	A_3	A_3	A_3	A_9	A_3
	2	A_2	A_2	A_2	A_2	A_{13}	A_2
	3	A_5	A_8	A_5	A_5	A_5	A_5
Three lowest rank	14	A_{12}	A_{14}	A_{12}	A_{12}	A_7	A_{15}
	15	A_1	A_1	A_1	A_1	A_4	A_{14}
	16	A_{16}	A_{16}	A_{16}	A_{16}	A_8	$A_1$6

5.3 Step C

For completing the evaluation process (Celen, 2014), we calculate the Pearson correlation between ranking alternative values from Table 4 and the results are depicted in Table 8. As mentioned before, we also tested Spearman correlation (Wang and Luo, 2010) to compare the result of Spearman correlation with the results of the Pearson one. With this comparison, we further ensure robustness and consistency in the assessment approach for evaluating normalisation techniques.

The reasoning for choosing Spearman's rank correlation coefficient is that it is a non-parametric test to define the degree of association between two variables without any assumption about distribution of data. Spearman's correlation coefficient is usually a good method to define the association and strength of a relationship between two sets of data and variables with ordinal scale in the problem. Spearman's correlation coefficient (q_s) is defined as (Chakraborty and Yeh, 2009):

$$q_s = 1 - 6\frac{\sum_{i=1}^{m} D_i^2}{m(m^2 - 1)}$$

where D_i is the difference between ranks r_i and r_i' and m is the number of alternatives; q_s value lies between –1 and +1.

In this evaluation, for all pairs of normalisation techniques, we calculated their correlation and also the average k_s value to determine the mean ranking agreement among them (Wang and Luo, 2010), as shown in Table 8.

Observing the results of Spearman and Pearson correlation and their mean ks value (Table 8), we see that max, linear and vector normalisation (N_1, N_3 and N_4) have equal importance because they display the same highest average value (P = 0.753 and S = 0.720). Fuzzification (N_6) gets the second place and logarithmic normalisation technique (N_5) is the worst technique for TOPSIS. It should be highlighted that calculating correlation between rankings of alternatives was not discriminative for both the Person and Spearman correlation methods because it displayed a draw for the three best techniques.

Another important conclusion from this assessment analysis is that fuzzification normalisation (N_6), introduced in this work, and is not really appropriate for usage with TOPSIS method.

For the evaluation of step C, we also use Pearson and Spearman correlation to check closeness between alternatives, as shown in Table 9. As it can be seen in Table 9, Pearson and Spearman correlation point to N_4 (vector normalisation) as the best technique for TOPSIS between proposed techniques, because it has the highest mean k_s value (P = 0.690 and S = 0.998), while N_5 (logarithmic normalisation) is the worst normalisation technique for TOPSIS (lowest mean k_s value (0.136) with Pearson correlation and N_6 (fuzzification) is the worst one with using Spearman correlation.

Table 8 Step C – Pearson (P) and Spearman (S) correlation between rankings of alternatives (see online version for colours)

	N1		*N2*		*N3*		*N4*		*N5*		*N6*		*Mean ks value*		*Rank*	
	*P**	*S***	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*
N1			0.808	0.783	1	1	1	1	0.244	0.143	0.714	0.676	0.753	0.720	1	1
N2	0.808	0.783			0.808	0.783	0.808	0.783	–0.323	–0.50	0.641	0.593	0.548	0.488	3	3
N3	1	1	0.808	0.783			1	1	0.244	0.143	0.714	0.676	0.753	0.720	1	1
N4	1	1	0.808	0.783	1	1			0.244	0.143	0.714	0.676	0.753	0.720	1	1
N5	0.244	0.143	–0.323	–0.50	0.244	0.143	0.244	0.143			0.032	–0.096	0.088	–0.033	4	4
N6	0.714	0.676	0.641	0.593	0.714	0.676	0.714	0.676	0.032	–0.096			0.563	0.505	2	2

Notes: *Pearson correlation coefficient
**Spearman correlation coefficient

Table 9 Step C – Pearson (P) and Spearman (S) correlation between closeness of alternatives (see online version for colours)

	N1		*N2*		*N3*		*N4*		*N5*		*N6*		*Mean ks value*		*Rank*	
	P	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*	*P*	*S*
N1			0.835	0.999	0.992	0.999	0.999	1	0.234	0.998	0.366	0.996	0.685	0.998	2	3
N2	0.835	0.999			0.776	0.999	0.844	0.999	–0.224	0.998	0.552	0.997	0.556	0.998	4	2
N3	0.992	0.999	0.776	0.999			0.989	0.999	0.266	0.997	0.267	0.995	0.658	0.998	3	4
N4	0.999	1	0.844	0.999	0.989	0.999			0.232	0.998	0.386	0.996	0.690	0.998	1	1
N5	0.234	0.998	–0.224	0.998	0.266	0.997	0.232	0.998			0.170	–0.096	0.136	0.779	6	5
N6	0.366	0.996	0.552	0.997	0.267	0.995	0.386	0.996	0.170	–0.096			0.348	0.777	5	6

Summarising, with the results of our assessment approach (Tables 6 to 10), we can claim, that vector normalisation (N_4) is the best normalisation technique for TOPSIS with this input data and between proposed techniques, the max (N_1) is ranked second, sum (N_3) is ranked third, and logarithmic normalisation (N_5) is the worst one. There is no consensus for max-min (N_2), and Fuzzification (N_6) as the fourth and fifth normalisation techniques. Further, with our evaluation approach, we not only validated the Chakraborty and Yeh (2009) result but also offered other candidate solutions for normalisation techniques, as for example using max (N1) (ranked second) and also identified the worst solution (logarithmic), which definitively should not be used as normalisation technique in TOPSIS. It should be noted that the ranking of techniques did not completely prove the requirement for normal distributions (condition 2 of step B in the Sig metric) and this test should be improved in the future.

6 Conclusions

Normalisation is an inseparable part of the decision-making process because we need to obtain dimensionless units for calculating the final rating per alternative. This preliminary study demonstrated the effects of using six common and well-known normalisation techniques.

We compared those six normalisation techniques using a small illustrative example and performed a consistent and robust assessment to determine which technique is more appropriate for TOPSIS method. The example is used for demonstrative purposes of the assessment process proposed in this study.

Our proposed assessment approach is quite robust and included three steps:

a determining a RCI

b performing an analysis and evaluation of normalisation techniques consistency with three conditions

c realising a comparative study between ranking of alternatives using Pearson and Spearman correlations to determine the mean value for each normalisation technique.

We demonstrated that vector normalisation technique (N_3) seems to be the most suitable for TOPSIS method and logarithmic normalisation technique is the worst one. This result is in accordance to other literature results (Celen, 2014; Pavlicic, 2011) but here we proposed a more robust and consistent assessment process, which provided a ranking for six well-known normalisations techniques to help decision maker to make more informed decisions. Researchers and practitioners should take these results in consideration when they normalise their data in MCDM application with TOPSIS method.

As future work we plan to apply the same assessment approach to other well-known MCDM methods (besides TOPSIS) – as for example, weighted average, ELECTRE or AHP – to determine which is the more adequate normalisation technique for each MCDM method. Also, we plan to perform more simulations in order to generalise our preliminary conclusion about the most suitable technique for each method. Another issue to be improved is the normal distribution requirement, which needs further investigation to ensure normal distributions are mandatory.

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