

10-15-2019

De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree

Avishek Chakraborty

Shreyashree Mondal

Said Broumi

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Chakraborty, Avishek; Shreyashree Mondal; and Said Broumi. "De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree." *Neutrosophic Sets and Systems* 29, 1 (2020). https://digitalrepository.unm.edu/nss_journal/vol29/iss1/1

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact amywinter@unm.edu, lsloane@salud.unm.edu, sarahrk@unm.edu.



De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree

Avishek Chakraborty^{1,4}, Shreyashree Mondal², Said Broumi³

¹ Department of Basic Science, Narula Institute of Technology, Agarpara, Kolkata-700109, India.
avishek.chakraborty@nit.ac.in

² Department of Information Technology, Narula Institute of Technology, Agarpara, Kolkata-700109, India
shreyashreemanai@gmail.com

³ Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955,
Sidi Othman, Casablanca, Morocco, broumisaid78@gmail.com

⁴ Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, India.

Abstract: In this current era, neutrosophic set theory is a crucial topic to demonstrate the ambiguous information due to existence of three disjunctive components appears in it and it provides a wide range of applications in distinct fields for the researchers. Generally, neutrosophic sets is the extended version of crisp set, fuzzy set and intuitionistic fuzzy sets to focus on the uncertain, hesitant and ambiguous datas of a real life mathematical problem. Demonstration of pentagonal neutrosophic number and its classification in different aspect is focused in this research article. Manifestation of de-neutrosophication technique of linear pentagonal neutrosophic number using removal area method has been developed here which has a remarkable impact in crispfication of pentagonal neutrosophic number. Afterthat, utilizing this invented result, a minimal spanning tree problem has been solved in pentagonal neutrosophic environment. Comparison analysis is done with the other established method in this article and this noble design will be beneficial for the researchers in neutrosophic domain in future.

1. Introduction

Currently, one of the eminent experimental studies of this era is on the subject of unpredictability and indeterminateness. On this aspect, Conception of Fuzzy set [1] has come up with an efficient way to work on. The theory of uncertainty plays an important role to deal with different issues relating to structure modelling in engineering domain, to do statistical calculation, in the field of social science and in any sort of real life problems relating to decision making and networking. After the invention of fuzzy set theory, researchers from several fields developed triangular [2, 3], trapezoidal [4], pentagonal [5] fuzzy number and its applications in various field of research. Professor Atanassov [6] put forward the concept of intuitionistic fuzzy sets where he considered both the idea of membership and non-membership functions. Later, in 2007 Liu F [7], merged the idea of triangular fuzzy set and intuitionistic set and created triangular intuitionistic fuzzy set. Further, Ye [8] familiarized with a basic concept on trapezoidal intuitionistic fuzzy set which includes both the truthiness and falseness membership function which are trapezoidal number in nature. Disjunctive interesting models in science and technology are developed day by day due to the invention of uncertainty theory.

In year 1995, Smarandache proposed the concept of neutrosophic sets, which was published in 1998 [9], comprised of three distinct logical components: i) truthfulness, ii) skepticism, iii) falsity. Due to the presence of hesitation component this theory gave a high impact in different kind of research domain. Further, Wang et al. [10] proposed single valued neutrosophic sets; Ye [11] formulated the concept of simplified Neutrosophic Sets, and Peng et. al. [12, 13] introduced some ideas on novel operations and aggregation operators. Recently, the concept of several forms of triangular and trapezoidal neutrosophic numbers having membership functions that are dependent or independent was manifested by Chakraborty et.al [14, 15]. In 2015, R. Helen [16] manifested the idea of pentagonal fuzzy number and A.Vigin [17] utilized it in neural network. T.Pathinathan [18] provided with the conception of reverse order triangular, trapezoidal and pentagonal fuzzy number. Several researches on neutrosophic arena were published in different fields like multi criteria decision making [19-26], graph theory [27-31], optimization techniques [32, 33] etc. Recently (2019), Chakraborty A [34] manifested the concept of pentagonal neutrosophic number and its classification component wise and applied it in solving a transportation problem in neutrosophic domain. Demonstration of pentagonal neutrosophic fuzzy number and its de-Neutrosophication value using removal area technique has been developed in this article, moreover it is applied on graph theory problem to evaluate the minimal spanning tree.

In this current epoch, neutrosophic set theory is applied in different sections of graph theory to evaluate the minimum path. Minimal spanning tree is one of the extremely vital concepts in the field of graph theory. Single valued neutrosophic minimal spanning tree and clustering method associated with it was originated by Ye [35]. Mandal & Basu [36] introduced similarity measure in optimum spanning tree problems related with neutrosophic arena. Mullai et.al [37] formulated minimum spanning tree problem in bipolar neutrosophic domain. Further, Broumi et.al [38, 39] manifested the concept of shortest path problem in neutrosophic graphs. Later, Broumi et.al [40] generated the perception of decision-making problem with the help of interval valued neutrosophic number and Kandasamy [41] developed double-valued neutrosophic sets and their application in minimum spanning tree problems. Currently, Broumi et.al [42] formulated neutrosophic shortest path for solving Dijkstra algorithm in graph theory. A few published articles [43-50] are addressed here related with neutrosophic domain which plays an important role in uncertainty research arena. Recently, in 2017 F. Smarandache developed a concept namely Plithogenic set, which has a great impact in current research arena and its is applied in hospital care system [51], IoT based problem [52], multi criteria decision making problem [53], cancer related problems [54], fractal programming problem [55], hybrid MCDM problem [56,57] and forecasting problems [58] etc.

1.1 Motivation

The invention of uncertainty theory plays a vital role in formulation of real-life scientific mathematical model, structural modelling in engineering domain, multi criteria oriented medical diagnosis problem etc. Recently, a question will arise if someone choose pentagonal neutrosophic number in any field of research then what will be the crispification value of this said number? How can we convert a pentagonal neutrosophic number equivalent to a crisp number in logical and scientific way? How can we generated some motivating approach in de-neutrosophication technique? Again, The concept of minimal spanning tree is a very well known concept in

mathematics field. Now, generally we considered crisp numbers in place of weight in a spanning tree problem. But, suppose the exact value of the weights are unknown to us and decision maker's mind is in dilemma in case of putting the exact weights. Thus, it is a conception of neutrosophic number which contains truth, falsity and hesitation components. Here we consider pentagonal neutrosophic numbers to allocate the weights of a spanning tree problem. Now, question will arise at once how can we tackle this problem in neutrosophic environment? From this aspect we shall try to built up this article.

The following table discusses the measurement of uncertainty, vagueness and hesitation of four disjunctive types of Minimal Spanning Tree including crisp environment, fuzzy environment, intuitionistic fuzzy environment and pentagonal neutrosophic environment.

Edge Parameters in case of Minimal Spanning Tree Problem	Measurement of Uncertainty	Measurement of Hesitation	Measurement of Vagueness
Crisp Number	×	×	×
Crisp Interval Valued Number	×	×	×
Fuzzy Number	Can Determine	×	×
Interval Valued Fuzzy Number	Can Determine	×	×
Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Interval Valued Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Pentagonal Neutrosophic Number	Can Determine	Can Determine	Can Determine

From the above table, it is observed that only pentagonal neutrosophic environment can tackle the impreciseness, hesitation and truthiness in a membership function of a uncertain number, which is more reliable, logical and realistic for a decision maker. Thus, we consider our minimal spanning tree model in neutrosophic arena and all the edges of the graph as pentagonal neutrosophic number all the graph.

Advantage and Restrictions of disjunctive categories of set

The below table will shows us the advantage and restrictions of different kind of parameters in our real life mathematical problems.

Disjunctive Categories of Set/Number	Advantages	Restrictions
Crisp Number	Determine the accurate value of a realistic problem perfectly.	Cannot determine the uncertainty information of a realistic problem.
Fuzzy Number	Can describe the uncertainty information of a realistic problem.	Cannot describe the hesitation & falsity information of a realistic problem.
Intuitionistic Fuzzy Number	Can determine the uncertainty & falsity information of a realistic problem.	Cannot determine the hesitation information of a realistic problem.
Pythagorean Fuzzy Number	Can deal with the uncertainty & falsity information of a realistic problem.	Cannot deal with the hesitation information of a realistic problem.
Neutrosophic Fuzzy Number	Can describe the uncertainty, falsity & hesitation information of a realistic problem.	Cannot describe the incomplete weight information of a realistic problem.

1.2 Contribution

In this research article, researchers are primarily focused on pentagonal neutrosophic fuzzy number and its properties. A very engrossing question will arise among the researchers from all around the world that how a neutrosophic number can be transformed into a crisp number? From the last century, researchers are tried to develop lots of new methods associated with the de-Neutrosophication technique for crispification. Here, we generate the idea of crispification of pentagonal neutrosophic fuzzy number is enlarged using removal area skill. Nowadays, researchers are giving their attention to solve the problem of minimal spanning tree in neutrosophic arena. By utilizing the idea of newly generated de-Neutrosophication skill on pentagonal neutrosophic number field, we can able to tackle the problems on minimal spanning tree. Lastly, comparison analysis is done with the established methods to show the importance of this algorithm.

1.3 Novelties

Several research articles had already published in different journals on neutrosophic arena. Researches from different domain applied this concept in distinct areas also. The conception of pentagonal neutrosophic number is totally new in research domain. Thus it can be extended into different fields and can be applied into various research arenas. However a few numbers of articles has been developed in pentagonal neutrosophic environment till now. Thus, our motivation and target is to try to sketch out some unpublished points that are described below.

- Formulation of linear pentagonal neutrosophic number and its classification.
- De-Neutrosophication technique of linear pentagonal neutrosophic number.
- Application in minimal spanning tree problem.

1.4 Structure of the paper

In this research article section 1 contains introduction and literature survey of neutrosophic number, section 2 covers mathematical preliminaries, section 3 admits a de-neutrosophication technique of linear pentagonal neutrosophic fuzzy number, section 4 covers minimal spanning tree problem in neutrosophic environment, section 5 shows comparison table and lastly section 6 contains the conclusion part of the total research work.

2. Mathematical Preliminaries

Definition 2.1: Fuzzy Set: [1] A set \tilde{C} , is denoted as $\tilde{C} = \{(x, \mu_{\tilde{C}}(x)) : x \in X, \mu_{\tilde{C}}(X) \in [0, 1]\}$ and is generally represented by $(x, \mu_{\tilde{C}}(x))$, where $x \in$ the crisp set X and $\mu_{\tilde{C}}(X) \in$ the interval $[0, 1]$, then set \tilde{C} is called an intuitionistic fuzzy set.

Definition 2.2: Intuitionistic Fuzzy Set (IFS): A set [6] \tilde{P} , is defined as $\tilde{P} = \{(x; [\tau(x), \varphi(x)]) : x \in X\}$, where $\tau(x) : X \rightarrow [0, 1]$ is named as the truth membership function which indicate the degree of assurance, $\varphi(x) : X \rightarrow [0, 1]$ is named the falsity membership and $\tau(x), \varphi(x)$ satisfies the following the relation

$$0 \leq \tau(x) + \varphi(x) \leq 1.$$

Definition 2.3: Neutrosophic Set: [9] A set \tilde{nA} is called a neutrosophic set if $\tilde{nA} = \{(x; [\rho_{\tilde{nA}}(x), \sigma_{\tilde{nA}}(x), \omega_{\tilde{nA}}(x)]) : x \in X\}$, where $\rho_{\tilde{nA}}(x) : X \rightarrow [0, 1]$ is said to be the truth membership function, $\sigma_{\tilde{nA}}(x) : X \rightarrow [0, 1]$ is said to be the indeterminacy membership function and $\omega_{\tilde{nA}}(x) : X \rightarrow [0, 1]$ is said to be the falsity membership function.

$\rho_{\tilde{nA}}(x), \sigma_{\tilde{nA}}(x) \& \omega_{\tilde{nA}}(x)$ exhibits the following relation:

$$-0 \leq \rho_{\tilde{nA}}(x) + \sigma_{\tilde{nA}}(x) + \omega_{\tilde{nA}}(x) \leq 3 +$$

Definition 2.4: Single-Valued Neutrosophic Set: A Neutrosophic set \tilde{nA} in the definition 2.1 is said to be a Single-Valued Neutrosophic Set (\tilde{SnA}) if x is a single-valued independent variable. $\tilde{SnA} = \{(x; [\alpha_{\tilde{SnA}}(x), \beta_{\tilde{SnA}}(x), \gamma_{\tilde{SnA}}(x)]) : x \in X\}$, where $\alpha_{\tilde{SnA}}(x), \beta_{\tilde{SnA}}(x) \& \gamma_{\tilde{SnA}}(x)$ denoted the concept of trueness, indeterminacy and falsity memberships function respectively.

If there exist three points $p_0, q_0 \& r_0$ for which $\alpha_{\tilde{SnA}}(p_0) = 1, \beta_{\tilde{SnA}}(q_0) = 1 \& \gamma_{\tilde{SnA}}(r_0) = 1$, then the \tilde{SnA} is called neut-normal.

\tilde{SnS} is called neut-convex, which follows the relation:

$$\alpha_{\tilde{SnA}}(\delta p_1 + (1 - \delta)p_2) \geq \min\{\alpha_{\tilde{SnA}}(p_1), \alpha_{\tilde{SnA}}(p_2)\}$$

$$\beta_{\tilde{SnA}}(\delta p_1 + (1 - \delta)p_2) \leq \max\{\beta_{\tilde{SnA}}(p_1), \beta_{\tilde{SnA}}(p_2)\}$$

$$\gamma_{\tilde{SnA}}(\delta p_1 + (1 - \delta)p_2) \leq \max\{\gamma_{\tilde{SnA}}(p_1), \gamma_{\tilde{SnA}}(p_2)\}$$

where $p_1 \& p_2 \in \mathbb{R}$ and $\delta \in [0, 1]$

Definition 2.5: Single-Valued Pentagonal Neutrosophic Number: A Single-Valued Pentagonal Neutrosophic Number (\tilde{S}) is defined and described as $\tilde{S} = \{[(g^1, h^1, i^1, j^1, k^1); \rho], [(g^2, h^2, i^2, j^2, k^2); \sigma], [(g^3, h^3, i^3, j^3, k^3); \omega]\}$, where $\rho, \sigma, \omega \in [0, 1]$. The

truth membership function $(\theta_{\tilde{s}}): \mathbb{R} \rightarrow [0, \rho]$, the indeterminacy membership function $(\phi_{\tilde{s}}): \mathbb{R} \rightarrow [\sigma, 1]$ and the falsity membership function $(\psi_{\tilde{s}}): \mathbb{R} \rightarrow [\omega, 1]$ are given as:

$$\theta_{\tilde{s}}(x) = \begin{cases} \theta_{\tilde{s}r_1}(x) & g^1 \leq x < h^1 \\ \theta_{\tilde{s}r_2}(x) & h^1 \leq x < i^1 \\ \rho & x = i^1 \\ \theta_{\tilde{s}r_2}(x) & i^1 \leq x < j^1 \\ \theta_{\tilde{s}r_1}(x) & j^1 \leq x < k^1 \\ 0 & \text{otherwise} \end{cases}, \quad \phi_{\tilde{s}}(x) = \begin{cases} \phi_{\tilde{s}r_1}(x) & g^2 \leq x < h^2 \\ \phi_{\tilde{s}r_2}(x) & h^2 \leq x < i^2 \\ \sigma & x = i^2 \\ \phi_{\tilde{s}r_2}(x) & i^2 \leq x < j^2 \\ \phi_{\tilde{s}r_1}(x) & j^2 \leq x < k^2 \\ 1 & \text{otherwise} \end{cases}$$

$$\psi_{\tilde{s}}(x) = \begin{cases} \psi_{\tilde{s}r_1}(x) & g^3 \leq x < h^3 \\ \psi_{\tilde{s}r_2}(x) & h^3 \leq x < i^3 \\ \omega & x = i^3 \\ \psi_{\tilde{s}r_2}(x) & i^3 \leq x < j^3 \\ \psi_{\tilde{s}r_1}(x) & j^3 \leq x < k^3 \\ 1 & \text{otherwise} \end{cases}$$

3. De-Neutrosophication of a Linear Neutrosophic Pentagonal Number

On development of the de-Neutrosophication technique, results can be generated into a crisp number according to the results of pentagonal neutrosophic number and its membership functions. Researchers from all around the globe are concerned to know what shall be the crisp value associating the pentagonal neutrosophic number having membership function? By the passing days, they have continuously developed some convenient means to change a fuzzy number to a crisp number and some of these approaches are discussed below:

1. BADD (basic defuzzification distributions)
2. BOA (bisector of area)
3. CDD (constraint decision defuzzification)
4. COA (center of area)
5. COG (center of gravity)
6. ECOA (extended center of area)
7. EQM (extended quality method)
8. FCD (fuzzy clustering defuzzification), etc.

On this pentagonal neutrosophic arena, researches had an ambiguity in finding the suitable method of changing the pentagonal neutrosophic number to a crisp number. There are three distinct membership functions present in pentagonal neutrosophic number. To transform a neutrosophic number to a crisp number, "removal area method" is proposed on this article.

On this pentagonal neutrosophic arena, researches had an ambiguity in finding the suitable method of changing the pentagonal neutrosophic number to a crisp number. There are three distinct membership functions present in pentagonal neutrosophic number. To transform a neutrosophic number to a crisp number, "removal area method" is proposed on this article.

Suppose, we consider a linear pentagonal neutrosophic number as follows

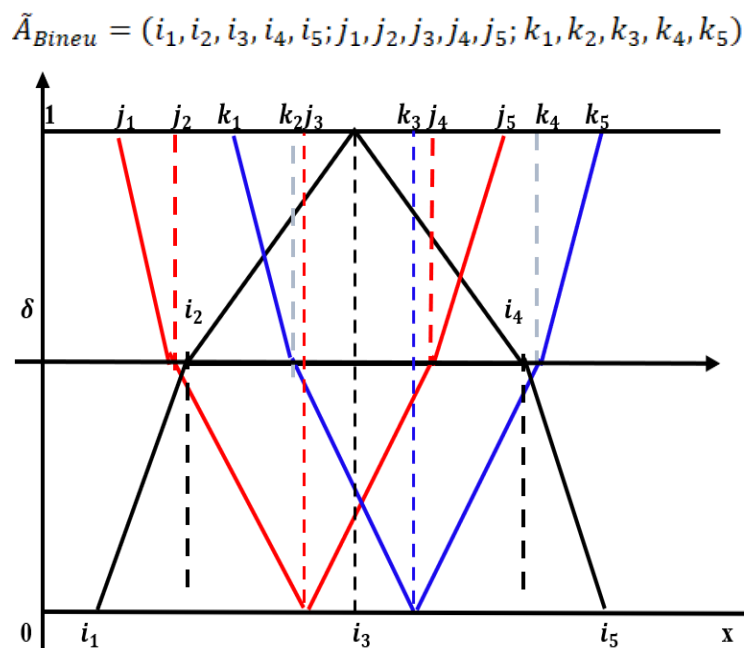


Figure 3.1: Graphical representation of Linear Pentagonal Neutrosophic Number.

Description of above figure: On the above figure we focused on the graphical presentation of linear pentagonal neutrosophic number. The black lined pentagonal represent the truth membership function. Red lined pentagonal represents the falsity membership function and blue lined pentagonal shows indefiniteness membership function of the number. In this, τ follows the relation $0 \leq \tau \leq 1$. The pentagonal number can be altered to triangular neutrosophic number if $\tau = 0$ or 1 .

Let us assume a real number $s \in R$ and a fuzzy number \check{P} for black line specified pentagons, area of the left side distribution of \check{P} w.r.t s , is $A_{Neu_l}(\check{P}, s)$ that indicates the zone fenced by s and the left side of the fuzzy number \check{P} . Proceeding in this way, the right zone area of \check{P} w.r.t s is $A_{Neu_r}(\check{P}, s)$. Considering a real number $s \in R$ along with the fuzzy number \check{Q} for the left most top and inverted pentagon, then area of left side of \check{Q} wrt s is $A_{Neu_l}(\check{Q}, s)$ is described as the area bounded by l and the left portion of the fuzzy number \check{Q} . For the second time, the area of right side of Q wr.t l is $A_{Neu_r}(\check{Q}, s)$. A fuzzy number \check{R} for the right most top and inverted pentagon, then left side removal of \check{R} w.r.t s is $A_{Neu_l}(\check{R}, s)$ is described by the area bounded by s and the left side of the fuzzy number \check{R} . similarly, the right portion removal of \check{R} w.r.t s is $A_{Neu_r}(\check{R}, s)$.

Mean is described as $A_{Neu}(\check{P}, s) = \frac{A_{Neu_l}(\check{P}, s) + A_{Neu_r}(\check{P}, s)}{2}$, $A_{Neu}(\check{Q}, s) = \frac{A_{Neu_l}(\check{Q}, s) + A_{Neu_r}(\check{Q}, s)}{2}$,

$$A_{Neu}(\check{R}, s) = \frac{A_{Neu_l}(\check{R}, s) + A_{Neu_r}(\check{R}, s)}{2}$$

Then, we quantified the de-neutrosophication value of a linear pentagonal neutrosophic number as,

$$A_{Neu}(\widetilde{D_{Pen}}s) = \frac{A_{Neu}(\check{P}, s) + A_{Neu}(\check{Q}, s) + A_{Neu}(\check{R}, s)}{3}$$

For $s = 0$,

$$A_{Neu}(\check{P}, 0) = \frac{A_{Neu l}(\check{P}, 0) + A_{Neu r}(\check{P}, 0)}{2}, A_{Neu}(\check{Q}, 0) = \frac{A_{Neu l}(\check{Q}, 0) + A_{Neu r}(\check{Q}, 0)}{2}, A_{Neu}(\check{R}, 0) = \frac{A_{Neu l}(\check{R}, 0) + A_{Neu r}(\check{R}, 0)}{2}$$

Thus, $A_{Neu}(\widetilde{D_{Pen}}, 0) = \frac{A_{Neu}(\check{P}, 0) + A_{Neu}(\check{Q}, 0) + A_{Neu}(\check{R}, 0)}{3}$

Here, we take $\check{X} = (i_1, i_2, i_3, i_4, i_5), \check{Y} = (j_1, j_2, j_3, j_4, j_5), \check{Z} = (k_1, k_2, k_3, k_4, k_5)$

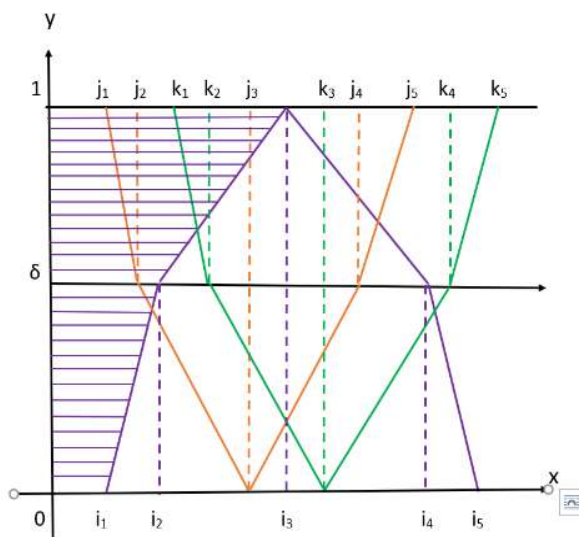


Figure 3.1(a)

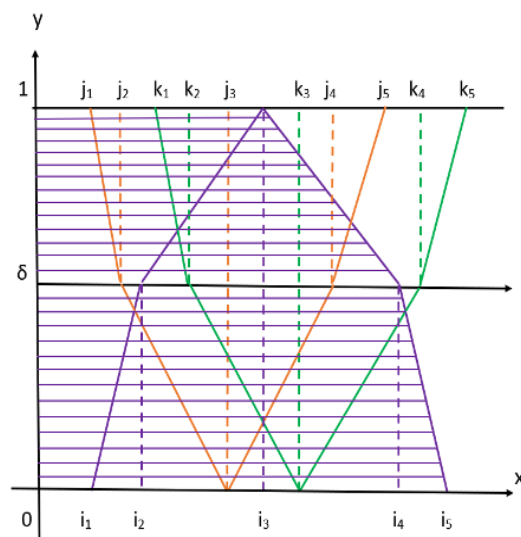


Figure 3.1(b)

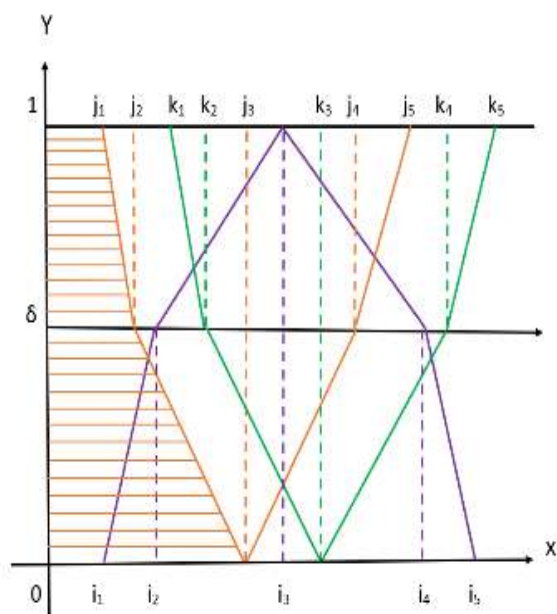


Figure 3.2(a)

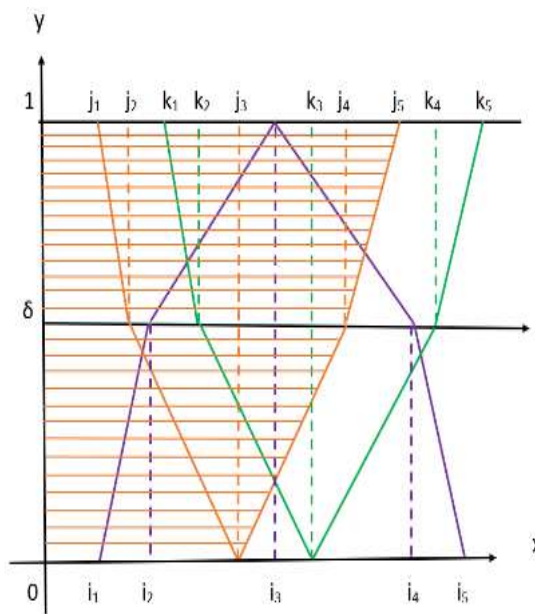


Figure 3.2(b)

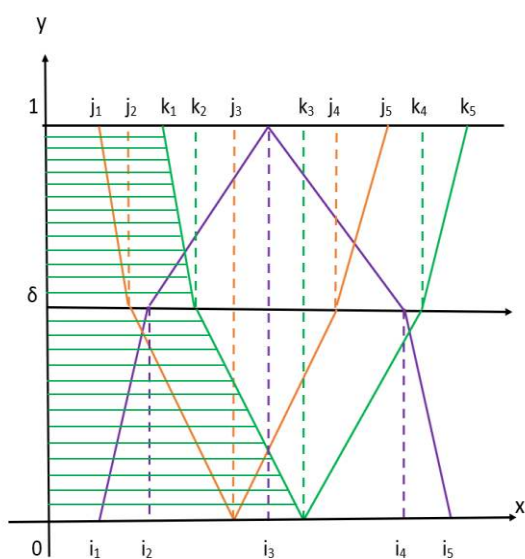


Figure 3.3(a)

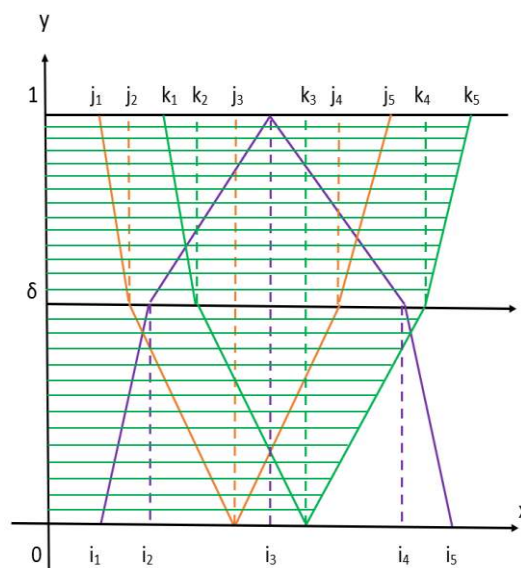


Figure 3.3(b)

Then,

$$A_{Neu_l}(\check{P}, 0) = \text{Area of Figure 3.1(a)} = \frac{(i_1+i_2)\delta}{2} + \frac{(i_2+i_3)(1-\delta)}{2}$$

$$A_{Neu_r}(\check{P}, 0) = \text{Area of Figure 3.1(b)} = \frac{(i_4+i_5)\delta}{2} + \frac{(i_3+i_4)(1-\delta)}{2}$$

$$A_{Neu_l}(\check{Q}, 0) = \text{Area of Figure 3.2(a)} = \frac{(j_1+j_2)(1-\delta)}{2} + \frac{(j_2+j_3)\delta}{2}$$

$$A_{Neu_r}(\check{Q}, 0) = \text{Area of Figure 3.2(b)} = \frac{(j_3+j_4)\delta}{2} + \frac{(j_4+j_5)(1-\delta)}{2}$$

$$A_{Neu_l}(\check{R}, 0) = \text{Area of Figure 3.3(a)} = \frac{(k_1+k_2)(1-\delta)}{2} + \frac{(k_2+k_3)\delta}{2}$$

$$A_{Neu_r}(\check{R}, 0) = \text{Area of Figure 3.3(b)} = \frac{(k_3+k_4)\delta}{2} + \frac{(k_4+k_5)(1-\delta)}{2}$$

$$\text{Hence, } A_{Neu}(\check{P}, 0) = \frac{\frac{(i_1+i_2)\delta}{2} + \frac{(i_2+i_3)(1-\delta)}{2} + \frac{(i_4+i_5)\delta}{2} + \frac{(i_3+i_4)(1-\delta)}{2}}{2},$$

$$A_{Neu}(\check{Q}, 0) = \frac{\frac{(j_1+j_2)(1-\delta)}{2} + \frac{(j_2+j_3)\delta}{2} + \frac{(j_3+j_4)\delta}{2} + \frac{(j_4+j_5)(1-\delta)}{2}}{2}, \quad A_{Neu}(\check{R}, 0) = \frac{\frac{(k_1+k_2)(1-\delta)}{2} + \frac{(k_2+k_3)\delta}{2} + \frac{(k_3+k_4)\delta}{2} + \frac{(k_4+k_5)(1-\delta)}{2}}{2}$$

$$\text{So, } A_{Neu}(\widetilde{D_{Pen}}, 0) = \frac{(i_1+i_2+i_4+i_5+j_2+2j_3+j_4+k_2+2k_3+k_4)\delta + (i_2+2i_3+i_4+j_1+j_2+j_4+j_5+k_1+k_2+k_4+k_5)(1-\delta)}{12} \dots\dots\dots(1)$$

Table 3.1: Numerical computation of De-Neutrosophication value

Sl. No.	Pentagonal Neutrosophic Number	De-Neutrosophication value
1	(1,2,3,4,5;0.5,1.5,2.5,3.5,4.5;2.2,2.8,3.7,4.5,6)	3.091667
2	(0.5,1.5,2.5,3.5,4.5;0.3,1.3,2.3,3.3,4.3;1.8,2.8,3.8,4.8,5.8)	2.86667
3	(0.7,1.7,2.5,3.5,4.7;0.5,1.5,2.2,3.2,4;1.7,2.7,3.7,4.7,5.7)	2.86250
4	(1.2,2.2,3.2,4.2,5.2;1,2,3,4,5;2.5,3.5,4.5,5,6.5)	3.52500
5	(1,4,7,10,13;0.5,3.5,6.5,9.5,12.5;4.5,7.5,9,12,14.5)	7.66667

4. Minimal Spanning Tree in Pentagonal Neutrosophic Environment

Spanning Tree: Let, G is a graph and T is a subgraph of G. If T is a connected graph having no circuits and covers all vertices of G, then T is called a spanning tree.

Minimal Spanning Tree: A spanning tree which contains the least weight in G is defined as minimal spanning tree. Let us consider a graph in pentagonal neutrosophic domain. Here we developed an algorithm to search out the minimal spanning tree where the weights are pentagonal neutrosophic numbers. Thus this is a problem of neutrosophic graph.

Algorithm:

- Construct an adjacency matrix of the graph.
- Utilize de-Neutrosophication technique and construct crisp matrix.
- Select the least weight and if there is a tie in selection of least weight then take any one edge from the given graph.
- From the edges that are left behind select an edge containing the least edge that doesn't form a loop with the previous established figure.
- Continue this process until all vertices will be covered.
- Stop.

4.1 Illustrative Example:

To acquire a minimal spanning tree of the following graph

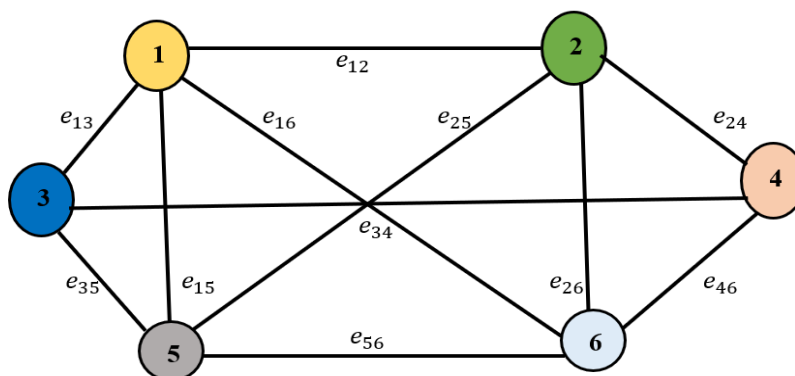


Figure 4.1.1: A Graph with pentagonal neutrosophic number Weight Edges

Table 1: The values of weights related with edges

Edges	Pentagonal Single-Valued Neutrosophic Weights
e_{12}	$\langle 0.5, 1.15, 2.25; 0.3, 0.7, 1.2, 1.6, 2.2; 0.8, 1.3, 1.8, 2.3, 3 \rangle$
e_{13}	$\langle 0.7, 1.2, 1.8, 2.4, 3; 0.6, 1.14, 2.25; 1.15, 2.25, 3.5 \rangle$
e_{15}	$\langle 0.3, 0.8, 1.4, 2.2, 6; 0.2, 0.7, 1.2, 1.8, 2.2; 0.5, 1.15, 2.4, 3 \rangle$
e_{16}	$\langle 1.15, 2.25, 3; 0.7, 1.2, 1.8, 2.2, 2.6; 1.2, 1.6, 2.2, 2.8, 3.5 \rangle$
e_{25}	$\langle 0.8, 1.4, 2.2, 6, 3.2; 0.6, 1.2, 1.8, 2.2, 2.8; 1.1, 1.8, 2.4, 2.8, 3.5 \rangle$
e_{26}	$\langle 0.6, 1.15, 2.25; 0.4, 0.8, 1.2, 1.8, 2.2; 0.8, 1.4, 2.2, 4, 3 \rangle$
e_{24}	$\langle 0.9, 1.4, 2.2, 5, 3; 0.6, 1.2, 1.6, 2.1, 2.4; 1.2, 1.5, 2.3, 2.8, 3.5 \rangle$
e_{34}	$\langle 1.1, 1.5, 1.9, 2.3, 2.7; 0.8, 1.2, 1.7, 2.1, 2.4; 1.4, 1.8, 2.2, 2.6, 3 \rangle$
e_{46}	$\langle 0.7, 1.1, 1.3, 1.6, 2; 0.6, 0.9, 1.2, 1.5, 1.8; 1.1, 1.4, 1.8, 2.2, 2.5 \rangle$
e_{35}	$\langle 0.8, 1.2, 1.5, 1.8, 2.4; 0.5, 0.9, 1.3, 1.7, 2.1; 1.1, 1.4, 1.8, 2.2, 2.6 \rangle$
e_{56}	$\langle 1.2, 1.5, 1.8, 2.4, 2.6; 0.9, 1.3, 1.7, 2.2, 3; 1.4, 1.8, 2.2, 2.5, 2.8 \rangle$

Step 1: The associated adjacency matrix of figure 1 is given as follows:

$$A = \begin{bmatrix} \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & \langle 0.5,1.15,2.25; 0.3,0.7,1.2,1.6,2.2; 0.8,1.3,1.8,2.3,3 \rangle & \langle 0.7,1.2,1.8,2.4,3; 0.6,1.14,2.25; 1.15,2.25,3.5 \rangle & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & \langle 0.3,0.8,1.4,2.2,6; 0.2,0.7,1.2,1.8,2.2; 0.5,1.15,2.4,3 \rangle & \langle 1.15,2.25,3; 0.7,1.2,1.8,2.2,2.6; 1.2,1.6,2.2,2.8,3.5 \rangle \\ \langle 0.5,1.15,2.25; 0.3,0.7,1.2,1.6,2.2; 0.8,1.3,1.8,2.3,3 \rangle & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & - & \langle 0.9,1.4,2.2,5,3; 0.6,1.2,1.6,2.1,2.4; 1.2,1.5,2.3,2.8,3.5 \rangle & \langle 0.8,1.4,2.2,6,3.2; 0.6,1.2,1.8,2.2,2.8; 1.1,1.8,2.4,2.8,3.5 \rangle & \langle 0.6,1.15,2.25; 0.4,0.8,1.2,1.8,2.2; 0.8,1.4,2.2,4,3 \rangle \\ \langle 0.7,1.2,1.8,2.4,3; 0.6,1.14,2.25; 1.15,2.25,3.5 \rangle & - & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & \langle 1.1,1.5,1.9,2.3,2.7; 0.8,1.2,1.7,2.1,2.4; 1.4,1.8,2.2,2.6,3 \rangle & \langle 0.8,1.2,1.5,1.8,2.4; 0.5,0.9,1.3,1.7,2.1; 1.1,1.4,1.8,2.2,2.6 \rangle & - \\ \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & \langle 0.9,1.4,2.2,5,3; 0.6,1.2,1.6,2.1,2.4; 1.2,1.5,2.3,2.8,3.5 \rangle & \langle 1.1,1.5,1.9,2.3,2.7; 0.8,1.2,1.7,2.1,2.4; 1.4,1.8,2.2,2.6,3 \rangle & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & - & \langle 0.7,1.1,1.3,1.6,2; 0.6,0.9,1.2,1.5,1.8; 1.1,1.4,1.8,2.2,2.5 \rangle \\ \langle 0.3,0.8,1.4,2.2,6; 0.2,0.7,1.2,1.8,2.2; 0.5,1.15,2.4,3 \rangle & \langle 0.8,1.4,2.2,6,3.2; 0.6,1.2,1.8,2.2,2.8; 1.1,1.8,2.4,2.8,3.5 \rangle & \langle 0.8,1.2,1.5,1.8,2.4; 0.5,0.9,1.3,1.7,2.1; 1.1,1.4,1.8,2.2,2.6 \rangle & - & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle & \langle 1.2,1.5,1.8,2.4,2.6; 0.9,1.3,1.7,2.2,3; 1.4,1.8,2.2,2.5,2.8 \rangle \\ \langle 1.15,2.25,3; 0.7,1.2,1.8,2.2,2.6; 1.2,1.6,2.2,2.8,3.5 \rangle & \langle 0.6,1.15,2.25; 0.4,0.8,1.2,1.8,2.2; 0.8,1.4,2.2,4,3 \rangle & - & \langle 0.7,1.1,1.3,1.6,2; 0.6,0.9,1.2,1.5,1.8; 1.1,1.4,1.8,2.2,2.5 \rangle & \langle 1.2,1.5,1.8,2.4,2.6; 0.9,1.3,1.7,2.2,3; 1.4,1.8,2.2,2.5,2.8 \rangle & \langle 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 \rangle \end{bmatrix}$$

Step 2: Using the De-neutrosophic value, the associated matrix becomes

$$A = \begin{bmatrix} 0 & 1.504 & 1.788 & - & 1.433 & 1.983 \\ 1.504 & 0 & - & 1.933 & 2.012 & 1.570 \\ 1.788 & - & 0 & 1.917 & 1.542 & - \\ - & 1.933 & 1.917 & 0 & - & 1.433 \\ 1.433 & 2.012 & 1.542 & - & 0 & 1.900 \\ 1.983 & 1.570 & - & 1.433 & 1.900 & 0 \end{bmatrix}$$

Step 3: After examining, the least value is 1.433. So, the edge is selected which is connected with the nodes (1, 5) and thus labelled it. This procedure is repeated till the final spanning tree is found.

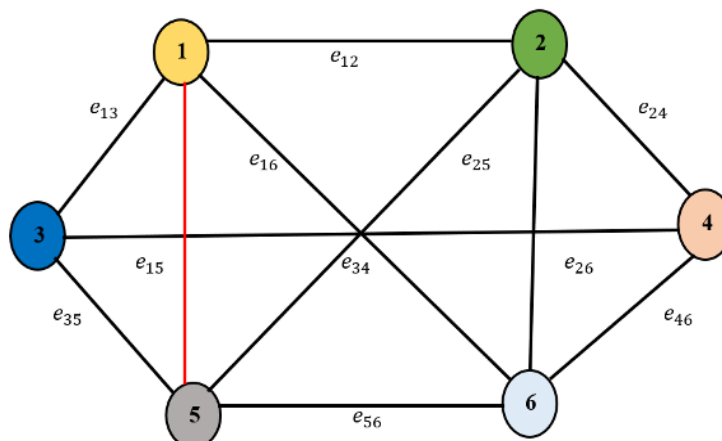


Figure 4.1.2 : Diagrammatic Presentation of Step 3

Step 4: After studying, the least value is 1.433 among the remaining weighted edges. Therefore the edge is selected connecting the nodes (4,6) and label it.

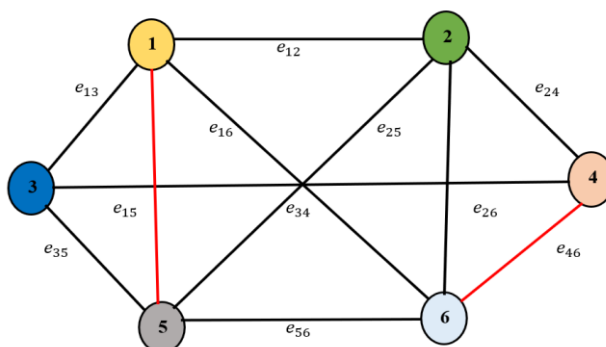


Figure 4.1.3: Diagrammatic Presentation of Step 4

Step 5: After examining, the least value is 1.504 amongst the remaining weighted edges. The edge is selected connecting nodes (1, 2) and thus marked it.

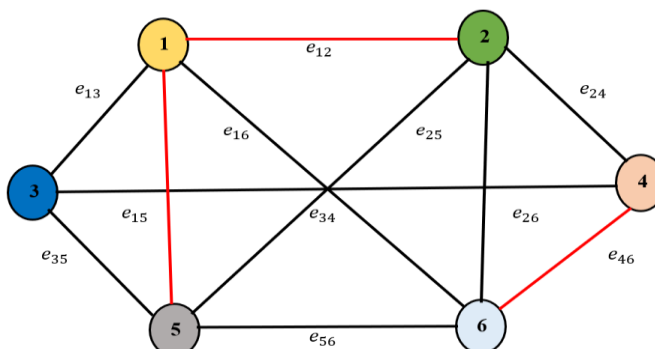


Figure 4.1.4: Diagrammatic Presentation of Step 5

Step 6: Examined that the least value is 1.542 among the remaining weighted edges. Hence, the edge is selected connecting with nodes (3, 5) and labelled it.

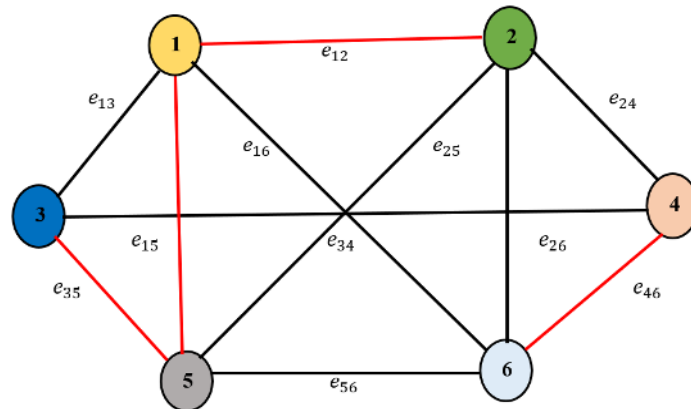


Figure 4.1.5: Diagrammatic Presentation of Step 6

Step 6: Examined that the least value is 1.570 out of the remaining weighted edges. Therefore, the edge is selected connecting the nodes (2,6) and marked it.

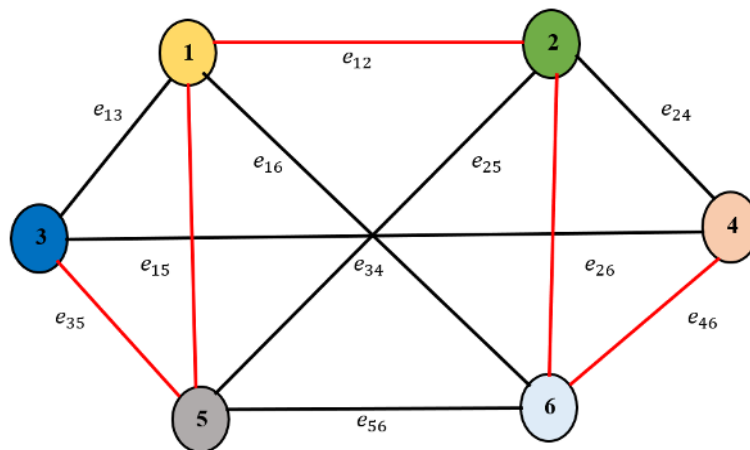


Figure 4.1.6: Diagrammatic Presentation of Step 7

Step 7: After examining all the nodes are joined and if more edges are to be joined it will form a circuit in the figure formed and as stated by the definition of a spanning tree it must not form any circuit but also all the nodes must be connected. Thus, the ultimate minimal spanning tree is followed:

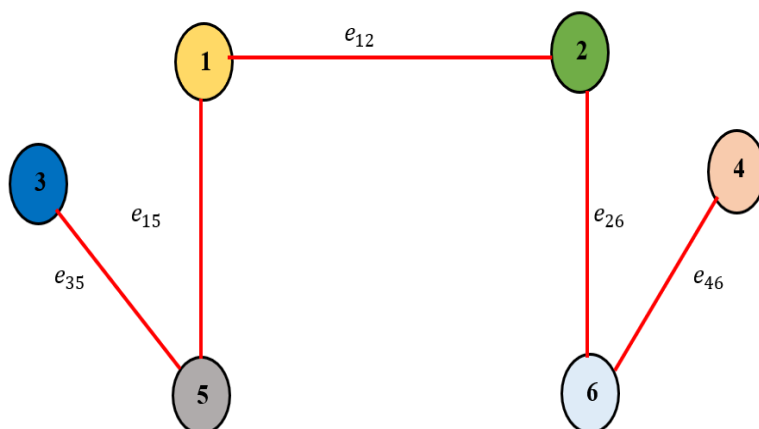


Figure 4.1.7: Diagrammatic Representation of ultimate minimal spanning tree

The least weight of the graph is – $(1.433 + 1.433 + 1.5041.542+1.570) = 7.482$ units.

5. Comparison:

Here, we compare our work with Mullai’s [37] established algorithm. According to previous concept, the required minimal spanning tree can be obtained from the following steps.

- Step 1: Let $S_1 = \{1\}$ then $\bar{S}_1 = \{2,3,4,5,6\}$
- Step 2: Let $S_2 = \{1,5\}$ then $\bar{S}_2 = \{2,3,4,6\}$
- Step 3: Let $S_3 = \{1,5,2\}$ then $\bar{S}_3 = \{3,4,6\}$
- Step 4: Let $S_4 = \{1,5,2,3\}$ then $\bar{S}_4 = \{4,6\}$
- Step 5: Let $S_5 = \{1,5,2,3,6\}$ then $\bar{S}_5 = \{4\}$
- Step 6: Let $S_6 = \{1,5,2,3,6,4\}$ then $\bar{S}_6 = \{\phi\}$

The Required spanning tree is

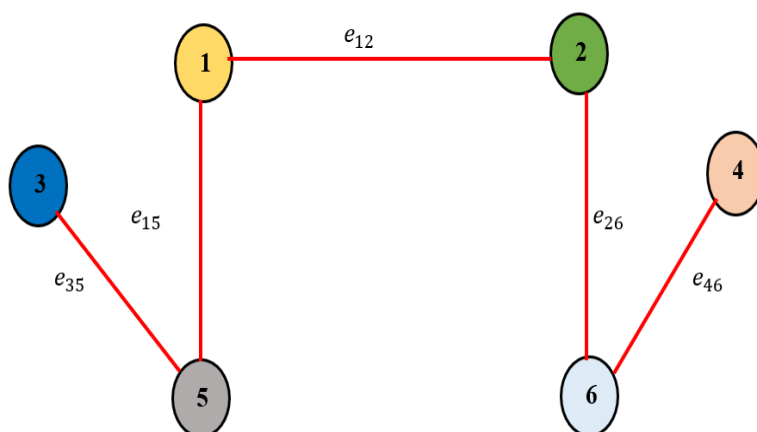


Figure 5.1: Diagrammatic Presentation of minimal spanning tree

Discussion: There is a contrast among the proposed approach and Mullai's technique is that Mullai's formulation is based on edges which are repeatedly evaluated at every steps of the algorithm which leads to the increase of time complexity. However, our technique relating with Matrix can be skillfully handled by utilizing Matlab software. In Mullai's method we need to consider each steps one by one manually but in our proposed method we can solve it using the help of computational software available in mathematics field with the help of computer as it is totally based on matrix concept. Thus we can claim that it will more useful and short time taking approach than any other established algorithm in this research domain.

6. Conclusion

In this research article, the concept of pentagonal neutrosophic number has been developed in a different aspect. Demonstration of De-Neutrosophication method utilizing the removal area technique has been introduced here for conversion of a pentagonal neutrosophic number into a real number. Further, this result is applied in the field of graph theory to evaluate the minimal spanning tree of a general graph. Comparison analysis is done with the established method which gave a crucial impact in this article for the evaluation of minimal spanning tree. Since, no work has been developed in this field so we can claim that this is the best method. Though the stated algorithm able to analyze the solutions of minimal spanning tree problem in pentagonal neutrosophic domain but more reliable, logical and short time taking algorithm maybe established in this field such that it can gives us much more fast, accurate and exact results after the total computation. Thus, these are the limitations of this stated algorithm in neutrosophic scenario.

In future, researchers can developed some interesting algorithms using pentagonal neutrosophic number in various fields like multi criteria decision making problem, image processing problem, pattern recognition problem, cloud computing problem and other mathematical modeling problems. Again, researcher may develop some new structural formulations of pentagonal neutrosophic number in different aspects. Also, researchers can compare this work with the new invented concept in pentagonal neutrosophic environment.

Reference

1. Zadeh LA; Fuzzy sets. Information and Control, 8(5): (1965),338- 353.
2. Yen, K. K.; Ghoshray, S.; Roig, G.; A linear regression model using triangular fuzzy number coefficients, fuzzy sets and system, doi: 10.1016/S0165-0114(97)00269-8.
3. Chen, S. M.; Fuzzy system reliability analysis using fuzzy number arithmetic operations, fuzzy sets and system, doi: 10.1016/0165-0114(94)90004-3.
4. Abbasbandy, S. and Hajjari, T.; A new approach for ranking of trapezoidal fuzzy numbers; Computers and Mathematics with Applications, 57(3)(2009), 413-419,
5. Chakraborty, A.; Mondal, S.P.; Ahmadian, A.; Senu, N.; Dey, D.; Alam, S.; and Salahshour, S.; The Pentagonal Fuzzy Number: Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problem, Symmetry, 11(2), 248,
6. Atanassov K ; Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20: (1986);87-96.
7. Liu F, Yuan XH ; Fuzzy number intuitionistic fuzzy set. Fuzzy Systems and Mathematics, 21(1): (2007); 88-91.
8. Ye J ; prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multi criteria decision making, Neural Computing and Applications, 25(6): (2014);1447-1454.
9. Smarandache, F. A unifying field in logics neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth. 1998.

10. H. Wang, F. Smarandache, Q. Zhang and R. Sunder raman, Single valued neutrosophic sets, Multi space and Multi structure 4 (2010), 410–413.
11. Yun Ye, Trapezoidal neutrosophic set and its application to multiple attribute decision-making, Neural Comput& Applic (2015) 26:1157–1166 DOI 10.1007/s00521-014-1787-6.
12. Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.(2015); Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision making problems. International Journal of Systems Science.
13. Peng, J.J.; Wang, J.Q.; Wu, X.H.; Zhang, H.Y.(2014); Chen, X.H. The fuzzy cross entropy intuitionistic indeterminacy fuzzy sets and their application in multi-criteria decision making, International Journal of Systems Science. 46(13), 2335–2350.
14. Chakraborty, A.; Mondal, S. P.; Ahmadian, A.; Senu, N.; Alam, S.; and Salahshour, S.; Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their Applications, Symmetry, Vol-10, 327; doi:10.3390/sym10080327.
15. Chakraborty, A.; Mondal, S. P.;Mahata, A.; Alam, S.; Different linear and non-linear form of Trapezoidal Neutrosophic Numbers, De-Neutrosophication Techniques and its Application in time cost optimization technique, sequencing problem; RAIRO Operation Research, doi:10.1051/ro/2019090.
16. R.Helen and G.Uma, A new operation and ranking on pentagon fuzzy numbers, Int Jr. of Mathematical Sciences & Applications, Vol. 5, No. 2, 2015, pp 341-346.
17. A.Vigin Raj and S.Karthik, Application of Pentagonal Fuzzy Number in Neural Network, International Journal of Mathematics And its Applications, Volume 4, Issue 4 (2016), 149-154.
18. T.Pathinathan and K.Ponnivalavan, Reverse order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers, Annals of Pure and Applied Mathematics, Vol. 9, No. 1, 2015, 107-117.
19. Abdel-Basset, M., Manogaran, G., Gamal, A., &Smarandache, F. (2019). A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, 43(2), 38.
20. S. Maity, A.Chakraborty, S.K De, S.P.Mondal, S.Alam, A comprehensive study of a backlogging EOQ model with nonlinear heptagonal dense fuzzy environment, Rairo Operations Research, <https://doi.org/10.1051/ro/2018114>, 2018.
21. Nabeeh, N. A., Abdel-Basset, M., El-Ghareeb, H. A., &Aboelfetouh, A. (2019). Neutrosophic multi-criteria decision making approach for iot-based enterprises. *IEEE Access*, 7, 59559-59574.
22. X. Zhao. TOPSIS method for interval-valued intuitionistic fuzzy multiple attribute decision making and its application to teaching quality evaluation. *Journal of Intelligent and Fuzzy Systems*, 26 (2014), 3049–3055.
23. Chakraborty, A.; Mondal, S. P.; Alam, S.; Ahmadian, A.; Senu, N.; De, D. and Salahshour, S.; Disjunctive Representation of Triangular Bipolar Neutrosophic Numbers, De-Bipolarization Technique and Application in Multi-Criteria Decision-Making Problems,Symmetry,11(7), (2019).
24. J. Ye. Similarity measures between interval neutrosophic sets and their applications in multi criteria decision making. *Journal of Intelligent and Fuzzy Systems*, 26 (2014), 165–172.
25. S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. *Neutrosophic Sets and Systems*, 17 (2017), 20-28.
26. S. Ye, and J. Ye. Dice similarity measure between single valued neutrosophic multi sets and its application in medical diagnosis. *Neutrosophic Sets and System*, 6 (2014), 49-54.
27. S. Broumi, A. Bakali, M. Talea, Prem Kumar Singh, F. Smarandache, Energy and Spectrum Analysis of Interval-valued Neutrosophic graph Using MATLAB, *Neutrosophic Set and Systems*, vol. 24, Mar 2019, pp. 46-60.
28. P. K. Singh, Interval-valued neutrosophic graph representation of concept lattice and its (α, β, γ) -decomposition, *Arabian Journal for Science and Engineering*, Year 2018, Vol. 43, Issue 2, pp. 723-74
29. S. Broumi, F. Smarandache, M. Talea and A. Bakali. An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. *Applied Mechanics and Materials*, vol.841,2016, 184-191.
30. S. Broumi, M. Talea, A. Bakali, F. Smarandache. Single Valued Neutrosophic, *Journal of New Theory*. N 10. 2016, pp. 86-101.
31. S. Broumi, M. Talea, A. Bakali, F. Smarandache. On Bipolar Single Valued Neutrosophic Graphs. *Journal of Net Theory*. N11, 2016, pp. 84-102.

32. M. MULLAI AND S. BROUMI, Neutrosophic Inventory Model without Shortages, Asian Journal of Mathematics and Computer Research, 23(4): 214-219,2018.
33. YANG, P., AND WEE, H., Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. Production Planning and Control, 11, 2000,474-480.
34. Chakraborty.A, Broumi.S, Singh,P.K; Some properties of Pentagonal Neutrosophic Numbers and its Applications in Transportation Problem Environment, Neutrosophic Sets and Systems, vol.28,2019,pp.200-215.
35. Ye, J. (2014). Single valued neutrosophic minimum spanning tree and its clustering method. . *J. Intell.Syst.* , 311–324 .
36. Mandal, K., & Basu, K. (2016). Improved similarity measure in neutrosophic environment and its application in finding minimum spanning tree. . *J. Intell. Fuzzy Syst.* , 1721-1730.
37. Mullai, M., Broumi, S., & Stephen, A. (2017). Shortest path problem by minimal spanning tree algorithm using bipolar neutrosophic numbers. . *Int. J. Math. Trends Technol.* , 80-87.
38. Broumi, S., Bakali, A., Talea, M., Smarandache, F., & Kishore Kumar, P. (2017). Shortest path problem on single valued neutrosophic graphs. . International Symposium on Networks, Computers and Communications (ISNCC), (p. In Press).
39. Broumi, S., Talea, M., Smarandache, F., & Bakali, A. (2016). Single valued neutrosophic graphs:degree, order and size. . IEEE International Conference on Fuzzy Systems, (pp. 2444-2451).
40. Broumi, S., Smarandache, F., Talea, M., & Bakali, A. (2016). Decision-making method based on the interval valued neutrosophic graph. *IEEE* , 44-50.
41. Kandasamy, I. (2016). Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm. . *J. Intell. Syst.* , 1-17.
42. Broumi, S., Bakali, A., Talea, M., Smarandache, F., & Vladareanu, L. (2016). Applying Dijkstra algorithm for solving neutrosophic shortest path problem. Proceedings on the International Conference onAdvanced Mechatronic Systems. Melbourne, Australia.
43. K. Mondal, and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34. [130]
44. K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.
45. Abdel-Basset, M., Saleh, M., Gamal, A., &Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452.
46. Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108, 210-220.
47. J. Ye. Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers. Journal of Intelligent and Fuzzy System 30 (2016), 1927–1934.
48. Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., &Aboelfetouh, A. (2019). Utilizing neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise Information Systems*, 1-21.
49. S. Broumi, P. K. Singh, M. Talea, A. Bakali, F. Smarandache and V.Venkateswara Rao, Single-valued neutrosophic techniques for analysis of WIFI connection, Advances in Intelligent Systems and Computing Vol. 915, pp. 405-512,DOI: 10.1007/978-3-030-11928-7_36.
50. Mahata A., Mondal S.P, Alam S, Chakraborty A., Goswami A., Dey S., Mathematical model for diabetes in fuzzy environment and stability analysis- Journal of of intelligent and Fuzzy System, doi: <https://doi.org/10.3233/JIFS-171571>.
51. Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A Novel Model for Evaluation Hospital Medical Care Systems Based on Plithogenic Sets. Artificial Intelligence in Medicine, 101710.
52. Abdel-Basset, M., Manogaran, G., Gamal, A., & Chang, V. (2019). A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT. IEEE Internet of Things Journal.
53. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry*, 11(7), 903.
54. Abdel-Basset, M., & Mohamed, M. (2019). A novel and powerful framework based on neutrosophic sets to aid patients with cancer. *Future Generation Computer Systems*, 98, 144-153.

55. Abdel-Basset, M., Mohamed, M., & Smarandache, F. (2019). Linear fractional programming based on triangular neutrosophic numbers. *International Journal of Applied Management Science*, 11(1), 1-20.
56. Abdel-Basset, M., Atef, A., & Smarandache, F. (2019). A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, 57, 216-227.
57. Abdel-Basset, M., Gamal, A., Manogaran, G., & Long, H. V. (2019). A novel group decision making model based on neutrosophic sets for heart disease diagnosis. *Multimedia Tools and Applications*, 1-26.
58. Abdel-Basset, M., Chang, V., Mohamed, M., & Smarandache, F. (2019). A Refined Approach for Forecasting Based on Neutrosophic Time Series. *Symmetry*, 11(4), 457.

Received: June 05, 2019. Accepted: October 11, 2019